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34721

HW-24137
Physics and Mathematics

This document classified by
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REACTIVITY EFFECTS OF VERY INHOMOGENEOUS ENRICHED LOADINGS

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Price $ 0.25

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

February 24, 1953

HANFORD ATOMIC PRODUCTS OPERATION
RICHLAND, WASHINGTON

Operated for the Atomic Energy Commission by the
General Electric Company under Contract # W-31-109-Eng-52

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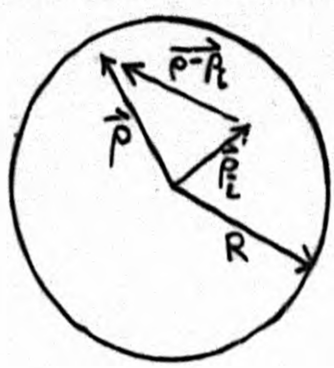
I. SUMMARY

The reactivity effects of one and of two highly enriched fuel rods placed into a natural uranium lattice thermal pile are investigated according to the small source model. (See HW-24282 by W. A. Horning.) This model seems well suited to problems involving very inhomogeneous loadings. Assuming moderator constants and blackness values for the pile fuel elements, the resonance escape probabilities for those neutrons arising from fissions in the enriched rods are calculated to be, in the case of a single enriched rod loaded centrally into the pile, 0.855, and for the case of two enriched rods loaded centrally into the pile, 0.868.

II. THE SMALL-SOURCE ANALYSIS

A. Small-Source Description of a Large Pile

Consider an infinitely long pile, right circular cylindrical, of radius R. Fuel rods are loaded parallel to the pile axis in a square array. Assuming that the pile is several slowing down lengths in radius, and that the fuel rods are small enough to be treated as line sinks, the thermal neutron diffusion equation is:



(1)

$$D_0\left(\nabla^2-\frac{1}{L^2}\right)nv = \sum_i Q_i\,\delta(|\vec{\rho}-\vec{\rho}_i|) - \sum_i \frac{p_i h_1 Q_i e^{-\frac{|\vec{\rho}-\vec{\rho}_i|^2}{4\theta}}}{4\pi\theta}$$

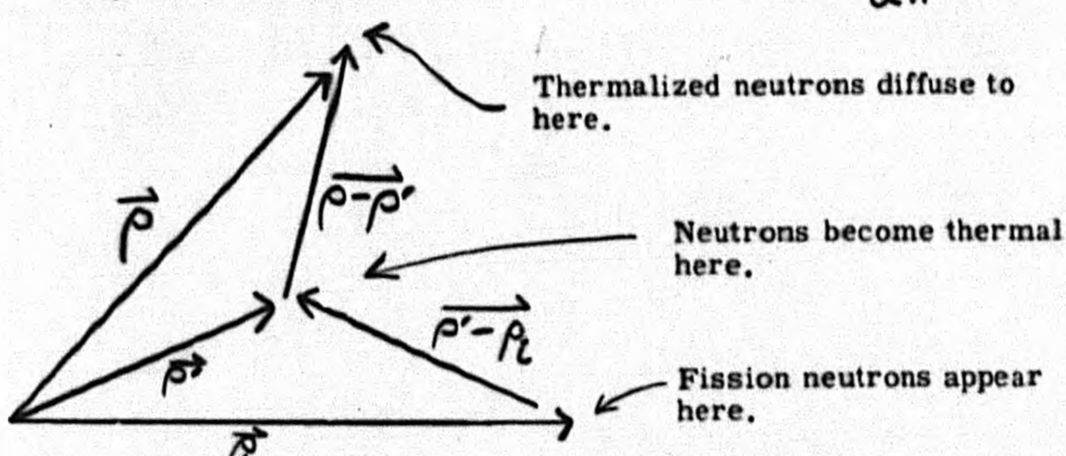
[redacted]

where the symbols are defined in the glossary of notation. Assuming that the pile face is many migration areas (so that infinite slowing down and diffusion kernels may be be used without appreciable error due to the extrapolation length at the pile face):

$$\text{(2)} \quad \varphi = nv = -\sum_i \frac{Q_i}{2\pi D_0} K_0(\lambda|\vec{\rho}-\vec{\rho}_i|) + \sum_i \frac{p_i h_i Q_i L^2 F(|\vec{\rho}-\vec{\rho}_i|)}{4\pi\Theta D_0}$$

where the sums extend over all the fuel rods in the pile and where $F(|\vec{\rho}-\vec{\rho}_i|)$ is the source function defined as:

$$F(|\vec{\rho}-\vec{\rho}_i|) = \lambda^2 \oint e^{-\frac{|\vec{\rho}'-\vec{\rho}_i|^2}{4\Theta}} K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\, \frac{d\vec{\rho}'}{2\pi}$$



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* The Gaussian kernel gives the source distribution of the thermal neutrons and the K_0 factor describes the thermal diffusion of neutrons. $\oint$ signifies integration over <u>all space</u>. $d\vec{\rho} = \rho\, d\rho\, d\phi$

The function $F(|\vec{\rho}-\vec{\rho}_i|)$, considered as a function of ρ, is bell shaped, centered about $\rho = \rho_i$.

