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INTERACTIONS AND LIFETIMES OF K MESONS

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ABSTRACT

The lifetimes of K^+ (not including τ^+) and K^- mesons from the Bevatron have been measured by use of a nuclear emulsion technique. The values found are $\tau_{K^+} = 1.01 {}^{+0.33}_{-0.21} \times 10^{-8}$ sec and $\tau_{K^-} = 0.95 {}^{+0.36}_{-0.25} \times 10^{-8}$ sec. The equivalence of these lifetimes (within the statistical errors) supports the hypothesis that the K^+ and K^- mesons are charge conjugates of each other.

A study of interactions in flight of K^- mesons has yielded a mean free path in nuclear emulsion of $\lambda_{K^-} = 23 {}^{+6}_{-5}$ cm, which is consistent with a geometric cross section. The products of the 21 interactions in flight observed include charged π mesons, charged Σ hyperons, and an excited fragment. No inelastically scattered K^- mesons were found. All the interactions were consistent with the conservation of "strangeness". Elastic scattering of K^- mesons is discussed.

The interactions of K^- mesons at rest in nuclear emulsion are discussed in the light of the data thus far published. From the π-meson and the Σ-hyperon energy spectra it is concluded that nearly all the interactions may be accounted for by single-nucleon capture of K^- mesons by the processes allowed by the conservation of strangeness, and that most of these primary reactions yield Σ hyperons rather than Λ^0 hyperons. A small percentage of the interactions may be due to capture by two nucleons. It is shown that if charge independence is assumed, the frequencies of charged π mesons and charged Σ hyperons are accounted for, and that in the cases in which Σ hyperons are produced the $T = 1$ isotopic spin state contributes appreciably though it is not necessarily dominant.

INTERACTIONS AND LIFETIMES OF K MESONS

I. INTRODUCTION

The discovery of V particles in 1947 in cloud chamber cosmic ray studies[1] and the discovery of the τ meson in 1949 in nuclear emulsion plates exposed to high-altitude cosmic rays[2] have opened a new era in fundamental particle research. Since then so many new particles have been found that it has been necessary to classify them phenomenologically.[3] A particle of mass between that of a π meson and a nucleon is called a K meson, while one with a mass between that of a nucleon and a deuteron is called a hyperon. The K mesons are further classified according to their decay products. The following types of decay of K mesons have been observed:[4]

τ^{+}	$K^{+} \rightarrow \pi^{+} + \pi^{+} + \pi^{-}$
τ'^{+}	$K^{+} \rightarrow \pi^{+} + \pi^{0} + \pi^{0}$
$K^{+}_{\pi 2}$	$K^{+} \rightarrow \pi^{+} + \pi^{0}$
$K^{+}_{\mu 2}$	$K^{+} \rightarrow \mu^{+} + \nu$
$K^{+}_{\mu 3}$	$K^{+} \rightarrow \mu^{+} + \pi^{0} + (\nu)$
K^{+}_{e3}	$K^{+} \rightarrow e^{+} + (?) + (?)$
θ^{0}	$K^{0} \rightarrow \pi^{+} + \pi^{-}$
	$(K^{0} \rightarrow \pi^{0} + \pi^{0})$
τ^{-}	$K^{-} \rightarrow \pi^{-} + \pi^{-} + \pi^{+}$
	$(K^{-} \rightarrow \pi^{-} + \pi^{0} + \pi^{0})$
$K^{-}_{\pi 2}$	$K^{-} \rightarrow \pi^{-} + \pi^{0}$
$K^{-}_{\mu 2}$	$(K^{-} \rightarrow \mu^{-} + \nu)$
$K^{-}_{\mu 3}$	$(K^{-} \rightarrow \mu^{-} + \pi^{0} + \nu)$
K^{-}_{e3}	$K^{-} \rightarrow e^{-} + (?) + (?)$

(The decay modes in parentheses have not actually been identified but are expected by charge symmetry.)

The question immediately arises whether this great array of decay types is due to several modes of decay of a single variety of particles or to the decay of two or more different types of particles. Analyses of the angle and energy correlation of the decay products of τ^+ and τ'^+ mesons by the method of Dalitz[5] have suggested that there are at least two types of K^+ mesons, differing from each other by either spin or parity or both. These analyses suggest that the τ^+ meson has a spin of 0 or 2 and odd parity, whereas if the $K_{\pi 2}$ meson has an even spin it must have even parity. If this were true one would expect that some of the other properties of the particles would also differ. Recent accurate mass measurements of K^+ particles have shown that their masses are the same within about two electron masses for the more abundant modes of decay and that the masses agree within experimental error in all cases.[6] Measurements of the K^- meson mass are in agreement with that found for K^+ mesons within the experimental error of a few electron masses.[7] Lifetime measurements and observations of the interactions of K mesons may lead to further information on this point.

Hyperons of three different mass groups have been found. The approximate masses (in electron mass units) and the decay modes are as follows:[8]

Mass in M_e	Particle
2181	$\Lambda^0 \rightarrow p + \pi^-$ $\rightarrow n + \pi^0$
2327	$\Sigma^+ \rightarrow p^+ + \pi^0$ $\searrow n + \pi^+$
2298 to 2332	$(\Sigma^0 \rightarrow \Lambda^0 + \gamma)$
2341	$\Sigma^- \rightarrow n + \pi^-$
2582	$\Xi^- \rightarrow \Lambda^0 + \pi^-$

To explain why these new "strange" particles, i.e., K mesons and hyperons, have a long lifetime ($\sim 10^{-10}$ sec) and yet are produced in great abundance in high-energy interactions between nucleons and between pions and nucleons, it has been proposed that they are produced only in association with one another.[9] That is, more than one of these particles must be produced at the same time. This idea has become generally accepted because no direct evidence against it has been found and two strange particles are frequently seen to be produced in the same nuclear reaction. From the correlation of particle types produced at the interactions of high-energy protons and pions with matter, a scheme has been suggested in which a new quantum number is introduced.[10] Any particle that may take part in a fast reaction is assigned a small integral number S, as follows

K^+, K^0	S = +1
K^-, $\bar{K}^0$; Λ^0; Σ^+, Σ^0, Σ^-,	S = -1
Ξ^0, Ξ^-	S = -2
p, n; π^+, π^0, π^-	S = 0

(It is to be noticed that two types of neutral K mesons and a neutral Ξ^0 hyperon have been introduced.) Then it is proposed that in fast reactions ($\sim 10^{-23}$ sec), such as production of strange particles or their interactions with nuclei, the total S must be conserved. For slow reactions ($\sim 10^{-10}$ sec) such as the decay of particles, the selection rule is $\Delta S = \pm 1$. All cases of associated production of these particles that have been observed follow these proposed rules. Investigation of interactions of these particles with nuclei will provide a further test.

This paper is a report on measurements of the lifetimes of K^+ and K^- mesons and interactions in flight of K^- mesons produced by the Bevatron and detected by the nuclear emulsion "stack" technique. There is also included a more general discussion of the interactions of K^- mesons with nuclei.

II. MEAN LIFETIMES OF K MESONS

A. General Experimental Methods

Relatively intense beams of artificially produced charged K mesons have recently become available.[11] They have made possible the study of K^+ and K^- mesons under controlled conditions. In this experiment a copper target in the west tangent tank of the Bevatron was bombarded with 6.2-Bev-protons. Particles produced at 90° to the incident proton beam direction were focused by a magnetic quadrupole system consisting of three quadrupoles with an aperture of 2 inches. The particles then passed through an analyzing magnet (with appropriate shielding) which selected particles of a given momentum and cut down extraneous background tracks. Stacks of Ilford G.5 nuclear emulsion were placed at the focus of the particle beam. The total distance of travel from the target to the detector was on the order of 3 meters in all exposures. Each of the stacks contained from 50 to 130 pellicles of emulsion 4 by 7 or 2 by 4 inches and 600 μ thick. They were oriented so that the particle tracks were parallel to the emulsion layers and were in the direction of the long dimension of the stack. The only major difference between the K^+ and K^- meson exposures was that the currents were reversed in the focusing and analyzing magnets. (A small compensation was applied in the analyzer magnet current to make up for the difference between the two cases due to the stray magnetic field from the Bevatron.) A diagram of the experimental arrangement is shown in Fig. 1.

After processing by a modified "Bristol" development, the plates were inspected with high-resolution microscopes by an "along the track" scanning technique. (As there are some differences in the arrangements and techniques used in different parts of the experiment, these are discussed under the particular sections in which they apply.)

