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EFFECT OF A DECELERATING GRID ON CURRENT FROM AN ION SOURCE

G. F. Bing, C. S. Gardner, and T. G. Northrop

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ABSTRACT

A theoretical criterion is given for the conditions under which the ions from an ion source may be decelerated by a decelerating grid, following the accelerating grid which extracts ions from the emitter, without causing a reduction in the current supplied by the source.

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INTRODUCTION

An ion source consisting of a plane emitter of ions is taken as the zero potential; it is followed by a grid G_1 maintained at a negative potential $-V_1$, which extracts ions from the emitter. The following work shows under what conditions it is possible to place after G_1 a second grid G_2, at a smaller negative potential $-V_2$, so that the ions will be decelerated, without reducing the current. (See Fig. 1.) This problem is essentially the same as that of the triode, which has been analyzed by Salzberg and Haeff[1] and by Fay, et al.[2] A good summary exists in Spangenberg.[3] The following is a derivation of the results in a form useful for this problem.

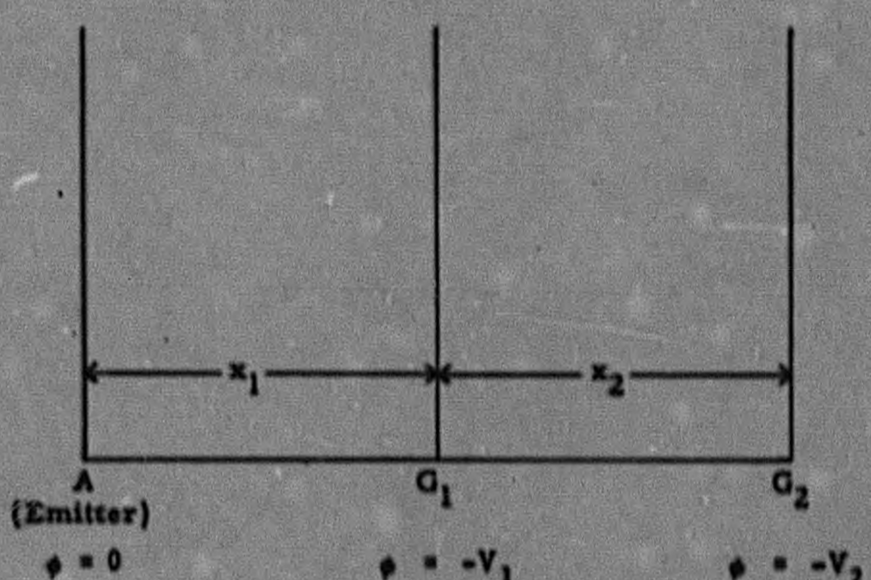

Fig. 1. Schematic diagram of ion source.

We make the following assumptions:

(1) The current from the anode A is space-charge limited.

(2) No returning current is emitted at G_2.

(3) No ions traverse G_1 more than twice (i.e., no Kurz-Barkhausen oscillator effect).

(4) That G_1 and G_2 determine the potential in their own planes. This cannot be exactly true unless they have no open spaces, in which case none of the ions would be transmitted. Diagrams showing actual potential distributions near grids are shown in Spangenberg[3] (see p. 262, especially). A grid actually gives a fairly uniform but lower potential at a plane spaced somewhat from itself. The 190 volt contour on p. 262 of Spangenberg is an example. Thus the grid potential which is effective in determining currents may be somewhat different from its actual potential.

Then there are three possible types of solution, according to the behavior of the negative potential, V(x), as indicated in Figs. 2a, 2b, and 2c.

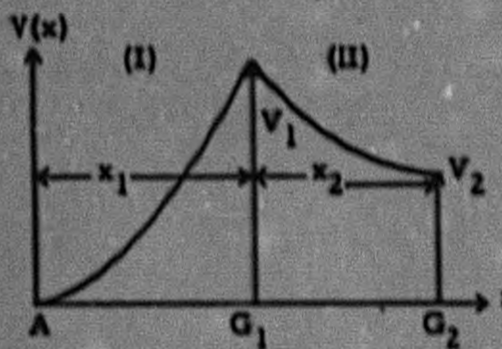

Fig. 2a Voltage plot for type 1 solution.