The Q_i are determined by application of their definition,

$$(3) \qquad Q_i = 2\pi a \beta_i J_{in}(|\overrightarrow{\rho - \rho_i}|)$$

where a is the radius of the fuel rods, the β_i are rod blacknesses,* and $J_{in}(|\overrightarrow{\rho - \rho_i}|)$ is the total thermal neutron current density flowing toward the i^{th} fuel rod, evaluated at the rod surface. Since

$$J_{in} = \frac{\varphi}{4} + \frac{D_0}{2} \vec{\nabla}\varphi \cdot \hat{r}$$

($\hat{r}$ is the unit radial vector normal to the fuel rod surface), expression (3) becomes:

$$(4) \qquad \frac{Q_i}{2\pi a \beta_i} = \frac{1}{8\pi D_0}\left[-Q_i K_0(\lambda a) - \sum_{\substack{j \\ j\neq i}} Q_j K_0(\lambda|\overrightarrow{\rho_j-\rho_i}|) + \frac{p_i h_i Q_i L^2 F(a)}{2\theta} + \sum_{\substack{j \\ j\neq i}} \frac{p_j h_j Q_j L^2 F(|\overrightarrow{\rho_i-\rho_j}|)}{2\theta} \right] + \frac{\lambda K_1(\lambda a)}{4\pi} Q_i$$

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* Rod blackness is defined as the fraction of those neutrons headed toward a fuel rod, at the rod surface, which is actually absorbed in the rod.

[redacted]

where the terms at the i^{th} fuel rod are written in detail and the only gradient term of importance is retained. This is a set of linearly independent equations, the order of which is equal to the number of fuel rods. For the usual thermal pile, where all fuel rods are identical, all $\beta_j = \beta_0$ (say), $h_j = h_0$, $p_j = p_0$. Replacing the sums in (4) by integrals (b^2 is the area of a lattice cell), the lattice function Q_i becomes a point function $Q(\rho)$, defined by the following integral equation (the terms are in the same order as in equation (4):

$$(5) \quad \frac{Q(\rho)}{2\pi a \beta_0} = \frac{1}{8\pi D_0}\Bigg[-Q(\rho)K_0(\lambda a) - \oint Q(\rho')K_0(\lambda|\rho-\rho'|)\frac{d\vec{\rho}'}{b^2} + \int_{i^{th}\,\mathrm{cell}} Q(\rho')K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\frac{d\vec{\rho}'}{b^2} + \frac{p_0 h_0 L^2 F(a)}{2\theta}\,Q(\rho) + \frac{p_0 h_0 L^2}{2\theta}\oint Q(\rho')F(|\vec{\rho}-\vec{\rho}'|)\frac{d\vec{\rho}'}{b^2} - \frac{p_0 h_0 L^2}{2\theta}\int_{i^{th}\,\mathrm{cell}} Q(\rho')F(|\vec{\rho}-\vec{\rho}'|)\frac{d\vec{\rho}'}{b^2}\Bigg] + \frac{\lambda K_1(\lambda a)}{4\pi}\,Q(\rho)$$

where two integrals, one over all space and one over the i^{th} cell, replace a single sum $\sum_{j \neq i}$ over all cells but the i^{th} cell, the i^{th} cell being centered about the point ρ.

[redacted]

It is found that $Q(\rho) = A\,J_0(\mathcal{H}\rho)$ is the solution of the integral equation which is regular over the pile, as the uniformity of fuel loading requires. Upon performing the integrations, the following equation for $\mathcal{H}$ is obtained:

$$\text{(6)}\quad \frac{1}{2\pi a\beta_0} = \frac{1}{4}\left\{-\frac{1}{2\pi D_0}\left[\frac{2\pi}{b^2(\mathcal{H}^2+\lambda^2)} + K_0(\lambda a) - \frac{2\pi\left\{\rho_0\left[\mathcal{H}J_1(\mathcal{H}\rho_0)K_0(\lambda\rho_0) - \lambda J_0(\mathcal{H}\rho_0)K_1(\lambda\rho_0)\right] - 1\right\}}{b^2(\mathcal{H}^2+\lambda^2)}\right] + \frac{p_0 h_0 e^{-\mathcal{H}^2\theta}}{D_0 b^2(\mathcal{H}^2+\lambda^2)}\right\} + \frac{\lambda K_1(\lambda a)}{4\pi}$$