B. K^+-Meson Mean Lifetime

A measurement of the mean lifetime of K^+ mesons has been carried out by making use of their decay in flight in nuclear emulsion.[12] Emulsion stacks were exposed in the K^+ meson beam as described in

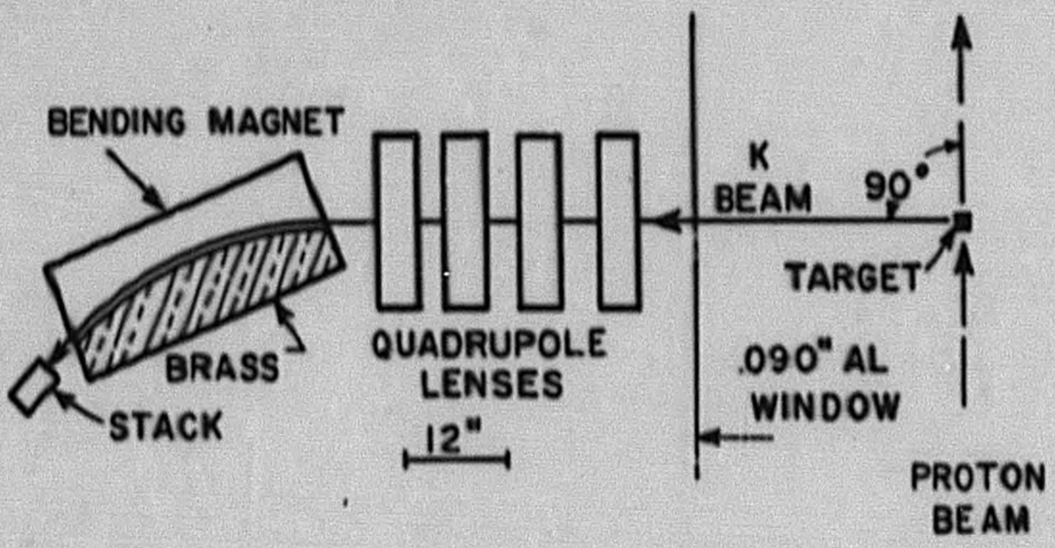

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Fig. 1. Experimental arrangement.

Section II-A. Exposures were made with two different momentum-acceptance bands, positive particles of 390 to 450 Mev/c and of 335 to 360 Mev/c. The particles traveled a distance of 2.7 m from the target to the detector. In such an exposure the protons, K$^+$ mesons, and π^+ mesons (all of the same momentum) have different ranges in the emulsion stack, increasing in that order. The protons stop within a few millimeters of where they enter the edge of the plate. The length of the plates is such that the K$^+$ mesons travel several centimeters from the edge of entrance and stop a few centimeters before reaching the far edge of the plate. The range of the π mesons is so great that they leave the far end of the stack, and there is no appreciable change in the grain density of their tracks. The π mesons are close to minimum grain density and are therefore very useful for calibration purposes.

The scanning technique used is as follows. In the region of the plate just beyond where the protons stop, tracks are chosen on the basis of grain density. K-particle tracks have about twice the minimum grain density. Tracks between 1.8 and 3 times minimum grain density are picked and followed through the stack. (They are followed with the aid of a grid system contact-printed on the bottom side of each emulsion layer.[13]) Nearly all tracks selected in this way turn out to be K particles or τ mesons, except for a contamination of about 15% caused by stray protons, π mesons scattered into the stack, and prongs of stars formed in the emulsion.

A K$^+$ meson, after entering the stack, may do any of three things. It may decay in flight, interact in flight with a nucleus of the emulsion, or come to the end of its range and decay at rest. For identification, the masses of the particles that decayed or interacted in flight have been measured by the multiple-scattering and grain-count technique. Particles coming to rest have been identified by the presence or absence of decay products and by their ranges. Of the events due to K$^+$ particles in flight, 19 have been found in which there is a single outgoing track, of grain density less than that of the incoming K$^+$ meson. If any of these events were due to interactions with emulsion nuclei in flight, one would expect to find some stars with a lightly ionizing track coming out together with one or more black evaporation prongs. No such stars were observed. Also, none of the interactions in flight so far seen give off a visible L meson. It therefore seems reasonable to identify all events of this type as the decay of K^+ mesons in flight.

The mean lifetime is obtained from N, the number of decays in flight observed, and $T = \Sigma t_i$, the total proper slowing-down time of all the K^+ mesons followed (where t_i is the slowing-down time of each meson followed from where it is picked up to where it decays or interacts in flight, or comes to rest in the emulsion). The mean lifetime is $\tau_K = \frac{T}{N}$. Excluding the track length due to τ^+ mesons, a total of 31.6 meters of K^+ meson track length has been followed. The corresponding total proper slowing-down time was calculated by use of the tables of Barkas and Young,[14] and was found to be 19.2×10^{-8} sec. The mass of the K^+ meson was taken as equal to that of the τ^+ meson for this calculation. Since decays in flight near the end of a track may not be readily identified, the proper time spent in the last 2 mm before stopping has not been included. From the 19 decays in flight observed we find a mean lifetime for K^+ mesons of

$$\tau_{K^+} = 1.01 \begin{smallmatrix} +0.33 \\ -0.21 \end{smallmatrix} \times 10^{-8} \text{ sec} .$$

The error given is the statistical standard deviation combined with a 10% uncertainty in the length of track scanned.

In the course of the experiment 2.0 meters of τ^+ meson track was followed, which corresponds to a total proper slowing-down time of 1.2×10^{-8} sec. One decay in flight of a τ^+ meson has been observed. This suggests an upper limit of 6.7×10^{-8} sec and a lower limit of 0.36×10^{-8} sec for the τ^+-meson mean lifetime. These limits are confidence limits for 68% probability (see Appendix I).

This was the first measurement of the K^+ mean lifetime in which artificially produced K mesons were used. Since it was completed more accurate measurements have been performed, making use of counting techniques. The result of this experiment is in agreement with these more recent measurements. (Results are discussed more fully in Section II-D.)

C. K^--Meson Mean Lifetime

A mean lifetime for K^- mesons has been determined by the method used for the K^+ meson lifetime which was discussed in the preceding section.[15] Stacks of emulsion were exposed to the K^--meson beam. Particles having momenta of from 285 to 415 Mev/c were incident on the emulsion stacks in the various exposures used for this experiment. The distance traveled by the particles from the target to the emulsion was about 3 meters in all cases. In these exposures the ranges of the K^- mesons are such that they stop in the stack while the π^- mesons of the same momentum pass on through the stack.

The plates were scanned for K^- interactions in flight and at rest, and for decays in flight. Tracks of grain density appropriate to K^- particles of the selected momentum were found near the edge of the plate where they entered and were followed until they decayed in flight, interacted in flight, or came to rest in the emulsion. All tracks that did not come to rest were identified by a mass measurement using the multiple-scattering and grain-count technique. An event was interpreted as a decay in flight if there was only one outgoing prong and if the prong had a grain density less than that of the incoming K^- particle. (No event with an associated "blob" was found that otherwise would have been called a decay in flight.) An event so interpreted could also possibly be an interaction in flight of a K^- meson and a nucleus with a lightly ionizing π^- meson emerging. In K^- interactions at rest in emulsion less than 3% of all the stars were found to be of this nature (8 out of 325). As the nucleus would be expected to be in a more highly excited state after interactions in flight than after interactions at rest, the proportion of stars with a single pion and with no other associated tracks or "blobs" would be even smaller. It is estimated that certainly less than 15% of the events we have taken to be decays in flight may have been interactions in flight. (This corresponds to 3% of the observed interactions in flight.) No decay in flight of a τ^- meson was seen.

As before, the mean lifetime is $\tau_{K^-} = \frac{T}{N}$, where N is the number of decays in flight observed, and $T = \sum_i t_i$ is the sum of the proper slowing-down times for each K^- track from where it is first picked up to where it decays or interacts in flight, or (if the particle comes to rest) to 2 mm from the end of its track. The last 2 mm of a stopping track is not included because a decay in flight would be difficult to identify in this region. In the 19.2 meters of track followed, 13 decays in flight were found. The corresponding total proper slowing-down time is 12.4×10^{-8} sec. This yields a K^--meson mean lifetime

$$\tau_{K^-} = 0.95 {}^{+0.36}_{-0.25} \times 10^{-8} \text{ sec} .$$

The error quoted is from the confidence limits for 68% probability on 13 events (see Appendix I); other errors are negligible in comparison. The tables from Barkas and Young[14] were used to calculate T.

D. Discussion of K-Meson Lifetime Measurements

In the introduction it is mentioned that there may be two or more types of K^+ mesons. In particular it has been suggested that the τ^+ meson and the θ^+ meson are not the same kind of particle. If this is so it is to be expected that their mean lives may be markedly different. Mean lives in radioactive decay are well known to be extremely steep functions of the energy involved in the decay and of the particular mode of decay. In this experiment, if particles corresponding to two or more different mean lives are involved, the lifetimes that have been measured are averages of the type

$$\tau_K = \left(\sum_i \frac{a_i}{\tau_i} \right)^{-1} ,$$

where a_i is the fraction of the particles entering the stack associated with a mean lifetime of τ_i. If there were particles with a lifetime of 0.3×10^{-8} sec or less they would be strongly discriminated against in these measurements because of the possibility of their decay in the time of flight between the target and the emulsion stack. In the measurement of τ_{K^+}, less than 3% of such particles leaving the target would arrive at the detector, while in the measurement of τ_{K^-} less than 1% of such particles would arrive.

