UCRL-3587

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

SINGLE-PARTICLE STATES IN A SPHEROIDAL NUCLEAR POTENTIAL

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(Thesis)

November 16, 1956

Printed for the U. S. Atomic Energy Commission

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ABSTRACT

In order to provide an additional tool for the study of the low-energy properties of highly nonspherical nuclei, a computation of eigenstates has been made for a particle moving in a spheroidal potential well under the influence of a strong spin-orbit interaction. The nucleus has been regarded as a collection of independently moving particles bound in the same potential well, in a manner similar to that in the shell model. A simplified nucleon Hamiltonian has been used. Particles have been treated as bound in an isotropic harmonic oscillator potential to which three perturbations have been applied. These consist of the spin-orbit interaction, a nonisotropic deformation term, and a truncation term that acts to flatten the bottom of the potential well. Approximate solutions have been obtained by an exact diagonalization of those submatrices of the perturbation Hamiltonian that connect only harmonic oscillator states within the same major oscillator shell. Eigenvalues and eigenfunction expansion coefficients are presented in tabular form as functions of the deviation of the potential shape from sphericity.

To illustrate the applicability of the single-particle eigenstates to nuclear systems, a study has been made of the ground-state spins and magnetic moments of a large number of deformed nuclei.

A close correlation is shown to exist in most instances between the empirical data and the predicted spins and magnetic moments.

I. INTRODUCTION

The past several years have seen a great improvement in our knowledge and understanding of the low-energy properties of nuclei, largely through the successes of the shell model[1,2] and the collective nuclear model.[3,4,5] In its simplest formulation, the shell model treats the nucleus as a system of noninteracting nucleons moving in a spherically symmetric potential and under the influence of strong spin-orbit forces. The potential seen by each particle is assumed to be an average of the specific interaction of that nucleon with each of the others, and any residual interactions beyond the isotropic average are taken as negligible. With this simple picture it has been possible to account for those discontinuities in various systematic properties of nuclei associated with the so-called magic numbers, to understand much of the rather haphazard succession of ground-state spins of nuclei, and to give better semiquantitative descriptions of such processes as isomeric transitions and beta decays.[6]

Among the notable qualitative and quantitative failures of the shell model have been its inability to account for the large and predominantly positive quadrupole moments and the rapid electric-quadrupole transitions[5] occurring in many of the heavier nuclei. The collective nature of the large quadrupole moments was first realized by Townes et al.[7] and by Rainwater,[8] who proposed the explanation that, because of their nonisotropic motion, nucleons beyond a closed spherical configuration could cause a polarization of the nuclear core, with the result that the nuclear mass distribution would become strongly nonspherical. Rainwater further proposed that the shell-model treatments be extended to include motion in nonspherical potentials, in order to obtain quadrupole-moment contributions from a majority of nucleon states. However, it was A. Bohr[3,9] who first struck at the essence of the trouble with the realization that because the nucleons move in a potential that they themselves create, there are collective motions of the particles, which in turn affect the potential seen by any one. With the formulation of these ideas into a theory, a partial return was made to the older liquid-drop picture of the nucleus.[10] In the unified model as first proposed,[3] the nucleus is treated as a core consisting of an irrotational, incompressible fluid that can undergo surface oscillations (i.e., collective vibrations), plus a group of nucleons that lies beyond the core and moves in a potential determined by the core shape. This nucleon group is, therefore, dynamically coupled to the core. The degrees of freedom of the collective motions are treated as unrelated to those of the particles. There is thus a neglect of the implicit constraint conditions that arise from the fact that the collective modes are the result of correlated individual-particle motions.[*11,12] As a consequence of the coupling between the particle and collective motions, the angular momentum of the individual particles is no longer a conserved quantity,[5,9] because the spherical symmetry of the potential in which they move has been destroyed. Rather there is now a sharing and exchange of angular momentum between each particle and the core.

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* Because the particles in this model move in a nonspherical nonstatic potential, a correlation has been introduced into their motion as regarded from the laboratory fixed coordinates.

The outstanding achievements of this theory have been in dealing with nuclei whose neutron and proton numbers are far removed from the magic numbers. This is the region of "strong coupling," where nuclear cores can assume stable spheroidal shapes of rather large eccentricities. When the collective-model Hamiltonian is written in terms of variables of the intrinsic coordinate system of the spheroidal mass distribution, the oscillations altering the degree of ellipticity of these nuclei are found to be of minor importance in the low-lying states, so that these nuclei may be treated as a rotating ellipsoidal mass and potential distribution, with the extra-core nucleons moving therein. Further, as the rotational frequencies are, in general, small compared with the intrinsic nucleon frequencies, the rotational couplings of the nucleons may be neglected and the system treated adiabatically, with the particles coupled to the vibrations only through the fact that the potential they "see" is now ellipsoidal. As was noted by Bohr,[9] this decomposition into independent vibrational, rotational, and particle motions is in many ways reminiscent of the treatment of complex molecules. However, this is unlike the treatment of molecules in that the vibrational component of motion is of least importance in the consideration of the heavy deformed nuclei.

That the nuclear mass distribution may be taken as spheroidal* is evidenced by the simplicity of the rotational spectra of excited states of the deformed nuclei.[13] These rotational energies are of the

* The suggestion has been made that some nuclei may prefer a pear-shaped density distribution,[14] though spheroidal distributions are usually favored.

form

$$E_{rot} = \hbar^2/2\mathcal{J} \left[I(I+1) - I_0(I_0+1)\right] \quad ,$$

where I_0 and I are the spins of the ground and excited states of the nucleus, respectively.† The presence of a single moment of inertia $\mathcal{J}$ indicates that two of the three principal moments of inertia of the mass distribution must be equal and the third very small, thereby establishing the existence of an axis of symmetry in the nucleus. Further evidence comes from measured branching ratios for decays to states within a single rotational band.

It has long been known that deformation is intimately related to the shell structure of the nucleus,[15,16] and that near closed major shells, no stable deformations are predicted, whereas midway between shell closures the opposite is true. Furthermore, a hydrodynamical core has been shown to be unrealistic by its prediction of rotational excitations of energies three to five times as great as are observed.[17] As an alternative to the assumption of a hydrodynamical core, the entire nucleus can be treated as a collection of particles moving in a single deformed potential.[8,18,19,20] This is a return to the original proposal by Rainwater, and it is this sort of model that is considered

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† This relation for the rotational energy holds for even-even nuclei and for odd-A nuclei in which the last odd particle has an intrinsic z component of angular momentum other than one-half. In this latter event, an additional term of form

$$\frac{\hbar^2}{2\mathcal{J}}\, a \left[(-1)^{I+\frac{1}{2}}(I+\tfrac{1}{2}) - (-1)^{I_0+\frac{1}{2}}(I_0+\tfrac{1}{2})\right]$$

appears.

in the investigation reported here. When the vibrational degrees of freedom are neglected, as mentioned above, the nucleus is treated as a symmetric top rotator of moment of inertia $\mathcal{J}$, in whose spheroidal rotating field the nucleons move, unaffected by centripetal and Coriolis forces. For odd-A nuclei, such a system may be described by a product-state function

$$\Psi_{k,\Omega}^{I,M,K} = \frac{1}{\sqrt{2}}\left[D_{M,K}^{I}\,\Psi_{k,\Omega} + (-1)^{I-\frac{1}{2}} D_{M,-K}^{I}\,\Psi_{k,-\Omega}\right] ,$$

whose exact form is the result of the symmetries attributed to the nucleus.[3] The functions $D_{M,K}^{I}$, describing the collective rotations, form a $(2I+1)$-dimensional representation of the rotation group and are the well-known solutions of the wave equation for the symmetric top,[21] with total angular momentum I, z component M, and projection along the intrinsic z axis, K. The quantity $\Psi_{k,\Omega}$ is the antisymmetrized product wave function of the single-particle nucleon states, which are coupled to give an intrinsic z component of angular momentum Ω. The index k summarizes the other particle quantum numbers describing the state. As stated above, particle angular momentum is not conserved in this model, but, because of the symmetry about the intrinsic z axis, Ω is a constant of the motion for each particle state. This type of model may be criticized on many grounds beyond the omission of certain interactions between particle and collective motions. (These may not always be neglected with impunity, as is illustrated by the necessity of including a direct particle-rotational coupling in the treatment of W^{183}.[22]) The principal defects are, of course, the replacement of the strongly correlative two-body interactions by a uniform potential well in which the nucleons move independently of one another, and the treatment of particle and collective motions as separate entities of unrelated origin, with the result that there are several degrees of freedom beyond the 3A degrees of the true nucleus. However, it is not the object of this work to investigate these matters, but, from the phenomenological standpoint, to see what may be obtained on a simplified basis in spite of them. The successful application of such models to many nuclear problems has established their merit.

The principal task to which this investigation is devoted is the construction of a set of single-particle states in a spheroidal potential that will be applicable to the nuclear model described above. The potential that has been chosen for this purpose is velocity-independent* and, as has been implied, possesses equipotential surfaces consisting of confocal ellipsoids having two equal semiaxes. The exact choice is like that made by Nilsson[19] in a similar calculation, but contains a number of essential differences. A discussion of this potential and the solution of the single-particle equation is presented in the following section, where some comparisons with Nilsson's work are also made. Section III contains a more precise consideration of the rotational model employed in this work, together with the derivation of formulas to be used in the applications that are presented in the final Section IV. There are a great number of

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* In keeping with the traditional way in which shell-model type calculations are performed, velocity-dependent potentials[23,24] are not considered here.

applications in which the particle states may be employed, covering a wide range of complexity and varying from spin assignments to the calculation of gamma transition rates and beta-decay ft values. Examples of several of the simpler of these have been considered and are presented to display the applicability of the $\Psi_{k,\Omega}$ functions to the interpretation of physical data and to assess the degree of success of the model.

II. THE SINGLE-PARTICLE STATES

In attempting to devise a model for the calculation of eigenstates applicable to spheroidally deformed nuclei, one must make a compromise between a desire for simplicity and a desire for a high degree of realism with respect to the choice of the collective nuclear potential. As is well known from shell-model and optical-model investigations,[6,25,26] the average single-particle nuclear potential in heavy nuclei is composed of a well, $V(\vec{r})$, of relatively constant depth over most of the nuclear volume, which drops off to zero in a small fraction of the nuclear radius, plus a strong spin-orbit interaction term, $U(\vec{r}, \vec{p}, \vec{s})$. The quantities $\vec{r}$, $\vec{p}$, and $\vec{s}$ are respectively the position vector of a particle relative to the nuclear center of mass, the particle momentum operator, and the spin operator. If the assumptions are made that the nuclear mass distribution follows the average nuclear potential closely and that nuclear matter is essentially incompressible, the condition that the shape of the nucleus is spheroidal imposes the restriction that V is a spatial function of $\sigma = \sqrt{D(x^2 + y^2) + z^2/D^2}$ only. The nuclear volume is then independent of the deformation parameter D, as necessitated by the assumption of incompressibility, and the single-particle Hamiltonian takes on the form

$$H_p = -\frac{\hbar^2}{2\mu}\nabla^2 + V(\sigma) + U(\vec{r}, \vec{p}, \vec{s}) , \qquad \text{(II-1)}$$

where μ is the nucleon mass.

The simplest soluble potential possessing some of the features required of $V(\sigma)$ is that of the anisotropic harmonic oscillator. This potential contains two unrealistic features. It is not sufficiently flat-bottomed and it represents an infinite well rather than a finite one. As a result of both the rounded bottom and the infinite walls, the zero-deformation ordering of the energy levels does not correspond to the empirical shell-model arrangement (minus the spin-orbit splitting), as is necessary to make the spheroidal model consistent with the spherical model. The high angular-momentum states within each of the principal harmonic oscillator shells are found to be too high with respect to the states of low angular momentum. Also, the uppermost particle states within a nucleus would be described by wave functions that would be more tightly bound than is physically expected. On the other hand, the harmonic-oscillator solutions are particularly tractable, and because they comprise a complete set of discrete states, their use in perturbation treatments is uncomplicated by the necessity of including continuum states, which occur with finite wells. A still more useful feature is that because the harmonic oscillator represents a diffuse infinite well, it does not fix the nuclear-well depth and radius a priori; thus all results may be applied to any desired nucleus. This is particularly important because the strongly deformed nuclei, to which these results are most applicable, cover a wide range of mass numbers.

Because of the advantages described above, the potential $V(\sigma)$ has been chosen as follows:

$$V(\sigma) = \tfrac{1}{2}\mu\,\omega^2\sigma^2 + V_D(\sigma) \quad , \qquad \text{(II-2)}$$

where

$$V_b(\sigma) = -\tfrac{1}{2}\mu\omega^2\sigma^2 + \tfrac{1}{2}\mu\omega^2\sigma_0^2 \qquad (\text{for } 0 \leqslant \sigma \leqslant \sigma_0)$$

$$= 0 \qquad (\text{for } \sigma > \sigma_0). \qquad \text{(II-3)}$$

Here ω is the oscillator frequency, which is fixed when a nuclear radius is specified. The potential $V_b(\sigma)$, which is dependent on the single parameter σ_0, has the effect of raising the low angular-momentum states relative to those of high angular momentum, as is necessary to achieve the shell-model level ordering at zero deformation. It has the further effect of removing some of the coupling to the symmetry axis of $V(\sigma)$, since for σ less than σ_0, $V(\sigma)$ is uniform and thus is indistinguishable from a spherical potential. In this sense, as in others, the deviation from sphericity is primarily a surface effect.* An estimate of the magnitude of this decoupling property of $V_b(\sigma)$ may be gained from the fact that in general it reduces the deformation-dependent part of the matrix element of $V(\sigma)$ by amounts ranging from 10% to 50%, though in some instances it increases the off-diagonal elements.

The potential described above is similar to that used by Nilsson[19] in a recently published calculation. The principal difference lies in that his reordering of the energy spectrum of the oscillator was accomplished by the addition to the Hamiltonian of a

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* The fact that the moments of inertia of deformed nuclei are less than the rigid-body moments indicates that not all the nucleons take part in rotational excitations. It is primarily the nucleons near the surface that appear to partake in the rotational motion.

term of the form $f(N)\ell^2$ instead of a term such as $V_b(\sigma)$. Here $f(N)$ is a numerical function of the oscillator shell number N, and ℓ is the orbital angular-momentum operator. It may be noted that Nilsson's term does not contain the decoupling features of $V_b(\sigma)$. This fact has served as one of the motivations for undertaking the study presented here, as it is hoped that this difference and several others may result in a somewhat more realistic set of eigenstates.

The spin-orbit interaction that has been used here is

$$U(\vec{r}, \vec{p}, \vec{s}) = -2 \lambda \hbar \omega \vec{\ell} \cdot \vec{s} , \qquad \text{(II-4)}$$

where λ is an adjustable parameter and $\vec{\ell}$ and $\vec{s}$ are the orbital and spin angular-momentum operators for a particle, respectively. Some comments are necessary with regard to this interaction. If the spin-orbit energy consisted of a Thomas-like term[27], as is often assumed, we would have

$$U = -2 g\hbar\, \vec{s} \cdot \nabla V(\sigma) \times \vec{p}$$

$$= -2 g\hbar \mu \omega^2 D\, \vec{s} \cdot \vec{\ell} - 2 g\hbar \mu \omega^2 \left(\frac{1}{D^2} - D \right) z\, \vec{s} \cdot \vec{k} \times \vec{p} - 2 g\hbar\, \vec{s} \cdot \nabla V_b \times \vec{p} .$$

On this basis, what has been done in using only Eq. (II-4) is to neglect the coupling between the spin-orbit and deformation energies on the ground that the deformation is small, to assume that $g\omega = \lambda$ is a constant in order to avoid the necessity of specifying ω before wave functions can be calculated, and finally, to disregard the V_b term, which does away with the spin-orbit interaction in the central region of the nucleus. This last omission may be in part justified qualitatively by noting that it causes the innermost nucleons to "see" too much spin-orbit interaction, but the same is true for the outer particles because the $U(\vec{r}, \vec{p}, \vec{s})$ term has not been set equal to zero beyond the nuclear radius. Thus a small value for λ gives all particles a more or less proper spin-orbit energy.

The wave equation

$$H_p \Psi = E \Psi \tag{II-5}$$

is not immediately soluble. Approximate perturbation solutions have therefore been obtained to first order. For this purpose, the Hamiltonian has been separated into an unperturbed term

$$H_0 = -\hbar^2/2\mu \nabla^2 + \tfrac{1}{2}\mu\omega^2 r^2 \tag{II-6}$$

and a perturbation term

$$H' = \tfrac{1}{2}\mu\omega^2(D - 1)r^2 + \tfrac{1}{2}\mu\omega^2\left(\frac{1}{D^2} - D\right)z^2 + V_b(\sigma) - 2\lambda\hbar\omega\vec{\ell}\cdot\vec{s} \; . \tag{II-7}$$

The unperturbed Hamiltonian has the well-known solution

$$|n, \ell, j, \Omega\rangle = R_{n,\ell}(\alpha r)\mathcal{Y}_{\ell,j,\Omega}(\theta, \phi, s) \; , \tag{II-8}$$

$$E_{n,\ell,j,\Omega} = \hbar\omega(2n + \ell - \tfrac{1}{2}) \; . \tag{II-9}$$

Here n represents the number of radial nodes of $R_{n,\ell}$, including the node at infinity; ℓ and j the orbital and total angular-momentum quantum numbers, respectively; and Ω the projection of j along the z axis. The parameter α is $\sqrt{\mu\omega/\hbar}$. The functions $R_{n,\ell}$ may be expressed in terms of Laguerre polynomials.[28] Those radial functions of interest in this calculation have been listed in Appendix B. The angular functions $\mathcal{Y}_{\ell,j,\Omega}$ may be written in terms of the normalized spherical harmonics[29] $Y_{\ell,m}(\theta, \phi)$, and the spin functions $\chi_{\frac{1}{2},\pm\frac{1}{2}}$ as

$$\mathcal{Y}_{\ell,j,\Omega} = \sum_{m=-\ell}^{\ell} Y_{\ell,m}\Big[\delta_{m,\Omega-\frac{1}{2}}\, C_{\ell,\frac{1}{2}}(j,\Omega;\, m,\tfrac{1}{2})\,\chi_{\frac{1}{2},\frac{1}{2}}$$

$$+\ \delta_{m,\Omega+\frac{1}{2}}\, C_{\ell,\frac{1}{2}}(j,\Omega;\, m,-\tfrac{1}{2})\,\chi_{\frac{1}{2},-\frac{1}{2}}\Big]\ , \qquad \text{(II-10)}$$

where the C's are the appropriate Clebsch-Gordan coefficients.

