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DESIGN CONSIDERATIONS FOR A LATTICE TEST PILE

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I. ABSTRACT

There probably exist lattice designs considerably better than the one currently in use in Hanford piles. In current practice, the experimental study of a proposed pile lattice requires the building of an exponential pile. Exponential piles built during the past two years have made possible advances in lattice design which probably justify several times the trouble required to build them.

An exponential pile is, however, an ambitious project. It requires about a year to build. It costs in excess of $100,000. Once such a pile is built it yields the buckling of a single type of lattice and is then torn down. Because of the large number of variables in a pile lattice, a quite large number of lattices must be explored before an optimum can be found. The present paper proposes a lattice test pile, (or LTP) as an alternate to the continuing program of building up and tearing down of exponential piles. Such a test pile may be expected to yield not only the buckling of a proposed lattice but other pile constants as well. It gives more complete information about the lattice tested. The materials

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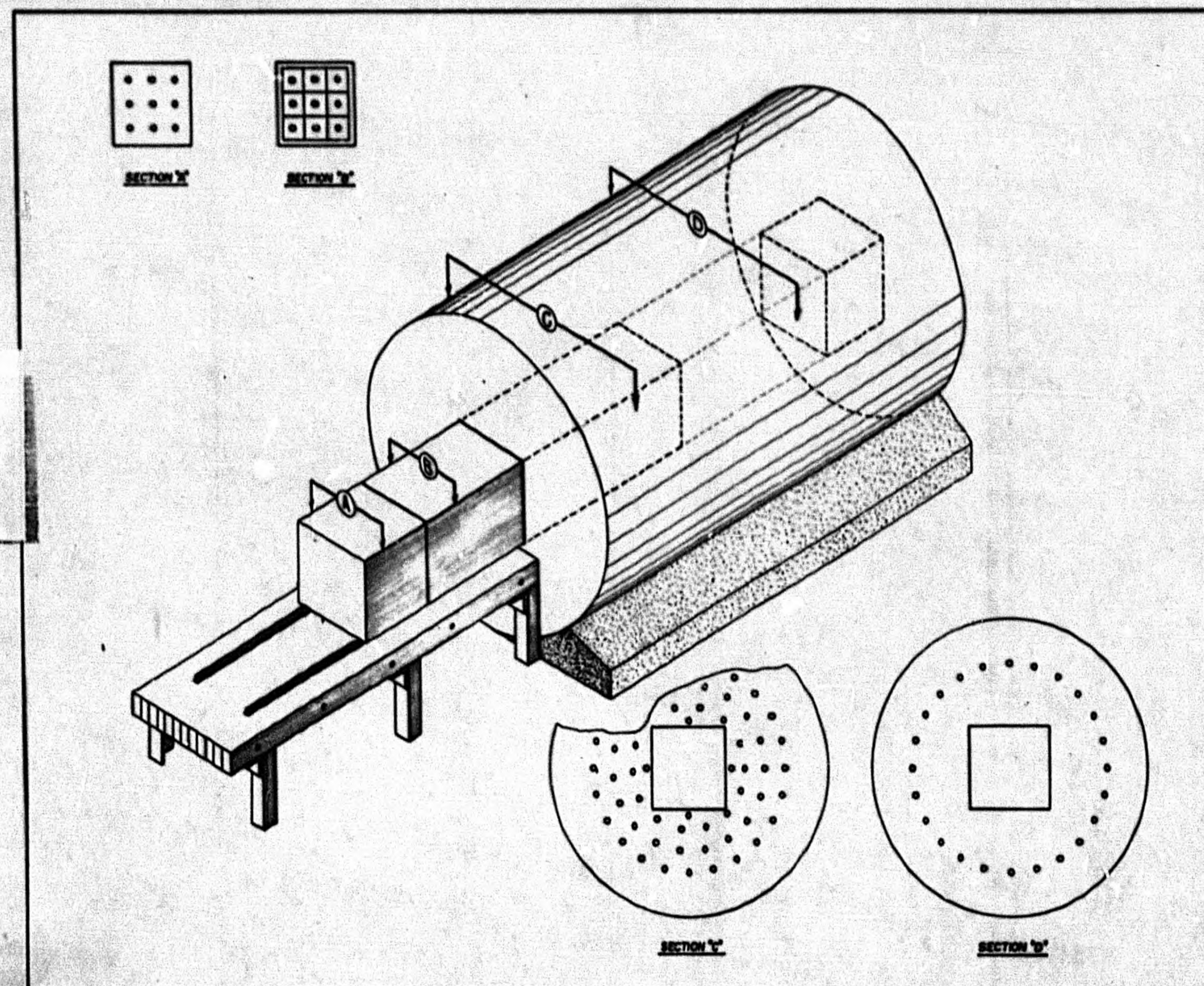