It is of interest to compare these results with lifetimes for K^+ mesons determined by other methods. Techniques using counters, cloud chambers, and nuclear emulsions have been used to measure mean lives of K^+ mesons originating in cosmic rays, in the Bevatron, and in the Cosmotron.[12, 16-22] (Evidence from experiments indicating the existence of a much shorter lifetime is discussed in the next paragraph.) The methods and results are summarized in Table 1. For purposes of comparison Fig. 2 shows the results graphically. The data from all measurements of the K^+ mean lifetime using artificially produced mesons are in agreement, except that the value from Harris, Orear, and Taylor for a mixture of K^+ mesons appears to be slightly low. Recent accurate measurements of the lifetime of artificially produced τ^+ mesons are in agreement, within the experimental errors, with the lifetimes found for various other "pure" decay modes shown here.[23] Crussard et al.[6] have found that K^+ mesons with a probable time of flight of $\sim 5 \times 10^{-10}$ sec have essentially the same proportions of the various K^+ modes of decay as do K^+ mesons from the Bevatron meson beam,[6,24] which have a time of flight of $\sim 10^{-8}$ sec prior to detection. Also Widgoff et al.[25] and Biswas et al.[26] have shown that these decay mode ratios are not changed significantly by nuclear scattering of the K^+ mesons. Thus there is no evidence from the lifetime measurements that the τ^+ and θ^+ mesons have different mean lives. The cosmic-ray measurements by the Princeton group (Mezzetti and Keuffel, and Robinson) are consistently lower than the more accurate of the measurements made on artificially produced K^+ mesons.[16, 19] The significance of this is not apparent.

The K^- meson lifetime found in this experiment agrees with all the K^+ lifetime measurements within the quoted error. The equality of the lifetimes together with the equality of the masses of the K^+ and K^- mesons (within experimental error) lends strong support to the current assumption that these particles are charge conjugates of each other.

All the measurements considered thus far except those by Crussard et al. have delay times between creation and detection that are much larger than 10^{-9} sec.[6] Therefore K particles of shorter lifetime would not be detected in these experiments. Cloud-chamber cosmic-ray measurements at Princeton have yielded a lifetime for both K^+ and

Table I

Mean lifetimes of K⁺ and K⁻ mesons reported by various experimenters

Authors and reference number	Source of K mesons	Method of detection	Type of K meson	Lifetime (sec.)	Approx. proper time delay prior to detection (sec.)
(A) L. Mezzetti and J. Keuffel[16]	Cosmic rays	Counters	K^+ (except τ^+)	$0.97 \pm 0.10 \times 10^{-8}$	1.2×10^{-9}
(B) Barker, Binnie, Hyams, Rout, and Sheppard[17]	Cosmic rays	Counter; and cloud chamber	K^+ (except τ^+)	$1.08 {}^{+0.36}_{-0.22} \times 10^{-8}$	0.5×10^{-9}
(C) Iloff, Chupp, Goldhaber, Goldhaber, Lannutti, Pevsner, and Ritson[12] (and this work)	Bevatron	Emulsions	K^+ (except τ^+)	$1.01 {}^{+0.33}_{-0.23} \times 10^{-8}$	1×10^{-8}
(D) L. Alvarez and S. Goldhaber[18]	Bevatron	Emulsions	τ^+	$1.0 {}^{+0.7}_{-0.3} \times 10^{-8}$	0.2×10^{-8}
(E) R. Robinson[19]	Cosmic rays	Counters	K^+ (except τ^+)	$0.805 \pm 0.064 \times 10^{-8}$	1.2×10^{-9}
(F) Harris, Orear, and Taylor[20]	Bevatron	Emulsions	τ^+	$0.8 {}^{+0.5}_{-0.2} \times 10^{-8}$	1.9×10^{-8}
			K^+ (except τ^+)	$0.7 {}^{+0.15}_{-0.1} \times 10^{-8}$	
(G) Alvarez, Crawford, Good, and Stevenson[21]	Bevatron	Counters	$K^+_{\mu 2}$	$1.4 \pm 0.2 \times 10^{-8}$	1.6×10^{-8}
			$K^+_{\pi 2}$	$1.1 \pm 0.2 \times 10^{-8}$	
			K^+	$1.3 \pm 0.1 \times 10^{-8}$	
(H) V. Fitch and R. Motley[22]	Cosmotron	Counters	$K^+_{\mu 2}$	$1.17 {}^{+0.08}_{-0.07} \times 10^{-8}$	2×10^{-8}
			$K^+_{\pi 2}$	$1.21 {}^{+0.11}_{-0.17} \times 10^{-8}$	
(I) Iloff, Goldhaber, Goldhaber, Lannutti, Gilbert, Violet, White, Fournet, Pevsner, Ritson, and Widgoff[13] (and this work)	Bevatron	Emulsions	K^-	$0.95 {}^{+0.[illegible]}_{-0.25} \times 10^{-8}$	1.9×10^{-8}
(J) R. Motley and V. Fitch[23]	Cosmotron	Counters	τ^+	$1.17 {}^{+0.08}_{-0.07} \times 10^{-8}$	2×10^{-8}
(K) Hoang, Kaplon, and Yekutieli[24]	Cosmotron	Emulsions	$\tau^+ + \tau'^+$	$1.30 \pm 0.11 \times 10^{-8}$	2.2×10^{-8}
			$K^+_{\mu 3}$	$0.88 \pm 0.23 \times 10^{-8}$	
			K^+_e	$1.44 \pm 0.46 \times 10^{-8}$	

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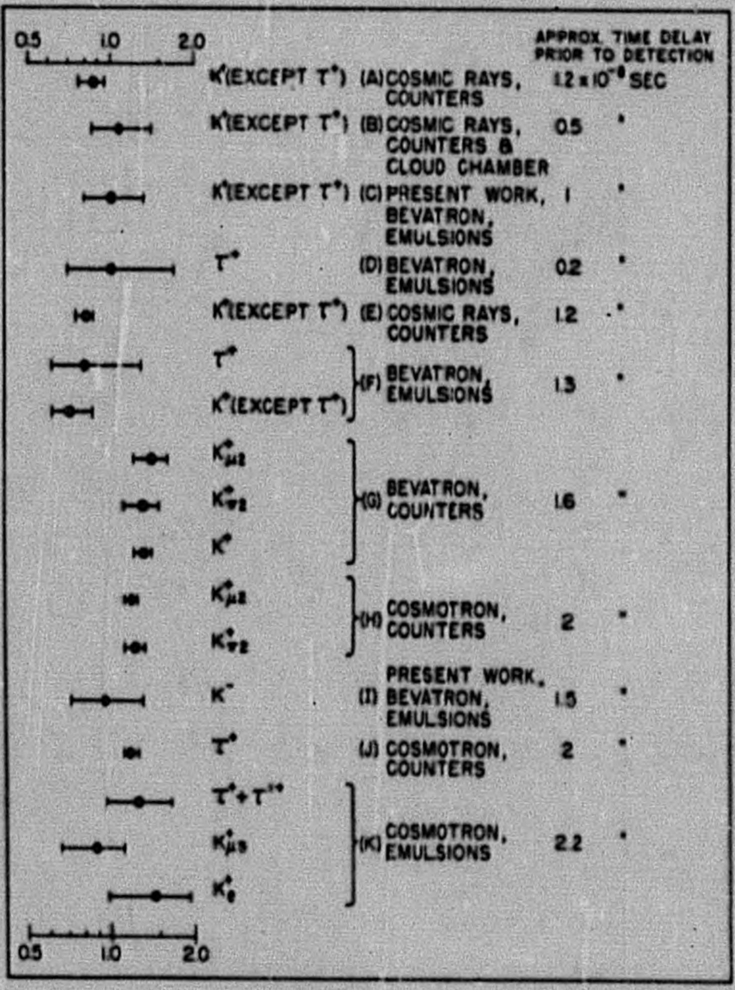

Fig. 2. Mean lifetimes of K^+ and K^- mesons reported by various experimenters. (The time is plotted on a logarithmic scale.) References are given in Table I.

K^- mesons that is much shorter than those we have just considered.[27] They obtain $\tau_{K^+} = 5.2^{+3.3}_{-1.5} \times 10^{-10}$ sec and $\tau_{K^-} = 4.2^{+3.8}_{-1.2} \times 10^{-10}$ sec. The errors are confidence limits for 50% probability. In similar measurements Trilling and Leighton have found evidence for short-lived negative V particles (which may or may not be K particles) but not for positive V particles.[28] Their result is $T_{V^-} = 1.3 \pm 0.6 \times 10^{-10}$ sec. Fretter, Friesen, and Lagarrique also observe no short-lived component for positive K mesons.[29] They find a mean life $\tau_{K^+} = 6.7^{+\infty}_{-5.5} \times 10^{-9}$ sec.