The evaluation of matrix elements may be carried out with particular ease in this representation. The matrix elements of the perturbation Hamiltonian have been calculated in Appendix A. The resulting expression is

$$\frac{1}{\hbar\omega}\left\langle n, \ell, j, \Omega \,|H'|\, n', \ell', j', \Omega'\right\rangle \ =$$

$$\tfrac{1}{2}(D-1)\,\delta_{\ell,\ell'}\,\delta_{j,j'}\,\delta_{\Omega,\Omega'}\Big[\delta_{n,n'}(2n+\ell-\tfrac{1}{2}) - I_{n,\ell\,|\,n',\ell'}\Big]$$

$$-\tfrac{1}{2}\,\delta_{\ell,\ell'}\,\delta_{j,j'}\,\delta_{\Omega,\Omega'}\Big[I_{n,\ell\,|\,n',\ell'} - \varepsilon^2\int_0^{\varepsilon} R_{n,\ell}(\rho)\,\rho^2 R_{n',\ell'}(\rho)\,d\rho\Big]$$

$$-\lambda\,\delta_{n,n'}\,\delta_{\ell,\ell'}\,\delta_{j,j'}\,\delta_{\Omega,\Omega'}\Big[\ell\,\delta_{j,\ell+\frac{1}{2}} - (\ell+1)\,\delta_{j,\ell-\frac{1}{2}}\Big]$$

$$+\ \tfrac{1}{2}\Big(\frac{1}{D^2} - D\Big)\,\delta_{\Omega,\Omega'}\sum_{m=-\ell}^{\ell}\Big[\left\langle R_{n,\ell}(\alpha r)\,\middle|\,\alpha^2 r^2\,\middle|\,R_{n',\ell'}(\alpha r)\right\rangle - I_{n,\ell|n',\ell'}\Big]$$

Eq. (II-11) Cont.

$$\times \left\{ \delta_{\ell,\ell'+2} \sqrt{\frac{\left[(\ell-1)^2 - m^2\right]\left[\ell^2 - m^2\right]}{(2\ell-3)(2\ell+1)(2\ell-1)^2}} \right.$$

$$+ \delta_{\ell,\ell'}\left[\frac{(\ell+1)^2 - m^2}{(2\ell+1)(2\ell+3)} + \frac{\ell^2 - m^2}{(2\ell-1)(2\ell+1)}\right]$$

$$\left. + \delta_{\ell,\ell'-2} \sqrt{\frac{\left[(\ell+2)^2 - m^2\right]\left[(\ell+1)^2 - m^2\right]}{(2\ell+1)(2\ell+5)(2\ell+3)^2}} \right\}$$

$$\times \left\{ \delta_{m,\Omega-\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega\,;\,m,\tfrac{1}{2})\, C_{\ell',\frac{1}{2}}(j',\Omega\,;\,m,\tfrac{1}{2}) \right.$$

$$\left. + \delta_{m,\Omega+\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega\,;\,m,-\tfrac{1}{2})\, C_{\ell',\frac{1}{2}}(j',\Omega\,;\,m,-\tfrac{1}{2}) \right\} , \qquad \text{(II-11)}$$

where

$$\left\langle R_{n,\ell}(\alpha r) \,\middle|\, \alpha^2 r^2 \,\middle|\, R_{n',\ell'}(\alpha r) \right\rangle = \int_0^\infty R_{n,\ell}(\rho)\,\rho^4 R_{n',\ell'}(\rho)\,d\rho$$

and

$$I_{n,\ell|n',\ell'} \equiv \int_0^\varepsilon R_{n,\ell}(\rho)\,\rho^4\, R_{n',\ell'}(\rho)\,d\rho \;.$$

The right side of Eq. (II-11) is independent of the parameter α, which specifies the nuclear radius; it is fixed when λ, ε, and D are given. Furthermore, only states of the same Ω and of the same parity are connected by H'. Therefore the matrix elements between states of adjacent oscillator-energy levels will all vanish, because the parities of degnerate oscillator states are always the same and are opposite to those of states of adjacent energies. Because states of nonadjacent oscillator levels are well separated in energy, their matrix elements do not contribute greatly to the energy or wave function of a perturbed state. In the following, only matrix elements between states in the same oscillator shell are considered as nonnegligible. This amounts to making computations within the framework of first-order perturbation theory.

For want of a better notation, the eigenstates of the total Hamiltonian have been labeled by the deformation parameter δ, to be discussed later, and the quantum numbers of the state to which it reduces when all perturbations are turned off. Thus we have

$$\Psi_{n,\ell,j,\Omega}(\delta) \rightarrow |n, \ell, j, \Omega\rangle$$

when we let H' go to zero. Of these four indices, only Ω remains a good quantum number of Ψ for $\delta \neq 0$, although ℓ still determines the parity in the usual way.* The state $\Psi_{n,\ell,j,\Omega}$ can be expanded in terms of the eigenfunctions of the unperturbed Hamiltonian that have the same parity and the same value of Ω ;

$$\Psi_{n,\ell,j,\Omega}(\delta) = \sum_{\substack{\{n',\ell',j'\} \\ \ell' \equiv \ell (\mathrm{mod}\ 2)}} d_{n',\ell',j',\Omega}(k,\Omega)\, |n', \ell', j', \Omega\rangle , \qquad \text{(II-12)}$$

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* The parity is even for ℓ even and odd for ℓ odd.

where k represents the indices $\{n, \ell, j\}$ of $\Psi_{n,\ell,j,\Omega}$. Neglect of the contributions to Ψ from states of different oscillator shells imposes the restriction $2n' + \ell' = 2n + \ell$ on the summation. This will be noted, hereafter, by a prime attached to the summation sign and to Ψ:

$$\Psi_{n,\ell,j,\Omega}(\delta) \cong \Psi'_{n,\ell,j,\Omega}(\delta) =$$

$$\sum'_{\{n',\ell',j'\}} d_{n',\ell',j',\Omega}(k,\Omega) \left|n', \ell', j', \Omega\right\rangle . \qquad \text{(II-13)}$$

For $\Psi'_{n,\ell,j,\Omega}$ to be an approximate eigenfunction of the total Hamiltonian, we must have

$$(H_0 + H') \sum'_{\{n',\ell',j'\}} d_{n',\ell',j',\Omega}(k,\Omega) \left|n', \ell', j', \Omega\right\rangle =$$

$$\hbar\omega\left[(2n + \ell - \tfrac{1}{2}) + E'_{n,\ell,j,\Omega}\right]$$

$$\times \sum'_{\{n',\ell',j'\}} d_{n',\ell',j',\Omega}(k,\Omega) \left|n', \ell', j', \Omega\right\rangle .$$

The degeneracy of the oscillator functions leads to the result

$$H_0 \Psi'_{n,\ell,j,\Omega} = \hbar\omega(2n + \ell - \tfrac{1}{2})\Psi'_{n,\ell,j,\Omega} ,$$

so that we obtain

$$\sum_{\{n',\ell',j'\}}{}' d_{n',\ell',j',\Omega}(k,\Omega) H' \left| n', \ell', j', \Omega \right\rangle =$$

$$\hbar\omega E'_{k,\Omega} \sum_{\{n',\ell',j'\}}{}' d_{n',\ell',j',\Omega}(k,\Omega) \left| n', \ell', j', \Omega \right\rangle .$$

(II-14)

Multiplication of Eq. (II-14) on the left in turn by each member of the degenerate set of states corresponding to $\left| n, \ell, j, \Omega \right\rangle$ results in the usual secular equations from which the expansion coefficients d and the perturbation energies E' can be obtained. This set of equations is most conveniently written as

$$\sum_{\{n',\ell',j'\}}{}' d_{n',\ell',j',\Omega} \left\langle n, \ell, j, \Omega \right| H' \left| n', \ell', j', \Omega \right\rangle = E' d_{n,\ell,j,\Omega} ,$$

(II-15)

where the various indices range over the quantum numbers of the degenerate set of states in question.

As was mentioned earlier, in order that this model be consistent with the shell model, it must result in the same predictions as the shell model when we have D = 1. In particular, it must give the empirical shell-model level ordering at zero deformation. This can be accomplished by a suitable choice of the respective spin-orbit and truncation parameters, λ and ε . For this purpose, D = 1 level diagrams were made for a wide selection of λ's and ε's. These were compared with the semiempirical ordering of neutron levels* obtained by Klinkenberg[30] as a best shell-model fit for the spins of odd-neutron nuclei.† It was found that only for values of ε between about 2.2 and 2.6 could the over-all features of Klinkenberg's diagram be approximated. A single value of $\varepsilon = 2.4$ has been used in computations, herein, which in turn required a choice of $\lambda = 0.045$. The Klinkenberg level scheme and that obtained here with the above choice of λ and ε are shown in Fig. 1.

The use of a single choice of λ and ε for all particle states involves two important assumptions. The first is that the well "seen" by any nucleon is the same as that seen by any other, and the second is that it is meaningful to prescribe a single ordering of nucleon states in which the lower ones remain unaltered by the addition of more particles

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* The proton-level ordering is only slightly different from that of the neutrons. The calculated eigenstates should thus apply about equally well to odd-neutron and odd-proton nuclei.

† Selection of the undeformed energy spectrum by comparison with that given by Klinkenberg is not a completely justifiable procedure. As already stated, his arrangement of levels was proposed on the basis of a study of spins and magnetic moments of odd-A nuclei and a correlation with shell-model assignments for these quantities. Such a procedure has no validity in the region of strong deformation where these quantities are related to the single particle states in a manner different from that used in the shell model. However, though the Klinkenberg level ordering might be expected to be unreliable between the higher closed shells, it should approximate the level structure well near the major shell extremities.

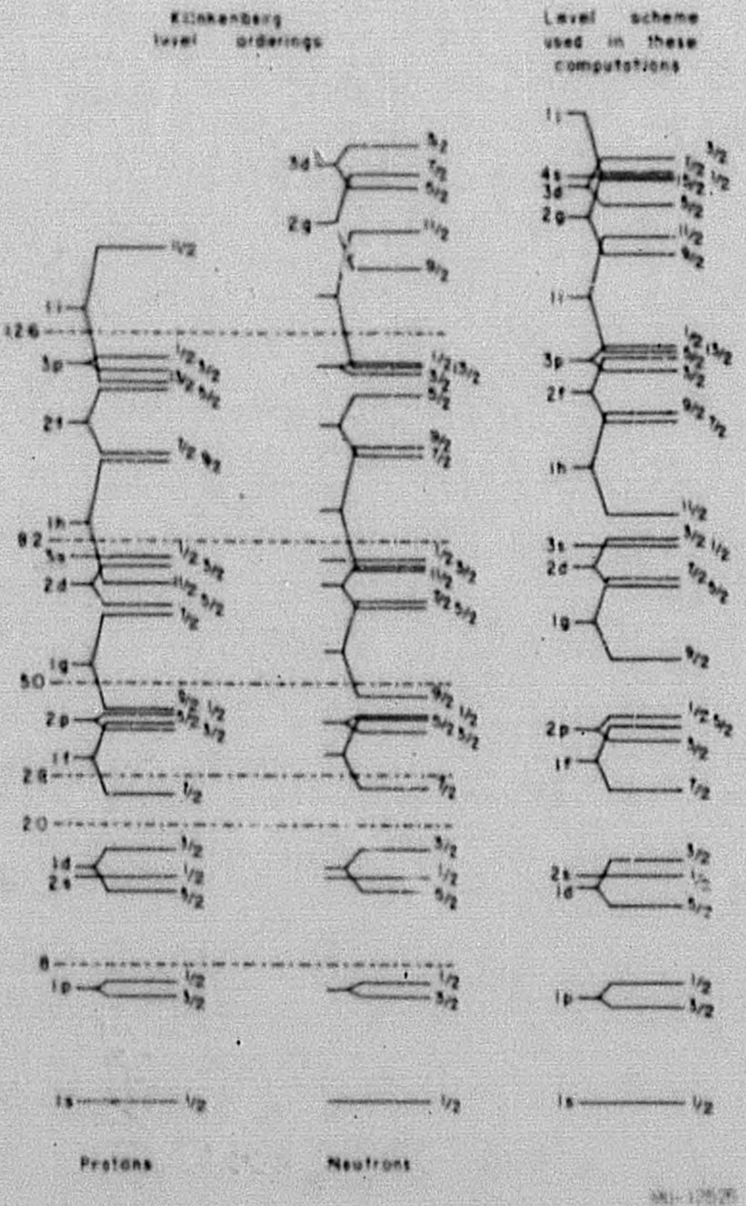

Fig. 1. Comparison of the Klinkenberg level ordering for shell-model states with the zero-deformation ordering employed in the present calculation.

(I would like to express my appreciation to Dr. Klinkenberg for permitting the reproduction of his level diagram in this report.)

to a nucleus. Both these assumptions are undoubtedly inaccurate, but they are nonetheless retained in order to avoid the introduction of an excessive number of arbitrary parameters.

The solution of the secular Eq. (II-15) has been carried out at several different deformations by an exact iterative procedure on the IBM-650 digital computor at the University of California Radiation Laboratory at Livermore. Eigenstates have been obtained through the eighth oscillator shell. The deformations are listed in terms of the more customary parameter $\delta = (b - a)/R$, where b is the symmetry semiaxis of the spheroidal nuclear mass distribution, a is the semiaxis perpendicular to b, and R is the average nuclear radius. The quantity δ may be related to D as follows.

If the spheroidal surface bounding the nuclear volume is given by $p^2 = D(x^2 + y^2) + z^2/D^2$, we then have $a = p/\sqrt{D}$ and $b = pD$. Furthermore, for small deformations, the average nuclear radius is $R = \frac{1}{3}(2a + b)$. Thus we obtain

$$\delta = \frac{pD - p/\sqrt{D}}{\frac{1}{3}\left[2p/\sqrt{D} + pD\right]} = \frac{3(D^{3/2} - 1)}{D^{3/2} + 2}$$

and (II-16)

$$D^{3/2} = \frac{3 + 2\delta}{3 - \delta} .$$

The nucleus is prolate for $\delta > 0$ and oblate for $\delta < 0$. The values of δ at which solutions have been obtained are $\delta = \pm 0.1$, ± 0.2, and ± 0.4. States of the total Hamiltonian of plus and minus Ω are degenerate, other quantum numbers being equal, and those of $-\Omega$ are related to the $+\Omega$ states, given in Eq. (II-12), by

$$\Psi_{n,\ell,j,-\Omega}(\delta) = e^{i\beta} \sum_{\{n',\ell',j'\}} (-1)^{j'-\frac{1}{2}} d_{n',\ell',j',\Omega}(k,\Omega) \left| n',\ell',j',-\Omega \right\rangle \tag{II-17}$$

where β is an arbitrary phase factor.* It has been necessary therefore to solve only the secular equation for states of positive Ω. Tables

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* The total Hamiltonian is invariant under the intrinsic coordinate transformation $z \rightarrow -z$, $y \rightarrow -y$. This is equivalent to letting $\theta \rightarrow \pi - \theta$, $\phi \rightarrow -\phi$, $\chi_{\frac{1}{2},\frac{1}{2}} \rightarrow \chi_{\frac{1}{2},-\frac{1}{2}}$, and $\chi_{\frac{1}{2},-\frac{1}{2}} \rightarrow \chi_{\frac{1}{2},\frac{1}{2}}$. The function obtained by making these substitutions in Eq. (II-12) must, therefore, also be an eigenfunction of H. Such a transformation affects only the $\mathcal{Y}_{\ell,j,\Omega}(\theta, \phi, s)$ functions. In particular, Eq. (II-10) transforms to

$$\mathcal{Y}_{\ell,j,\Omega} \rightarrow \sum_{m=-\ell}^{\ell} Y_{\ell,m}(\pi - \theta, -\phi) \left\{ \delta_{m,\Omega-\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega;\, m,\tfrac{1}{2}) \chi_{\frac{1}{2},-\frac{1}{2}} + \delta_{m,\Omega+\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega;\, m,\tfrac{1}{2}) \chi_{\frac{1}{2},\frac{1}{2}} \right\} .$$

Employing the properties of the spherical harmonics and the Clebsch-Gordan coefficients[29]

$$Y_{\ell,m}(\pi - \theta, -\phi) = (-1)^{\ell}\, Y_{\ell,-m}(\theta, \phi)$$

$$C_{\ell,\frac{1}{2}}(j,\Omega;\, m,s) = (-1)^{j-\ell-\frac{1}{2}}\, C_{\ell,\frac{1}{2}}(j,-\Omega;\, m,-s)$$

gives the result

$$\mathcal{Y}_{\ell,j,\Omega}(\theta, \phi, s) \rightarrow \sum_{m=-\ell}^{\ell} (-1)^{\ell} Y_{\ell,-m}(\theta, \phi)$$

$$\times \left\{ \delta_{-m,-\Omega-\frac{1}{2}} (-1)^{j-\ell-\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,-\Omega;\, m,\tfrac{1}{2}) \chi_{\frac{1}{2},\frac{1}{2}} \right.$$

$$\left. + \delta_{-m,-\Omega+\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,-\Omega;\, m,-\tfrac{1}{2}) \chi_{\frac{1}{2},-\frac{1}{2}} \right\}$$

$$= (-1)^{j-\frac{1}{2}} \mathcal{Y}_{\ell,j,-\Omega}(\theta, \phi, s) \ .$$

Thus we obtain

$$\Psi_{n,\ell,j,\Omega}(\delta) \rightarrow \sum_{\{n',\ell',j'\}} (-1)^{j'-\frac{1}{2}} d_{n',\ell',j',\Omega}(k,\Omega) \, |n',\ell',j',-\Omega\rangle$$

$$= e^{-i\beta} \Psi_{n,\ell,j,-\Omega}(\delta) \ ,$$

where the phase factor β may be chosen arbitrarily. The choice of β is discussed further in the following section.

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of eigenvalues and eigenfunction coefficients, $d_{n,\ell,j,\Omega}$, are given in Appendix B, together with a plot of energies $E_{n,\ell,j,\Omega}/\hbar\omega$ as a function of the deformation. The eigenstates have been listed with the assumption that no crossing of levels of the same Ω and from the same oscillator shell may occur. The possible crossings, or lack thereof, among states of like Ω and parity coming from different oscillator shells has been disregarded completely. Subject to the approximations made in this paper, taking account of such crossings would at most necessitate a trivial change in the labeling of some states, and would only complicate the manner in which the wave-function coefficients have been tabulated.

A few remarks may be made regarding the deformed level structure and the eigenfunctions. At deformations of $|\delta| > 0.1$, all semblance of a shell structure has vanished. It is this complex mixing of energy levels that is the cause of the large stable deformations. This mixing also illustrates some of the difficulties that would be encountered in a finite-well perturbation calculation, in which one would find many unbound zero-deformation states coming down out of the continuum to become bound at some $|\delta| > 0$. Furthermore it prevents the separation of the states of a nucleus into a definite unambiguous core surrounded by a small number of extra-core particles in the region of large distortion. This in turn tends to prevent such investigations as the testing of the strong-coupling scheme for the assignment of nuclear spins by the introduction of two-body forces between the extra-core particles.

In a comparison of our level diagram with that by Nilsson, it is found that the general features of both are very similar. This is, of course, to be expected. Some differences in the ordering of states do occur aside from those which may be directly attributed to a difference of choice of the zero-deformation arrangements in the two schemes. A more evident difference in the lower portion of the diagram is that the states of our calculation have been less affected by the deformation than are Nilsson's. This is because of the nature of $V_b(\sigma)$, which has already been discussed.

The wave functions obtained in this calculation are about equally divided among those whose properties vary little with deformation and those which exhibit rapid changes because of strong interactions among the unperturbed states. The general structures of the energy diagram and of the state functions appear to be relatively stable with respect to small changes in the matrix elements, as was learned in the course of correcting several solutions obtained with erroneous matrix coefficients. This perhaps further justifies the omission of matrix elements between nonadjacent oscillator shells in the solution of the secular equation.

It is shown in Section IV that the single-particle states constructed in the manner described above may be successfully applied to the interpretation of many physical properties of nuclei in the regions of large deformation.

III. DERIVATION OF FORMULAS

A number of equations and other relations are used repeatedly in the applications contained in the following section. These have all appeared previously in various publications dealing with the properties of nonspherical nuclei.[3,5,19,31] However, we considered it worth while to rederive these expressions here instead of merely stating them in order to clarify their origin and the nature of the model that has been employed in this work.