PAGE 2

MB-2507

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used in a lattice test pile, mostly graphite, are roughly equal in bulk to those in a single exponential pile. An inventory of 20 kg, very roughly, of enriched uranium would be required to run the proposed lattice test pile. It would cost as much as a "few" exponential piles, exclusive of uranium; further study is needed to show whether this "few" means a factor of two or five. Once built, a lattice test pile would be a permanent installation.

By far the greatest advantage of a lattice test pile is that it measures pile lattices with a much greater speed than can exponential piles; greater by a factor of ten is a conservative guess. This is possible largely because the test sample of the new lattice required for a lattice test pile is from 1/30 to 1/50 as large as in an exponential pile. It seems likely that lattice improvements for production piles could be found and put into use, with the aid of a lattice test pile, many years before they could be found with the aid of an exponential pile only.

The analysis of experimental readings taken with an LTP is more complex than with an exponential pile; a feature which has retarded the development of ideas on the LTP, whose basic idea is several years old. This greater complexity arises from the importance of edge effects in the small sample of test lattice used in the lattice test pile. A method of allowing for these edge effects is presented in what follows.

II. DESCRIPTION OF A LATTICE TEST PILE

The principal properties of a neutron pile are commonly expressed in terms of certain pile constants such as buckling, reactivity, diffusion lengths, etc. Most of these pile constants are in principle determined as soon as one knows the design of a single cell in the lattice of fuel rods and moderator which comprises most of the bulk of a pile. The problem of pile lattice design may be phrased: What lattice has the most desirable pile constants? Current theories can give only a partial answer to this problem. The more reliable pile constants, now a days, are experimentally measured.

The current method of measuring pile constants involves the use of exponential piles. A typical such pile is a graphite cube about 12 feet high, built up of accurately drilled and milled graphite bars. An exponential pile, although small compared to a production pile is still truly elephantine. After the measurement of one buckling has been made with such a pile, its usefulness is over, and it is torn down. An exponential pile may be criticized for its bulk and expense. A more serious objection to such a pile is that it is not entirely adequate. As customarily used, it yields one pile constant quite well, the buckling, and not very much more. For example, it does not measure the conversion ratio of the lattice of which it is built. The fact is that the utility of the pile lattice can only be fully judged after several pile constants are known, not all of which can be given by an exponential experiment.

The purpose of the present essay is to describe a lattice test pile which may be smaller and cheaper than an exponential pile, and which in addition may yield several pile constants descriptive of this lattice instead of one or two. The general idea of such a pile has been discussed from time to time at Hanford for several months. This general idea is that the test pile should consist of two parts, a core and a driver. The core might be a set of nine uranium rods with associated moderator, the entire core forming a comparatively small sample of the lattice to be tested. Its dimensions could be about 2' x 2' x 6'. This core, which would have to be made anew whenever work started on a new type of lattice, would weigh roughly 1/50 as much as an exponential pile which is built anew in current practice when measurements start on a new lattice. The driver would consist of a cylinder of graphite about six feet in diameter and about seven feet long with an axial hole for admitting the core. In addition the driver would be drilled for the "driver rods", which could be made of an alloy containing enriched uranium. These driver rods, parallel to the axis of the driver would lie on a cylinder of about four feet in diameter. This cylinder would have distributed over each base, a set of "leveling slugs". By a leveling slug is meant a short length of rod, roughly four inches in length, and of the same type as that used for the driver rods.