There is a possibility that the τ and θ mesons have quite different lifetimes but that the lifetimes measured are not identified with the correct particle. Orear and Lee have suggested that one of these two types of mesons is heavier than the other and that in addition to its normal decay mode (or modes) it has a sizable branching ratio for decay into a γ-ray and the other of the two types.[30] If this then again immediately decays with a much shorter lifetime we see events that have the decay products of the second type, but the apparent lifetime of these events is that of the first type. A search for the γ ray (or γ rays) from the process

$$\tau^+ \rightarrow \theta^+ + \gamma\ (+\gamma)$$

has been carried out by Alvarez et al.[31] They report that such γ rays of energy greater than 0.5 Mev are absent. As was mentioned before, it has been suggested on the basis of angle- and energy-correlation measurements of the decay products that the spin and parity assignment of the τ^+ meson is 0 (-). An experiment by Osher and Moyer at the Bevatron (in which γ rays arising from points displaced from the target are detected) supports the proposal that θ^0 mesons decay by the process $\theta^0 \rightarrow \pi^0 + \pi^0$ as well as by $\theta^0 \rightarrow \pi^+ + \pi^-$. This decay into two identical bosons requires that the θ meson must have even spin and thus even parity. Both the 0 (-) to 0 (+) and the 0 (-) to 2 (+) transitions require mass differences of 2 to 3 Mev to proceed rapidly. The conclusion is, then, that the process $\tau \rightarrow \theta + \gamma\ (+\gamma)$ does not occur fast enough (if at all) to explain the lifetime dilemma. Also it is to be expected that some of the particles of short lifetime would be produced directly from the target, so that measurements capable of detecting very short lifetimes may throw some light on the problem. The cosmic-ray evidence on charged K particles of very short lifetime, which was considered in the preceding paragraph, is too inconsistent to offer much help at present. Another proposal is that of Weinstein, who suggests that the two types of particles may be converted into each other by interaction with the atomic electric and magnetic fields in passing through matter.[32] Thus the same proportion of each of the two types of K mesons would be present so long as the K particles are passing through or at rest in matter, and therefore the same average lifetime would always be measured. To test this idea it would be necessary to carry out lifetime measurements in a vacuum. If the τ^+ meson has spin and parity 0 (-) and the θ^+ meson has spin and parity 0 (+) or 2 (+), the couplings with the atomic electric and magnetic fields are too weak to account for the equivalence of the lifetimes by this mechanism.[33] Lee and Yang have considered the possibility that parity may not be conserved in weak interactions such as meson decay.[34] In this case the τ^+ and θ^+ mesons may be just two different decay modes of the same particle, which then must of course have a single mass value and a single lifetime. Experiments are suggested in which the lack of parity conservation would result in the observation of certain types of asymmetries in the angular distributions of the decay products of strange particles. There are as yet no published experimental results that test this possibility. Schwinger has considered a dynamical theory of strange particles which predicts the existence of pairs of particles that are eigenstates of the parity reflection operator[35] (as has been suggested by Lee and Yang[36]). Such eigenstates consist of equal mixtures of the two states with definite parity. If then the dominant decay mode ($K \rightarrow \mu + \nu$) preserves parity reflection symmetry, the particles will exhibit lifetimes that do not differ greatly even if the minor modes of decay are characterized by states of definite parity. (Since each of the minor decay modes comes from a state of definite parity these states represent particles of slightly different mass.) These minor modes of decay will cause only a relatively small shortening of the lifetime of each of the particles of definite parity. Formulas are given for decay-mode ratios as functions of time and of the lifetime of the two particles of definite parity. This scheme may be tested by measurement of the decay-mode ratios for K^+ mesons that have traveled a large number of mean lifetimes after production, or by very accurate measurements of the τ^+- and θ^+-meson mean lifetimes.

III. INTERACTIONS IN FLIGHT OF K$^-$ MESONS

A. Procedure and Results

An investigation of the interactions of K$^-$ mesons in flight in nuclear emulsion has been carried out.[37] The experimental details, including the exposures and the scanning technique, are described in Sections II-A and II-C of this report. The plates were scanned by an "along the track" technique, and all interactions in flight were identified by grain count and multiple-scattering measurements. (Decays in flight were eliminated by their identification as described in Section II-C.) The K$^-$ mesons on entering the emulsions have momenta between 280 and 355 Mev/c. Because their ranges are less than the length of the emulsion stack, interactions are recorded from these momenta all the way down to zero momentum. For practical purposes, since it is difficult to determine whether an event that occurs at a residual range of 2 mm or less is an event in flight or at rest, there is an experimental lower bound of 16 Mev for the energy of the K$^-$ mesons causing the events accepted. (In calculation of the mean free path, of course, the last 2 mm of the tracks of K particles that come to rest in the emulsion must be excluded from the total path length followed.) Table II shows the amount of track scanned in various energy intervals.

A total of 21 interactions in flight due to K$^-$ mesons have been found and analyzed. The products of these interactions include π mesons, hyperons, and an excited fragment, in addition to the usual heavy evaporation tracks and higher-energy protons. No star was seen in which a K$^-$ meson emerged along with other products or in which a K$^-$ meson was inelastically scattered. Also no elastic scatters at angles greater than 40° were found. (It is to be noted that in nuclear emulsion it is often impossible to tell whether a particle has undergone an elastic scattering or has lost a small amount of energy upon being scattered. Therefore some inelastic scatterings with small energy loss are nearly always included among "elastic" scatters.) In 2.8 m of track inspected for 20° to 40° scatters six such events were found. Scattering in this angular region may be accounted for by diffraction and Coulomb effects. Table III contains a more detailed description of each of the interactions.

Table II

Length of track scanned in energy intervals	
Energy interval (Mev)	**Track length (meters)**
16 - 30	0.52
30 - 40	0.51
40 - 50	0.62
50 - 60	0.73
60 - 70	0.81
70 - 80	0.83
80 - 90	0.60
90 - 100	0.24
100 - 110	0.04
Total	4.90

Table III

Detailed description of K^- interactions in flight

	Energy of K at interaction (Mev)	Prong No.	Range	Energy of Prong (Mev)	Identity of Prong	Comments
1.	85 ± 5	1	180 μ	5	(p)	
		2	440 μ	9	(p)	
		3	---	55	π	
		Pion rest energy		140		
		Binding energy		16		
		Total		225 Mev		
2.	87 ± 5	1	1.8 mm	20	(p)	
		2	>15.9 mm	> 68	(p)	
		3	370 μ	8	(p)	
		4	28.8 mm	42	π^-	Ends. Gives 1-prong σ star.
		5	110 μ	4	(p)	
		6	230 μ	6	(p)	
		7	75 μ	3	(p)	
		Pion rest energy		140		
		Binding energy		48		
		Total		>339 Mev		

Table III (cont.)

	Energy of K at interaction (Mev)	Prong No.	Range	Energy of Prong (Mev)	Identity of Prong	Comments
3.	77 ± 6	1	62 μ	2.6	(p)	
		2	1.5 mm	18	(p)	
		Binding energy		16		
			Total	36 Mev		
4.	70 ± 6	1	716 μ	12	(p)	
		Binding energy		8		
			Total	20 Mev		
5.	43 ± 7	Disappearance in flight.				Short electron track associated.
6.	71 ± 6	1	11 μ	0.8	(p)	
		2	110 μ	3.7	(p)	
		3	60 μ	2.7	(p)	
		4	38 μ	1.9	(p)	
		Binding energy		32		
			Total	41 Mev		

Table III (cont.)

	Energy of K at interaction (Mev)	Prong No.	Range	Energy of Prong (Mev)	Identity of Prong	Comments
7.	72 ± 6	1	---	≥100	(π)	Energy by grain count.
		2	480 μ	9	(p)	
		Pion rest energy		140		
		Binding energy		8		Short recoil associated.
			Total	257 Mev		
8.	69 ± 6	1	22 mm	84	(p)	
		2	29 mm	96	(p)	
		Binding energy		16		
			Total	196 Mev		
9.	75 ± 6	1	18 mm	> 88	Σ	Decays in flight into pion.
		2	80 μ	1.2	(p)	
		3	12 μ	0.9	(p)	
		4	450 μ	8.5	(p)	
		5	1250 μ	15.5	(p)	
		Difference between Σ rest energy and proton rest energy		251		
		Binding energy		40		
			Total	>405 Mev		

Table III (cont.)