In the simplified nuclear model that is used in this investigation, all collective modes of motion other than rotation have been neglected. The nucleus is assumed to consist of a rigidly rotating spheroidal potential in which the nuclear particles are bound. The density distribution of the nucleons is assumed to be of the same shape as the potential, and to have associated with it a set of moments of inertia for the rotational motion, produced in some unspecified way. The particle motion is further assumed to be independent and unaffected by the rotational motion when viewed from the rotating coordinate system. The total Hamiltonian of this system is most easily written in terms of "intrinsic" coordinates referring to the principal axes of the nuclear mass distribution. If $\vec{R}$ is the rotational angular-momentum operator and $\vec{J}$ the angular-momentum operator of the particles in the rotating coordinate system, the total Hamiltonian for this system is

$$H_T = H'_p + \hbar^2 \left[\frac{R_1^2}{2\mathcal{J}_1} + \frac{R_2^2}{2\mathcal{J}_2} + \frac{R_3^2}{2\mathcal{J}_3} \right] . \qquad \text{(III-1)}$$

The term H'_p will be defined presently. The moments of inertia, $\mathcal{J}_k$, are related in the following manner:

$$\mathcal{J}_1 = \mathcal{J}_2 = \mathcal{J} , \qquad \mathcal{J}_3 \ll \mathcal{J} .$$

Equation (III-1) may be rewritten in terms of the components of the total angular-momentum operator $\vec{I} = \vec{R} + \vec{j}$. This results in the expression

$$H_T = \left(H'_p + \frac{\hbar^2 j^2}{2\mathcal{J}}\right) + \frac{\hbar^2}{2\mathcal{J}_3}(I_3 - j_3)^2 + \frac{\hbar^2}{2\mathcal{J}}\left[I^2 - I_3^2 - j_3^2\right]$$

$$- \frac{\hbar^2}{2\mathcal{J}}\left[I_+ j_- + I_- j_+\right] , \qquad \text{(III-2)}$$

where $I_+ = I_1 + iI_2$ and $I_- = I_1 - iI_2$, with similar relations for j_+ and j_-. Use has been made of the commutation relations $\left[I_k, j_\ell\right] = 0$ for components of $\vec{I}$ and $\vec{j}$ in the intrinsic coordinate system. The quantity $H'_p + \frac{\hbar^2 j^2}{2\mathcal{J}}$ refers only to the particle motion and is set equal to the particle Hamiltonian H_p given by Eqs. (II-6) and (II-7). The form of H_p is such that it excludes all effects of rotation on the intrinsic motion of the nucleons. Furthermore, the fourth term of H_T, which couples the rotational motion to that of the particles, is assumed to be effective only insofar as it has diagonal matrix elements for solutions of the remainder of the Hamiltonian, i.e., it contributes to the energy of the system only and not to a coupling between eigenstates of the first three terms of H_T. (In the following discussions the Hamiltonian H_T will be treated as if this assumption were precise.)

For the system described by the Hamiltonian Eq. (III-2), j_3 and I_3 are both constants of the motion. This is true for j_3 because the particle potential is axially symmetric, and for I_3 because the mass distribution is symmetric about the intrinsic z axis and therefore behaves like a symmetric top rotator.[3,5] The eigenfunctions of Eq. (III-2) are the products of the state vectors of its two basic parts, H_p and $\frac{\hbar^2}{2\mathcal{J}}\vec{I}^2$. Using the notation of Section I, we have

$$H_p \Psi_{k,\Omega} = Ep(k,\Omega)\Psi_{k,\Omega} ,$$

$$J_3 \Psi_{k,\Omega} = \Omega \Psi_{k,\Omega} ,$$

and

$$\frac{\hbar^2}{2\mathcal{J}} \vec{I}^2 D^I_{M,K} = \frac{\hbar^2 I(I+1)}{2\mathcal{J}} D^I_{M,K} ,$$

$$I_3 D^I_{M,K} = K D^I_{MK} .$$

The quantity $\Psi_{k,\Omega}$ is the eigenfunction of the A-nucleon Hamiltonian, H_p. It is labled by the quantum number Ω, which is the net intrinsic z component of angular momentum of all the particles, and by some set of other quantum numbers that have been lumped into the index k. The $D^I_{M,K}$ are normalized functions of the Eulerian angles that describe the position of the intrinsic rotating axes of the mass distribution with respect to some space-fixed coordinate system.* The term I represents

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* See, for example, Principles of Mechanics by Synge and Griffith. The expressions for the space-fixed coordinates in terms of those of the body-fixed system are given below. The space-fixed coordinates are primed.

$$x' = x\left[\cos\phi_e \cos\theta_e \cos\Psi_e - \sin\phi_e \sin\Psi_e\right]$$
$$+ y\left[-\cos\phi_e \cos\theta_e \sin\Psi_e - \sin\phi_e \cos\Psi_e\right] + z\cos\phi_e \sin\theta_e$$

$$y' = x\left[\sin \phi_e \cos \theta_e \cos \Psi_e + \cos \phi_e \sin \Psi_e\right]$$

$$+ y\left[- \sin \phi_e \cos \theta_e \sin \Psi_e - \cos \phi_e \cos \Psi_e\right] + z \sin \phi_e \sin \theta_e$$

$$z' = - x \sin \theta_e \cos \Psi_e + y \sin \theta_e \sin \Psi_e + z \cos \theta_e \ .$$

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the total angular momentum of the system, and M and K are the z components of angular momentum along the space-fixed and body-fixed axes. Both $\Psi_{k,\Omega}$ and $D_{M,K}^{I}$ are degenerate with respect to changes of sign of any of the quantum numbers Ω, M, and K. The state functions for the complete system have the form

$$\Psi_{k,\Omega}^{I,M,K} = \frac{1}{\sqrt{2}}\left[D_{M,K}^{I}\,\Psi_{k,\Omega} + (-1)^{I-\frac{1}{2}}\,D_{M,-K}^{I}\,\Psi_{k,-\Omega}\right]. \qquad \text{(III-3)}.$$

(The phase factor $(-1)^{I-\frac{1}{2}}$, which appears between the two terms of the combined wave function, Eq. (III-3), is dependent on the value of the arbitrary phase parameter β, which is discussed in Section II, page 25. In the present considerations, β has been set equal to zero. However, it should be noted that in the table of wave-function coefficients of Appendix B, some of the states are listed with $\beta = \pi$.) The appearance of terms of both signs K and Ω in the above equation stems from the invariance of the nuclear shape under various rotations and reflections of the intrinsic coordinate axes.[3] Because there is no definite orientation of the nuclear mass distribution in space, no physical significance can be attributed to a specification of this orientation. The wave function describing the complete system must therefore be invariant under such transformations as a rotation of 180° about the nuclear symmetry axis, or a simultaneous inversion of the intrinsic y and z axes. These matters have been discussed more fully by Bohr. Only the results of performing the symmetry operations described above are given. The first, i.e., the 180° rotation about the symmetry axis, restricts the value of $(K - \Omega)$ to even integers, $0, \pm 2, \pm 4$, etc.. The second inverts the intrinsic z axis but retains the right-handedness of the internal coordinate system. As a consequence, the quantum numbers K and Ω change sign and appropriate phase factors appear that restrict the wave function to the specific form given by Eq. (III-3). Another important property of this model is that the rotational functions $D_{M,K}^{I}$ do not contribute to the ground-state parity of the system and therefore the parity is determined solely by that of the particle wave functions, $\Psi_{K,\Omega}$.

Three quantities will be of interest for the applications of the following section. They are the expectation value of the total Hamiltonian, the quadrupole moment, and the magnetic moment. Relations for these are derived below in the order mentioned.

A. Rotational Energy

The expectation value of the total Hamiltonian is

$$\langle H_T \rangle \equiv \langle \Psi_{k,\Omega}^{IMK} | H_T | \Psi_{k,\Omega}^{IMK} \rangle =$$

$$\tfrac{1}{2} \langle D_{M,K}^{I} \Psi_{k,\Omega} | H_T | D_{M,K}^{I} \Psi_{k,\Omega} \rangle + \tfrac{1}{2}(-1)^{I-\frac{1}{2}} \langle D_{M,-K}^{I} \Psi_{k,-\Omega} | H_T | D_{M,K}^{I} \Psi_{k,\Omega} \rangle$$

$$+ \text{ terms with } \{K, \Omega\} \rightarrow \{-K, -\Omega\} .$$

Inserting H_T, and writing $E_p(k,\Omega) \equiv \langle \Psi_{k,\pm\Omega} | H_p | \Psi_{k,\pm\Omega} \rangle$, gives

$$\langle H_T \rangle = E_p + \frac{\hbar^2}{2\mathcal{J}_3}(-\Omega)^2 + \frac{\hbar^2}{2\mathcal{J}}\left[I(I+1) - K^2 - \Omega^2\right]$$

$$- \tfrac{1}{2}(-1)^{I-\frac{1}{2}} \frac{\hbar^2}{2\mathcal{J}} \Big[\langle D^I_{M,-K} \Psi_{k,-\Omega} | I_+ j_- + I_- j_+ | D^I_{M,K} \Psi_{k,\Omega} \rangle$$

$$+ \text{ a term with } \{K,\Omega\} \rightarrow \{-K,-\Omega\}^2 \Big] . \qquad \text{(III-4)}$$

Making use of the properties of I_+ and I_-, i.e.,

$$I_+ D^I_{M,K} = \sqrt{(I+K)(I-K+1)}\; D^I_{M,K-1} ,$$

$$I_- D^I_{M,K} = \sqrt{(I-K)(I+K+1)}\; D_{M,K+1} ,$$

one obtains

$$\langle D^I_{M,-K} \Psi_{k,-\Omega} | I_+ j_- + I_- j_+ | D^I_{M,K} \Psi_{k,\Omega} \rangle =$$

$$(I+\tfrac{1}{2})\, \delta_{K,\frac{1}{2}} \langle \Psi_{k,-\Omega} | j_- | \Psi_{k,\Omega} \rangle . \qquad \text{(III-5)}$$

(The commutation rules for the components of I in the rotating, body-fixed system are

$$[I_\lambda, I_\mu] = -i\, \varepsilon_{\lambda\mu\sigma} I_\sigma$$

rather than the usual

$$[I_\lambda, I_\mu] = i\, \varepsilon_{\lambda\mu\sigma} I_\sigma .$$

As a consequence, the roles of I_+ and I_- are interchanged, I_+ becoming the lowering operator, and I_- the raising operator.)

The final term of Eq. (III-4), which has the signs of K and Ω reversed with respect to those of (III-5) the preceding term, can be shown to give the result of Eq. (III-5); thus it is only necessary to consider $\langle \Psi_{k,-\Omega} | J_- | \Psi_{k,\Omega} \rangle$ in detail. For even-even nuclei $K = 0$, so that Eq. (III-5) makes no contribution to the energy. In odd-A nuclei, which are of primary interest in this investigation, all but the last odd nucleon are assumed to occupy the degenerate particle states of plus and minus Ω in pairs. Only the last unpaired nucleon, which partly occupies a state of $+\Omega$ and partly a state of $-\Omega$, can contribute to Eq. (III-5). The term $\Psi_{k,\Omega}$ may therefore be thought of as referring only to the state of the unpaired particle of an odd-A nucleus. By the use of Eq. (II-12) and (II-17), which give the expansions of $\Psi_{k,\Omega}$ and $\Psi_{k,-\Omega}$ in terms of eigenfunctions of $\vec{J}^2$, $\langle \Psi_{k,-\Omega} | J_- | \Psi_{k,\Omega} \rangle$ may be written as

$$\langle \Psi_{k,-\Omega} | J_- | \Psi_{k,\Omega} \rangle =$$

$$\sum_{\{n,\ell,j\}} \sum_{\{n',\ell',j'\}} (-1)^{j-\frac{1}{2}} d_{n,\ell,j,\Omega} d_{n',\ell',j',\Omega}$$

$$\times \cdot \langle n,\ell,j,-\Omega | J_- | n,\ell,j,\Omega \rangle$$

$$= \delta_{\Omega,\frac{1}{2}} \sum_{\{n,\ell,j\}} (-1)^{j-\frac{1}{2}} (j+\tfrac{1}{2}) d^2_{n,\ell,j,\Omega} \qquad \text{(III-6)}$$

The quantity

$$a(k,\Omega) \equiv \sum_{\{n',\ell',j'\}} (-1)^{j'-\frac{1}{2}} (j'+\tfrac{1}{2}) d^2_{n',\ell',j',\Omega}(k,\Omega) \qquad \text{(III-7)}$$

is called the decoupling constant.[5,31] It is seen to arise from the effect of the Coriolis force, i.e., $(I_+ j_- + I_- j_+)$, on a particle in a state having $\Omega = \frac{1}{2}$, and represents a measure of the destruction of the coupling of the particle to the axis of nuclear symmetry.

The expectation value of the total Hamiltonian may be written in a final form of

$$\langle H_t \rangle = E_p + \frac{\hbar^2}{2\mathcal{J}_3}(K - \Omega)^2 + \frac{\hbar^2}{2\mathcal{J}}\left[I(I+1) - K^2 - \Omega^2\right]$$

$$+ (-1)^{I+\frac{1}{2}} \frac{\hbar^2}{2\mathcal{J}}(I + \tfrac{1}{2})\,\delta_{\Omega,\frac{1}{2}} \cdot \delta_{K,\frac{1}{2}}\, a \; .$$

(III-8)

The existing data on nuclear rotation spectra indicate that the term $\frac{\hbar^2}{2\mathcal{J}_3}(K - \Omega)^2$ does not contribute to the observed low-lying rotational states. This indicates that $\mathcal{J}_3$ must be very small, and that $K = \Omega$ for the low-energy rotations. Each particle configuration, characterized by $\Psi_{K,\Omega}$, has associated with it a set of rotational states of different spins I. If I_0 is the spin of the ground state of a nucleus and I the spin of a state in the rotational band, the excitation energy of the latter is given by

$$E_{rot} = \frac{\hbar^2}{2\mathcal{J}}\left[I(I+1) - I_0(I_0+1)\right]$$

$$+ \delta_{K,\frac{1}{2}}\,\delta_{\Omega,\frac{1}{2}}\,\frac{\hbar^2}{2\mathcal{J}}\, a \left[(-1)^{I+\frac{1}{2}}(I+\tfrac{1}{2}) - (-1)^{I_0+\frac{1}{2}}(I_0+\tfrac{1}{2})\right] .$$

(III-9)

For even-even or odd-A nucleon configurations with $\Omega \neq \frac{1}{2}$, the energy is seen to be a minimum for $K = \Omega = I$. If we have $\Omega = \frac{1}{2}$, the ground-state spin depends on the value of the decoupling constant a. In this instance, the expectation value of the Hamiltonian is

$$\langle H_T \rangle = E_p + \frac{\hbar^2}{2\mathcal{J}}\left[I(I+1) - \frac{3}{4} + (-1)^{I+\frac{1}{2}}(I+\tfrac{1}{2})\,a\right] .$$

The value of I for which $\langle H_T \rangle$ is a minimum, i.e., the ground-state spin, is seen to be a function of a. By inserting various values for I into the bracketed term $\left[I(I+1) - \frac{3}{4} + (-1)^{I+\frac{1}{2}}(I+\frac{1}{2})\right]$, and comparing the resulting quantities with one another, we can obtain the range of a that produces a particular ground-state spin.[5,19] These ranges are:

Ground-State Spin	Limits of a
$I = 1/2$	$-1 < a < 4$
$I = 3/2$	$-6 < a < -1$
$I = 5/2$	$4 < a < 8$
$I = 7/2$	$-10 < a < -6$
$I = 9/2$	$8 < a < 12$

B. Quadrupole Moment

The quadrupole moment of a nucleus is the sum of the contributions to this moment from each of the occupied proton states. The quadrupole operator in the space-fixed coordinate system, denoted by primes, is then

$$Q_{op} = \sum_{i=1}^{Z} (3z'^2 - r'^2)_i \quad . \qquad \text{(III-10)}$$

This may be rewritten in terms of the intrinsic particle coordinates and functions of the Eulerian angles to give

$$Q_{op} = \sum_{i=1}^{Z} \Big[3(x^2 \sin^2\theta_e \cos^2\Psi_e + y^2 \sin^2\theta_e \sin^2\Psi_e + z^2 \cos^2\theta_e)$$

$$- 6(xy \sin^2\theta_e \sin\Psi_e \cos\Psi_e + xz \sin\theta_e \cos\theta_e \cos\Psi_e$$

$$- yz \sin\theta_e \cos\theta_e \sin\Psi_e) - r^2 \Big]_i \; .$$

The spectroscopic quadrupole moment is the expectation value of Q_{op} for that state of the system with $M = I$. The expectation values of the cross terms in Q_{op} must vanish because of the axial symmetry of the charge distribution. For the same reason we have

$$\langle \Psi_{k,\pm\Omega} \mid \sum_{i=1}^{Z} x_i^2 \mid \Psi_{k,\pm\Omega} \rangle = \langle \Psi_{k,\pm\Omega} \mid \sum_{i=1}^{Z} y_i^2 \mid \Psi_{k,\pm\Omega} \rangle ,$$

(III-11)

where $\Psi_{k,\Omega}$ is the total particle wave function. From these relations and with a small amount of algebraic manipulation, the quadrupole moment may be written in the form

$$Q = \Big\langle \sum_{i=1}^{Z} \Big[3(\tfrac{1}{2}x^2 \sin^2\theta_e + \tfrac{1}{2}y^2 \sin^2\theta_e + z^2 \cos^2\theta_e) - r^2 \Big]_i \Big\rangle_{M=I}$$

$$= \Big\langle (\tfrac{3}{2}\cos^2\theta_e - \tfrac{1}{2}) \sum_{i=1}^{Z} (3z^2 - r^2)_i \Big\rangle_{M=I}$$

(III-12)

Inserting the state function (III-3) into Eq. (III-12) and noting that the two resulting expectation values for the respective functions $D^{I}_{I,K}\, \Psi_{K,\Omega}$ and $D^{I}_{I,-K}\, \Psi_{K,-\Omega}$ are equal, we obtain

$$Q = \left\langle D^{I}_{I,K} \,\middle|\, \tfrac{3}{2}\cos^2\theta_e - \tfrac{1}{2} \,\middle|\, D^{I}_{I,K} \right\rangle Q_0 \quad ,$$

where Q_0, the intrinsic quadrupole moment of the nucleus, is given by

$$Q_0 = \left\langle \Psi_{K,\Omega} \,\middle|\, \sum_{i=1}^{Z} (3z_i^2 - r_i^2) \,\middle|\, \Psi_{K,\Omega} \right\rangle . \qquad \text{(III-13)}$$

The evaluation of the projection factor

$$\left\langle D^{I}_{I,K} \,\middle|\, \tfrac{3}{2}\cos^2\theta_e - \tfrac{1}{2} \,\middle|\, D^{I}_{I,K} \right\rangle$$

can be most easily accomplished by use of the analytic expression for $D^{I}_{I,K}$,[36]

$$D^{I}_{I,K} = \left[\frac{(2I+1)\,(2I)!}{8\pi^2\, \Gamma(I+K+1)\,\Gamma(I-K+1)} \right]^{\frac{1}{2}} e^{\,iI\phi_e + iK\psi_e}$$

$$\times \left[\tfrac{1}{2}(1 - \cos\theta_e) \right]^{\frac{1}{2}(I-K)} \left[\tfrac{1}{2}(1 + \cos\theta_e) \right]^{\frac{1}{2}(I+K)}$$

By making some simple changes of variable and using the definition of the beta function, one can obtain

$$\left\langle D^{I}_{I,K} \,\middle|\, \tfrac{3}{2}\cos^2\theta_e - \tfrac{1}{2} \,\middle|\, D^{I}_{I,K} \right\rangle = \frac{(2I+1)\,(2I)!}{\Gamma(I+K+1)\,\Gamma(I-K+1)}$$

$$\times \int_0^1 \left[6y^2 - 6y + 1 \right] (1-y)^{I-K}\, y^{I+K}\, dy$$

$$= \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} \quad . \qquad \text{(III-14)}$$

For nuclei other than those having a last odd particle in a state of $\Omega = \frac{1}{2}$, the ground state is always characterized by $K = I$. Therefore, for these cases, the measured and intrinsic quadrupole moments in the nuclear ground state are related by the equation

$$Q = \frac{I(2I-1)}{(I+1)(2I+3)} Q_0 \quad . \qquad \text{(III-15)}$$

This expression is consistent with the fact that nuclei of spin zero or one-half can have no directly measurable electric quadrupole moment. For those nuclei with $\Omega = K = \frac{1}{2} \neq I$, Eq. (III-14) must be used without change in relating Q_0 to Q.