where ρ_0 is the radius of a cell equivalent in area to b^2. $\mathcal{H}^2$ may be recognized as the usual pile radial buckling. By solving relation (6) for $\mathcal{H}$, it may be determined at what pile radius the sink density (and neutron flux) vanishes. Relation (6) is therefore the pile critical relation according to small-source theory. Note that the small-source method yields the same cell-averaged flux as ordinary one-group theory for this type of pile.

B. Small-Source Description of a Pile with One Enriched Rod*

The pile model here is identical to that of section A, except that along the axis of the pile there is an enriched fuel rod instead of a natural uranium fuel rod.

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* For purposes of this discussion, an enriched fuel rod is one containing an alloy of U^{235} and Al. The alloy content is, by weight, 95.65% Al, of 2S quality, 4.06% U^{235}, .30% U^{238}. The alloy density is 2.792 g/cm^3. However it may be seen that the analysis would apply, with small changes in detail, to any other kind of fuel rod placed at the pile axis.

Denoting the sink strength of the enriched column by Q_e, the rod multiplication by h_e, the resonance escape probability to be associated with neutrons arising from fissions in the enriched rod by p_e **, the boundary conditions may be seen to be:

For the j^{th} fuel rod ($j \neq e$, the enriched rod):

$$(7) \quad \frac{Q_j}{2\pi a \beta_0} = \frac{1}{8\pi D_0} \left\{ \begin{aligned} & - Q_j K_0(\lambda a) - Q_e K_0(\lambda \rho) \\ & - \sum_{\substack{i \\ i\neq e \\ i \neq j}} Q_i K_0(\lambda |\vec{\rho_j} - \vec{\rho_i}|) \\ & + \frac{p_e h_e Q_e L^2}{2\theta} F(\rho) + \sum_{\substack{i \\ i\neq e}} \frac{p_0 h_0 L^2 Q_i}{2\theta} F(|\vec{\rho_j} - \vec{\rho_i}|) \end{aligned} \right\} + \frac{Q_j}{4\pi} \lambda K_1(\lambda a)$$

For the enriched rod:

$$(8) \quad \frac{Q_e}{2\pi a \beta_e} = \frac{1}{8\pi D_0} \left\{ \begin{aligned} & - Q_e K_0(\lambda a) - \sum_{\substack{i \\ i\neq e}} Q_i K_0(\lambda |\vec{\rho_i} - \vec{a}|) \\ & + \frac{p_e h_e L^2}{2\theta} Q_e F(a) \\ & + \sum_{\substack{i \\ i\neq e}} \frac{p_0 h_0 L^2}{2\theta} Q_i F(|\vec{\rho_i} - \vec{a}|) \end{aligned} \right\} + \frac{Q_e}{4\pi} \lambda K_1(\lambda a)$$

** The resonance escape probability for the neutrons originating in fissions in the enriched rod must be somewhat greater than p for surrounding lattice, because of the absence of U^{238} (with its resonance capture levels) in the enriched fuel rod.

Clearly, this method of separation of the resonance escape probabilities is not fully accurate, since there should be a shading of p from a high value down to the lattice value, moving away from the enriched rod. However, the present technique, used consistently, gives good results and has the advantage of simplicity.

Replacing the sums by integrals as in section A (the terms are in the same order as in equations (7) and (8):

$$(9)\quad \frac{Q(\rho)}{2\pi a\beta_0} = \frac{1}{8\pi D_0}\left\{\begin{aligned} &-K_0(\lambda a)\,Q(\rho) - K_0(\lambda\rho)\,Q_e \\ &-\oint Q(\rho')\,K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\,\frac{d\vec{\rho}'}{b^2} \\ &+\int_{i^{th}\,cell} Q(\rho')\,K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\,\frac{d\vec{\rho}'}{b^2} \\ &+\int_{e^{th}\,cell} Q(\rho')\,K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\,\frac{d\vec{\rho}'}{b^2} \\ &+\frac{p_0 h_0 L^2}{2\theta}\oint Q(\rho')\,F(|\vec{\rho}-\vec{\rho}'|)\,\frac{d\vec{\rho}'}{b^2} \\ &-\frac{p_0 h_0 L^2}{2\theta}\int_{e^{th}\,cell} Q(\rho')\,F(|\vec{\rho}-\vec{\rho}'|)\,\frac{d\vec{\rho}'}{b^2} \\ &+\frac{p_e h_e L^2}{2\theta}\,Q_e\,F(\rho) \end{aligned}\right\}$$

$$+\frac{Q(\rho)}{4\pi}\,\lambda K_1(\lambda a)$$

the expression valid for $\rho > \rho_0$ (because the central pile cell is enriched).