Bringing the pile to critical would be effected through radial motion of the driver rods out from or in toward the axis of the pile, and through axial motion of the leveling slugs. Each driver rod would be mounted in an eccentrically bored tube of graphite, about eight inches in diameter which would be mounted in a graphite hole in the driver, with sliding fit. The rotation, in phase, of the eccentric tubes would vary continuously the radius of the cylinder containing the driver rods, without making voids in the graphite near these rods. The leveling slugs in the end of the driver would be divided into three concentric rings, the slugs in any one ring being rigidly connected. The slugs in any one ring could be moved along the length of the pile independently of those in the other rings; these rings would move in dead-end holes in which there would usually be voids.

In operation the rings of leveling slugs in the driver rods would be adjusted by trial and error until the pile was critical and at the same time had zero longitudinal buckling everywhere on its equatorial section. Due to end and edge effects, there would probably not exist any adjustments of the leveling rings which produced axial leveling of both the fast and thermal fluxes throughout the length of the core. These edge effects die down in about one migration length or two feet. A two foot length of the core at each of its ends is present simply to provide space in which end effects may die away; it acts similarly to a guard ring in a precision condenser for electric charge. It is only the mid section of the core to which a detailed analysis will be applied, with the simplifying assumption of no axial variation of neutron density. After the pile has been simultaneously leveled and brought to critical, then a reading of the radius

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out to the driver rods gives in principle one pile constant, or at least one relation between pile constants. In what follows, a mathematical procedure for interpreting this setting will be outlined.

III. INTERPRETATION OF READINGS ON A LATTICE TEST PILE

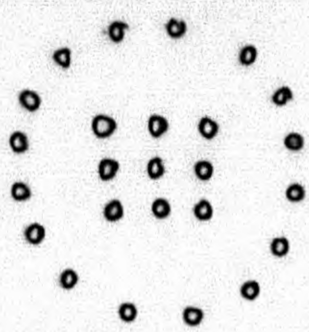

Fig II

A schematic cross section of the pile is given in Figure II. Each small circle represents a section of a metal rod. The remainder of the section is through graphite. We will apply the small source model for analyzing the neutron flux, HW-24282. Within the graphite

$$(1)\quad (\nabla^2-\lambda^2)n = \sum_i \frac{Q_i}{D}\delta(\vec{\rho}-\vec{\rho}_i) - \frac{q}{D}$$

In this equation, n is the thermal neutron density, $\lambda^{-1}=L$ the diffusion length in graphite, D the diffusion constant, q the slowing down density. Q_i is the sink density for the i'th rod; that is the number of neutrons absorbed per second per cm of length by this rod. The δ functions in (1) are approximately correct because the radius of a uranium rod is small compared to the distance between adjacent rods. The age equation reads

$$\nabla^2 q = \frac{\partial q}{\partial \theta}$$

We put

$$\bar{q} = \int_0^\infty e^{-\theta\varphi^2} q(\theta)\,d\theta$$

so that

$$(2)\quad (\nabla^2-\varphi^2)\bar{q} = -q(0) = -\sum_i S_i\,\delta(\vec{\rho}-\vec{\rho}_i)$$

where S_i is the source strength of the i'th rod, that is the number of neutrons produced per thermal neutron absorbed in the i'th rod. We write

$$S_i = h_0 Q_i \quad \text{or} \quad S_i = h_1 Q_i$$

according as the i'th rod is in the driver or core. h_0 is called the slug multiplication for a driver rod and similarly for h_1. The solution of (2) is