C. Magnetic Moment

The following discussion is restricted to the evaluation of the magnetic moment of an odd-A nucleus. It is assumed that all particle states are filled in pairs, so that only the last odd nucleon contributes directly to the nuclear magnetic moment.

In the present model, the nuclear magnetic moment may be separated into two parts. The first is the usual contribution of the odd nucleon. The second is a collective contribution coming from the rotation of the total charge distribution. The magnetic-moment operator may be written as

$$\mu_{op} = \vec{\mu} \cdot \vec{k}' = (g_R \vec{R} + g_l \vec{l} + g_s \vec{s}) \cdot \vec{k}' \quad , \qquad \text{(III-16)}$$

where g_ℓ and g_s are the orbital and spin gyromagnetic ratios of the odd nucleon, $\hat{k}'$ is the unit vector in the space-fixed z direction, and g_R is the gyromagnetic ratio of the collectively rotating charge distribution. An estimate of the magnitude of g_R can be made by computing the gyromagnetic ratio for a classical uniformly charged spheroid of effective moment of inertia $\mathcal{J}$ that is rotating about an axis perpendicular to the axis of symmetry with a constant angular velocity $\vec{\omega}$. If we assume the mass distribution to be uniform and distributed over the same volume V as is the charge, the matter and charge densities ρ_m and ρ_e for a nucleus of A particles, of which Z is the number of protons, are*

$$\rho_m = \frac{mA}{V} \qquad \text{and} \qquad \rho_e = \frac{Ze}{V}$$

The moment of inertia and the magnetic moment are respectively

$$\mathcal{J} = \int_V \rho_m (y^2 + z^2)\, f(\bar{r})\, d\tau$$

and

$$\vec{\mu} = \int_V \frac{\rho_e}{2}\, \vec{\omega}\, (x^2 + y^2) f(\bar{r})\, d\tau \ ,$$

where $f(\bar{r})$ is a factor that can account for slippage in the motion. For a rigid body $f(\bar{r}) = 1$. Because the angular velocity is related to the angular momentum $\hbar\vec{R}$ by

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* For odd-proton nuclei, Z should be replaced by $Z - 1$ since the odd particle is being considered separately. However, both Z and A are large for the strongly deformed nuclei, so that this correction can be neglected.

$$\vec{\omega} = \frac{\hbar \vec{R}}{\mathcal{J}} ,$$

a combination of the above relations gives

$$\vec{\mu} = \frac{Z}{A} \frac{e\hbar}{2m} \vec{R} .$$

The gyromagnetic ratio g_R is thus Z/A .

Because of the rapid precession of the magnetic-moment vector about the total angular momentum, and the precession of the angular-momentum vector about the $\vec{k}'$ axis, the measured magnetic moment may be written as

$$\mu = \langle \mu_{op} \rangle_{M=I} = \frac{\langle \vec{\mu} \cdot \vec{I} \rangle_{M=I}}{I(I+1)} \langle \vec{I} \cdot \vec{k}' \rangle_{M=I} = \frac{\langle \vec{\mu} \cdot \vec{I} \rangle_{M=I}}{I+1} , \qquad \text{(III-27)}$$

where $\vec{\mu} \cdot \vec{I}$ is given by

$$\vec{\mu} \cdot \vec{I} = g_R \vec{I}^2 + (g_\ell - g_R) \vec{I} \cdot \vec{j} + (g_s - g_\ell) \vec{I} \cdot \vec{S} .$$

In the following, the vector components are assumed to refer to the principal axes of the nuclear density distribution. The evaluation of the various expectation values proceeds in the same manner as in Part A of this section.

$$\langle \vec{j} \cdot \vec{I} \rangle_{M=I} = \langle I_3 j_3 + \tfrac{1}{2} I_+ j_- + \tfrac{1}{2} I_- j_+ \rangle_{M=I}$$

$$= K\Omega + \frac{(-1)^{I-\frac{1}{2}}}{4} \Big\{ \langle D_{I,K}{}^{I} \psi_{k,\Omega} \,|\, I_+ j_- + I_- j_+ \,|\, D_{I,-K}{}^{I} \psi_{k,-\Omega} \rangle$$

$$+ \langle D_{I,-K}{}^{I} \psi_{k,-\Omega} \,|\, I_+ j_- + I_- j_+ \,|\, D_{I,K}{}^{I} \psi_{k,\Omega} \rangle \Big\} .$$

Here $\Psi_{k,\Omega}$ refers to the state of the last particle only. These expectation values have already been evaluated and are given by Eqs. (III-5) and III-6). Therefore we have

$$\langle \vec{J}\cdot\vec{I} \rangle_{NaI} = K\Omega + \frac{(-1)^{I-\frac{1}{2}}}{2} (I + \tfrac{1}{2}) \delta_{\Omega,\frac{1}{2}} \delta_{K,\frac{1}{2}} a(k,\Omega) \; . \tag{III-18}$$

The final expectation value, $\langle \vec{S}\cdot\vec{I} \rangle_{NaI}$, is similar to but somewhat more complex than the preceding one:

$$\langle \vec{S}\cdot\vec{I} \rangle_{NaI} = \langle S_3 I_3 + \tfrac{1}{2} I_+ S_- + \tfrac{1}{2} I_- S_+ \rangle_{NaI}$$

$$= \frac{K}{2} \left\{ \langle \Psi_{k,\Omega} | s_3 | \Psi_{k,\Omega} \rangle - \langle \Psi_{k,-\Omega} | s_3 | \Psi_{k,-\Omega} \rangle \right\}$$

$$+ \frac{(-1)^{I-\frac{1}{2}}}{4} (I + \tfrac{1}{2}) \delta_{K,\frac{1}{2}} \left\{ \langle \Psi_{k,-\Omega} | s_- | \Psi_{k,\Omega} \rangle \right.$$

$$\left. + \langle \Psi_{k,\Omega} | s_+ | \Psi_{k,-\Omega} \rangle \right\} .$$

It may easily be shown that we have

$$\langle \Psi_{k,\Omega} | s_3 | \Psi_{k,\Omega} \rangle = -\langle \Psi_{k,-\Omega} | s_3 | \Psi_{k,-\Omega} \rangle ,$$

therefore

$$\langle \vec{S}\cdot\vec{I} \rangle_{NaI} = K \langle \Psi_{k,\Omega} | s_3 | \Psi_{k,\Omega} \rangle$$

$$+ \frac{(-1)^{I-\frac{1}{2}}}{2} (I + \tfrac{1}{2}) \delta_{K,\frac{1}{2}} \, \mathrm{Re} \langle \Psi_{k,-\Omega} | s_- | \Psi_{k,\Omega} \rangle .$$

The evaluation of the remaining matrix elements involves a straightforward application of the expansion of the $\Psi_{k,\Omega}$ functions as given by Eq. (II-12). One obtains

$$\langle \Psi_{k,\Omega} | s_3 | \Psi_{k,\Omega} \rangle =$$

$$\frac{1}{2} \sum_{\{n',\ell',j'\}} d^2_{n',\ell',j',\Omega}(k,\Omega) \Big\{ C^2_{\ell',\frac{1}{2}}(j',\Omega;\Omega-\tfrac{1}{2},\tfrac{1}{2})$$

$$- C^2_{\ell',\frac{1}{2}}(j',\Omega;\Omega+\tfrac{1}{2},-\tfrac{1}{2}) \Big\}$$

$$+ \frac{1}{2} \sum_{\{n',\ell',j'\}} \sum_{j''\neq j'} d_{n',\ell',j',\Omega}(k,\Omega) d_{n',\ell',j'',\Omega}(k,\Omega)$$

$$\times \Big\{ C_{\ell',\frac{1}{2}}(j',\Omega;\ \Omega-\tfrac{1}{2},\tfrac{1}{2})\, C_{\ell',\frac{1}{2}}(j'',\Omega\ ;\Omega-\tfrac{1}{2},\tfrac{1}{2})$$

$$- C_{\ell',\frac{1}{2}}(j',\Omega;\Omega+\tfrac{1}{2},-\tfrac{1}{2})\, C_{\ell',\frac{1}{2}}(j'',\Omega;\Omega+\tfrac{1}{2},-\tfrac{1}{2}) \Big\} ,$$

where the $C_{\ell,\frac{1}{2}}(j,\Omega;\ m,\pm\frac{1}{2})$ are the Clebsch-Gordan coefficients for the addition of an angular momentum of magnitude ℓ to an angular momentum of $\frac{1}{2}$. By use of the explicit form of the Clebsch-Gordan coefficients, this can be simplified to

$$\langle \Psi_{k,\Omega} | s_3 | \Psi_{k,\Omega} \rangle = \sum_{n',\ell'} \frac{\Omega}{2\ell'+1} \Big\{ d^2_{n',\ell',\ell'+\frac{1}{2},\Omega}(k,\Omega)$$

$$- d^2_{n',\ell',\ell'-\frac{1}{2},\Omega}(k,\Omega) \Big\}$$

$$- \sum_{n',\ell'} \frac{\sqrt{(2\ell'+1)^2 - 4\Omega^2}}{2\ell'+1}\, d_{n',\ell',\ell'+\frac{1}{2},\Omega}(k,\Omega)\, d_{n',\ell',\ell'-\frac{1}{2},\Omega}(k,\Omega). \tag{III-19}$$

Similarly

$$\langle \Psi_{k,-\Omega} | s_- | \Psi_{k,\Omega} \rangle = \delta_{\Omega,\frac{1}{2}} \sum_{n',\ell'} \frac{(-1)^{\ell'}}{2\ell' + 1}$$

$$\times \left\{ (\ell' + 1) d^2_{n',\ell',\ell'+\frac{1}{2},\frac{1}{2}}(k, \tfrac{1}{2}) + \ell' \, d^2_{n',\ell',\ell'-\frac{1}{2},\frac{1}{2}}(k, \tfrac{1}{2}) \right\}$$

$$- 2 \delta_{\Omega,\frac{1}{2}} \sum_{n',\ell'} (-1)^{\ell'} \frac{\sqrt{\ell'(\ell' + 1)}}{2\ell' + 1} d_{n',\ell',\ell'+\frac{1}{2},\frac{1}{2}}(k,\tfrac{1}{2})$$

$$\cdot d_{n',\ell',\ell'-\frac{1}{2},\frac{1}{2}}(k, \tfrac{1}{2}) \; . \qquad \text{(III-20)}$$

When the various expectation values given above are combined to obtain the magnetic moment, three cases arise. These are:

a. $\underline{\Omega = K = I \neq \frac{1}{2}}$

$$\mu = g_R \frac{I}{I+1} + g_\ell \frac{I^2}{I+1} + \frac{(g_s - g_\ell)}{2} \frac{I}{I+1}$$

$$\times \sum_{n,\ell} \left\{ \frac{2I}{2\ell + 1} \left[d^2_{n,\ell,\ell+\frac{1}{2},I} - d^2_{n,\ell,\ell-\frac{1}{2},I} \right] \right.$$

$$\left. - 2 \frac{\sqrt{(2\ell + 1)^2 - 4I^2}}{(2\ell + 1)} d_{n,\ell,\ell+\frac{1}{2},I} \, d_{n,\ell,\ell-\frac{1}{2},I} \right\}$$

$$\text{(III-21)}$$

b. $\underline{\Omega = K = \frac{1}{2} \neq I}$

$$\mu = g_R I + (g_\ell - g_R) \frac{1}{4(I+1)} \left\{ 1 + (-1)^{I-\frac{1}{2}} (2I+1)\, a \right\}$$

$$+ (g_s - g_\ell) \frac{1}{4(I+1)} \sum_{n,\ell} \frac{1}{2\ell+1} \left\{ d^2_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}} - d^2_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} - 4\sqrt{\ell(\ell+1)}\; d_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}} d_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} \right\}$$

$$+ (g_s - g_\ell) \frac{(-1)^{I-\frac{1}{2}}}{4(I+1)} (2I+1) \sum_{n,\ell} \frac{(-1)^\ell}{2\ell+1} \left\{ (\ell+1) d^2_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}} + \ell d^2_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} - 2\sqrt{\ell(\ell+1)}\; d_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}} d_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} \right\} .$$

(III-22)

c. $\underline{\Omega = K = I = \frac{1}{2}}$

$$\mu = \frac{1}{3} g_R (1 - a) + \frac{1}{6} g_\ell (1 + 2a)$$

$$+ \frac{1}{6} (g_s - g_\ell) \sum_{n,\ell} \frac{1}{2\ell+1} \left\{ \left[1 + (-1)^\ell\, 2(\ell+1) \right] d^2_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}} - \left[1 - (-1)^\ell\, 2\ell \right] d^2_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} - 4\sqrt{\ell(\ell+1)} \left[1 + (-1)^\ell \right] d_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}}\, d_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} \right\} .$$

(III-23)

In the event that the particle state of Case c is of odd parity, all ℓ values in the above sum are odd, and the magnetic moment reduces to an especially simple form:

$$\mu = \frac{1}{3} g_R(1 - a) + \frac{1}{6} g_\ell(1 + 2a) - \frac{1}{6}(g_s - g_\ell)$$

$$\times \sum_{n,\ell} \left\{ d^2_{n,\ell,\ell+\frac{1}{2},\frac{1}{2}} + d^2_{n,\ell,\ell-\frac{1}{2},\frac{1}{2}} \right\} .$$

Because the $\Psi_{k,\Omega}$ functions are normalized, we have

$$\sum_{n,\ell} \left\{ d^2_{n,\ell,\ell+\frac{1}{2},\Omega} + d^2_{n,\ell,\ell-\frac{1}{2},\Omega} \right\} = 1 ,$$

and

$$\mu = \frac{1}{3} g_R(1 - a) + \frac{1}{3} g_\ell(1 + a) - \frac{1}{6} g_s . \qquad \text{(III-24)}$$

The magnetic moments of deformed odd-parity nuclei with spins of $\frac{1}{2}$ depend on the form of the particle wave function only through the decoupling constant. Since both the magnetic moment μ and the decoupling constant a are directly measurable quantities, Eq. (III-24) can be used as a rather sensitive check on the assumptions that have gone into the calculation of the magnetic moments.

IV. APPLICATIONS TO DEFORMED NUCLEI

In this section, a number of simple applications of the single-particle states discussed above to nuclei in the region of large deformation are presented. No attempt has been made at a thorough and exhaustive study of these nuclei. Rather, a few nuclear properties have been selected, and an attempt made to understand these in terms of the model discussed in the preceding sections. Comparisons between the experimental and theoretical values of the quantities chosen for consideration have been made for a large group of deformed nuclei. This has been done in order to gauge the over-all degree of success of the present calculations in describing ground-state properties of deformed nuclei.

The physical quantities that may be considered fall into two categories. Those in the first group are independent of the nuclear radius, or, more specifically, independent of the radial parts of the functions in terms of which the spheroidal eigenfunctions have been expanded. Examples of this type of quantity are the ground-state spins and parities, the magnetic moments, and the decoupling constants. All these may be obtained by use of the relative energy-level ordering of particle states or by the eigenfunction expansion coefficients only. The second category consists of all expectation values that are dependent on the radial character of the particle wave functions, and are thus functions of the as yet unspecified parameter $\alpha = \sqrt{\mu\omega/\hbar}$.*

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* The quantity α can be obtained for a given nucleus by computing $\langle r^2 \rangle = \frac{1}{A} \sum_{i=1}^{A} \langle r^2 \rangle_i$, and setting this equal to $3/5 \left[1.2 \times 10^{-13} A^{1/3} \right]^2$. The function $\langle r^2 \rangle$ is independent of deformation for the approximate wave functions presented here.

In the first part of this section an attempt is made to correlate the ground-state spins I of about thirty odd-A nuclei with orbitals of the level-ordering diagram that have $\Omega = I$. No differentiation has been made between the level orderings of the neutrons and protons in this process. Following this, the results of magnetic-moment calculations for the above nuclei are presented and comparison is made with the observed values. Some comparisons have been made between corresponding results from Nilsson's and Gottfried's calculations and those obtained here. Other nuclear properties, such as the nature of the excitation spectrum and electric-dipole transition rates, have a bearing on the selection of the orbital assigned to a given nucleus, especially when it is necessary to distinguish between several particle states that appear to fit the spin and magnetic-moment data equally well. The consideration of all such properties has been neglected. There is thus an appreciable amount of uncertainty and ambiguity in some of the orbital assignments and computations.

A. Spins of Odd-A Nuclei

The first and simplest application that can be made with the spheroidal orbitals is to attempt to correlate the observed spins of odd-A nuclei in the region of strong deformation with the states of the energy-level diagram. Each energy state of a spheroidal potential will have a two-fold degeneracy corresponding to orientations $+\Omega$ and $-\Omega$ of the component of particle angular momentum along the intrinsic symmetry axis. In the strong-coupling approximation, which is assumed valid for the nuclei to be considered here, these states are filled pairwise by nucleons of each type, insuring a cancellation of all Ω

contributions except from possible unpaired particles. The ground-state spin of a deformed nucleus is in general given by $I = \left| \sum_{k=1}^{A} \Omega_k \right|$, except when this sum equals one-half. Even-even nuclei all have zero spin, whereas nuclei of odd mass numbers have spins equal to the Ω of the unpaired nucleon, provided $\Omega \neq \frac{1}{2}$. If $\Omega = \frac{1}{2}$, a collective rotational state of the nucleus could become the nuclear ground state, causing I to be possibly greater than one-half. This possibility is neglected in the first considerations of spin assignments. Also neglected is the possibility that strong "pairing" forces may sometimes cause the unpaired particle to be other than the last, as specified by the energy-level ordering.

Spin assignments have been made by taking the level ordering as literally as possible. Investigation of nuclear spins entails the assignment of approximate values of the deformation parameter, δ, to the nuclei under consideration (which is discussed presently). With δ chosen, a search of the energy-level scheme is made for states of Ω equal to the measured nuclear spin in the vicinity of this deformation and the occupation number corresponding to that of the last unpaired nucleon. The states listed are not always at exactly the prescribed nucleon occupation level; they are sometimes just above or just below it. This is not felt to be a serious defect of the model, for we are primarily interested in describing the over-all systematics of the strongly deformed nuclei rather than the characteristics of any one nucleus. The availability of a state of the proper Ω in the correct region of the level diagram indicates that complete agreement of the sort desired could be obtained by a small change in the model potential or the a priori level ordering at $\delta = 0$, applicable to the nucleus in question, without making an appreciable change in the broad features of the level diagram or in the orbital wave functions. A definite failure of the scheme is evident only when no level of $\Omega = I$ can be found for the odd nucleon, and even here the possibility that we have $\Omega = \frac{1}{2}$ and $I > \Omega$ must first be investigated. For a proper assignment of ground-state orbitals to the last unpaired nucleons, a comparison of the pattern of particle excitations of the nucleus with that predicted by the level diagram should be made. However, this has not been done here.