(10)

$$\frac{Q_e}{2\pi a \beta_e} = \frac{1}{4}\left[-\frac{Q_e}{2\pi D_0}K_0(\lambda a) - \oint \frac{Q(\rho')}{2\pi D_0} K_0(\lambda\cancel{|\vec{\rho}\,'-\vec{a}|})\cancel{\frac{d\vec{\rho}\,'}{b^2}} \right.$$

$$+ \int_{e^{th}\,\mathrm{cell}} \frac{Q(\rho')}{2\pi D_0} K_0(\lambda|\vec{\rho}\,'-\vec{a}|)\frac{d\vec{\rho}\,'}{b^2}$$

$$+ \oint \frac{p_0 h_0 L^2}{4\pi\theta D_0} Q(\rho') F(|\vec{\rho}\,'-\vec{a}|)\frac{d\vec{\rho}\,'}{b^2}$$

$$\left. - \int_{e^{th}\,\mathrm{cell}} \frac{p_0 h_0 L^2}{4\pi\theta D_0} Q(\rho') F(|\vec{\rho}\,'-\vec{a}|)\frac{d\vec{\rho}\,'}{b^2} \right]$$

$$+ \frac{Q_e \lambda K_1(\lambda a)}{4\pi}$$

If the sink function $Q_1(\rho) = A J_0(\varkappa\rho) + B Y_0(\varkappa\rho)$ is tried, equation (9) becomes (the terms are in the same order as in equation (9)):

$$(11)\quad \frac{Q_1(\rho)}{2\pi a\beta_0} = \frac{1}{4}\left(-\frac{1}{2\pi D_0}\left[\begin{array}{l} K_0(\lambda a)\,Q_1(\rho) + Q_e\,K_0(\lambda\rho) \\ + N\left\{Q_1(\rho) + \frac{2}{\pi}B\,K_0(\lambda\rho)\right\} \\ - N\,P(\rho_0)\,Q_1(\rho) \\ - N\left\{S(\rho_0)A + T(\rho_0)B + \frac{2B}{\pi}\right\}K_0(\lambda\rho) \end{array}\right] + \frac{L^2}{4\pi\theta D_0}\left[\begin{array}{l} 2\lambda^2\theta p_0 h_0 N e^{-\kappa^2\theta} A\,J_0(\kappa\rho) \\ + \frac{N p_0 h_0 \lambda^2 B}{b^2}\int e^{-\frac{\rho'^2}{4\theta}}\left[\begin{array}{l} Y_0(\kappa|\vec{\rho}-\vec{\rho}'|) \\ + \frac{2}{\pi}K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\end{array}\right]d\vec{\rho}' \\ - N p_0 h_0\left\{S(\rho_0) + T(\rho_0)B + \frac{2B}{\pi}\right\}F(\rho) \\ + p_e h_e\,Q_e\,F(\rho) \end{array}\right]\right)$$

$$+ \frac{\lambda K_1(\lambda a)}{4\pi}\,Q_1(\rho)$$

where the symbols are defined in the glossary of notation. Clearly if B $\longrightarrow$ O, the pile approaches the uniformly loaded lattice status. That is, the parameter B is a measure of the strength of the enriched rod. It may be shown that $F(\rho)$ is asymptotic to $2\lambda^2\theta\, e^{\lambda^2\theta}\, K_0(\lambda\rho)$ and that $\frac{1}{2\pi}\int e^{-\rho'^2/4\theta}\, Y_0(\kappa|\vec{\rho}-\vec{\rho}'|)\, d\vec{\rho}'$ is asymptotic to $2\lambda^2\theta\, e^{-\kappa^2\theta}\, Y_0(\kappa\rho)$ . . To a first approximation, the asymptotic forms may be substituted for the integrals. This is equivalent to neglecting very small terms proportional to $\sim K_0\left(\frac{\rho}{\sqrt{\theta}}\right)$. Then the ρ-dependence of the expression is a linearly independent combination of $Q_1(\rho)$ and of $K_0(\lambda\rho)$.

