Twisted Waveguide Accelerating Structure

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Abstract. A hollow waveguide with a uniform cross section may be used for accelerating charged particles if the phase velocity of an accelerating mode is equal to or less than the free-space speed of light. Regular straight hollow waveguides have phase velocities of propagating electromagnetic waves greater than the free-space speed of light. If the waveguide is twisted, the phase velocities of the waveguide modes become slower. The twisted waveguide structure has been modeled and computer simulated in 3-D electromagnetic solvers to show the slow-wave properties for the accelerating mode.

INTRODUCTION

Slow-wave structures are used in many radio frequency (rf) applications including charged-particle accelerators. A slow-wave structure employs reactive loadings in a hollow waveguide to slow down the phase velocity of electromagnetic fields in a specific mode to be used. However, a question can be raised: “Is it possible to have a slow-wave hollow waveguide structure with a uniform cross section?” The question is not only academic but also practical; there exist interest for finding rf structures that can be inexpensive to manufacture especially for high-energy linear accelerators. The cost reduction can be important for large-scale accelerators, such as the next-generation linear colliders.

The above question sounds useless if one recalls the fundamental dispersion relations of waves in hollow waveguides. However, the above question could be answered in an unexpected way that is discussed and shown in the following. In a twisted noncircular hollow waveguide, the phase velocities of propagating modes can be slower than the velocities in a straight waveguide. A twisted waveguide has been usually a twisted rectangular waveguide section with a slow pitch angle. This type of waveguide has been implemented in waveguide circuits for simple plumbing purposes with no attention to their phase properties. In this paper, a slow-wave TM_{01}-like mode in a twisted waveguide that is useful for acceleration of charged particles is discussed.

Since the twisted waveguide structure has a uniform cross section, it may be built without welding or brazing many parts unlike the conventional disk-loaded accelerating cavity structures. A special extrusion technique or electroforming may be used for mass production of the waveguide and can lower the manufacturing cost significantly. The uniform smooth cross section along the direction of propagation in the structure means that higher-order modes may have lower impedances and can exit more easily. The reduction of the higher-order mode power harmful to the particle beam may deliver better beam properties.
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**TWISTED WAVEGUIDE**

Figure 1 shows a section of twisted rectangular waveguide. Intuitively, the fields in such a waveguide are seen as twisted along the guide. For computer simulation and any accurate analysis of the structure, a short section of the twisted waveguide shown in the figure must be used. A short section of a long twisted waveguide structure must have end walls that can satisfy boundary conditions for the twisted electric and magnetic fields. That means the end walls are orthogonal to the twisted surfaces; the end-wall surfaces must also be twisted, thus they are no longer flat 2-D planes.

![Figure 1](image)

**FIGURE 1.** Geometry of a section of twisted waveguide. (a) a waveguide section with twisted port surfaces, (b) cross section of the waveguide. (A rectangular shape is shown, but it can be arbitrary.)

The twisted field vectors of a mode in the waveguide are assumed normal to the conductor surface for E-field, and tangential to the surface for H-field. These vectors will have a tilt angle, $\theta(r)$, comparing with the fields in a straight waveguide, which is a function of the radial distance $r$. Effective height of the waveguide can be approximated as

$$ a'(r) = a \sin \theta(r) . $$

Therefore, if a rectangular waveguide is twisted, the twisting effectively squeezes the waveguide height down to a lower height as $r$ increases. This suggests that the cross section defined in the $x$-$y$ plane of a regular rectangular coordinate system must be modified to have an effective cross section similar to that of a rectangular waveguide. This goal may be achieved if the cross section in the $x$-$y$ plane has a bow-tie shape as shown in Figure 2. The fields near the narrow walls of the waveguide are stretched as the waveguide is twisted. Therefore, the volume of the waveguide is increased when the waveguide height is increased with the angle $\theta(r)$.