A more serious ambiguity occasionally occurs when several states of the same Ω fall in close proximity and seem to apply equally well as contenders for the odd-nucleon state. In such cases some other criterion must be used if a particular orbital is to be singled out as the correct one.

Two groups of nuclei, those of the lanthanides and of the actinides, have been considered. In each, the odd-proton and odd-neutron nuclei have been listed separately. In most cases, assignments of equilibrium deformation have been made by assuming the nuclei to have a uniform charge distribution over a spheroidal region of average radius $R = 1.2 \times 10^{-13} A^{1/3}$ cm, and relating the intrinsic quadrupole moment Q_0 obtained thereby to the spectroscopic value Q, through the equation

$$Q = \frac{I(2I - 1)}{(I + 1)(2I + 3)} Q_0 .$$

With these approximations, for a spheroid of semiaxes $a_x = a_y = p D^{-\frac{1}{2}}$ and $a_z = p D$ represented by $p^2 = D(x^2 + y^2) + z^2/D^2$ and charge density $\rho_e = \dfrac{Z e}{\frac{4}{3}\pi p^3}$, one obtains

$$Q_0 = \frac{\rho_e}{e\cdot 10^{24}} \int_0^{2\pi} d\phi \int_{-pD}^{pD} dz \int_0^{D^{-\frac{1}{2}}\sqrt{p^2 - z^2/D^2}} (2 z^2 - \rho^2)\rho \, d\rho$$

$$= \frac{2}{5} Z p^2 (D^2 - 1/D)\cdot 10^{-24} .$$

Using the previous definition of Section II, i.e.,

$$\delta = \frac{pD - pD^{-\frac{1}{2}}}{R} \qquad \text{and} \qquad D^{3/2} = \frac{3 + 2\delta}{3 - \delta} ,$$

where $R \simeq \frac{1}{3}(2 a_x + a_z)$, one gets

$$Q_0 = \frac{4}{3} Z\cdot 10^{-24} R^2 \delta (1 + \frac{1}{6}\delta) .$$

The further simplifying approximation has been made that $|\delta| \sim 0.3$, so that $(1 + \frac{1}{6}\delta)$ can be replaced by 1.05 when we have $\delta > 0$ and by 0.95 when we have $\delta < 0$. The final combination of all quantities gives

$$\delta = \frac{1}{Z A^{2/3}} \frac{(I + 1)(2I + 3)}{I(2I - 1)} Q \begin{cases} 82.67 & (\text{for } \delta > 0) \\ 91.37 . & (\text{for } \delta < 0) \end{cases}$$

This relation is of course invalid if $I = \frac{1}{2}$. Optically measured values for Q are used when available. In other case, values of Q_0 obtained directly from E2 transition probabilities have been used.

These are considered less reliable. Where no knowledge of Q or Q_0 is had, levels of $I = \Omega$ have been sought in the deformation range $0.1 \lesssim \delta \lesssim 0.2$. The computed values of δ must not be taken too seriously because the measured quadrupole moments may be considerably in error.

Tables A, B, C, and D given below list the following spin information according to isotope: The first three columns contain the experimental values of Q and I along with a number referring to the source of the data. References on magnetic-moment measurements are also given here. Column four gives our computed value for δ. A parenthesis about the number indicates that the deformation has been obtained by using transition-rate data rather than optically measured quadrupole moments. In columns five and six, the spins and orbital numbers of states that could qualify for the last unpaired nucleon are given. In those instances where the state occurs at the exact occupation number and near the listed deformation, the state number has been bracketed. Finally, in columns seven and eight the orbital number has been repeated if it is found to correspond to one of the state assignments that have been made by use of the schemes of Nilsson[32,33] and Gottfried.[20] A line has been drawn through those locations in columns seven and eight where the nucleus in question has not been studied with the Nilsson or Gottfried level orderings.

An examination of the accompanying tables shows that a state of the proper Ω to give a nuclear spin of $I = \Omega$ can invariably be found in the desired region of the energy-level diagram. In many cases more than one such state is available, so that an immediate unambiguous assignment of an orbital for the last odd nucleon is

Table A.

Odd Proton: 150 < A < 190

Nucleus	$Q_{exp.}$	$I_{exp.}$	Ref.	δ*	$I_{th.}$*	Orbital Number†	Comparison with Nilsson	Comparison with Gottfried
$_{63}Eu^{151}$	1.2	5/2	37	0.16	5/2+	[25]	25	
$_{63}Eu^{153}$	2.5	5/2	37	0.32	5/2+	26		
$_{65}Tb^{159}$		3/2	38	(0.36)	3/2+, 3/2−	28, 46		
$_{67}Ho^{165}$	−|2|	7/2	39	0.18, (0.32)	7/2+, 7/2−	[23], (39) ?		39
$_{69}Tm^{169}$		1/2	40	(0.31)	1/2+, 1/2−	35, 32		
$_{71}Lu^{175}$	5.7	7/2	41, 42	0.48, (0.33)	7/2+, 7/2−	[23], 39	23	39
$_{73}Ta^{181}$	6.0	7/2	42	0.44, (0.26)	7/2−, 7/2+	[30], 23	23	23
$_{75}Re^{185}$	2.8	5/2	42	0.27	5/2−, 5/2+	42, 25		
$_{75}Re^{187}$	2.6	5/2	42	0.25, (0.20)	5/2−, 5/2+	42, 25		
$_{77}Ir^{191}$	1.5	3/2	43	.24	3/2+	[30], 48	30	48
$_{77}Ir^{193}$	1.5	3/2	43	.24	3/2+	[30], 48	30	48

*See text for definitions. †A line has been drawn through those orbital numbers which have been rejected on the basis of magnetic moment calculations.

Table B.

Odd Neutron: $150 < A < 190$

Nucleus	$Q_{exp.}$	$I_{exp.}$	Ref.	δ*	$I_{th.}$*	Orbital Number†	Comparison with Nilsson	Comparison with Gottfried
$_{64}Gd^{155}$	−1.1	3/2	44	0.25	3/2−, 3/2+	[47], 72	47	47
$_{64}Gd^{157}$	−1.0	3/2	44	0.22	3/2+, 3/2−	[72], 47	47	47
$_{66}Dy^{161}$		5/2	50		5/2−, 5/2−, 5/2+	[43], ~~[illegible]~~, 67	44	———
$_{66}Dy^{163}$		5/2	50		5/2+, 5/2−	[[illegible]], [43]	———	———
$_{68}Er^{167}$		7/2	45	large	7/2+	~~[illegible]~~, (60, 6[illegible])		
$_{70}Yb^{171}$		1/2	46		1/2−, 1/2+	[55], ~~[illegible]~~	55	
$_{70}Yb^{173}$	3.9	5/2	46, 47	0.42	5/2−, 5/2+	[43], 67	43	43
$_{72}Hf^{177}$	~3	7/2	48, 49	−.23, (.22)	7/2+, 7/2−, 7/2−	~~[illegible]~~, 41, ~~[illegible]~~	41	41
$_{72}Hf^{179}$	~3	9/2	48, 49	−.2	9/2−, 9/2+	~~[illegible]~~, (60)	60	60
$_{74}W^{183}$		1/2	51	(0.29)	1/2+, 1/2−	~~[illegible]~~, [54]	54	———
$_{76}Os^{187}$		1/2	52		1/2+, 1/2+	~~[illegible]~~, [79], ~~[illegible]~~	54	
$_{76}Os^{189}$	2.0	3/2	53, 54	.33	3/2−, 3/2+	[50], 74	50	———

*See text for definitions. †A line has been drawn through those orbital numbers which have been rejected on the basis of magnetic moment calculations.

Table C.

Odd Proton: A > 220

Nucleus	$Q_{exp.}$	$I_{exp.}$	Ref.	δ *	$I_{th.}$ *	Orbital Number †	Comparison with Nilsson	Comparison with Gottfried
${}_{89}Ac^{227}$	-1.7	3/2	55	-0.24	3/2-, 3/2+	48, 30		———
${}_{91}Pa^{231}$		3/2	42		3/2-, 3/2+	[47], 72	47, 72	———
${}_{93}Np^{237}$		5/2	56		5/2-, 5/2-, 5/2+	~~[24]~~, 43, [78]		———
${}_{93}Np^{239}$		1/2	57		1/2+, 1/2-	~~[70]~~, 52		———
${}_{95}Am^{241}$	4.9	5/2	58	0.31	5/2-, 5/2+, 5/2-	~~[23]~~, ~~[62]~~, 44	44	———
${}_{95}Am^{243}$	4.9	5/2	58	0.31	5/2-, 5/2+, 5/2-	~~23~~, ~~62~~, 44	44	———

*See text for definitions. †A line has been drawn through those orbital numbers which have been rejected on the basis of magnetic moment calculations.

Table D.

Odd Neutron: A > 220

Nucleus	$Q_{exp.}$	$I_{exp.}$	Ref.	δ*	$I_{th.}$*	Orbital Number†	Comparison with Nilsson	Comparison with Gottfried
$_{92}U^{233}$		5/2	42		5/2+, 5/2−, 5/2−	[68], [45], [100]	———	———
$_{92}U^{235}$		7/2	59		7/2−, 7/2+	[95], 65, 64	———	———
$_{94}Pu^{239}$		1/2	56, 60		1/2−, 1/2+	[56], 82	———	———
$_{94}Pu^{241}$		5/2	56		5/2+, 5/2−	68, 100	———	———

*See text for definitions. †A line has been drawn through those orbital numbers which have been rejected on the basis of magnetic moment calculations.

impossible. This ambiguity of choice becomes worse as the occupation number of the odd nucleon becomes larger, because energy levels become denser in the upper portion of the level diagram. This is considered to be a rather serious defect of the present model. What is chiefly to be noted is that a very high degree of correlation exists between the spins of the deformed odd-A nuclei and the states of the energy-level diagram obtained in the present calculations. However, results are not sufficiently clear-cut to make reliable predictions of nuclear spins possible. The orbital assignments are discussed further in the following section on magnetic moments.

A comparison between the present orbital assignments and those made with the Nilsson or Gottfried level orderings shows that although there are many cases of agreement, there are similarly many instances in which the assigned orbitals are different, i.e., they originate from states of different angular momentum at zero deformation. The origin of these differences lies primarily in the differences in the $\delta = 0$ level ordering chosen by the various authors. The disagreement is particularly evident in the odd-proton nuclei because the level ordering in this particular work has been chosen to resemble that of the neutron arrangement rather than that of the proton arrangement. This apparent lack of consistency among the various spheroidal-well calculations is considered to be another defect of this type of model in general, but one which can be resolved when sufficient comparisons with experiments have been made according to the several models.

B. Magnetic Moments of Odd-A Nuclei

The calculation of nuclear magnetic moments acts as a somewhat more stringent test of the applicability of the present model to deformed nuclei than does the assignment of spins. It serves further as a criterion for rejecting some of the orbitals that have been selected as possible states for the unpaired nucleon of odd-A nuclei. The magnetic moments of a large number of spheroidal odd-A nuclei have been measured, especially in the rare earth region; in a considerable number of cases, however, the precision is not very great. Moments of those nuclei for which data are available have been computed by use of Eqs. (21) through (24) of Section III and the orbital assignments of Part A of this section. The results of these calculations have been presented in graphical form in the following pages. For each nucleus considered, a plot has been made of theoretical magnetic moments predicted by the assigned orbitals, as a function of the deformation. Except where specific knowledge to the contrary was available, all nuclei have been considered to be prolate and therefore moments for $\delta > 0$ only have been given. Orbitals yielding magnetic moments that differ widely from the measured values, or with improper decoupling constants, have been rejected and excluded from the graphs. Each plot contains horizontal lines showing the mean observed moment, and when sufficient space is available, the nearest Schmidt-line* and Dirac-line moments. The Dirac-line moment is equivalent to the Schmidt-line moment, except that the gyromagnetic ratios of ideal Dirac nucleons are used in place of the observed values, i.e., for the proton the