Energy of K at interaction (Mev)	Prong No.	Range	Energy of Prong (Mev)	Identity of Prong	Comments
10. 24 ± 10	1	59 μ	2	(p)	
	2	34 μ	1.8	(p)	
	3	55 μ	1.9	(p)	
	4	17 μ	1.0	(p)	
	Binding energy		32		
		Total	39 Mev		
11. 45 ± 7	1	26 μ	5	Excited fragment	Has two dark prongs (a) 35 μ (b) 50 μ
	2	7.7 mm	45	(p)	
	3	---	102	(p)	Energy and identity by multiple scattering and grain count
	Binding energy		24		
		Total	176 Mev		
12. 41 ± 7	1	8 μ	0.7	(p)	
	2	1.3 mm	16	(p)	
	3	88 μ	3.6	Σ^+	Decays by $\Sigma^+ \rightarrow p + \pi^0$.
	4	---	73±11	π	Energy by grain count.
	Pion rest energy		140		
Difference between Σ rest energy and proton rest energy			251		
	Binding energy		24		
		Total	508 Mev		

Table III (cont.)

	Energy of K at interaction (Mev)	Prong No.	Range	Energy of Prong (Mev)	Identity of Prong	Comments
13.	39 ± 7	Disappearance in flight.				
14.	27 ± 10	1	180 μ	5	(p)	
		2	130 μ	4	(p)	
		3	800 μ	12	(p)	
		4	1080 μ	14	(p)	
		Binding energy		32		
			Total	67 Mev		
15.	63 ± 6	1	27 mm	94	(p)	
		2	30 μ	1.7	(p)	
		Binding energy		24		Two short "recoil" tracks associated.
			Total	120 Mev		
16.		Disappearance in flight.				
17.		Disappearance in flight.				

Table III (cont.)

	Energy of K at interaction (Mev)	Prong No.	Range	Energy of Prong (Mev)	Identity of Prong	Comments
18.		Disappearance in flight.				
19.	32 ± 9	1	---	~100	(π)	
		Pion rest energy		140		Short electron track associated.
			Total	~240		
20.	52 ± 7	1	1.6 mm	18	(p)	
		Binding energy		8		
			Total	26 Mev		
21.	50 ± 7	1	6 μ	0.5	(p)	
		2	12 μ	0.9	(p)	
		Binding energy		16		4-μ recoil associated.
			Total	17 Mev		

In order to find the 21 interactions in flight 4.9 meters of K^- meson track were followed. (The last 2 mm of all stopped K tracks is excluded, as explained previously. Also the first 3 mm of each track has not been included, because the primary particle causing such an event could not easily be identified as a K particle. Events occurring in the first 3 mm of a track, of course, have not been counted.) This leads to a mean free path for K^- mesons in nuclear emulsion of

$$\lambda_{K^-} = 23^{+6}_{-5}\ \text{cm},$$

which is to be compared to the geometrical mean free path in nuclear emulsion of

$$\lambda_G = 31\ \text{cm}.$$

(This geometrical mean free path is assigned on the basis of a nuclear radius $R = 1.3 \times 10^{-13} A^{1/3}$ cm for all the constituents of the emulsion.)

B. Discussion of K^--Meson Interactions in Flight and Comparison with K^+-Meson Interactions in Flight

The most striking difference between the interactions of K^- mesons in flight and those of K^+ mesons[38-41] is that π mesons, hyperons, and excited fragments frequently appear among the products in the former, whereas in the latter they do not appear. All the K^+-meson interactions observed may be interpreted as elastic or inelastic scattering, or as charge-exchange scattering of the incident K^+ meson by a nucleus. These reactions, if assumed to be due to one nucleon in the nucleus, correspond to Reactions 1 through 3 of Section IV. In contrast, the K^--meson interactions frequently exhibit so large a visible energy release that the absorption of the K^- meson is required to account for all the energy seen. The occurrence of π mesons and hyperons among the products of K^- interactions is evidence for the absorption type of reactions, such as the single-nucleon Reactions 7 through 13 of Section IV (in which a π meson is produced together with a Λ or Σ hyperon), which appear to be responsible for most of the stars caused by K^- mesons interacting at rest. (As is pointed out in the discussion in Section IV of stars due to K^- mesons captured at rest, reactions--such as those with two or more nucleons--are not excluded.) This contrast in the behavior of K^- and K^+ mesons is, of course, just what is to be expected if the conservation of the "strangeness" number is accepted, since these Reactions 1 through 13 are just those (of a K meson with a single nucleon) which are allowed by this conservation requirement. (These reactions and the strangeness scheme are discussed in more detail in Section IV.) Thus the concept of conservation of strangeness, which was derived from experiments on the production of strange particles, has produced selection rules for the interactions of K^- and K^+ mesons which are now confirmed by the experimental data.

It is interesting to note that none of the 21 interactions in flight of K^- mesons that we have observed could be interpreted as inelastic K^- meson scattering, or as elastic scattering of a K^- meson through an angle greater than 40°. (Elastic scattering through angles less than 40° may be due to Coulomb and diffraction scattering.) From this observation an upper limit of 9% may be set for the ratio of the number of inelastic scatters to the total number of reactions (except elastic scattering through an angle of 40° or less). This is the confidence limit for 84% probability. (See Appendix I.) These reactions of K^- mesons, together with charge-exchange scattering (Reactions (4) through (6) of Section IV-A), are allowed by the selection rules derived from the conservation of strangeness. We may make an estimate of an upper limit for the frequency of inelastic charge-exchange scattering, if we assume that the scattering is due to interaction with a single nucleon, and also impose charge independence for these Reactions (4) through (6). For simplicity, the numbers of protons and of neutrons in the nucleus are assumed to be equal. Then, with the worst possible case of interference between the products from the singlet and triplet isotopic spin states taken into account, the ratio of charge-exchange to non-charge-exchange scattering must be less than 2 to 1. From this we find that inelastic charge-exchange scattering should occur in less than 18% of K^- interactions with nuclei in nuclear emulsions. The limit given is again the confidence limit for 85% probability.

The mean free path for K^- mesons in nuclear emulsion found in this experiment agrees with those found in similar experiments.[42] These results are shown in Table IV. Combining all the results, we may find a more accurate mean free path for K^- mesons in nuclear emulsion:

$$\lambda_{K^-} = 28.0 \, {}^{+4.4}_{-3.4} \text{ cm} .$$

It is equal, within the statistical error, to the geometric mean free path (λ_G = 31 cm; see Section III-A). This is to be compared with the mean free path for K^+ mesons in nuclear emulsion,[38] which is

$$\lambda_{K^+} = 95 \pm 16 \text{ cm} .$$

The difference in the K^-- and K^+-meson mean free paths is not surprising, for, as we have seen, the reactions involved in the two cases are not the same.

A mean free path of 47 ${}^{+31}_{-17}$ cm was found for "elastic" scattering of K^- mesons in nuclear emulsion through angles of 20° to 40°. For "elastic" scattering of K^+ mesons[38] in the same angular interval the mean free path is 160 ${}^{+42}_{-32}$ cm. The statistics of these results are very poor; however, the K^- mean free path for such scattering appears to be shorter than that for K^+ mesons. This is to be expected if the scattering in this angular interval is largely due to a diffraction effect, because the total K^--meson mean free path is about 1/3 that for K^+ mesons. Simple calculations on diffraction from a black disk show this effect to be of reasonable size. A more exact calculation should be made with the aid of an optical model consisting of a "black" sphere (of radius equal to the geometric radius of the nucleus) surrounded by a Coulomb potential.

The number of K^- interactions in flight observed is much too small to reach any conclusions from the frequencies of the various types of products. (Much of the discussion of Section IV could be applied to interactions in flight.) Further work on this problem, leading to a much larger statistical sample, should yield more information concerning the reactions involved and would, of course, increase greatly the accuracy of the measured quantities reported herein.

Table IV

Mean free paths of K^+ mesons in nuclear emulsion reported by various authors

Authors	K path length scanned (m)	No. of interactions	Mean free path (cm)
J. Hornbostel and E. Salant[42]	1.52	9	17 ± 5
D. M. Fournet and M. Widgoff[42]	8.68	24	$36.2^{+9.2}_{-6.1}$
This work	4.9	21	$23\ ^{+6}_{-5}$
Total	15.1	54	$28.0^{+4.4}_{-3.4}$

IV. CONSIDERATIONS OF THE INTERACTIONS OF K MESONS IN NUCLEI

To bring order to the data on the copious associated production and long lifetimes of the strange particles (K mesons and hyperons), several schemes have been proposed.[10] These all have certain simple ideas in common. Interactions involving these particles are divided into three classes:

1. Fast interactions (~ 10^{-22} sec)
   (e.g., direct production);

2. Electromagnetic interactions (~ 10^{-17} sec)
   (e.g., those involving γ rays);

3. Slow interactions (~ 10^{-10} sec)
   (e.g., decay).

All particles that may enter into "fast" interactions are assigned a small integral number, positive or negative. Here we follow the notation of Gell-Mann and call this number S. Each of these particles is also assigned an isotopic spin. It is then proposed that the number S must be conserved in fast interactions, whereas it must change by ± 1 in slow interactions. The strangeness number and isotopic spin assignments are as follows:

	S	T
K^+, K^0	+1	1/2
p, n	0	1/2
π^+, π^0, π^-	0	1
K^-, $\bar{K}^0$	-1	1/2
Λ^0	-1	0
Σ^+, Σ^0, Σ^-	-1	1
Ξ^0, Ξ^-	-2	1/2

Associated production follows automatically from this scheme, and it leads to selection rules for production that have been verified experimentally (i.e., thus far no violations have been observed and those reactions which are expected have been observed).