Equating to zero the coefficient of $Q_1(\rho)$, there is obtained simply the relation (6). Equating to zero the coefficient of $K_0(\lambda\rho)$, there is obtained a relation among the several parameters. Substituting $Q_1(\rho)$ into equation (9), there is found another relation among the parameters (the terms are in the same order as in equation (10)):

$$
\frac{Q_e}{2\pi a \beta_e} = \frac{1}{4}\left\{ -\frac{1}{2\pi D_0}\left[\begin{array}{l} N\left\{A J_0(\kappa a) + B Y_0(\kappa a) + \frac{2B}{\pi} K_0(\lambda a)\right\} \\ + K_0(\lambda a)\, Q_e \\ - N K_0(\lambda a)\left\{A S(\rho_0) + B T(\rho_0) + \frac{2B}{\pi}\right\} \end{array} \right] + \frac{L^2}{4\pi\Theta D_0}\left[\begin{array}{l} 2N\lambda^2\Theta\, p_0 h_0 e^{-\kappa^2\Theta} A J_0(\kappa a) \\ + N p_0 h_0 B \lambda^2 \oint e^{-\rho'^2/4\Theta}\, Y_0(\kappa|\vec{\rho}' - a|)\frac{d\vec{\rho}'}{2\pi} \\ + \left(\frac{2}{\pi}\right) N p_0 h_0 B\, F(a) \\ + p_e h_e Q_e\, F(a) \\ - N p_0 h_0 F(a)\left\{A S(\rho_0) + B T(\rho_0) + \frac{2B}{\pi}\right\} \end{array} \right] \right\} + \frac{\lambda K_1(\lambda a)}{4\pi} Q_e \tag{12}
$$

In (12), F(a) and $\oint e^{-\rho'^2/4\Theta}\, Y_0(\kappa|\vec{\rho}' - a|)\, d\vec{\rho}'$ are calculated accurately, by numerical techniques.

It is convenient to regard A as a normalization constant. Of the several ways in which these equations may be applied, the one chosen here is the calculation of p_e, the effective resonance escape probability for

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neutrons arising from the enriched rod. Pile critical measurements give $\mathcal{H}$ and B/A*, and the rod blacknesses are also determined experimentally. The h_e is well-calculable. Knowing $\mathcal{H}$, the $p_o h_o$ product may be found from equation (6). Conventional values are assumed for the moderator parameters.

C. Small-Source Description of a Pile With Two Enriched Rods

In this case, two enriched columns were located at the ends of a diagonal across the central lattice cell of the pile, replacing the two ordinary fuel columns. Analysis very similar to that of section B, case I, yields the following expressions:

$$(13)\quad \frac{Q(\rho)}{2\pi a \beta_0} = \frac{1}{4}\left\{ -\frac{1}{2\pi D_0}\left[\begin{aligned} &K_0(\lambda a)\,Q(\rho)+2Q_e K_0(\lambda\rho)\\ &+N\left\{Q_1(\rho)+\frac{2}{\pi}K_0(\lambda\rho)\right\}\\ &-NP(\rho_0)\,Q(\rho)\\ &-2N\left\{AS(\rho_0)+BT(\rho_0)+\frac{2B}{\pi}\right\}K_0(\lambda\rho)\end{aligned}\right] + \frac{L^2}{4\pi\theta D_0}\left[\begin{aligned} &2\lambda^2\theta N p_0 h_0 e^{-\mathcal{H}^2\theta} A J_0(\mathcal{H}\rho)\\ &+N p_0 h_0 B\lambda^2 \int e^{-\frac{\rho'^2}{4\theta}}\left[\begin{aligned}&Y_0(\mathcal{H}|\vec{\rho}-\vec{\rho}'|)\\&+\frac{2}{\pi}K_0(\lambda|\vec{\rho}-\vec{\rho}'|)\end{aligned}\right]d\vec{\rho}'\\ &-2N\left\{AS(\rho_0)+BT(\rho_0)+\frac{2B}{\pi}\right\}F(\rho)\\ &+2p_e h_e Q_e F(\rho)\end{aligned}\right]\right\} + \frac{\lambda K_1(\lambda a)}{4\pi}\,Q(\rho)$$

* Since the neutron flux and the sink density expressions are proportional over all the reactor, for $\rho > \rho_0$: $B/A = -J_0(\mathcal{H}R)/Y_0(\mathcal{H}R)$ where R is the pile critical radius.