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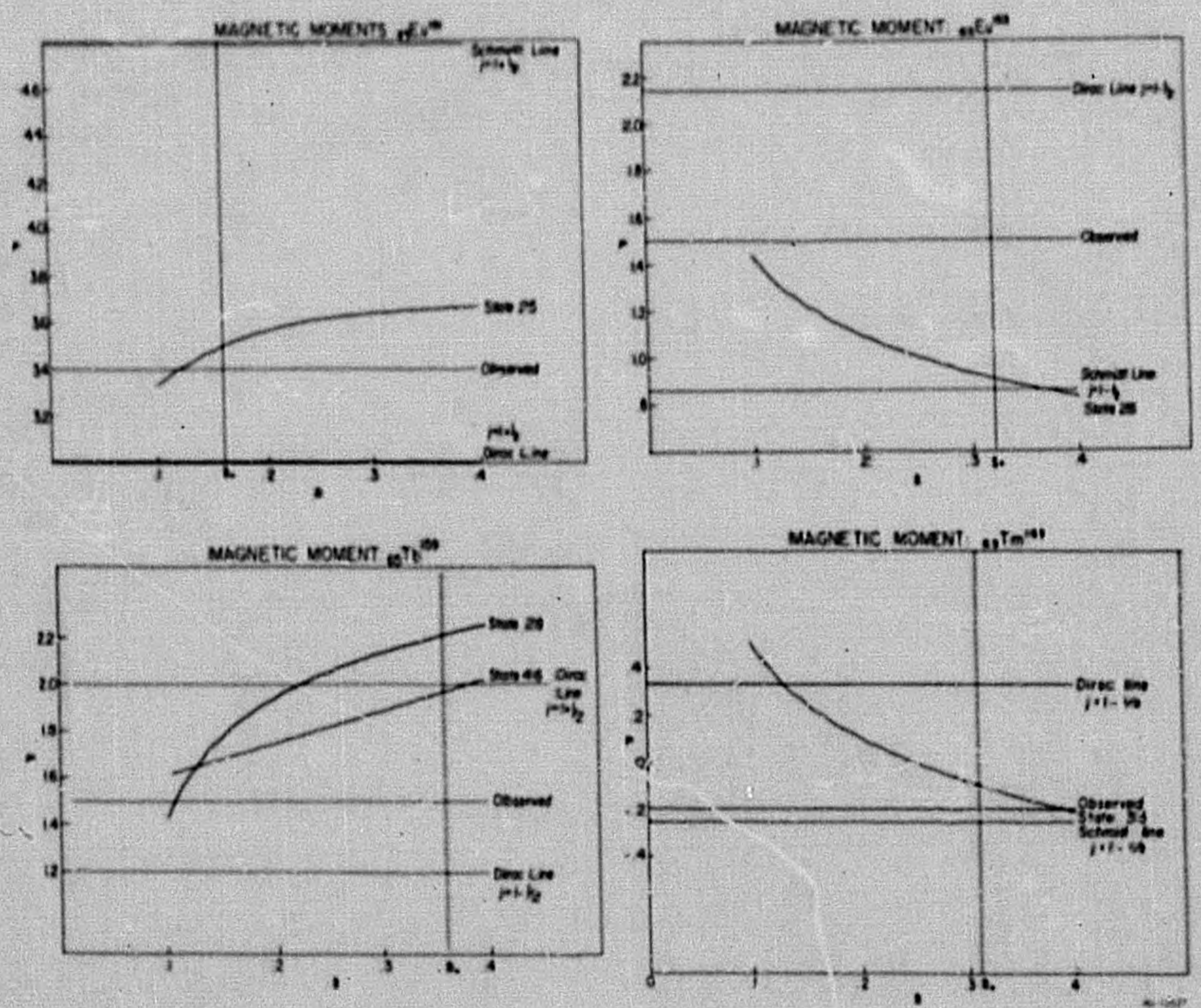
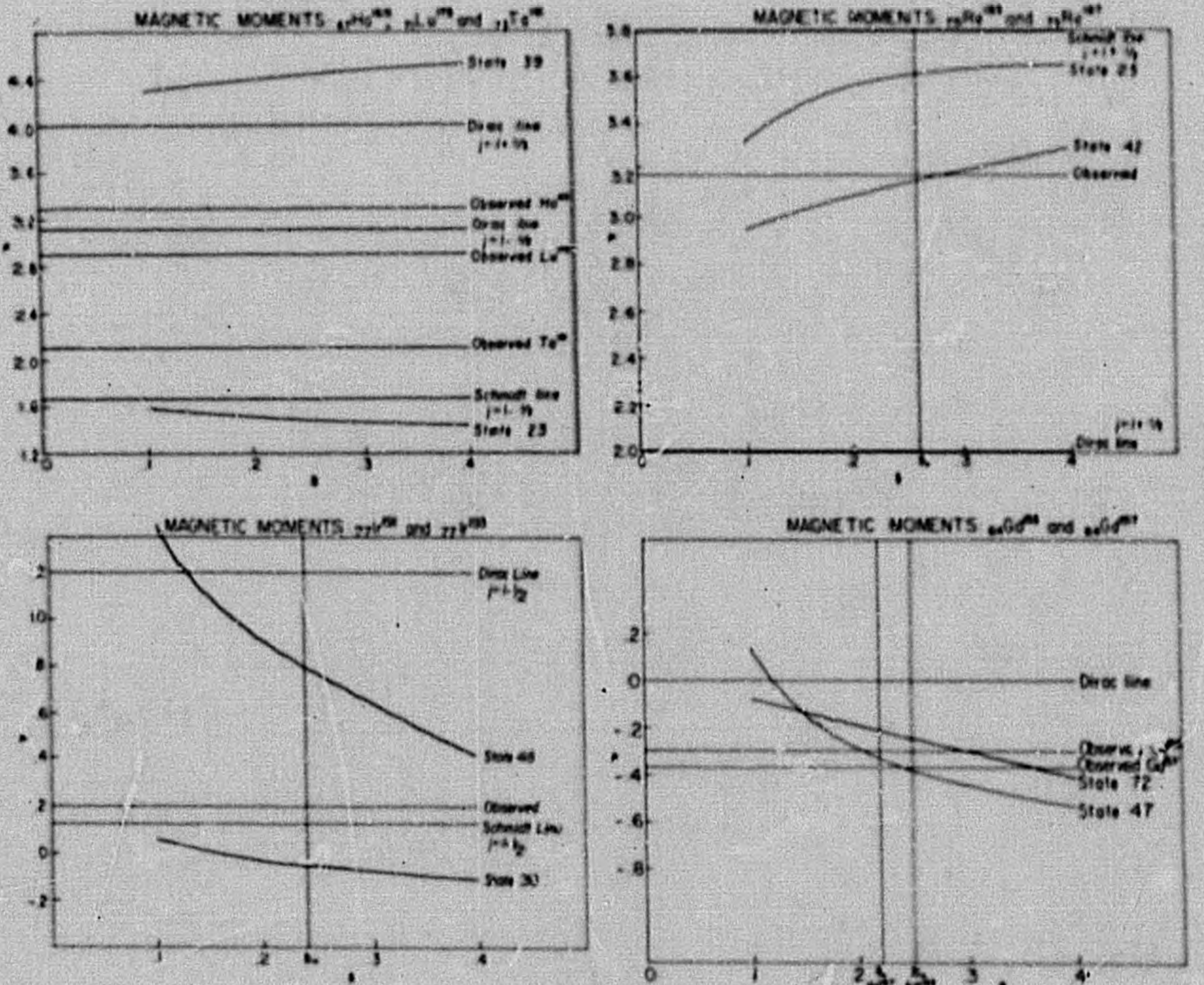
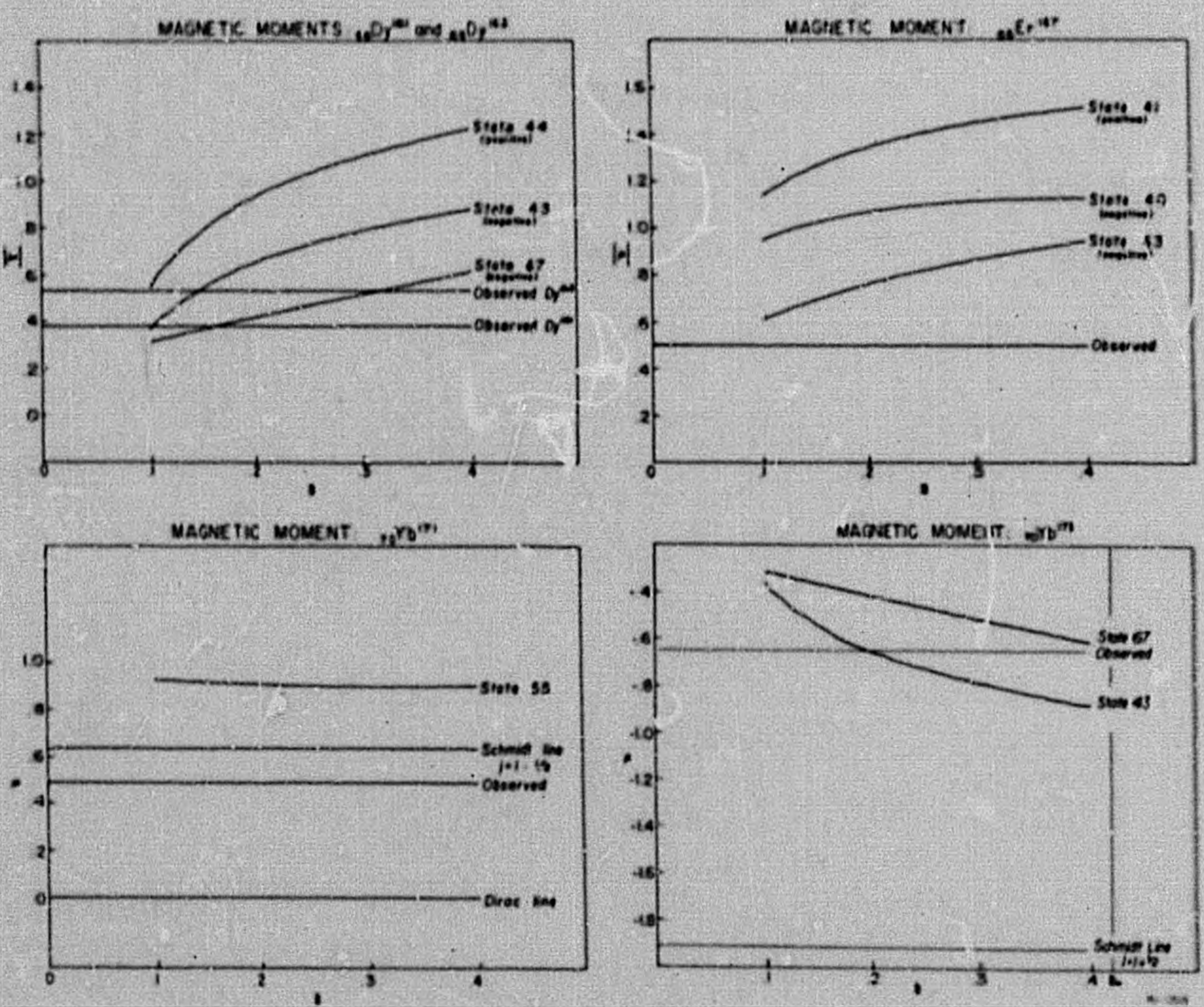
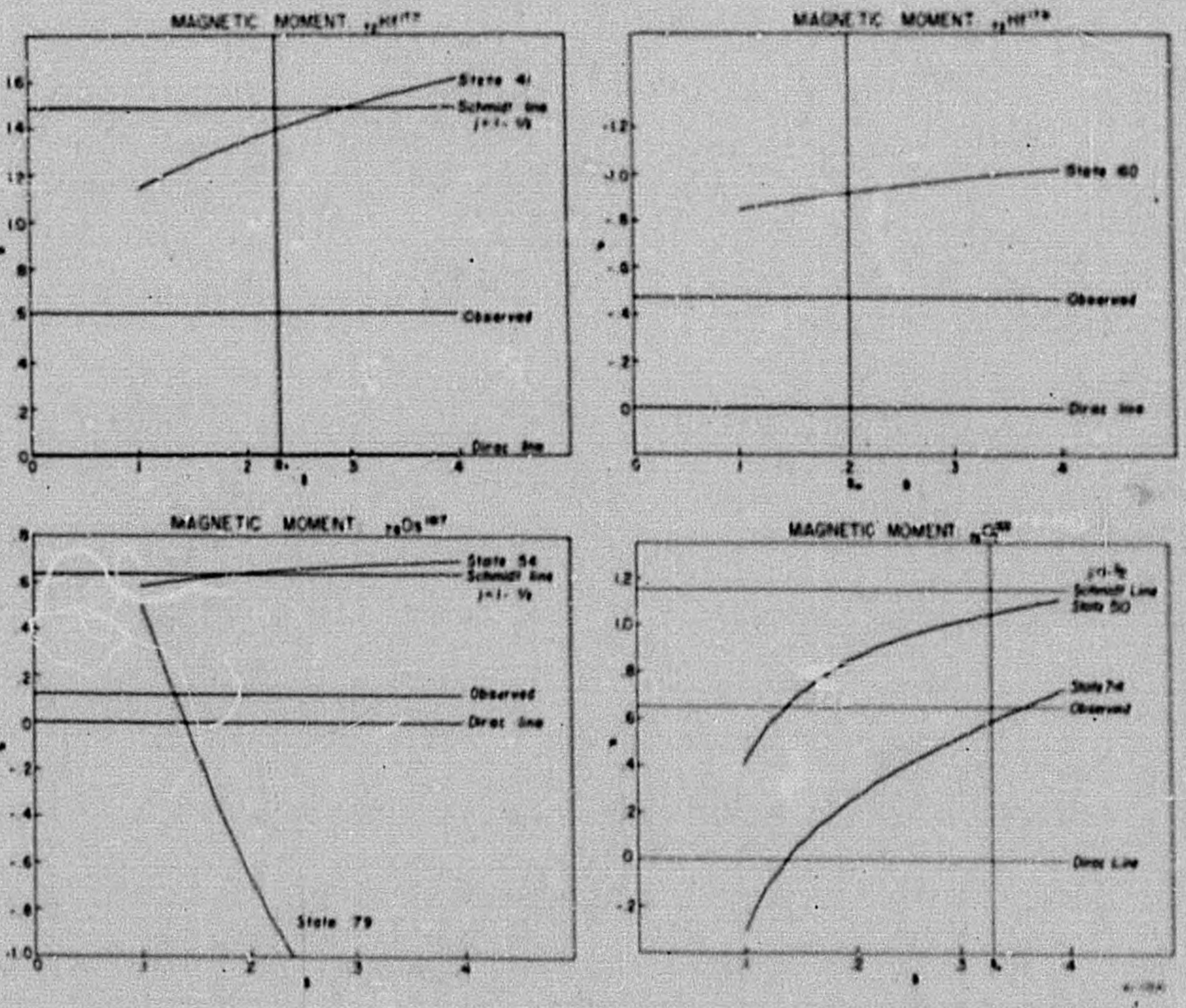
* As was noted by Gottfried[20], because l and j are no longer good quantum numbers, the nearness of a magnetic moment to a given Schmidt line no longer gives an indication of the parity of a nucleus as it does in the shell model.


MAGNETIC MOMENTS
Schmidt Line
State 25
Observed
Dirac Line
MAGNETIC MOMENT
Dirac Line
Observed
Schmidt Line
State 25
MAGNETIC MOMENT
State 29
State 46
Dirac Line
Observed
Dirac Line
MAGNETIC MOMENT
Dirac line
Observed
State 34
Schmidt line

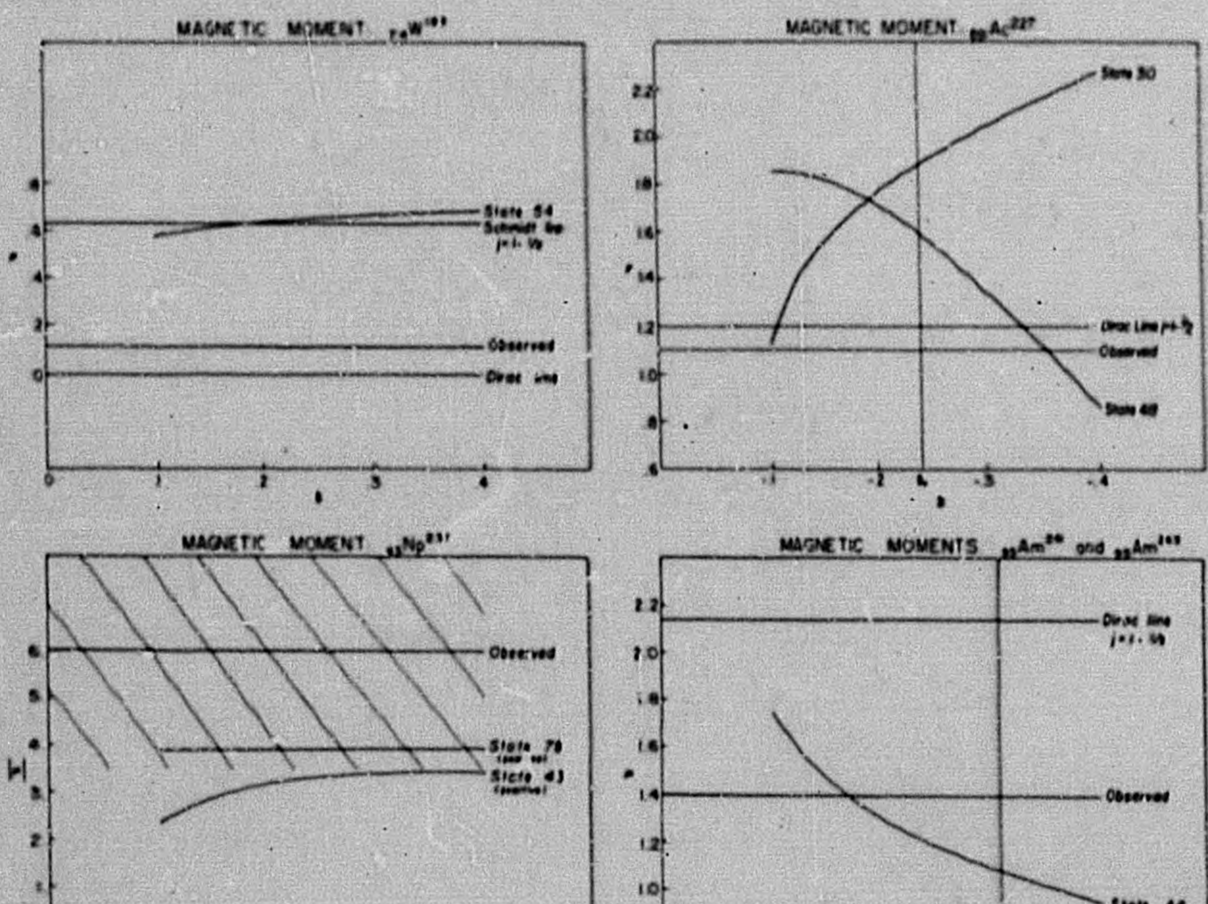

MAGNETIC MOMENTS
State 39
Dirac line
Observed
Schmidt line
State 23
MAGNETIC MOMENTS
Schmidt line
State 23
State 42
Observed
Dirac line
MAGNETIC MOMENTS
Dirac Line
State 48
Observed
Schmidt Line
State 30
MAGNETIC MOMENTS
Dirac line
State 72
State 47

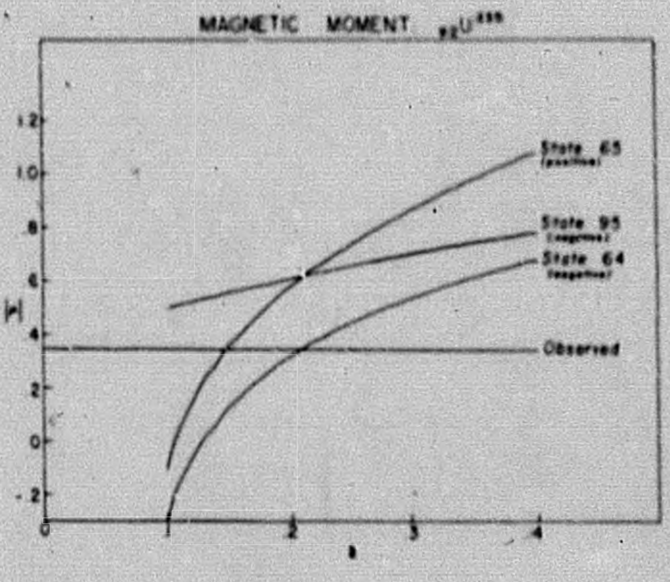


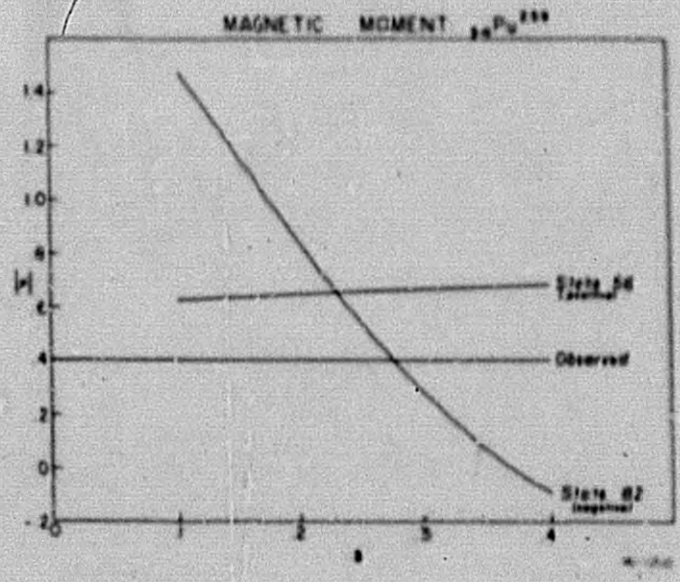
MAGNETIC MOMENTS Dy and Dy
State 44
State 43
State 67
Observed Dy
Observed Dy
MAGNETIC MOMENT Er
State 41
State 40
State 53
Observed
MAGNETIC MOMENT Yb
State 55
Schmidt line
Observed
Dirac line
MAGNETIC MOMENT Yb
State 67
Observed
State 43
Schmidt Line



MAGNETIC MOMENT
State 41
Schmidt line
Observed
MAGNETIC MOMENT
State 60
Observed
MAGNETIC MOMENT
State 54
Schmidt line
Observed
Dirac line
State 79
MAGNETIC MOMENT
Schmidt Line
State 50
Observed
Dirac Line


MAGNETIC MOMENT
State 54
Schmidt line
Observed
Dirac line
MAGNETIC MOMENT
State 50
Dirac Line
Observed
State 48
MAGNETIC MOMENT
Observed
State 79
State 43
MAGNETIC MOMENTS
Dirac line
Observed
State 44
Schmidt line


MAGNETIC MOMENT
State 65
(positive)
State 95
(negative)
State 64
(negative)
Observed


MAGNETIC MOMENT
Observed
State 82
(negative)

[redacted]

Dirac gyromagnetic ratio is $g_p = 2$, and for the neutron $g_n = 0$. All moments are given in nuclear magnetrons. The estimated deformations of the various nuclei have been noted on the graphs as vertical lines labeled with the symbol δ_c. In those cases in which only the absolute magnitude of the magnetic moment is known, the word "negative" or "positive" beneath the state numbers indicates the signs of the moments arising from the orbitals.

An examination of the graphs shows that for most of the nuclei considered, an orbital can be found in the vicinity of the proper deformation and odd-particle occupation number that gives a magnetic moment very close to the observed value, or at least substantially better than the Schmidt-line moment. There are too many examples here for individual comment on each. However, some remarks are necessary concerning several of the nuclei. The orbital No. 25 has been assigned to $_{63}Eu^{151}$. This is a state of positive parity, unlike the Nilsson or Gottfried assignments in which negative parity states have been suggested. The predicted magnetic moment is seen to agree exceptionally well with the experimental moment. The great difference between the magnetic moments of $_{63}Eu^{153}$ and $_{63}Eu^{151}$ is also exhibited by the orbitals selected for these nuclei, although the computed $_{63}Eu^{153}$ moment is somewhat smaller than is observed. For a number of nuclei, all assigned orbitals predict magnetic moments in very poor agreement with the empirical values. This is particularly true for $_{67}Ho^{165}$, $_{71}Lu^{175}$, $_{70}Yb^{171}$, $_{72}Hf^{187}$, $_{74}W^{183}$, and $_{94}Pu^{241}$. The theoretical moment of $_{94}Pu^{241}$ has not been shown. In the tungsten isotope, the poor agreement can be attributed to the failure of the adiabatic treatment of the particle motion as independent of the rotation.[22]

The only orbital of $\Omega = 9/2$ that might reasonably be assigned to $_{72}Hf^{179}$ is No. 38. However, this gives a magnetic moment of the wrong sign and of a different magnitude from that observed. The next closest $\Omega = 9/2$ state is No. 60, coming from the $i_{13/2}$ configuration. Although it is quite far from the correct particle occupation number, its magnetic moment has been calculated and found to be in better agreement with the experimental value. This tends to indicate the necessity for lowering the $i_{13/2}$ states with respect to the others in the energy-level diagram. Another interesting nucleus is $_{93}Np^{237}$. The measured moment is $|6.0 \pm 2.5|$. The large degree of uncertainty of this value has been indicated on the graph by light diagonal lines. Orbital No. 43 is the only $\Omega = 5/2$ state attributable to $_{93}Np^{237}$ that gives a magnetic moment of magnitude even approaching the required value. However, a state of $\Omega = \frac{1}{2}$, No. 78, giving a ground-state spin of $I = 5/2$, is present at the correct proton occupation number for this nucleus, and its magnetic moment has been found to lie within the experimental errors of the empirical result. The suggestion that $_{93}Np^{237}$ might have an anomalous rotational spectrum and ground-state spin was first made by John O. Rasmussen.[34] Unfortunately, recent Coulomb-excitation studies of this nucleus, by J. O. Newton,[35] indicate that in spite of the better magnetic moment resulting from this assumption the odd proton of $_{93}Np^{237}$ is not in a state of $\Omega = \frac{1}{2}$.

To summarize the results of the spin assignments and the magnetic-moment calculations: it is felt that the agreement with the experimental data is sufficiently close in most cases to call the present model successful. Whether it has specific advantages over the procedures of Nilsson or Gottfried cannot be said without a more complete analysis of data relating to strongly deformed nuclei.

ACKNOWLEDGMENTS

I should like to acknowledge my gratitude to Dr. Alfred Reifman for suggesting this work and for many helpful discussions before his departure from the University of California Radiation Laboratory. It is a further pleasure to express my deep appreciation to Dr. David L. Judd for his continued encouragement and friendly advice throughout the past few years of my graduate studies. I would also like to thank the many other people from whose valued discussions and penetrating criticisms I have benefited throughout the course of this research, and in particular Drs. Warren Heckrotte and John O. Rasmussen. I am indebted to Mr. Harold Hanerfeld and Mr. James A. Baker for the diagonalization of the matrices that arise in this calculation, and for their kind assistance in the carrying out of many lesser computations. Finally, I would like to thank Miss Georgella Perry and all others whose help has been instrumental in the preparation of this report.