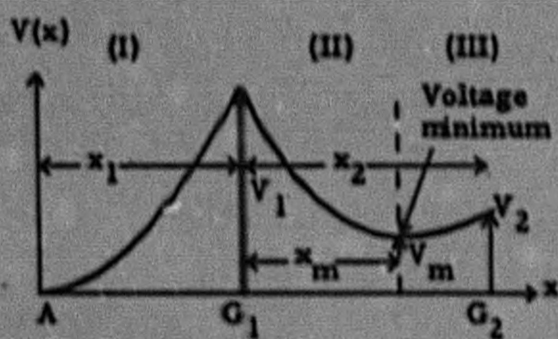

Fig. 2b Voltage plot for type 2 solution.

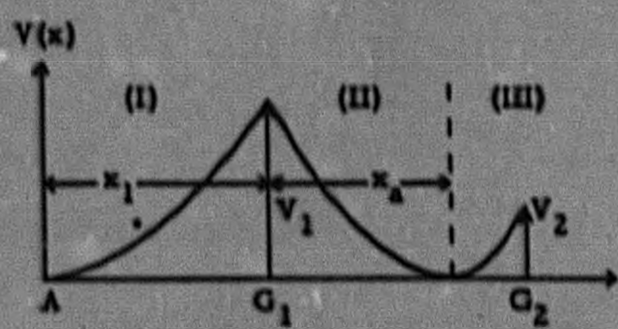

Fig. 2c Voltage plot for type 3 solution.

In the type 1 solution, (Fig. 2a) $V(x)$ is monotone between A and G_1, and between G_1 and G_2.

In the type 2 solution, (Fig. 2b) $V(x)$ has a minimum V_m at $x = x_1 + x_m$ between G_1 and G_2.

In the type 3 solution, (Fig. 2c) $V(x)$ is zero at a point $x = x_1 + x_a$ between G_1 and G_2.

In the type 1 and type 2 solutions, all the ions emitted pass through to G_2. The current is the full value given by Child's Law between A and G_1.

In the type 3 solution some of the emitted ions are turned around at the potential zero at $x = x_1 + x_a$, and only a fraction are transmitted to G_2. Here the current will be less than the value given by Child's Law as applied between A and G_1.

CONDITION FOR TYPE 3 SOLUTION

In the type 3 solution we have $dV/dx = 0$ at A, because the current is space charge limited, and also $dV\ dx = 0$ at $x = x_1 + xa$. Thus Child's Law applies in each of the three regions (I), (II), (III). (See Fig. 2c.)

Let I_1 be the current density emitted at A and I_2 the current density transmitted beyond $x = x_1 + x_a$ to G_2. Since the space charge effect of a current is independent of its direction, the "Child's Law" current in regions (I) and (II) is $2I_1 - I_2$, and we have

$$2I_1 - I_2 = \frac{a\,V_1^{3/2}}{x_1^2} \tag{1}$$

$$2I_1 - I_2 = \frac{a\,V_1^{3/2}}{x_a^2} \tag{2}$$

$$I_2 = \frac{a\,V_1^{3/2}}{(x_2 - x_a)^2} \tag{3}$$

where a is the Child's Law constant for the ions. From (1) and (2) we see that

$$x_a = x_1 \tag{4}$$

It follows that a type 3 solution is impossible unless $x_2 \geq x_1$. From (1), (3), (4), we have, since $2I_1 - I_2 = I_2 + 2(I_1 - I_2) > I_2$,

$$\frac{a\, V_1^{3/2}}{x_1^2} \geq \frac{a\, V_2^{3/2}}{(x_2 - x_1)^2}$$

so that we obtain, as the condition for the possibility of a type 3 solution,

$$(x_2/x_1) \geq 1 + \left(\sqrt{V_2/V_1}\right)^{3/2} \tag{5}$$

CONDITION FOR TYPE 1 SOLUTION

We now consider the type 1 solution. In region (I) (See Fig. 2a) we have

$$I = \frac{a\, V_1^{3/2}}{x_1^2}$$

and in region (II) we have Poisson's equation,

$$\frac{d^2V}{dx^2} = \text{const} \cdot \rho = \text{const} \cdot \frac{(\rho v)}{v} = \text{const} \cdot \frac{I}{\sqrt{(2e/M)\ V}}$$