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where $Q(\rho) = A\,J_0(\mu\rho) + B\,Y_0(\mu\rho)$ and Q_e refers to the sink strength of <u>each</u> enriched rod;

$$
(14)\quad \frac{Q_e}{2\pi a\beta_e} = \frac{1}{4}\left\{ -\frac{1}{2\pi D_0}\left[\begin{array}{l} N\left\{A J_0(\mu a) + B Y_0(\mu a) + \frac{2B}{\pi} K_0(\lambda a)\right. \\ + K_0(\lambda a)\, Q_e + K_0(\lambda\sqrt{2}b)\, Q_e \\ - N\left\{A S(\rho_0) + B T(\rho_0) + \frac{2B}{\pi}\right\} \\ \qquad \times \left\{K_0(\lambda a) + K_0(\lambda\sqrt{2}b)\right\} \end{array}\right] + \frac{L^2}{4\pi\theta D_0}\left[\begin{array}{l} 2\lambda^2\theta N p_0 h_0 e^{-\mu^2\theta} A J_0(\mu a) \\ + N p_0 h_0 B \lambda^2 \oint e^{-\rho'^2/4\theta} Y_0(\mu|\vec{\rho}\,'-a|)\frac{d\vec{\rho}\,'}{2\pi} \\ + \frac{2}{\pi} N p_0 h_0 B F(a) + p_e h_e Q_e F(a) \\ + p_e h_e Q_e F(\sqrt{2}b) \\ + p_0 h_0 N\left\{A S(\rho_0) + B T(\rho_0) + \frac{2B}{\pi}\right\} \\ \qquad \times \left\{F(a) + F(\sqrt{2}b)\right\} \end{array}\right] \right\}
$$

$$
+ \frac{\lambda K_1(\lambda a)}{4\pi} Q_e + \frac{\lambda K_1(\lambda\sqrt{2}b)}{4\pi} Q_e
$$

As in the case of the single enriched rod, the coefficient of $Q(\rho)$ yields just the relation (6). $F(\rho)$ and $\oint e^{-\rho'^2/4\theta} Y_0(\mu|\vec{\rho}-\vec{\rho}\,'|)\,d\vec{\rho}\,'$ are replaced by their asymptotic forms (in (13) only; not in (14)). The coefficient of $K_0(\lambda\rho)$ yields a relation among the parameters.

Since a pile critical measurement was performed for this case, also, it is possible to determine p_e for this case, proceeding precisely as in section A, Case I.

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III. NUMERICAL RESULTS

The following numerical values are used:

$D_0 = \dfrac{\lambda_{tr}}{3} = 0.95$ cm $\qquad \Theta = 358$ cm^2

$L = 1/\lambda = 50$ cm $\qquad b^2 = 452.5$ cm $= \pi \rho_0^2$

$a = 1.723$ cm $\qquad \beta_0 = 0.465 \qquad \beta_e = 0.315$

These blackness values are those at the surface of uranium columns, these columns being surrounded by a thin film of cooling water, the whole assembly being contained within an aluminum tube. Pile critical measurements give directly the critical radii, the values of R for the case of uniform loading and for lattices containing one and two enriched columns. Applying the definition of geometrical radial buckling ($\beta_r^2 = \dfrac{j_{0,1}}{R^2}$ where $j_{0,1}$ is the first root of the Bessel function $J_0(x)$ and R is the pile critical radius), the reactivity effects are determined in terms of buckling increments. The computations give:

Parameter	One Enriched Rod	Two Enriched Rods	Uniform Loading
Pile radial buckling	48.4×10^{-6} cm^2	52.3×10^{-6} cm^2	$\gamma^2 = 45.6 \times 10^{-6}$
Pile buckling increment	$+ 2.8 \times 10^{-6}$ cm^2	$+ 6.7 \times 10^{-6}$ cm^2	
p_e (finite pile)	0.855	0.868	
p_e (infinite pile)*	0.875	0.890	

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* Corrected by the factor $e^{\theta \gamma^2}$, where γ^2 is the total (radial plus 16×10^{-6} cm^2 axial) pile buckling.

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It must be emphasized that these are effective values of the resonance escape probabilities to be associated with neutrons arising from fissions in the enriched rods, under the assumption that surrounding natural uranium columns generate fission neutrons with the same resonance escape probability that they would have were the enriched rods not present. Consequently, the values of p listed above form ceiling estimates of the value of p for the natural metal lattice. To estimate this last number, notice that the pile buckling increments for the two-rod case stands to the increment for the one rod case as $\frac{6.7}{2.8} = 2.4$. One might then expect that the difference in p between the one enriched rod case and the homogeneous lattice would be about $\frac{1}{1.4}$ times the difference between the two enriched rod and one enriched rod cases, or $\frac{1}{1.4} \times (0.890 - 0.875) = 0.011$. According to this estimate, then for the natural uranium lattice $p_{\infty} =$ $0.875 - 0.011 = 0.864$.