This work was performed under the auspices of the U.S. Atomic Energy Commission.

APPENDIX A. MATRIX ELEMENTS OF THE PERTURBATION HAMILTONIAN

The matrix elements of the perturbation Hamiltonian

$$H' = \tfrac{1}{2}\mu\omega^2 r^2 (D - 1) + \tfrac{1}{2}\mu\omega^2 z^2\left(\frac{1}{D^2} - D\right) + V_b(\sigma) - 2\lambda\hbar\omega\vec{\ell}\cdot\vec{S}$$

may be obtained in a simple and straightforward manner in terms of the harmonic-oscillator wave functions $|n,\ell,j,\Omega\rangle$ described in Section II. The four terms of H' are considered individually below.

a. <u>Term $H'_1 = -2\lambda\hbar\omega\vec{\ell}\cdot\vec{S}$</u>

The evaluation of the matrix element of H'_1 may be accomplished without actual reference to the form of the function $|n,\ell,j,\Omega\rangle$ by noting that ℓ, S, and j are good quantum numbers for this state. The total angular-momentum operator for the particle state is $\vec{j} = \vec{\ell} + \vec{S}$. Squaring this gives

$\vec{j}^2 = \vec{\ell}^2 + \vec{S}^2 + 2\vec{\ell}\cdot\vec{S}$. Thus we have $2\vec{\ell}\cdot\vec{S} = \vec{j}^2 - \vec{\ell}^2 - \vec{S}^2$,

$$\langle n',\ell',j',\Omega'|H'_1|n,\ell,j,\Omega\rangle =$$

$$= -\lambda\hbar\omega\langle n',\ell',j',\Omega'|\vec{j}^2 - \vec{\ell}^2 - \vec{S}^2|n,\ell,j,\Omega\rangle$$

$$= -\lambda\hbar\omega\left[j(j+1) - \ell(\ell+1) - S(S+1)\right]\langle n',\ell',j',\Omega'|n,\ell,j,\Omega\rangle .$$

Insertion of $S = \frac{1}{2}$ and evaluation for the two possible cases, $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$, gives the final result

$$\langle n',\ell',j',\Omega'|H'_1|n,\ell,j,\Omega\rangle =$$

$$= -\lambda\hbar\omega\,\delta_{n,n'}\,\delta_{\ell,\ell'}\,\delta_{j,j'}\,\delta_{\Omega,\Omega'}\left[\ell\,\delta_{j,\ell+\frac{1}{2}} - (\ell+1)\,\delta_{j,\ell-\frac{1}{2}}\right].$$

b. Term $H'_2 = \frac{1}{2}\mu\omega^2(D-1)r^2$

The matrix element of H'_2 is

$$\langle n', \ell', j', \Omega' \,|\, H'_2 \,|\, n, \ell, j, \Omega \rangle =$$

$$\frac{1}{2}\hbar\omega\alpha^2 \int_0^\infty R_{n',\ell'}(\alpha r) r^2 R_{n,\ell}(\alpha r) r^2 \, dr \int \mathcal{Y}^*_{\ell',j',\Omega'} \mathcal{Y}_{\ell,j,\Omega} \, d\Omega$$

$$= \frac{1}{2}\hbar\omega\alpha^2 \, \delta_{\ell,\ell'} \, \delta_{j,j'} \, \delta_{\Omega,\Omega'} \int_0^\infty R_{n',\ell} \, r^4 \, R_{n,\ell} \, dr \quad ,$$

where the well-known orthogonality property of the $\mathcal{Y}_{\ell,j,\Omega}(\theta,\emptyset,s)$ functions has been used. The radial functions may be written in terms of Laguerre polynomials[28] as

$$R_{n,\ell}(\alpha r) = N_{n,\ell}(\alpha r)^\ell \, e^{-\frac{1}{2}\alpha^2 r^2} \, L_{n-1}^{\ell+\frac{1}{2}}(\alpha^2 r^2) \; ,$$

where

$$N^2_{n,\ell} = \frac{2\alpha^3 (n-1)!}{\left[\Gamma(n+\ell+\frac{1}{2})\right]^3} \qquad \text{and} \qquad \alpha^2 = \mu\omega/\hbar \; .$$

Furthermore, the Laguerre functions obey the orthogonality relations

$$\int_0^\infty \rho^{\ell+\frac{1}{2}} e^{-\rho} L_{n-1}^{\ell+\frac{1}{2}}(\rho) \, L_{n'-1}^{\ell+\frac{1}{2}}(\rho) \, d\rho =$$

$$\delta_{n,n'} \frac{\left[\Gamma(n+\ell+\frac{1}{2})\right]^3}{(n-1)!} \quad ,$$

$$\int_0^\infty \rho^{\ell+3/2} e^{-\rho} L_{n-1}^{\ell+\frac{1}{2}}(\rho) \, L_{n'-1}^{\ell+\frac{1}{2}}(\rho) \, d\rho =$$

$$\delta_{n,n'} \left[\Gamma(n+\ell+\tfrac{1}{2})\right]^3 \frac{2n+\ell-\frac{1}{2}}{(n-1)!} \quad .$$

Substitution of $\rho = \alpha^2 r^2$ in the radial integral of the H'_2 matrix element and use of the above expression for $R_{n,\ell}(\alpha r)$ yields

$$\langle n', \ell', j', \Omega' \,|\, H'_2 \,|\, n, \ell, j, \Omega \rangle =$$

$$\tfrac{1}{2}\hbar\omega\, \delta_{\ell,\ell'}\, \delta_{j,j'}\, \delta_{\Omega,\Omega'} \frac{N_{n',\ell} N_{n,\ell}}{2\alpha^3}$$

$$\times \int_0^\infty \rho^{\ell+\frac{3}{2}} e^{-\rho} L_{n-1}^{\ell+\frac{1}{2}}(\rho)\, L_{n'-1}^{\ell+\frac{1}{2}}(\rho)\, d\rho$$

$$= \tfrac{1}{2}\hbar\omega\, \delta_{n,n'}\, \delta_{\ell,\ell'}\, \delta_{j,j'}\, \delta_{\Omega,\Omega'} (2n + \ell - \tfrac{1}{2}) .$$

This computation is merely a proof of the virial theorem for the harmonic oscillator.

c. Term $H'_3 = \frac{1}{2}\mu\omega^2(\frac{1}{D^2} - D)z^2$

The matrix element of H'_3 is

$$\langle n', \ell', j', \Omega' \,|\, H'_3 \,|\, n, \ell, j, \Omega \rangle =$$

$$\tfrac{1}{2}\hbar\omega\alpha^2\left(\frac{1}{D^2} - D\right) \int_0^\infty R_{n',\ell'}(\alpha r) r^2 R_{n,\ell}(\alpha r) r^2\, dr$$

$$\times \int \mathcal{Y}^*_{\ell', j', \Omega'} \cos^2\theta\, \mathcal{Y}_{\ell, j, \Omega}\, d\Omega .$$

The integral over the angular coordinates can be carried out by inserting the expansion for the $\mathcal{Y}_{\ell,j,\Omega}$ functions given in Eq. (II-10):

$$\int \mathcal{Y}_{\ell',j',\Omega'} \cos^2\theta \, \mathcal{Y}_{\ell,j,\Omega} \, d\Omega =$$

$$\sum_{m'=-\ell'}^{\ell'} \sum_{m=-\ell}^{\ell} \int Y^*_{\ell',m'} \cos^2\theta \, Y_{\ell,m} \, d\Omega$$

$$\times \Big\{ \delta_{m',\Omega'-\frac{1}{2}} C_{\ell',\frac{1}{2}}(j',\Omega';\, m',\tfrac{1}{2}) \chi^*_{\frac{1}{2},\frac{1}{2}}$$

$$+ \delta_{m',\Omega'+\frac{1}{2}} C_{\ell',\frac{1}{2}}(j',\Omega';\, m',-\tfrac{1}{2}) \chi^*_{\frac{1}{2},-\frac{1}{2}} \Big\}$$

$$\times \Big\{ \delta_{m,\Omega-\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega;\, m,\tfrac{1}{2}) \chi_{\frac{1}{2},\frac{1}{2}}$$

$$+ \delta_{m,\ell-\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega;\, m,-\tfrac{1}{2}) \chi_{\frac{1}{2},-\frac{1}{2}} \Big\} .$$

Applying the orthogonality properties of the spin functions, $\chi_{\frac{1}{2},\pm\frac{1}{2}}$, and the well-known recursion relations for the spherical harmonics, one finds, by a lengthy but straightforward calculation,

$$\int \mathcal{Y}_{\ell',j',\Omega'} \cos^2\theta \, \mathcal{Y}_{\ell,j,\Omega} \, d\Omega =$$

$$\delta_{\Omega,\Omega'} \sum_{m=-\ell}^{\ell} \Big\{ \delta_{\ell,\ell'+2} \sqrt{\frac{\left[(\ell-1)^2 - m^2\right]\left[\ell^2 - m^2\right]}{(2\ell+1)(2\ell-3)(2\ell-1)^2}}$$

$$+ \delta_{\ell,\ell'} \left[\frac{(\ell+1)^2 - m^2}{(2+1)(2+3)} + \frac{\ell^2 - m^2}{(2\ \ -1)(2+1)} \right]$$

$$+ \delta_{\ell,\ell'-2} \sqrt{\frac{\left[(\ell+2)^2 - m^2\right]\left[(\ell+1)^2 - m^2\right]}{(2+1)(2+5)(2+3)^2}} \Big\}_{\text{Cont.}}$$

$$\times \left\{ \delta_{m,\Omega-\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega;\, m,\tfrac{1}{2}) C_{\ell',\frac{1}{2}}(j',\Omega;\, m,\tfrac{1}{2}) + \delta_{m,\Omega+\frac{1}{2}} C_{\ell,\frac{1}{2}}(j,\Omega;\, m,-\tfrac{1}{2}) C_{\ell',\frac{1}{2}}(j',\Omega;\, m,-\tfrac{1}{2}) \right\} .$$

The implicit form of the Clebsch-Gordan coefficients, $C_{\ell,\frac{1}{2}}(j,\Omega;\, m,\pm\frac{1}{2})$, has been retained. The explicit form for these may be found in Appendix A of Reference 29.

The radial integrals fall into two classes, one with $\ell = \ell'$ and the other with $\ell = \ell' + 2$. The first of these has already been evaluated in Part b of this appendix and is given by

$$\alpha^2 \int_0^\infty R_{n',\ell}(\alpha r) r^4 R_{n,\ell}(\alpha r) dr = \delta_{n,n'} \left[2n + \ell - \tfrac{1}{2} \right] .$$

The evaluation of the second radial integral with $\ell = \ell' + 2$ is somewhat more involved than that of the first. Only a restricted class of these, with $n' = n + 1$, is necessary for the computations of the investigation at hand. Without explaining here the details of the integration, one finds*

$$\alpha^2 \left\langle R_{n+1,\ell-2} \left| r^2 \right| R_{n,\ell} \right\rangle = + \sqrt{2n\left[2(n+\ell) - 1\right]}$$

---

* If one tries to evaluate this integral by use of the equations on page 785 of Methods of Theoretical Physics by Morse and Feshbach, the result will appear with a minus sign. This is because of a different choice of the phase relations among the Laguerre polynomials from that used here.

$$\langle n',\ell',j',\Omega' \mid H'_3 \mid n,\ell,j,\Omega\rangle =$$

$$\tfrac{1}{2}\hbar\omega\left(\frac{1}{D^2} - D\right)\delta_{\Omega,\Omega'}\langle R_{n',\ell'}(\alpha r) \mid \alpha^2 r^2 \mid R_{n,\ell}(\alpha r)\rangle$$

$$\times \sum_{m=-\ell}^{\ell}\left\{\delta_{\ell,\ell'+2}\sqrt{\frac{\left[(\ell-1)^2 - m^2\right]\left[\ell^2 - m^2\right]}{(2\ell+1)(2\ell-3)(2\ell-1)^2}}\right.$$

$$+ \delta_{\ell,\ell'}\left[\frac{\left[(\ell+1)^2 - m^2\right]}{(2\ell+1)(2\ell+3)} + \frac{\left[\ell^2 - m^2\right]}{(2 \quad -1)(2\ell+1)}\right]$$

$$\left. + \delta_{\ell,\ell'-2}\sqrt{\frac{\left[(\ell+2)^2 - m^2\right]\left[(\ell+1)^2 - m^2\right]}{(2\ell+1)(2\ell+5)(2\ell+3)^2}}\right\}$$

$$\times\left\{\delta_{m,\Omega-\frac{1}{2}}C_{\ell,\frac{1}{2}}(j,\Omega;m,\tfrac{1}{2})C_{\ell',\frac{1}{2}}(j',\Omega;m,\tfrac{1}{2})\right.$$

$$\left. + \delta_{m,\Omega+\frac{1}{2}}C_{\ell,\frac{1}{2}}(j,\Omega;m,-\tfrac{1}{2})C_{\ell',\frac{1}{2}}(j',\Omega;m,-\tfrac{1}{2})\right\} .$$

d. <u>Term $H'_4 = V_b(\sigma)$</u>

The truncation function, $V_b(\sigma)$, with $\sigma = \sqrt{D(x^2+y^2) + z^2/D^2}$, may be separated into three parts,

$$V_b(\sigma) = -\tfrac{1}{2}\mu\omega^2(r^2 - \sigma_0^2) - \tfrac{1}{2}\mu\omega^2 r^2(D-1) - \tfrac{1}{2}\mu\omega^2 z^2\left(\frac{1}{D^2} - D\right) \qquad (\text{for } 0 \leqslant \sigma \leqslant \sigma_0)$$

$$= 0 \qquad (\text{for } \sigma > \sigma_0) .$$

The second and third terms are identical to H'_2 and H'_3 over the range $0 \leqslant \sigma \leqslant \sigma_0$. In the evaluation of the matrix elements of H'_4, the approximation has been made that the bounding surface $\sigma = \sigma_0$, within which $V_b(\sigma)$ is nonzero, may be taken as spherical instead of spheroidal. This permits one to replace $V_b(\sigma)$ by

$$V'_b(r) = -\tfrac{1}{2}\mu\omega^2(r^2 - r_0^2) - \tfrac{1}{2}\mu\omega^2 r^2(D - 1) - \tfrac{1}{2}\mu\omega^2 z^2(\frac{1}{D^2} - D) \qquad (\text{for } 0 \leqslant r \leqslant r_0)$$

$$= 0\,, \qquad (\text{for } r > r_0)$$

and greatly simplifies all integrations over the truncation volume. The approximation does not alter the matrix elements significantly because the deviations from sphericity considered here are small.

Except for the radial integrals, which extend here over the finite interval $0 \leqslant r \leqslant r_0$, the evaluation of the matrix elements of H'_4 is the same as for H'_2 and H'_3, namely,

$$\langle n', \ell', j', \Omega' \,|\, H'_4 \,|\, n, \ell, j, \Omega \rangle \cong -\tfrac{1}{2}\hbar\omega\alpha^2 \delta_{\ell,\ell'} \delta_{j,j'} \delta_{\Omega,\Omega'} \times \int_0^{r_0} R_{n',\ell}(\alpha r)(r^2 - r_0^2) R_{n,\ell}(\alpha r) r^2\, dr$$

$$- \tfrac{1}{2}\hbar\omega\alpha^2 (D - 1)\, \delta_{\ell,\ell'} \delta_{j,j'} \delta_{\Omega,\Omega'} \times \int_0^{r_0} R_{n',\ell}(\alpha r) r^4 R_{n,\ell}(\alpha r)\, dr$$

$$- \tfrac{1}{2}\hbar\omega\alpha^2 (\frac{1}{D^2} - D) \langle \mathcal{Y}_{\ell',j',\Omega'} \,|\, \cos^2\theta \,|\, \mathcal{Y}_{\ell,j,\Omega} \rangle \times \int_0^{r_0} R_{n',\ell'}(\alpha r) r^4 R_{n,\ell}(\alpha r)\, dr\,.$$

If the substitutions $\rho = \alpha r$ and $\varepsilon = \alpha r_0$ are made, one obtains

$$\langle n',\ell',j',\Omega' \,|\, H'_4 \,|\, n,\ell,j,\Omega \rangle \cong$$

$$- \tfrac{1}{2}\hbar\omega\, \delta_{\ell,\ell'}\, \delta_{j,j'}\, \delta_{\Omega,\Omega'} \Big\{ I_{n',\ell\,|\,n,\ell}$$

$$- \varepsilon^2 \int_0^{\varepsilon} R_{n',\ell}(\rho)\,\rho^2\, R_{n,\ell}(\rho)\,d\rho \Big\}$$

$$- \tfrac{1}{2}\hbar\omega\,(D - 1)\,\delta_{\ell,\ell'}\, \delta_{j,j'}\, \delta_{\Omega,\Omega'}\, I_{n',\ell\,|\,n,\ell}$$

$$- \tfrac{1}{2}\hbar\omega\left(\frac{1}{D^2} - D\right) \langle \mathcal{Y}_{\ell',j',\Omega'} \,|\, \cos^2\theta \,|\, \mathcal{Y}_{\ell,j,\Omega} \rangle\, I_{n',\ell'\,|\,n,\ell'}$$

where

$$I_{n',\ell'\,|\,n,\ell} \equiv \int_0^{\varepsilon} R_{n',\ell'}(\rho)\,\rho^4\, R_{n,\ell}(\rho)\,d\rho \quad .$$

APPENDIX B. TABLES AND DIAGRAM

Tables I and II, respectively, contain the eigenvalues and eigenfunctions of the particle equation (II-5) described in Section II. The dimensionless eigenvalues $E/\hbar\omega$ are listed in Table I by their arbitrary orbital numbers for seven values of the deformation parameter: $\delta = 0, \pm 0.1, \pm 0.2, \pm 0.4$. The eigenfunctions are similarly listed according to their orbital numbers, and the values of Ω corresponding to their states. Each orbital number corresponds to two degenerate states, one of $+\Omega$ and one of $-\Omega$. The expansion coefficients of those harmonic oscillator states that occur in the spheroidal wave function are given in the rows of each eigenfunction tables. The oscillator state function $|n, \ell, j, \Omega\rangle$ are given in the left-hand column. The coefficients are listed for the seven values of δ given above. At $\delta = 0$, the wave function reduces to its unperturbed form.

The eigenfunction coefficients, $d_{n,\ell,j,\Omega}$, are given for the two signs of Ω. Where both a plus and a minus sign appear before a numerical value, the upper sign corresponds to $+\Omega$ and the lower to $-\Omega$. The relations between the expansions of the pairs of degenerate state functions that have been used in preparing these tables are:

$$\Psi'_{n,\ell,j,\Omega} = \sum_{\{n',\ell',j'\}}{}' d_{n',\ell',j',\Omega}(k,\Omega)\, |n',\ell',j',\Omega\rangle ,$$

$$\Psi'_{n,\ell,j,-\Omega} = (-1)^{j-\frac{1}{2}} \sum_{\{n',\ell',j'\}}{}' (-1)^{j'-\frac{1}{2}} d_{n',\ell',j',\Omega}(k,\Omega)\, |n',\ell',j',-\Omega\rangle .$$

(See Eq. (II-12) and (II-17) for a definition of terms.)