$$\bar{q} = \sum_i S_i \frac{K_0(\varphi|\vec{\rho}-\vec{\rho}_i|)}{2\pi}$$

Hence

$$(3) \quad n = -\sum_i \frac{Q_i}{2\pi D} K_0(\lambda|\vec{\rho}-\vec{\rho}_i|)$$

$$+\sum_i h_i \frac{Q_i}{D} \int \frac{K_0(\lambda|\vec{\rho}-\vec{\rho}'|)}{2\pi} d\vec{\rho}' \int \frac{\lambda^2+\varphi^2}{2\pi^2} \frac{K_0(\varphi|\vec{\rho}'-\vec{\rho}_i|)}{2\pi} d\varphi^2 \quad (3)$$

For all, or almost all cases of interest, the neutron buckling will be sufficiently small so that the change in sink strength as we go from rod to rod within the core will be very small. For the present, then, we write $Q_i = Q_1$ when the i'th rod is within the core. Similarly, we write as a good approximation $Q_i = Q_0$ when the i'th rod is within the driver. Equation (3) contains the two constants Q_1 and Q_0 as yet undetermined. To determine these constants we use the boundary conditions

$$(4) \quad Q_1 = \beta_1 \int_1 \vec{J}\cdot d\vec{A} \quad , \quad Q_0 = \beta_0 \int_0 \vec{J}\cdot d\vec{A}$$

β_0 is the blackness of the thermal rod, that is, the probability that a thermal neutron, having once entered such a rod will be absorbed before emerging into the graphite. The integral $\int_0$ is an integral over the surface of the unit length of the driver rod of the total neutron current. J is the neutron current, dA an area element. β_1 and $\int_1$ have corresponding meanings for a core rod. For J we write the expression from diffusion theory

$$J = \frac{nv}{4} + \frac{\lambda_t v}{6} \nabla n$$

We will use the relation

$$\sum K_0(\lambda|\vec{\rho}-\vec{\rho}_i|) \simeq 2\pi \frac{\rho_0}{b} I_0(\lambda\rho) K_0(\lambda\rho_0)$$

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where the summation on the left is over the driver rods, ρ is the diameter of the cylinder on which the booster rods lie, b is the distance between booster rods. Substituting in the first equation of (4) now gives

$$Q_1 = 2\pi a \rho_1 \Big[\frac{h_1}{4} \Big\{ \frac{-Q_1}{2\pi D}\Big(K_0(\lambda a) + \sum_i{}' K_0(\lambda \rho_i)\Big) - \frac{Q_0}{D}\frac{a}{4} I_0(\lambda a) K_0(\lambda \rho)$$

$$(5) \qquad + \frac{1}{D}\int \frac{K_0(\lambda|\vec\rho - \vec\rho\,'|)}{2\pi} \int \frac{e^{-\lambda\rho'}}{2\pi}\Big[h_0 Q_0 \sum_0 \frac{K_0(\lambda|\vec\rho\,' - \vec\rho_0|)}{2\pi} + h_1 Q_1 \sum_i \frac{K_0(\lambda|\vec\rho\,' - \vec\rho_i|)}{2\pi}\Big] d\rho'$$

$$+ \frac{l_v}{6}\frac{Q_1}{2\pi D}\lambda K_0'(\lambda a)$$

(5)

and the second equation of (4) becomes

$$(6) \qquad Q_0 = 2\pi a \rho_0 \Big[\frac{h_0}{4} \Big\{ \frac{-Q_1}{2\pi D} \sum_i K_0(\lambda|\vec\rho_0 - \vec\rho_i|) - \frac{Q_0}{2\pi D} K_0(\lambda a)$$

$$- \frac{Q_0}{2\pi D}\int_{\theta_1}^{\theta_2} K_0(\lambda|\vec\rho_0 - \vec\rho\,'|) \frac{\rho_0\, d\theta}{b}$$