where ρ is the charge density, v the speed of the ions, I the current density, (e/M) the charge-to-mass ratio of the ions. The constant is given by

$$\frac{d^2V}{dx^2} = \left(\frac{4I}{9a}\right) \cdot \frac{1}{\sqrt{V}} \tag{6}$$

where a is the Child's Law constant for the ions.

Multiplying (6) by $2(dV/dx)$ and integrating, we obtain

$$\left(\frac{dV}{dx}\right)^2 = \left(\frac{16I}{9a}\right)(\sqrt{V} - c)$$

where c is a constant of integration.

For $V_2 \leq V_1$, $\frac{dx}{dV} = -\left(\frac{3}{4}\right)\sqrt{\frac{a}{I}} \cdot \frac{1}{(\sqrt{V} - c)^{1/2}}$.

Integrating this over region (II), setting $I = a \frac{V_1^{3/2}}{x_1^2}$, we obtain

$$\frac{x_2}{x_1} = \frac{1}{V_1^{3/4}} \int_{V_2}^{V_1} (3/4) \frac{dV}{(\sqrt{V} - c)^{1/2}} \qquad (7)$$

$$= 1/V_1^{3/4} \left\{ (\sqrt{V_1} - c)^{1/2} (\sqrt{V_1} + 2c) - (\sqrt{V_2} - c)^{1/2} (\sqrt{V_2} + 2c) \right\}$$

$$= f(c)$$

f(c) can have values ranging from

$f(c) = 0$ at $c = -\infty$

to

$f(c) = (1 - \sqrt{V_2/V_1})^{1/2} (1 + 2\sqrt{V_2/V_1})$ at $c = \sqrt{V_2}$

And f(c) is moreover a monotone increasing function of c. Hence (7) will have exactly one solution if

$$0 < x_2/x_1 \leq (1 - \sqrt{V_2/V_1})^{1/2} (1 + 2\sqrt{V_2/V_1}) \qquad (8)$$

Similarly, $0 < x_2/x_1 \leq (\sqrt{V_2/V_1} - 1)^{1/2} (\sqrt{V_2/V_1} + 2)$ if $V_2 \geq V_1$.

This is the condition for a type 1 solution.

CONDITION FOR TYPE 2 SOLUTION

Here we have, in region I, as before,

$$I = \frac{a V_1^{3/2}}{x_1^2}$$

In regions II, III we have

$$\left(\frac{dV}{dx}\right)^2 = \left(\frac{16 I}{9 a}\right) (\sqrt{V} - c)$$

and here $c = \sqrt{V_m}$, since, at $V = V_m$, $dV/dx = 0$.

Hence

$$\left(\frac{dx}{dV}\right) = \pm (3/4) \frac{x_1}{V_1^{3/4}} \cdot \frac{1}{(\sqrt{V} - \sqrt{V_m})^{1/2}}$$

and in II,

$$\left(\frac{x_m}{x_1}\right) = \frac{1}{V_1^{3/4}} \int_{V_m}^{V_1} (3/4) \frac{dV}{(\sqrt{V} - \sqrt{V_m})^{1/2}}$$

In III,

$$\left(\frac{x_2 - x_m}{x_1}\right) = \frac{1}{V_1^{3/4}} \int_{V_m}^{V_2} (3/4) \frac{dV}{(\sqrt{V} - \sqrt{V_m})^{1/2}}$$