IV. ACCURACY OF RESULTS

Two relations among the parameters A, B, $p_e h_e$, β_e, and Q_e are obtained in section B, case I. One of these is equation (12), the other, the coefficient of $K_0(\lambda\rho)$ in equation (11), which is equated to zero. If Q_e is eliminated between the expressions, and the resultant relation differentiated, it is found that

$$(15) \quad \Delta p_e = 0.220\left(\frac{\Delta B}{B}\right) - 0.337\left(\frac{\Delta \beta_e}{\beta_e}\right) - 0.855\left(\frac{\Delta h_e}{h_e}\right)$$

for the one-rod case. If expressions (13) and (14) are treated similarly, for the two-rod case, there results:

$$(16) \quad \Delta p_e = 0.217\left(\frac{\Delta B}{B}\right) - 0.345\left(\frac{\Delta \beta_e}{\beta_e}\right) - 0.868\left(\frac{\Delta h_e}{h_e}\right)$$

Since (fortuitously) $\Delta B/B$ is very nearly equal to $\frac{\Delta \text{(reactivity increment)}}{\text{reactivity increment}}$ [the function $Y_o(x)$ proves to be essentially linear in the small range of x necessary - approximately the first root of $J_o(x)$], relations (15) and (16) provide the uncertainty in p_e (and ultimately in p_∞ for the natural uranium pile - see section III) for uncertainties in measurement of the reactivity increments, measurement of rod blackness, and computation of h_e, the rod multiplication. The reactivities were measured to better than one per cent. The computation of h_e, based upon very accurate knowledge of alloy materials of the enriched metal, is as accurate as the cross-sections of BNL-170 and AECU-2040. However, the blackness measurements very possibly may be low by as much as ten per cent.

One may question the values of θ, L, and D_o used in these computations. It must be pointed out, however, that with these quantities and the measured value of $\mathcal{H}^2$, a value of the product $p_o h_o$ was computed from relation (6). These numbers then are self-consistent; a change in θ, L, or D_o would cause a largely canceling change in $p_o h_o$.

V. APPROXIMATE COMPUTATIONS

A. Weighting Theorem

Knowing that the value of $h_e = 1.977$, that the critical radial buckling of the homogeneous lattice is 45.6×10^{-6} cm^2, that the buckling increment due to the insertion of one enriched rod is 2.8×10^{-6} cm^2, that the pile axial buckling is 16×10^{-6} cm^2, and that the area of one cell of the lattice is 452.5 cm^2, knowing that $f \sim .87$ and $M^2 \sim 650$ cm^2, one can compute $p_{e\infty}$ approximately by the weighting theorem as follows:

$$p_{e\infty} = \frac{1 + 650\left[2.8\times10^{-6}\,\dfrac{\int_{pile} n^2 dV}{\int_{cell} n^2 dV} + 16\times10^{-6} + 45.6\times10^{-6}\right]}{(1.977)(.87)}$$

[redacted]

where $n = A\, J_0(\sqrt{45.6 \times 10^{-6}}\, \rho) \cos(\sqrt{16 \times 10^{-6}}\, z)$.

This computation gives $p_e \simeq 0.855$, with an uncertainty of perhaps ten per cent. (Carried out for the case of two rods, where the buckling increment is 6.7×10^{-6} cm^2 and the cell integration is over two cells, a value of $p_e \simeq 0.905$ is obtained with uncertainty surely at least as great as for the case of one rod.) This uncertainty, involving M^2, f, and the actual flux in the enriched cell, is certainly not removable in any simple way. It must be remarked, however, that while one cannot, apparently, expect the weighting technique to predict accurately and with certainty the reactivity effects of strong lattice perturbations, the weighting theorem might be expected to yield quite good values for the change in strength of a perturbation upon moving it to a new location in a pile, providing its strength was well-known in its initial location.