$$+ \frac{1}{D}\int \frac{K_0(\lambda|\vec\rho_0 - \vec\rho\,'|)}{2\pi} d\rho' \int \frac{e^{-\lambda\rho'}}{2\pi}\Big\{h_0 Q_0 \sum_0 \frac{K_0(\lambda|\vec\rho\,' - \vec\rho_0|)}{2\pi} + h_1 Q_1 \sum_1 \frac{K_0(\lambda|\vec\rho\,' - \vec\rho_i|)}{2\pi}\Big\} d\rho'$$

$$+ \frac{l_v}{6}\frac{Q_0}{2\pi D}\lambda K_0'(\lambda a)$$

(6)

Since equations (5) and (6) are linear and homogeneous in Q_0 and Q_1, their descriminant say $D_9(h_1, \rho_1)$ must be zero. The subscript 9 refers to the fact that all 9 of the rod holes in the core are filled with metal rods. The equation

$$(7) \qquad D_9(h_1, \rho_1) = 0$$

gives one relation between the desired lattice constants h_1 and ρ_1. To get a second relation between the constants h_1 and ρ_1 the central rod

6335.7

hole in the core, for example, is now filled with a rod of graphite instead of metal and the pile brought to critical again, with a new radius out to the driver rod. The summation $\sum$ in (5) and (6) will now be over only 8 metal core rods instead of 9. Equations (5) and (6) will now yield the discriminant

$$D_8(h_1, \rho_1) = 0 \tag{8}$$

From the two equations (7) and (8) one may now solve for the pile constants h_1 and ρ_1.

The reader will have noticed that the equations (5) and (6) as well as the critical conditions (7) and (8) depend on h_0 and ρ_0, the rod multiplication and blackness of the driver rods. Before information about the core can be obtained from the critical conditions, h_0 and ρ_0 must be known. h_0 is quite simply derivable from the neutron cross sections of the material of the driver rods. To find ρ_0, the core space may be filled simply with graphite and the pile brought to critical. Then the critical condition will be (6) with Q_i set equal to zero; and this critical condition yields the value of ρ_0.

IV. DEDUCTION OF K^2 FROM h AND ρ

The slug multiplication h and the slug blackness ρ, whose measurement has been outlined in the preceding are not the lattice constants best known to nuclear engineers. They have nevertheless been used because of their direct occurence in that form of diffusion theory suitable for the description of a pile so small that edge effects are important. A better known equivalent pair of pile constants are the buckling K^2 and the thermal utilization f.

To calculate K^2, we consider a large pile consisting of graphite and fuel rods. Within the graphite we have from (3)

$$\begin{aligned} n = &-\sum_i \frac{Q_i}{2\pi D} K_0(\lambda|\vec r - \vec r_i|) \\ &+ h\sum_i \frac{Q_i}{D}\int \frac{K_0(\lambda|\vec r' - \vec r_i|)}{2\pi}\, d\vec r' \int \frac{e^{i\vec q\cdot(\vec r - \vec r')}}{2\pi} \frac{K_0(\tau|\vec q|)}{2\pi}\, d\vec q \end{aligned} \tag{9}$$

where the summation is over all the rods in the pile. Applying the boundary condition

$$Q_j = \rho\int_j \vec J\cdot d\vec A\,, \quad J = \frac{nv}{4} + \frac{\lambda v}{6}\frac{\partial n}{\partial \rho}$$

where the integration is over the surface of a unit length of the j'th rod, yields

$$\begin{aligned} Q_j = \frac{2\pi a}{4}\frac{v}{D}\rho\Big[&-Q_j\frac{K_0(\lambda a)}{2\pi} - \sum_i Q_i\frac{K_0(\lambda|\vec r_j - \vec r_i|)}{2\pi} \\ &+ h\int d\vec r'\,\frac{K_0(\lambda|\vec r' - \vec r_j|)}{2\pi}\Big[\sum_i Q_i \frac{K_0(\lambda|\vec r' - \vec r_i|)}{2\pi}\Big]\frac{e^{i\vec q\cdot\vec r'}\,d\vec q}{2\pi}\Big] \end{aligned} \tag{10}$$