Adding these equations and integrating, we obtain

$$\left(\frac{x_2}{x_1}\right) = \frac{1}{V_1^{3/4}} \left\{ (\sqrt{V_2} - \sqrt{V_m})^{1/2} (\sqrt{V_2} + 2\sqrt{V_m}) + (\sqrt{V_1} - \sqrt{V_m})^{1/2} (\sqrt{V_1} + 2\sqrt{V_m}) \right\} \quad (9)$$

$$= g(\sqrt{V_m})$$

The function $g(\sqrt{V_m})$ has a maximum at $\sqrt{V_m} = \dfrac{\sqrt{V_1 V_2}}{\sqrt{V_1} + \sqrt{V_2}}$,

where $g(\sqrt{V_m}) = (1 + \sqrt{V_2/V_1})^{3/2}$

Its minima are at

$\sqrt{V_m} = 0$, where $g(\sqrt{V_m}) = 1 + (\sqrt{V_2/V_1})^{3/2}$, for all values of V_2/V_1, and at $\sqrt{V_m} = \sqrt{V_2}$, (if $V_2 \leq V_1$) where

$$g(\sqrt{V_m}) = (1 - \sqrt{V_2/V_1})^{1/2} (1 + 2\sqrt{V_2/V_1})$$

or at $\sqrt{V_m} = \sqrt{V_1}$ (if $V_2 \leq V_1$), where $g(\sqrt{V_m}) = (\sqrt{V_2/V_1} - 1)^{1/2} (\sqrt{V_2/V_1} + 2)$

Hence for values of (x_2/x_1), up to $(1 + \sqrt{V_2/V_1})^{3/2}$, there will be a type 2 solution.

SUMMARY

These results are summarized by the diagram of Fig. 3.