These proposals lead directly to rules for the interactions of these particles with nucleons. Thus the following reactions of a charged K particle with a single nucleon are allowed:

$$K^{+} + p \rightarrow K^{+} + p\,, \tag{1}$$

$$K^{+} + n \rightarrow K^{+} + n\,, \tag{2}$$

$$K^{+} + n \rightarrow K^{0} + p\,, \tag{3}$$

$$K^{-} + p \rightarrow \bar{K}^{0} + n\,, \tag{4}$$

$$K^{-} + p \rightarrow K^{-} + p\,, \tag{5}$$

$$K^{-} + n \rightarrow K^{-} + n\,, \tag{6}$$

$$K^{-} + p \rightarrow \Lambda^{0} + \pi^{0}\,, \tag{7}$$

$$K^{-} + n \rightarrow \Lambda^{0} + \pi^{-}\,, \tag{8}$$

$$K^{-} + p \rightarrow \Sigma^{-} + \pi^{+}\,, \tag{9}$$

$$K^{-} + p \rightarrow \Sigma^{0} + \pi^{0}\,, \tag{10}$$

$$K^{-} + p \rightarrow \Sigma^{+} + \pi^{-}\,, \tag{11}$$

$$K^{-} + n \rightarrow \Sigma^{0} + \pi^{-}\,, \tag{12}$$

$$K^{-} + n \rightarrow \Sigma^{-} + \pi^{0}\,. \tag{13}$$

(Some reactions resulting in two pions are also allowed, but from phase-space arguments they are expected to contribute very little. Also, reactions involving capture by two or more nucleons are allowed. This is discussed again later.) Most of the data available on K-meson interactions are from the interactions of K mesons with complex nuclei in nuclear emulsions[37-49] (though Reactions (1), (5), (9), and (11) have all been seen to occur with a hydrogen nucleus in emulsion. Furthermore, evidence has recently become available from the hydrogen bubble chamber experiments by Alvarez et al.[50])

Let us consider a simple model for the interaction of a K^- meson at rest with a nucleus. Assume that the K^- meson interacts with a single nucleon in the nucleus. Reactions (7) through (13) are then allowed, producing a π meson together with a Λ^0 or Σ hyperon. (Reaction (4) is either forbidden by considerations of conservation of energy or is suppressed owing to the very small energy release involved.) Then, assuming a maximum Fermi energy for the nucleon of 20 Mev and allowing 20 Mev for the adiabatic removal of a nucleon from the nucleus, we may calculate maximum and minimum energy limits for the produced particles. In the case in which a Σ hyperon is produced this leads to a broad peak in the pion spectrum between the limits of 55 and 100 Mev, and an upper limit for the Σ energy of 45 Mev. If, instead, a Λ^0 hyperon is produced, the pion energy is between 110 and 175 Mev and the Λ^0 upper energy limit is 70 Mev. Of course some π mesons of lower energy are to be expected because of possible interaction with other nucleons before they have left the nucleus. This is especially true for those produced together with a Λ^0 hyperon, since pions of these energies have exceedingly high cross sections for scattering from nucleons. The lowest "Bohr orbit" of a K meson around a heavy nucleus is well inside the nucleus. Blatt and Butler have carried out calculations that suggest that if K^- mesons are captured in a time on the order of 10^{-22} sec in nuclear matter then most of the captures at rest in heavy nuclei take place from the 2p and 2s states, since the electromagnetic transition from an np state to the 1s state ($\sim 10^{-19}$ sec) is much less probable than direct nuclear capture from the 2p state ($\sim 10^{-21}$ sec).[51] Thus since the capture of a K^- meson is fast (in Section III A we have shown that K^- mesons in flight have approximately geometric cross sections with nuclei), it takes place from higher Bohr orbits ($n \geq 2$) or, in other words, near the surface of the nucleus. It is estimated roughly that about one-half of the produced π mesons will be captured in the nucleus before emerging (if we assume that those going toward the nucleus are captured and those going away escape). Then we expect that most of the π mesons that come out will not have interacted and therefore will retain their original energies. The experimental data from K^- mesons captured at rest in nuclear emulsion appear to fit this picture rather well.[42-49] Most of the observed $\pi^{\pm}$ mesons coming from these stars are within the energy range that corresponds to the production of Σ hyperons, giving a very pronounced peak in this region of the π-meson spectrum. There are a few charged pions that appear to have energies which would appear to be associated with Λ^0 production; however, the energies of π mesons are hard to measure in nuclear emulsion in this region, so that it is not possible to establish a definite peak. The energies of the charged Σ hyperons are nearly all below the calculated maximum. The few cases exceeding this maximum can be attributed to a two-nucleon capture of the K^- meson in such a reaction as

$$K^- + n + p \rightarrow \Sigma^- + p \qquad (14)$$

In all cases the various products and combinations of products observed in the stars formed by K^- mesons at rest seem to be compatible with a combination of the assumed primary reactions (i. e., Reactions (7) through (13)) with a small contribution from the reactions of the type represented by (14). It may be estimated from the pion and hyperon energy spectra that perhaps less than 10% of all the stars may be due to reactions involving two nucleons, and about 65% are due to a single-nucleon capture yielding a π meson and a Σ hyperon, while the remaining proportion of about 25% or less is due to a single-nucleon capture resulting in a π meson and a Λ^0 hyperon. Recent results on the interactions of K^- mesons with protons in a hydrogen bubble chamber by Albarez et al. confirm the predominance of the reactions in which Σ hyperons are produced over those in which Λ^0 hyperons are produced.[50] Λ^0 hyperons seen coming from K^- stars in nuclei may be due either to primary production of a pion and a Λ^0 or to the interaction of an originally produced Σ hyperon with a nucleon in the nucleus according to a reaction such as

$$\Sigma^+ + n \rightarrow \Lambda^0 + p .$$

The excited fragments that are frequently observed may also be due to Λ^0 hyperons from either source. High-energy γ rays associated with K^--meson capture may come from the decay of produced π^0 mesons or from the decay of Σ^0 hyperons by the suggested scheme $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. The latter is an electromagnetic interaction which conserves the number S and therefore is expected to have a very short lifetime ($\sim 10^{-20}$ sec).[39] The π^0 meson is known to have a lifetime of about 5×10^{-15} sec. Thus these high-energy γ rays should all have their points of origin very near to the star from which they come. Additional γ rays may come from points farther from the star, owing to the decay of π^0 mesons from Λ^0 or Σ^+ hyperons produced in the star. ($\Lambda^0 \rightarrow n + \pi^0$ or $\Sigma^+ \rightarrow p + \pi^0$.)

Now let us consider the effect of the assumption of charge independence on these reactions. For simplicity assume that the numbers of protons and neutrons in the nuclei involved are equal. (Actually in nuclear emulsion we have $\left(\frac{A}{Z}\right)_{Ave} = 2.2$.) The relative probabilities of the various reactions are given in Table V. These have meaning only within the group of reactions resulting in the same type of particles but with different charges, and may be used to calculate charge ratios of produced particles within that group. The reactions are divided into groups by horizontal lines. The ratios between groups depend, of course, on the as yet unknown strengths and types of interactions involved for each group. Table V gives the probabilities for pure T = 0 and T = 1 isotopic spin states and for a mixture of the two. The results for a mixture contain an interference term and therefore depend on a phase angle as well as on the singlet and triplet interaction amplitudes.

We have seen that Reactions (7) through (13) are responsible for nearly all the interactions of K^- mesons at rest. For Reactions (7) and (8), in which a pion and a Λ^0 hyperon are produced, the only isotopic spin state that can take part is the T = 1 state. Assuming an equal number of protons and neutrons to take part in the interactions gives a ratio of π^- to π^0 mesons of $\pi^-/\pi^0 = 2$. No π^+ mesons are formed in these reactions. In Reactions (9) through (13), in which a pion and a Σ hyperon result, both the isotopic spin states T = 0 and T = 1 may contribute. Here we find a ratio of charged pions to neutral pions $\pi^{\pm}/\pi^0 = 2$. This is independent of the isotopic spin state or mixture of isotopic spin states through which the reactions go, and is not affected by their interference. We therefore have an over-all ratio of charged to neutral pions of 2, which means that 2/3 of the pions initially formed

Table V

Probabilities of various reactions due to a K meson and a nucleon. (A, B, C, D are reaction amplitudes. Subscripts 0 and 1 refer to the T = 0 and T = 1 isotopic spin states.)