where the summation Σ' is over all rods except the j'th. Approximating sums by integrals gives

$$Q(\rho) = \frac{3\pi}{2}\frac{a}{\lambda_t}\rho\Big[-Q(\rho)\int_1 \frac{K_0(\frac{a}{L})-K_0(\frac{\rho'}{L})}{2\pi}\frac{d\vec{\rho}\,'}{4b^2} - \int Q(\rho')\frac{K_0\left(\frac{|\vec{\rho}-\vec{\rho}\,'|}{L}\right)}{2\pi}\frac{d\vec{\rho}\,'}{4b^2}$$

$$-\frac{2}{3}\frac{\lambda_t}{L}Q(\rho)\frac{K_0'(\frac{a}{L})}{2\pi} + h\int\frac{K_0(|\vec{\rho}-\vec{\rho}\,'|/L)}{2\pi}\int_{-i\infty}^{i\infty} Q(\vec{\rho}\,'')\frac{K_0(|\vec{\rho}\,'-\vec{\rho}\,''|\varphi)}{2\pi}\frac{d\vec{\rho}\,''}{4b^2}\,. \qquad (11)$$

$$e^{\theta\varphi^2}\frac{d\varphi^2}{2\pi i}\,d\vec{\rho}\,'.$$

This is an integral equation determining the yet undetermined function $Q(\rho)$. Put $Q(\rho) = J_0(\kappa\rho)$ where K is a constant to be determined. We need the integral formula

$$\int_0^z K_0(z)\,z\,dz = 1 - z\,K_1(z)$$

from which

$$\int_1 \frac{K_0(\frac{\rho'}{L})}{2\pi}\frac{d\vec{\rho}\,'}{4b^2} = \frac{L^2}{4b^2}\Big[1-\frac{\rho_1}{L}K_1\Big(\frac{\rho_1}{L}\Big)\Big]$$

where ρ_1 is the radius of the circle whose area equals that of a cell cross section. We have also

$$\int \frac{J_0(\kappa\rho')\,K_0(|\vec{\rho}-\vec{\rho}\,'|/L)}{2\pi}\frac{d\vec{\rho}\,'}{4b^2} = \frac{1}{4b^2}\frac{1}{\kappa^2+\frac{1}{L^2}}$$

We have

$$\int J_0(\kappa\rho'')\frac{K_0(|\vec{\rho}\,'-\vec{\rho}\,''|\varphi)}{2\pi}\,d\vec{\rho}\,'' = \frac{J_0(\kappa\rho')}{\kappa^2+\varphi^2}$$

The last integral in (11) thus becomes

$$\int\frac{K_0(|\vec{\rho}-\vec{\rho}\,'|/L)}{2\pi}J_0(\kappa\rho')\,d\vec{\rho}\,'\int_{-i\infty}^{i\infty}\frac{e^{\theta\varphi^2}}{\kappa^2+\varphi^2}\frac{d\varphi^2}{2\pi i}\frac{1}{4b^2}$$

$$= \frac{1}{4b^2}e^{-\kappa^2\theta}\frac{J_0(\kappa\rho)}{\kappa^2+\frac{1}{L^2}}$$

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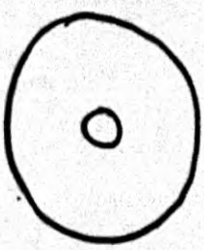
6335-9

Substituting in (11) yields

$$(12)\quad 1=\frac{3\pi}{2}\frac{a}{\lambda_t}\beta\Big[-\frac{K_0(a/L)}{2\pi}+\frac{L^2}{4b^2}\Big[1-\frac{\rho_1}{L}K_1\Big(\frac{\rho_1}{L}\Big)\Big]-\frac{1}{4b^2}\frac{1}{\kappa^2+\frac{1}{L^2}}$$
$$-\frac{2}{3}\frac{\lambda_t}{L}\frac{K_0'(a/L)}{2\pi}+\frac{e^{-\kappa^2\tau}}{4b^2}\frac{h}{\kappa^2+\frac{1}{L^2}}\Big]$$

which determines the buckling K^2 as a function of h, β and the lattice dimensions.