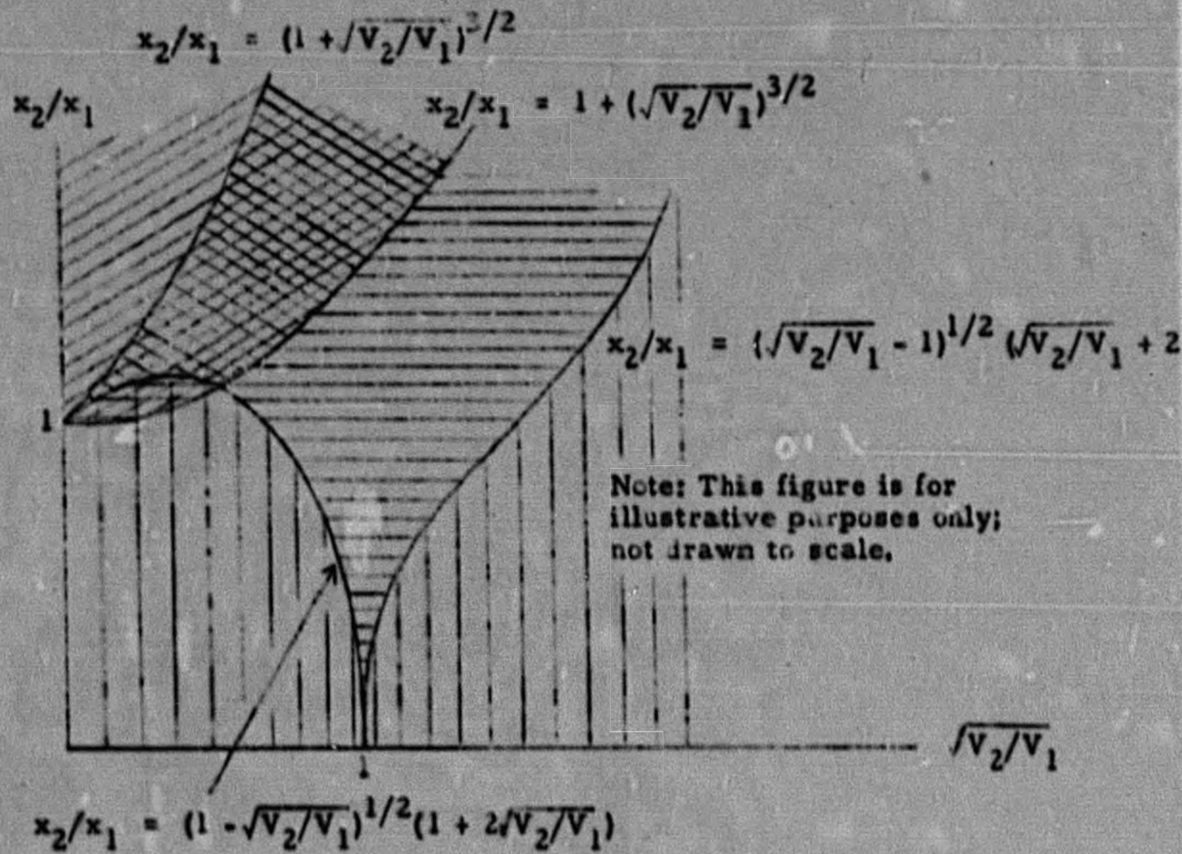

Fig. 3. Regions of variation of x_2/x_1 and V_2/V_1 for the three types of solution.

Further analysis shows the following:

In the vertically-shaded region there is exactly one type 1 solution.

In the horizontally-shaded region there is exactly one solution of type 2.

In the region shaded by lines, slanting upward to the right, there is exactly one type 3 solution.

In the region shaded by lines slanting downward to the right, there are exactly two type 2 solutions.

There are no other solutions.

Conclusions:

(1) The conditions $(x_2/x_1) < 1$

$(V_2/V_1) < 3/4$

are sufficient to ensure that (a) There is exactly one solution,

(b) This solution is of type 1.

Under these conditions the current reaching G_2 is given by

$$I = \frac{a V_1^{3/2}}{x_1^2}$$

and the energy of the ions by

$$E = e V_2.$$

(2) All ions reach G_2 if $x_2/x_1 < 1 + (\sqrt{V_2/V_1})^{3/2}$

EFFECT OF INTERCEPTION OF IONS BY G_1

We may consider the possibility that a certain fraction of ions passing G_1 in either direction are collected by G_1. The results remain qualitatively the same. Let β be the fraction of ions incident on G_1 which pass G_1. The equations for the type 3 solution now are

$$I_1 + \beta \left[\beta I_1 - I_2\right] = \frac{a V_1^{3/2}}{x_1^2} \qquad (1')$$

$$\beta I_1 + \left[\beta I_1 - I_2\right] = \frac{a V_1^{3/2}}{x_a^2} \qquad (2')$$

$$I_2 = \frac{a V_2^{3/2}}{(x_2 - x_a)^2} \qquad (3')$$

Because $\beta I_1 > I_2$, we have from (2'), (3')

$$\frac{V_1^{3/2}}{x_a^2} > \frac{V_2^{3/2}}{(x_2 - x_a)^2}$$

or

$$x_2/x_2 \geq 1 + (\sqrt{V_2/V_1})^{3/2}$$

By (1'), (2')

$$x_a/x_1 = \sqrt{\frac{(1+\beta^2)\, I_1 - \beta I_2}{2\beta I_1 - I_2}} > 1$$

Thus, as before, a type 3 solution will not occur if

$$(x_2/x_1) < 1 + (\sqrt{V_2/V_1})^{3/2}$$

The equation for the type 1 solution becomes

$$(x_2/x_1) = 1/\sqrt{\beta}\ \ f(c) \qquad (7')$$

so that the type 1 solution is possible if

$$(x_2/x_1) \leq 1/\sqrt{\beta}\ \left[f(c)\right]_{max} = 1/\sqrt{\beta}\ \ (1 - \sqrt{V_2/V_1})^{1/2}\ (1 + 2\sqrt{V_2/V_1})$$

The equation for the type 2 solution becomes

$$(x_2/x_1) = 1/\sqrt{\beta}\ \ g(\sqrt{V_m}) \qquad (9')$$

so that the type 2 solution is possible if

$$(x_2/x_1) \leq 1/\sqrt{\beta}\ \left[g(\sqrt{V_m})\right]_{max} = 1/\sqrt{\beta}\ \ (1 + \sqrt{V_2/V_1})^{3/2}$$

Our conclusions are then: (1) There will be exactly one solution of type 1, if $(x_2/x_1) < 1$ and $(V_2/V_1) < 3/4$; the current is given by β times the Child's Law current between A and G_1, and the energy by $E = eV_2$.

(2) All ions reach G_2 if $x_2/x_1 < 1 + (\sqrt{V_2/V_1})^{3/2}$.

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