Reaction	Pure T = 0	Pure T = 1	Mixture
$K^+ + p \rightarrow K^+ + p$	---	A_1^2	A_1^2
$K^+ + n \rightarrow K^+ + n$	$1/4\, A_0^2$	$1/4\, A_1^2$	$1/4\, A_0^2 + 1/4\, A_1^2 - 1/2\, A_0 A_1 \cos\phi_A$
$\rightarrow K^0 + p$	$1/4\, A_0^2$	$1/4\, A_1^2$	$1/4\, A_0^2 + 1/4\, A_1^2 + 1/2\, A_0 A_1 \cos\phi_A$
$K^- + p \rightarrow K^- + p$	$1/4\, B_0^2$	$1/4\, B_1^2$	$1/4\, B_0^2 + 1/4\, B_1^2 - 1/2\, B_0 B_1 \cos\phi_B$
$\rightarrow \bar{K}^0 + n$	$1/4\, B_0^2$	$1/4\, B_1^2$	$1/4\, B_0^2 + 1/4\, B_1^2 + 1/2\, B_0 B_1 \cos\phi_B$
$K^- + n \rightarrow K^- + n$	---	B_1^2	B_1^2
$K^- + p \rightarrow \pi^0 + \Lambda^0$	---	$1/2\, C_1^2$	$1/2\, C_1^2$
$K^- + n \rightarrow \pi^- + \Lambda^0$	---	C_1^2	C_1^2
$K^- + p \rightarrow \pi^+ + \Sigma^-$	$1/6\, D_0^2$	$1/4\, D_1^2$	$1/6\, D_0^2 + 1/4\, D_1^2 - 1/\sqrt{6}\, D_0 D_1 \cos\phi_D$
$\rightarrow \pi^0 + \Sigma^0$	$1/6\, D_0^2$	---	$1/6\, D_0^2$
$\rightarrow \pi^- + \Sigma^+$	$1/6\, D_0^2$	$1/4\, D_1^2$	$1/6\, D_0^2 + 1/4\, D_1^2 + 1/\sqrt{6}\, D_0 D_1 \cos\phi_D$
$K^- + n \rightarrow \pi^0 + \Sigma^-$	---	$1/2\, D_1^2$	$1/2\, D_1^2$
$\rightarrow \pi^- + \Sigma^0$	---	$1/2\, D_1^2$	$1/2\, D_1^2$

are charged. If, as before, we assume that 90% of the stars are due to these reactions (including all stars which have π mesons) and that 50% of the pions are absorbed before leaving the nucleus in which they were formed, it is apparent that about $2/3 \times 0.9 \times 0.5 = 0.30$ of all the stars are expected to emit visible π mesons in nuclear emulsion. This is in fortuitously good agreement with the experimental result of 30 ± 3%,[49] considering the roughness of the correction used for pion absorption.

Returning again to the reactions in which Σ hyperons are produced, let us consider the ratio of π^+ to π^- mesons. We find

$$\pi_\Sigma = \frac{\pi^+}{\pi^-} = \frac{a^2 - \sqrt{6}\, a \cos\phi + 3/2}{a^2 + \sqrt{6}\, a \cos\phi + 9/2}\,, \qquad \text{(A)}$$

where a is the ratio of the $T = 0$ reaction amplitude to the $T = 1$ reaction amplitude, and ϕ is the phase angle between the outgoing π^- mesons in the $T = 0$ and $T = 1$ states. It is seen then that if the reaction takes place purely through the $T = 0$ state then we have $\pi^+/\pi^- = 1$, while if it takes place purely through the $T = 1$ state we have $\pi^+/\pi^- = 1/3$. However, owing to the interference terms, if mixtures are allowed this ratio may have any value from $\pi^+/\pi^- = 0$ to $\pi^+/\pi^- = 3$. If we now combine the results from the reactions producing Λ^0 hyperons with those in which Σ hyperons are produced, we get a ratio

$$\frac{\pi^+}{\pi^-} = \pi_\Sigma \left(\frac{1 - \gamma}{1 + \gamma}\right), \qquad \text{(B)}$$

where γ is the ratio of stars in which the initial reaction produced a Λ^0 hyperon and a π meson to those in which the initial reaction results in a Σ hyperon and a π meson and π_Σ is given by Equation (A). This ratio should be changed very little by interaction of charged pions as they come out of the nucleus, because (from charge-independence considerations) the same proportion of each should be absorbed or charge-exchange scattered. Charge-exchange reactions of π^0 mesons before leaving the nucleus in which they were formed would tend to affect the ratio in such a way that it would be closer to 1. Similarly, we may also find the expected charge ratios for the Σ hyperons produced in the single-nucleon capture of K^- mesons. The ratio for the number of charged Σ hyperons to neutral Σ hyperons is $\Sigma^{\pm}/\Sigma^{0} = 2$, and is independent of the isotopic spin states involved. If we assume as before that 65% of the stars are due to a single nucleon K^- capture which results in a Σ hyperon, and that 50% of these hyperons are absorbed within the nucleus where the reaction occurs, we find that $2/3 \times 0.65 \times 0.5 = 0.22$ of the stars should have charged Σ hyperons emerging with an energy that is consistent with a single-nucleon reaction. The experimental result is somewhat clouded by the fact that Σ^- hyperons coming to rest in nuclear emulsion are often not recognized. The captures of Σ^- hyperons by nuclei are thought to be due mainly to the process $\Sigma^- + p \rightarrow \Lambda^0 + n$, since this is the only single-nucleon fast reaction that is allowed by the strangeness scheme (except for charge exchange, which if not forbidden by energy conservation, is certainly suppressed by phase-space limitations). Since the products of this reaction are both neutral and their combined kinetic energy (neglecting the binding energy of the proton in the nucleus) is only about 83 Mev, a large proportion of the stars produced have no visible prongs. Since Σ^- hyperons coming from K^- meson capture at rest are of rather low energy, those stopping with no visible prongs are not easily distinguished from protons in nuclear emulsion and thus are not included in the number of charged hyperons reported. The proportion of K^- stars in emulsion having distinguishable visible hyperons emerging is found by experiment to be 0.14 ± 0.02.[49] Using the experimental ratio of Σ^-/Σ^+ (uncorrected for the zero-prong Σ^- stars),[43] and assuming that the number of Σ^- hyperons reported should be increased by a factor of 2 to 2-1/2 (2.25 is used here),[45,48] we find the corrected experimental result for the proportion of all K^--capture stars that have visible (i.e., charged) hyperons coming out is 0.21 ± 0.3. (The error quoted is the statistical error only, and is not to be taken too seriously, because the size of the correction just discussed is not at all well established.) This is again in fortuitously good agreement with the predicted value of 0.22, considering the roughness of the corrections used. The agreement between the experimental numbers and the predicted numbers of both charged pions and charged hyperons from K^- stars lends support to the model used as well as to the strangeness scheme, and furnishes evidence to support charge independence in the interactions of strange particles.

We may also compute the ratio of positive to negative Σ hyperons expected from single-nucleon K^- capture,

$$\frac{\Sigma^+}{\Sigma^-} = \frac{\alpha^2 + \sqrt{6}\,\alpha \cos\phi + 3/2}{\alpha^2 - \sqrt{6}\,\alpha \cos\phi + 9/2}\,, \qquad \text{(C)}$$

where α is the ratio of singlet to triplet reaction amplitudes and ϕ is the phase angle between the outgoing pions in these two isotopic spin states. This ratio should not be changed by absorption and charge-exchange scattering of the charged Σ's, but the effect of charge-exchange scattering of the Σ^0 hyperons would be to bring the ratio closer to 1. Since many of the Σ hyperons are of rather low energy, there may also be some suppression of positive Σ's and enhancement of negative Σ's due to the Coulomb effect.