The twisted waveguide has greater volume by expanded space along the narrow walls. This structure may be treated as an equivalent loaded waveguide with either dielectric or ferromagnetic materials with radially nonuniform weighted permittivity
or permeability $\mu(y)$, respectively, considering the stretched path with the on-axis length [1]. An approximate expression for the change of propagation constant of a waveguide with material perturbation is given as [2]

$$\beta - \beta_0 \approx \omega \frac{\iint_0^s (\Delta \mu | H_0|^2 + \Delta \varepsilon | E_0|^2) ds}{\iint_0^s (E_0^* \times H_0 + E_0 \times H_0^*) \cdot u_0 ds},$$

(2)

where $\beta$ and $\beta_0$ are the perturbed and unperturbed phase constants respectively, $E_0$ and $H_0$ are the unperturbed electric and magnetic fields, and $^*$ denotes a complex conjugate. In a hollow waveguide, the stored magnetic energy is greater near the narrow walls of the waveguide. The above expression suggests that a decrease in the phase velocity may result in a loaded waveguide described above and equivalently in a twisted waveguide with a properly modified cross section. The envelope of the longitudinal cross section can be determined by transforming the radial distance $R(\phi, z=z_0)$ into $R'(z)$, where $\phi$ is the azimuth angle in a circular cylindrical coordinate system [1].

**SIMULATION**

Because existing commercial electromagnetic codes use orthogonal curvilinear coordinate systems, the twisted structure can not be modeled accurately. The twisted waveguide used here as an example has a cross section resembling a bow tie. The twisted waveguide has been modeled in the MAFIA code [3] using stacked waveguide slices and is shown in Figure 2. As mentioned in the previous section, the end walls used in the simulations do not satisfy the boundary conditions of the accelerating mode fields of a long twisted waveguide structure. A constant displacement angle between the slices determines the pitch of the twist. The displacement angle of the slices is adjusted until the free-space half wavelength of the TM_{01}-like resonance frequency becomes identical to the length of the twisted structure. This is the case for relativistic particles, $\beta_e=1$.

![Figure 2](image)

*FIGURE 2. A twisted waveguide modeled in MAFIA with a stack of slices turned by a constant angle.*
For a structure with a length of 5 cm, the angle was varied until a resonant frequency of 3 GHz was obtained from the desired TM$_{01}$-like mode. The fields in the simulated waveguide do not look like the fields in a regular straight hollow waveguide. In a regular rectangular waveguide, the magnetic field vectors of the TM$_{11}$ mode (comparable to TM$_{01}$ mode in a circular cylindrical waveguide) are lying on the transverse plane. However, the magnetic field of the present TM$_{01}$-like mode is twisted to conform to the true orthogonal walls of the twisted waveguide. That is, the $H_z$ portion of the magnetic field vector increases as radial distance r increases.

The computed properties of the waveguide modes are also shown in the following. Figure 3 shows the magnetic field of the TM$_{01}$-like mode in the bow-tie shaped twisted waveguide with $v_p=c$.

![FIGURE 3. Magnetic field of the TM$_{01}$-like mode in the bow-tie twisted waveguide.](image)

Figure 4 shows the electric field of the TM$_{01}$-like mode in the twisted waveguide with $v_p=c$. The electric field will have $E_z$ only on the beam axis but will have $E_\phi$ if $r \neq 0$. It is interesting to note that the cavity envelope and the field configurations are similar to those of the conventional iris-loaded rotationally symmetrical structure.

![FIGURE 4. Electric field of the TM$_{01}$-like mode in the bow-tie twisted waveguide.](image)
CONCLUSION

Computer simulations have been made to show that a twisted waveguide can support a slow-wave accelerating mode. Commercially available MAFIA and Agilent HFSS codes have been used, and the results of the MAFIA simulations have been presented in this paper. Result of HFSS also showed similar slow-wave properties. Although approximations are made in the modeling due to nonorthogonal end walls, it has been shown that such twisted waveguide structures can support an accelerating mode with $\beta_b \leq 1$. A slow wave with a specific $\beta_b$ at a frequency can be excited in a twisted waveguide by choosing an optimum shape of the cross section and a pitch angle.

The simulations verified that the phase velocity in the waveguide is equal to the free-space speed of light when the angle of twist is 180 degrees. For the specific example for 3 GHz, the waveguide made of copper can support a TM$_{01}$-like mode with $Q = 12,620$ and the shunt impedance $R_s = 258 \, M\Omega / m$.

The inner surface of the structure is smooth and free from any sharp corners. This may raise the field gradient limit. This also means that the structure is made without any irises or posts, which usually degrade the Q-factor of the usable accelerating mode. Prototype fabrication and bench measurements of the twisted waveguide are in progress. Investigation on the beam properties in the waveguide is also in progress. Development of new code for the specific coordinate system, which is conformal to the twisted structure, may be needed. Using this type of new code may aid in making more accurate assessments of rf and beam properties in the structure.

ACKNOWLEDGEMENT

This work was supported by the U. S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.

REFERENCES

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