B. Two-Region Pile

Knowing that the critical radial buckling of the pile not containing the enriched column is $\mathcal{H}^2 = 45.6 \times 10^{-6}$ cm^2, that $h_e = 1.977$, that the replacement of a natural uranium column by an enriched column causes a buckling increment of 2.8×10^{-6} cm^2, that the area of one lattice cell is 452.5 cm^2, that the pile axial buckling is 16×10^{-6} cm^2, knowing that $f \sim .87$ and $M^2 \sim 650$ cm^2, a two-region pile calculation may be performed. In the cell containing the enriched column (centered in the pile), the flux is $n_e = J_0(\mathcal{H}' \rho) \cos(.004 z)$. In the remainder of the pile $n = \left[A J_0(\mathcal{H} \rho) + B Y_0(\mathcal{H} \rho)\right]$ where B/A is determined as in section B, page 15. Applying the flux and current boundary conditions at the edge of an equivalent cell ($\pi \rho_0^2 = 452.5$), taking the ratio of the two relations thus found, and recalling B/A for the one-rod case = -0.07106, the following compatibility relation for $\mathcal{H}'$ is determined:

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$$\varkappa' \frac{J_1(\varkappa'\rho_0)}{J_0(\varkappa'\rho_0)} = \varkappa \frac{J_1(\varkappa\rho_0) - 0.07196\, Y_1(\varkappa\rho_0)}{J_0(\varkappa\rho_0) - 0.07196\, Y_0(\varkappa\rho_0)}$$

which gives $\varkappa' = 0.0246, \quad \varkappa'^2 = 604 \times 10^{-6}\ \text{cm}^2$

and $$p_{e_\infty} = \frac{1 + (\varkappa'^2 + 16 \times 10^{-6})M^2}{\eta f} = 0.816$$

Similarly, for the two-rod case, where boundary conditions are matched at a radius $\sqrt{2\rho_0}$ and B/A = -.16410, $p_{e\,\infty}$ = 0.821. These values must have about the same uncertainty as weighting theorem calculations.

VI. CONCLUSION

It is shown that conventional calculations of the reactivity effects of placing highly enriched rods of uranium into a pile require quite accurate knowledge of p, f, $\eta\epsilon$, and M^2 not only for the natural metal lattice, but for the enriched material placed into a lattice of natural metal (these quantities for the enriched rod certainly being different from the corresponding values in a large lattice made exclusively of enriched material).

The small-source method requires knowledge of p, $\eta\epsilon$, the moderator constants, the critical buckling of the natural metal lattice, and uranium rod blacknesses, a reduction in the number of those constants of a pile which are difficult to know precisely. Consequently, this model seems quite appropriate to this type of problem.

ACKNOWLEDGMENT

The author is indebted to Dr. W. A. Horning for suggesting this problem, for much of the formulation, and for many fruitful discussions.

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GLOSSARY OF NOTATION

D_o = thermal diffusion coefficient of pile moderator.

$L = 1/\lambda$ = thermal diffusion length of pile moderator.

Q_i = sink strength of fuel rod i per unit length of fuel rod.

p_i = resonance escape probability of neutrons arising from fission in the i^{th} rod.

h_i = number of fast neutrons arising per thermal fission in the i^{th} rod.

$\dfrac{e^{-|\vec{\rho}-\vec{\rho}_i|^2/4\theta}}{4\pi\theta}$ = age slowing down kernel for fission neutrons to (thermal) age θ — here a normalized (to unity) distribution function for thermal neutrons arising from fissions in the i^{th} rod.

§ signifies integration over all space.

$d\vec{\rho} = \rho d\rho d\theta$ = element of area in cylindrical coordinates.

$$F(|\vec{\rho}-\vec{\rho}_i|) = \lambda^2 \S\, e^{-|\vec{\rho}-\vec{\rho}_i|^2/4\theta} K_0(\lambda|\vec{\rho}-\vec{\rho}\,'|)\frac{d\vec{\rho}\,'}{2\pi}$$

= distribution of thermal neutrons arising from a line fission source at $\rho = \rho_i$.

$Q(\rho)$ = sink density per unit length of pile at radial position ρ.

$$P(\rho_0) = \rho_0\left[\varkappa J_1(\varkappa\rho_0)K_0(\lambda\rho_0) - \lambda J_0(\varkappa\rho_0)K_1(\lambda\rho_0)\right] + 1$$

$$S(\rho_0) = \rho_0\left[\varkappa J_1(\varkappa\rho_0)I_0(\lambda\rho_0) + \lambda J_0(\varkappa\rho_0)I_1(\lambda\rho_0)\right]$$

$$T(\rho_0) = \rho_0' \left[\mathcal{H}\, Y_1(\mathcal{H}\rho_0)\, I_0(\lambda\rho_0) + \lambda\, Y_0(\mathcal{H}\rho_0)\, I_1(\lambda\rho_0) \right]$$

a = fuel rod radius
β = fuel rod blackness
$\mathcal{H}$ = pile radial buckling

$\rho_0 = (b^2/\pi)^{1/2}$ — radius of a cylindrical cell equal in cross-sectional area to a lattice cell of side b.

b = length of an edge of the square lattice cell.

END

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