The experimental results on these particle-charge ratios are based on rather poor statistics and contain a number of possible biases. The π mesons whose charges have been identified are those which stopped in the emulsion stacks. This immediately causes a bias toward π mesons of low energy. We therefore assume that all the pions whose charges have been observed were produced in association with Σ hyperons. Nearly all the Σ hyperons produced by one-nucleon K^- capture would have energies such that they would stop within an emulsion stack. Thus there is little energy bias in their selection. Bias might be caused by the fact that the charge signs of hyperons that decay in flight often cannot be determined. However, since the mean lifetimes of Σ^+ and Σ^- hyperons differ by a factor of 2, (the Σ^+ having the shorter lifetime),[52] it is probable that nearly all the decays in flight are due to Σ^+ hyperons. A correction made on this basis should cause no serious error even though an appreciable number of the Σ hyperons do decay in flight. Negative Σ hyperons are discriminated against because they often come to rest in the emulsion and interact without causing visible prongs or "blobs." (This was discussed in some detail previously.) Therefore a correction factor of 2-1/4 is used (as before) for the number of Σ^- hyperons. Also, in order to eliminate the influence of the Coulomb effect on the charge ratios, we do not count hyperons and pions of less than 10 Mev. The data from Webb et al. (obtained from "along the track" scanning).[43] after these adjustments are made, yield the following charge ratios:

$$\pi_\Sigma = \frac{\pi^+}{\pi^-} = 0.30 \begin{smallmatrix} +0.41 \\ -0.18 \end{smallmatrix} ,$$

$$\Sigma = \frac{\Sigma^+}{\Sigma^-} = 0.57 \begin{smallmatrix} +0.50 \\ -0.26 \end{smallmatrix} .$$

Solving Equations (A) and (C) for a and ϕ, we find

$$a^2 = \frac{3}{2} \left[\frac{3\Sigma\,\pi_\Sigma + \Sigma + \pi_\Sigma - 1}{1 - \Sigma\,\pi_\Sigma} \right]$$

and

$$\cos\phi = \frac{a}{\sqrt{6}} \left[\frac{\Sigma - \pi_\Sigma}{3\Sigma\,\pi_\Sigma + \Sigma + \pi_\Sigma - 1} \right] .$$

Putting in the above values for π_Σ and Σ, we may obtain an upper limit for a:

$$0 \leqq a < 2.5 .$$

(This is the confidence limit for a probability of 0.84.) The value of ϕ cannot be determined at all, because of the very limited statistics. It is plain that the most that can be said from these data is that the $T = 1$ state does contribute to the reactions whereas the $T = 0$ state may or may not.

Similar considerations have been made independently by Koshiba, who suggests that the $T = 1$ state is the dominant one.[53] A collection of data from nuclear emulsion experiments obtained by both "along the track" and "area" scanning techniques was discussed. The Σ^- hyperons observed were corrected by a factor of 3.1 to make up for the Σ^-_p events that are not recorded, and Σ decays in flight were not counted because their charge signs were not known. It is now known that nearly all the decays in flight are due to Σ^+ hyperons, since they have a much shorter lifetime than the Σ^- hyperons. Taking this into account, I have re-analyzed the data considered by Koshiba. When his factor of 3.1 is used to correct the observed number of Σ^- hyperons, the following limit (confidence limit for a probability of 0.84) is obtained:

$$0 \leq a < 0.21 .$$

If instead a correction factor of 2-1/4 is used for the Σ^- hyperons the limit is

$$0 \leq a < 0.75 .$$

It is apparent that the result is highly dependent on the value assumed for the poorly known correction factor for Σ^- hyperons. Also, since these data include many hyperons of 10 Mev or less, the result is biased in favor of negatively charged hyperons (and thus in favor of the $T = 1$ state) by the Coulomb effect. It is clear that from the nuclear-emulsion data on pion and hyperon charge ratios that have been published up to this time, we may conclude only that at least an appreciable part of the K^--meson captures that produce Σ hyperons goes through the $T = 1$ state. The hydrogen bubble chamber data include examples of the reaction $K^- + p \to \Sigma^0 + \pi^0$.[50] Since this reaction takes place only in the $T = 0$ state it is evident that this state must also contribute to the reactions. Obviously many more data are needed to draw any more definite conclusions.

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APPENDIX

A problem often met in counting experiments is that of setting limits of error on a quantity whose value is estimated from a small number of counts. Sometimes one even wishes to set a limit with no counts at all. The usual limits of error used in counting experiments are given by the standard deviation; however, this has little meaning for small numbers. A more general (and commonly used) method for setting limits of error is that of confidence limits.[54] (Another commonly used method is that of fiducial limits, but in problems such as this, involving only one parameter, both methods give the same results.) The meaning of confidence limit for confidence coefficient q is as follows. Each time an experiment is performed we may set an upper confidence limit. If the experiment is repeated a large number of times, in a proportion q of those times the limit estimate falls above the actual value of the quantity being measured. (This is not the same as saying that the probability that the actual value lies below the confidence limit found from a particular one of the experiments is q, for the actual value is on either one side or the other of a particular limit.) A lower confidence limit may be defined in a similar way.

The quantity of interest in a counting experiment is usually the mean number of counts P expected. The probability of getting exactly η counts in a single performance of the experiment is given by the general term of the Poisson series:

$$\phi(\eta, P) = \frac{P^{\eta}}{\eta!} e^{-P} \qquad (\text{for } \eta = 1, 2, \cdots) .$$

Unfortunately, because the result of a single such experiment is always a discrete integral number, rather than one of a continuum of numbers, no completely satisfactory solution of the problem exists. If pairs of confidence limits are assigned corresponding to each possible experimental result η, the indices of the confidence limits must be changing functions of P, the quantity we are measuring, while what we would like is confidence limits with one index of constant value. As a practical solution Table VI gives confidence limits that are as close to the measured estimate of P as possible, with the condition that the confidence

Table VI

Confidence limits and errors to be assigned to small numbers obtained from counting experiments. The limits given are upper and lower confidence limits for confidence coefficient 0.8413. This results in a confidence interval of probability 1 - 2(1 - 0.8413) = 0.6826. The error is the difference between the confidence limit and the number of counts. (See Appendix.)

Number of counts	Confidence limits	Errors	Percent errors	Number of counts	Confidence limits	Errors	Percent errors
0	1.841	+1.841	---	13	17.70	+4.70	+36.2
	---	---	---		9.44	-3.56	-27.4
1	3.299	+2.299	+229.9	14	18.83	+4.83	+34.5
	0.1727	-0.827	- 82.7		10.30	-3.70	-26.4
2	4.637	+2.637	+131.8	15	19.96	+4.96	+33.1
	0.7083	-1.292	- 64.6		11.17	-3.83	-25.5
3	5.918	+2.918	+ 97.3	16	21.06	+5.06	+31.6
	1.368	-1.632	- 54.4		12.04	-3.96	-24.8
4	7.162	+3.162	+ 79.0	17	22.20	+5.20	+30.6
	2.086	-1.914	- 47.8		12.91	-4.09	-24.1
5	8.382	+3.382	+ 67.6	18	23.32	+5.32	+29.6
	2.842	-2.158	- 43.2		13.80	-4.20	-23.3
6	9.583	+3.583	+ 59.7	19	24.43	+5.43	+28.6
	3.623	-2.377	- 39.6		14.68	-4.32	-22.7
7	10.770	+3.770	+ 53.9	20	25'56	+5.56	+27.8
	4.421	-2.579	- 36.8		15.57	-4.43	-22.2
8	11.944	+3.944	+ 49.3	21	26.65	+5.65	+26.9
	5.236	-2.764	- 34.6		16.46	-4.54	-21.6
9	13.110	+4.110	+ 45.7	22	27.76	+5.76	+26.2
	6.058	-2.942	- 32.7		17.35	-4.65	-21.1
10	14.27	+4.27	+ 42.7	23	28.86	+5.86	+25.5
	6.90	-3.10	- 31.0		18.24	-4.76	-20.7
11	15.41	+4.41	+ 40.1	24	29.97	+5.97	+24.9
	7.73	-3.27	- 29.7		19.14	-4.86	-20.2
12	16.56	+4.56	+ 38.0	25	31.07	+6.07	+24.3
	8.59	-3.41	- 28.4		20.03	-4.97	-19.9

[illegible]

coefficient of each confidence limit is always equal to or greater than 0.8413 no matter what the value of P. Thus if an experiment is performed a large number of times and each time the confidence limits are found from this table, the limits enclose the actual value of the quantity being measured in more than $1 - 2(1 - 0.8413) = 0.6826$ of the cases, and these are the closest limits that will fulfill this condition for any value of P whatsoever. These limits are always at least as conservative as the standard deviation when η is large. (For $\eta = 0$ there is, of course, only an upper limit, and it is for $q = 0.8413$.) The standard deviation, which is usually used to set limits in cases where large samples are involved, gives limits (if the discrete Poisson distribution is replaced by a continuous Gaussian distribution) that are just the confidence limits for the same confidence coefficient as has been used here (i.e., $q = 0.8413$).

The values for the limits were found from the table by Molina.[55] Part 2 of this table gives values for

$$\sum_{i=N}^{\infty} \phi(i, P) = \sum_{i=N}^{\infty} \frac{P^i}{i!} e^{-P} .$$

The upper limit of P for a particular experimental result η is the value of a in this table for $c = \eta + 1$ and a probability of 0.8413 (a and c are symbols in Molina's table). The lower limit of P for a particular η is the value of a for $c = \eta$ and a probability of $1 - 0.8413 = 0.1587$. (It is to be noted that as η becomes large the difference between the lower limit and the mean approaches $\sqrt{\eta}$, while the difference between the mean and upper limit approaches $1 + \sqrt{\eta}$. This is a consequence of the difficulties discussed previously.) A similar treatment may be used for results that are expected to follow a binomial distribution.[56] For example, the errors of particle charge ratios may be treated in this manner.

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