V. DEDUCTION OF f FROM h AND β



To compute f in terms of h and β, we simulate a lattice cell by an "equivalent cylinder". Within the graphite of this cylinder

$$(13)\quad (\nabla^2-\lambda^2)n=-\frac{q}{D}+\frac{Q}{D}\delta(\vec{\rho}),\qquad \lambda\equiv L^{-1}$$

where the notation follows that of equation (1), but where q may be regarded as a constant throughout the cell. The solution of (13) is

$$n=-\frac{Q}{2\pi D}K_0(\lambda\rho)+\frac{q}{\lambda^2 D}+A\,I_0(\lambda\rho)$$

Let n' designate $\frac{\partial n}{\partial\rho}$ evaluated at the edge of equivalent cylinder, where $\rho=\rho_1$. Then the constant of integration A is determined by

$$-\frac{Q\lambda}{2\pi D}K'(\lambda\rho_1)+A\lambda I_0'(\lambda\rho_1)=n'$$

The boundary condition at the slug surface reads

$$(14)\quad Q=\beta\int\vec{J}\cdot d\vec{A}$$
$$\text{with } J=\frac{nv}{4}+\frac{\lambda_t v}{6}\frac{\partial n}{\partial\rho}$$

Substituting in (14) gives

$$(15)\quad Q = 2\pi a\beta\left\{\frac{v}{4}\left[\frac{Q}{2\pi D}K_0(\lambda a) + \frac{q}{\lambda^2 D} + \frac{I_0(\lambda a)}{I_0(\lambda\rho_1)}\left(\frac{Q}{2\pi D}K_0'(\lambda\rho_1) + \frac{qn'}{\lambda}\right)\right] - \frac{\lambda_t v}{6}\frac{Q\lambda}{2\pi D}K_0'(\lambda a)\right\}$$

which is a form of the pile critical condition. If k is the pile multiplication constant,

$$\kappa^2(L^2+\theta) = k-1$$

k also satisfies

$$k = \frac{Q + 2\pi\rho_1\frac{\lambda_t}{3}n'}{Q}, \text{ or } n' = Q\frac{k-1}{2\pi\rho_1\frac{\lambda_t v}{3}}$$

Since L^2 and θ have been assumed known from the beginning, and since the finding of κ^2 was outlined in the previous section, we may now regard k as known.

Also, by definition of f:

$$f = \frac{Q}{gV} \text{ or } g = \frac{Q}{fV}$$

where V is the graphite volume per unit length of a lattice cell. Substituting (15) from (16) and (17) finally yields f in terms of known quantities.

V. SUMMARY

The preceding analysis is approximate in several ways, the sink function used probably should be replaced by analogues derived from transport theory. An attempt is being made to find these analogues. The effect of resonance capture on the slowing down of neutrons has been ignored. Despite these imperfections the preceding analysis probably is more detailed than that usually used in pile engineering. In particular, it seems as accurate as that used in connection with exponential piles.

The effect of the inaccuracies in the pile model which has been used in the analysis will be reduced by the fact that the formulas contain several empirical constants, determined to fit quite accurate measurements. The lattice test pile could be calibrated with the aid of any lattice sample whose pile constants were known in advance.

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The conclusion is that if experimental work on pile lattice is to continue, then the building of a lattice test pile, along the lines described should be very seriously considered, (1) as a substantial economy, (2) as a more accurate means of making lattice measurements and (3) as a more complete way.

VI. ACKNOWLEDGEMENTS

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