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II. LARGE SYMMETRIC FORCES**

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STABILITY OF TRILATERAL FORCES: II. LARGE SYMMETRIC FORCES

Gregory H. Canavan

For large symmetric offensive forces, as for small, at few weapons per missile all forces are reserved, costs are constant, and configurations are stable. At many weapons per missile, no weapons are reserved, first strike costs decrease, fractionation is attractive, and stability degrades. These results are due to symmetries that would not be degraded by additional symmetric opponents.

A companion paper discusses first strike stability of symmetric trilateral configurations of modest forces with an extension of the theory developed for bilateral interactions.¹ This note extends that analysis to symmetric trilateral forces of arbitrary size. It predicts that for few weapons per missile, all forces are reserved; for many weapons per missile, none are reserved. The former leads to constant first strike costs and stability; the latter to decreasing costs, payload fractionation, and rapidly degrading stability at large fractionation. For a minimum pairwise metric, the composite index is the same as that for bilinear interactions; hence it should not be degraded by the addition of more opponents with symmetric forces and objectives.

Exchanges between symmetric forces are treated by decomposing them into bilinear interactions, which are treated by optimization techniques derived in companion notes.² Each side has M vulnerable missiles with m weapons each and N survivable missiles with n weapons each for a total of $W = mM + nN$ weapons. Each withholds a fraction $1 - g$ and commits gW weapons to the exchange at hand. For three symmetric forces, the fraction of forces withheld by each side is the same for each participant by symmetry. Thus, the side that strikes first delivers f gW weapons on the other's M vulnerable missiles and the rest in a first strike on military value targets of magnitude

$$F = (1 - f)g(mM + nN) = (1 - f)gW, \quad (1)$$

This counter force strike delivers an average of $r = fW/M$ weapons on each vulnerable missile, which gives them a survival probability $Q \approx q^r$, where $p = 1 - q$ is the missile single shot probability of kill. The second strike on value is then

$$S = g(QmM + nN). \quad (2)$$

where for simplicity it is assumed that the reserve fraction is the same for both first and second strike and vulnerable and survivable forces. First and second strikes are converted into costs through exponential approximations to the value of military value targets destroyed, assuming that each side has $1/k \sim 1,000$ value targets. The cost of damage to self and incomplete damage to other are joined with a weighting parameter L , which measures the attacker's relative preference for damage to other and prevention of damage to self.³ The costs of striking first and second are thus

$$C_1 \approx (1 - e^{-kS} + Le^{-kF}) / (1 + L), \quad (3)$$

$$C_2 \approx (1 - e^{-kF} + Le^{-kS}) / (1 + L), \quad (4)$$

and the conventional cost ratio stability index is

$$I = C_1 / C_2. \quad (5)$$

Results. While these equations are nonlinear, they can be solved by iteration for the g and f that minimize C_1 for any set of forces and m . Figure 1 compares the analytic estimate of the number of weapons allocated to each of $M = 250$ vulnerable missiles with that from the numerical solution of Eqs. (1)-(5). For $m = 3$ weapons per missile, the discrepancy is about 10%; it is much smaller for smaller m . The apparent discrepancy for $m < 1.75$ is an artifact of the numerical solution. The optimum $g = 0$ there, so the solution for f is not relevant.

Figure 2 shows g , f , r , and Q as functions of M . g switches essentially discontinuously from 0 to 1 at $m = 1.75$, in accord with the behavior analyzed in the previous note. f and Q trend downward gently in the region where g is non-zero. For large m the opponents are willing to put all of their forces at risk; for small m they are not willing to put any.

Figure 3 shows that the first strike F increases rapidly when m reaches 1.75 and more slowly thereafter. The second strike S jumps initially, but then falls slowly with Q .

Figure 4 shows the strike costs and index. The strike costs are constant and equal for $m < 1.75$, so I is unity there. For larger m , C_1 falls gradually and C_2 increases more rapidly, so I falls—very rapidly for $m = 1.5$ to 1.75 and more slowly for larger m . Since C_1 falls with m , there is an incentive to increase it by further fractionating payload. Conversely, an attempt to decrease m to increase I would increase C_1 , which would logically be opposed on military grounds.

The withhold for other opponents enters only in that symmetry dictates that it would be the same for all three players. Thus, if a four-sided interaction was similarly decomposed into binary and tertiary interactions, symmetry would also dictate the equality of their reserves. If the overall metric is one of the minimum value of the index for any interaction, that minimum would be the same for all sides—and equal to that calculated for binary interactions between symmetric forces—so the same overall stability index would be obtained for three, four, or any number of symmetric sides.

Summary and conclusions. The first strike stability of symmetric trilateral configurations is discussed with an extension of the theory developed for bilateral configurations and tested on symmetric moderate trilinear configurations earlier. The approach assumes a reserve for other interactions, optimizes the first strike cost for the interaction at hand given that reserve, and varies the reserve to find the overall minimum. The results extend to all force levels the conclusions found earlier for moderate forces: at few weapons per missile, all forces are reserved, producing constant costs and stability, and at many weapons, none are reserved, first strike costs decrease, fractionation becomes attractive, and stability degrading. For a minimum

pairwise metric, the composite index is the same as that calculated for individual pairs, which indicates the stability of symmetric triads is the same as that of symmetric dyads. Since these results follow largely from symmetry, they imply that stability would not be degraded by the addition of more opponents of symmetric forces and objectives.

References

¹. G. Canavan, "Stability of Trilinear Forces: I. Moderate Symmetric Forces," Los Alamos report LA-UR-98-draft, September 1998.

². G. Canavan, "Considerations in Missile Reductions and Dealerting," L. Kruger ed, Proceedings of the World Federation of Scientists Working Group on Missile Proliferation and Defense (World Federation of Scientists, Lausanne, 1998); Los Alamos National Laboratory Report LA-UR-98-1426, April 1988.

³. G. Kent and R. DeValk, "Strategic Defenses and the Transition to Assured Survival," RAND Report R-3369-AF, October, 1986.

Fig. 1. Comparison of analytic and numerical allocations

Fig. 2. Reserve, allocation, weapons per missile, and survivability

Fig. 3. First and second strikes vs m

Fig. 4. Strike costs and stability index