This corresponds to setting the phase factor δ (discussed in Section II, page 25 and Section III, page 32) equal to zero when $j-\frac{1}{2}$ is an even integer, and equal to π when $j-\frac{1}{2}$ is odd. Only the coefficients corresponding to $+\Omega$ should be used in the applications of the formulas given in Section III.

Table III contains the explicit forms of the normalized radial parts of the harmonic-oscillator wave functions, Eq. (II-8), used in the computations for this paper.

Table IV shows decoupling constants which have been calculated for those orbitals of $\Omega = \frac{1}{2}$ which fall within the range of particle occupation numbers applicable to the strongly deformed nuclei. These have been listed in Table IV, according to the orbital number of the particle state, for the three positive values of the deformation parameter: $\delta = 0.1$, 0.2, and 0.4.

Table V gives the particle occupation number associated with each orbital as a function of the deformation. The left column lists the number of neutrons or protons present in the nucleus. The orbital number and the value of Ω of the state present at each such occupation level are given for the six values of δ : ± 0.1, ± 0.2, and ± 0.4.

Energy-Level Diagram. A level diagram of the dimensionless eigenvalues, $E/\hbar\omega$, of the particle Hamiltonian, Eq. (II-6), (II-7), is given in Fig. 2, in this appendix. The energies $E/\hbar\omega$, plotted on an arbitrary scale, are shown as functions of the deformation parameter δ . The energy levels are labeled by the orbital numbers of the states followed by the value of the intrinsic z projection of angular momentum, Ω . The plus or minus sign after each Ω indicates the parity of the orbital.

Table I. Eigenvalues.

[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 3 (cont.)

[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table I (cont.)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
37	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
38	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
39	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
42	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
48	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
49	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
51	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 1 (cont.)

j(Y.T.) \ k	-3	-2	-1	0	1	2	3
52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
55	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
59	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
62	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
63	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
64	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
65	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
66	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
67	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 1 (cont.)

j(a) \ i	-3	-2	-1	0	1	2	3
69	7,7530	7,8398	7,8089	7,9079	7,9734	7,8605	7,7965
70	9,0935	8,5552	8,3401	8,3079	8,3547	8,5390	8,9383
71	8,7774	8,7143	8,0779	8,1456	8,2775	8,4110	8,7061
72	7,1379	7,1439	7,1745	7,1070	7,1958	7,3799	6,9960
73	8,1984	7,9997	8,0477	7,9970	7,8881	7,8113	7,7518
74	7,8380	7,9754	7,9580	7,9089	7,8160	7,7106	7,4148
75	9,3776	8,5081	8,7961	8,3079	8,2790	8,3593	8,4458
76	8,7731	8,3126	8,0778	8,1456	8,1575	8,1810	8,2987
77	9,8956	8,5633	8,5361	8,3706	8,1956	8,6608	9,0635
78	7,1116	7,5095	7,1540	7,1070	7,2947	7,1370	6,8397
79	8,1037	8,0088	8,0356	7,9920	7,8366	7,6758	7,5026
80	7,7319	7,9138	7,9675	7,9779	7,7734	7,5974	7,2127
81	9,8460	8,8181	8,1714	8,3079	8,3132	8,1556	8,7467
82	8,4464	8,2408	8,3578	8,1256	8,0416	7,9778	7,9571
83	10,1079	9,1369	8,4759	8,3706	8,1875	8,6616	9,0735
84	9,2871	8,5535	8,3640	8,2936	8,2816	8,2848	8,3333
85	7,5454	7,9833	8,0760	8,2814	8,5000	8,7321	9,2379

Table I (cont.)

I.E.K. \ δ	-.4	-.2	-.1	0	.1	.2	.4
86	7.9606	8.0979	8.1944	8.2914	8.4385	8.5716	8.9611
87	8.2956	8.5775	8.7573	8.9564	9.1689	9.3981	9.5985
88	7.9638	8.1718	8.2312	8.2914	8.3439	8.4750	8.6463
89	8.5580	8.8037	8.9415	8.9564	9.0828	9.2447	9.6590
90	8.4429	8.4814	8.6444	8.7747	8.9670	9.1653	9.4495
91	8.2605	8.2628	8.2830	8.2914	8.2779	8.2871	8.3054
92	9.0118	8.8135	8.9179	8.9564	9.0001	9.0866	9.4113
93	8.5508	8.7396	8.7196	8.7747	8.8765	8.9925	9.2146
94	9.4855	9.1318	9.1309	9.2497	9.4497	9.6714	10.1497
95	8.1749	8.3232	8.3273	8.2914	8.2221	8.1674	8.1099
96	8.9722	8.7354	8.9027	8.9564	8.9252	8.9336	9.1797
97	8.6066	8.7180	8.7793	8.7747	8.7964	8.8331	8.9465
98	10.2018	9.4350	9.2812	9.2697	9.3792	9.5556	10.0394
99	9.5766	9.0937	8.9950	9.1184	9.2902	9.4469	9.8149
100	8.2967	8.3697	8.3108	8.2914	8.1754	8.0590	7.8772
101	9.0453	8.9269	8.8938	8.9564	8.8616	8.7926	8.8259
102	8.6767	8.8785	8.8323	8.7747	8.7287	8.6977	9.5968

Table 1 (cont.)

State	-.4	-.2	-.1	0	.1	.2	.4
103	10.2419	9.3653	9.2673	9.2497	9.2875	9.3929	9.7734
104	9.5397	9.2108	9.1505	9.1184	9.1696	9.2681	9.4420
105	10.6955	9.7378	9.4606	9.4334	9.6250	9.8309	10.3152
106	8.3037	8.3981	8.3692	8.2814	8.1676	7.9978	7.8295
107	9.0597	8.9911	9.0135	8.9564	8.8127	8.6706	8.5674
108	8.7029	8.8730	8.8627	8.7747	8.6744	8.5748	8.3386
109	10.1579	9.4736	9.3136	9.2697	9.1952	9.2189	9.4702
110	9.6049	9.1913	9.1649	9.1184	9.0332	9.0530	9.1153
111	11.2796	10.3469	9.6456	9.4334	9.6045	9.8311	10.3421
112	10.7115	9.7000	9.4710	9.4101	9.4436	9.5896	9.9374
113	8.3428	8.4132	8.3784	8.2814	8.1323	7.9596	7.6198
114	9.1218	9.0091	9.0340	8.9564	8.7948	8.6006	8.3302
115	8.7462	8.9442	8.8790	8.7747	8.6467	8.4931	8.1355
116	10.1761	9.4537	9.3837	9.2697	9.1107	9.0346	9.0810
117	9.6223	9.2572	9.2168	9.1184	8.9900	8.8578	8.7370
118	11.2800	10.0674	9.6389	9.4334	9.4502	9.5702	9.9133
119	10.7397	9.7516	9.4799	9.4101	9.3016	9.3242	9.5711
120	11.9795	10.4312	9.8681	9.5451	9.7025	9.9261	10.4354

Table II. Eigenfunctions.

1. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\langle 1,0,\frac{1}{2},\pm\frac{1}{2}\vert$	1	1	1	1	1	1	1

2. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,1,\frac{3}{2},\pm\frac{3}{2}\rangle$	1	1	1	1	1	1	1

3. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,1,\frac{3}{2},\pm\frac{1}{2}\rangle$	.9731	.9953	.9990	1	.9993	.9975	.9922
$\vert 1,1,\frac{1}{2},\pm\frac{1}{2}\rangle$	± .2306	± .0969	± .0442	0	∓ .0376	∓ .0705	∓ .1249

4. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,1,\frac{3}{2},\pm\frac{1}{2}\rangle$	∓ .2306	∓ .0969	∓ .0442	0	± .0376	± .0705	± .1249
$\vert 1,1,\frac{1}{2},\pm\frac{1}{2}\rangle$	.9731	.9953	.9990	1	.9993	.9975	.9922

Table 11 (cont.)

5. $\Omega = \pm 5/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\langle 1, 2, 5/2, \pm 5/2 \rangle$	1	1	1	1	1	1	1

6. $\Omega = \pm 3/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\langle 1, 2, 5/2, \pm 3/2 \rangle$	.9566	.9939	.9988	1	.9993	.9976	.9930
$\langle 1, 2, 3/2, \pm 3/2 \rangle$	$\pm$ .2913	$\pm$ .1107	$\pm$ .0685	0	$\mp$ .0372	$\mp$ .0698	$\mp$ .1180

7. $\Omega = \pm 1/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\langle 1, 2, 5/2, \pm 1/2 \rangle$	$\mp$ .2913	$\mp$ .1107	$\mp$ .0685	0	$\pm$ .0372	$\pm$ .0698	$\pm$ .1180
$\langle 1, 2, 3/2, \pm 1/2 \rangle$	.9566	.9939	.9988	1	.9993	.9976	.9930

Table II (cont.)

8. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,5/2,\pm 1/2\rangle$	.7900	.9304	.9832	1	.9891	.9670	.9218
$\|1,2,3/2,\pm 1/2\rangle$	∓ .1475	∓ .0336	± .0012	0	∓ .0282	∓ .0665	∓ .1400
$\|1,0,1/2,\pm 1/2\rangle$	- .5951	- .3649	- .1823	0	.3442	.2458	.3616

9. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,5/2,\pm 1/2\rangle$	∓ .2938	∓ .1230	∓ .0438	0	∓ .0103	∓ .0608	∓ .3644
$\|1,2,3/2,\pm 1/2\rangle$	.7608	.9094	.9723	1	.9656	.8771	.7034
$\|2,0,1/2,\pm 1/2\rangle$	∓ .5786	∓ .3974	∓ .2296	0	± .2597	± .4764	± .6915

10. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,5/2,\pm 1/2\rangle$	.5382	.3452	.1770	0	- .1466	- .2474	- .3511
$\|1,2,3/2,\pm 1/2\rangle$	± .6320	± .4146	± .2337	0	∓ .2584	∓ .4757	∓ .6969
$\|2,0,1/2,\pm 1/2\rangle$	.5577	.8420	.9560	1	.9548	.8441	.6253

Table II (cont.)

11. $\Omega = \pm 7/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,3,7/2,\pm 7/2\rangle$	1	1	1	1	1	1	1

12. $\Omega = \pm 5/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,3,7/2,\pm 5/2\rangle$	.9345	.9923	.9986	1	.9992	.9976	.9934
$\|1,3,5/2,\pm 5/2\rangle$	$\pm$ .3560	$\pm$ .1238	$\pm$ .0522	0	$\mp$ .0393	$\mp$ .0698	$\mp$ .1148

13. $\Omega = \pm 5/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,3,7/2,\pm 5/2\rangle$	$\mp$ .3560	$\mp$ .1238	$\mp$ .0522	0	$\pm$ .0393	$\pm$ .0698	$\pm$ .1148
$\|1,3,5/2,\pm 5/2\rangle$	.9345	.9923	.9986	1	.9992	.9976	.9934

Table II (cont.)

14. $\Omega = \pm 1/2$

δ	$-.4$	$-.2$	$-.1$	0	$.1$	$.2$	$.4$
$\|1,3,7/2,\pm 1/2\rangle$	.6786	.8866	.9773	1	.9906	.9747	.9464
$\|1,3,5/2,\pm 1/2\rangle$	$\mp$ .1236	$\mp$ .0151	$\pm$ .0177	0	$\mp$ .0409	$\mp$ .0837	$\mp$ .1526
$\|2,1,3/2,\pm 1/2\rangle$	− .7240	− .4622	− .2111	0	.1301	.2072	.2846

15. $\Omega = \pm 1/2$

δ	$-.4$	$-.2$	$-.1$	0	$.1$	$.2$	$.4$
$\|1,3,7/2,\pm 1/2\rangle$	$\mp$ .3989	$\mp$ .1762	$\mp$ .0696	0	$\mp$ .0379	$\mp$ .1267	$\mp$ .2084
$\|1,3,5/2,\pm 1/2\rangle$	.7657	.9130	.9680	1	.8336	.5564	.3850
$\|2,1,3/2,\pm 1/2\rangle$	$\mp$ .5046	$\mp$ .3679	$\mp$ .2409	0	$\pm$ .5510	$\pm$ .8212	$\pm$ .8991

16. $\Omega = \pm 3/2$

δ	$-.4$	$-.2$	$-.1$	0	$.1$	$.2$	$.4$
$\|1,3,7/2,\pm 3/2\rangle$	.6167	.4276	.2001	0	− .1310	− .1840	− .2466
$\|1,3,5/2,\pm 3/2\rangle$	$\pm$ .6312	$\pm$ .4076	$\pm$ .2501	0	$\mp$ .5508	$\mp$ .8267	$\mp$ .9102
$\|2,1,3/2,\pm 3/2\rangle$	.4703	.8068	.9473	1	.8243	.5317	.3326

Table II (cont.)

17. $\Omega = \pm 1/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,3,3/2,\pm 1/2\rangle$	.7476	.9120	.9756	1	.9766	.9236	.8229
$\|1,3,5/2,\pm 1/2\rangle$	± .1022	± .0269	± .0093	0	∓ .0209	∓ .0549	∓ .1202
$\|2,1,1/2,\pm 1/2\rangle$	- .5522	- .3879	- .2164	0	.2122	.3701	.5258
$\|2,1,3/2,\pm 1/2\rangle$	∓ .3545	∓ .1303	∓ .0349	0	∓ .0278	∓ .0839	∓ .1783

18. $\Omega = \pm 1/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,3,3/2,\pm 1/2\rangle$	± .3674	± .2077	± .0649	0	∓ .0370	∓ .1308	∓ .2822
$\|1,3,5/2,\pm 1/2\rangle$	.7582	.8089	.8979	1	.9386	.8646	.7549
$\|2,1,1/2,\pm 1/2\rangle$	± .4760	± .5498	± .2935	0	± .2873	± .4759	± .5963
$\|2,1,3/2,\pm 1/2\rangle$	+ .2519	- .0157	- .3249	0	+ .1873	+ .0942	- .0826

Table II (cont.)

19. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,1,1/2,\pm 1/2\rangle$	.4800	.3196	.2060	0	-.2294	-.3531	-.4755
$\lvert 1,1,3/2,\pm 1/2\rangle$	∓.5300	∓.3959	∓.1148	0	±.2236	±.4034	±.5774
$\lvert 2,1,3/2,\pm 1/2\rangle$	.1083	.4816	.8470	1	.9320	.7222	.4453
$\lvert 2,1,1/2,\pm 1/2\rangle$	±.6906	±.7144	±.4764	0	∓.3962	∓.4370	∓.4094

20. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,3,1/2,\pm 1/2\rangle$	∓.2751	∓.1534	∓.0607	0	±.0317	±.0724	±.1306
$\lvert 1,3,3/2,\pm 1/2\rangle$	.3656	.4344	.4768	0	-.2618	-.2945	-.2970
$\lvert 2,1,1/2,\pm 1/2\rangle$	∓.6758	∓.5616	∓.3968	0	±.2406	±.1389	±.4262
$\lvert 2,1,1/2,\pm 1/2\rangle$	.5778	.6973	.8162	1	.9336	.8906	.9479

Table II (cont.)

21. $\Omega = \pm 9/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,+,9/2,\pm 9/2\rangle$	1	1	1	1	1	1	1

22. $\Omega = \pm 7/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,+,7/2,\pm 7/2\rangle$	.9173	.9917	.9986	1	.9993	.9977	.9941
$\|1,+,9/2,\pm 7/2\rangle$	∓ .3981	∓ .1284	∓ .0525	0	± .0380	± .0672	± .1082

23. $\Omega = \pm 7/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,+,7/2,\pm 7/2\rangle$	∓ .3981	∓ .1284	∓ .0525	0	± .0380	± .0672	± .1082
$\|1,+,9/2,\pm 7/2\rangle$	.9173	.9917	.9986	1	.9993	.9977	.9941

Table II (cont.)

24. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,3/2,\pm 5/2\rangle$	.6369	.8861	.9807	1	.9928	.9908	.9589
$\|1,4,7/2,\pm 5/2\rangle$	$\mp$ .0950	$\pm$ .0095	$\pm$ .0289	0	$\mp$ .0442	$\mp$ .0851	$\mp$ .1478
$\|2,2,5/2,\pm 5/2\rangle$	- .7651	- .4627	- .1933	0	.1111	.1757	.2421

25. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,3/2,\pm 5/2\rangle$	$\mp$ .4565	$\mp$ .2031	$\mp$ .0838	0	$\mp$ .0809	$\mp$ .1399	$\mp$ .1989
$\|1,4,7/2,\pm 5/2\rangle$	.7533	.9061	.9570	1	.4358	.3217	.2583
$\|2,2,5/2,\pm 5/2\rangle$	$\mp$ .4735	$\mp$ .3710	$\mp$ .2777	0	$\pm$ .8964	$\pm$ .9364	$\pm$ .9454

26. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,3/2,\pm 5/2\rangle$	.6213	.4153	.1767	0	- .0880	- .1362	- .2022
$\|1,4,7/2,\pm 5/2\rangle$	$\pm$ .6508	$\pm$ .4229	$\pm$ .2885	0	$\mp$ .8989	$\mp$ .9430	$\mp$ .9547
$\|2,2,5/2,\pm 5/2\rangle$	.4364	.8054	.9410	1	.4291	.3036	.2183

Table II (cont.)

27. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,9/2,\pm 3/2\rangle$	.6964	.9045	.9745	1	.9819	.9468	.8923
$\|1,4,7/2,\pm 3/2\rangle$	± .1684	± .0416	± .0204	0	∓ .0368	∓ .0813	∓ .1583
$\|2,2,5/2,\pm 1/2\rangle$	- .5634	- .4099	- .2218	0	.1850	.3091	.4318
$\|2,2,3/2,\pm 1/2\rangle$	∓ .4114	∓ .1102	∓ .0254	0	∓ .0158	∓ .0459	∓ .1000

28. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,7/2,\pm 1/2\rangle$	± .4733	± .3436	∓ .0118	0	∓ .1029	∓ .2206	∓ .3378
$\|1,4,9/2,\mp 1/2\rangle$	.6199	.5181	.8962	1	.7129	.5356	.4388
$\|2,2,5/2,\pm 3/2\rangle$	± .5432	± .7605	± .0808	0	± .6920	± .8146	± .8248
$\|2,2,3/2,\pm 3/2\rangle$	.3110	.1874	- .4361	0	- .0477	- .0296	- .1144

Table II (cont.)

29. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,5/2,\pm 3/2\rangle$	.4277	.1929	.2173	0	-.1579	-.2310	-.3190
$\|1,4,7/2,\pm 3/2\rangle$	$\mp$ .5490	$\mp$ .6339	$\pm$ .0664	0	$\mp$ .6732	$\mp$ .7986	$\mp$ .8329
$\|2,2,5/2,\pm 3/2\rangle$	-.1494	.1647	.9259	1	.6842	.4506	.2612
$\|2,2,3/2,\pm 3/2\rangle$	$\pm$ .7026	$\pm$ .7306	$\pm$ .3017	0	$\mp$ .2376	$\mp$ .3254	$\mp$ .3691

30. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,5/2,\pm 1/2\rangle$	$\mp$ .3287	$\mp$ .1632	$\mp$ .0542	0	$\mp$ .0316	$\mp$ .0405	$\mp$ .0744
$\|1,4,7/2,\pm 1/2\rangle$	.5348	.5727	.4383	0	-.2000	-.2624	-.2978
$\|2,2,5/2,\pm 1/2\rangle$	$\mp$ .6046	$\mp$ .4759	$\mp$ .2948	0	$\pm$ .1366	$\pm$ .1959	$\pm$ .2551
$\|2,2,3/2,\pm 1/2\rangle$	.4903	.6472	.8474	1	.9701	.9440	.9369

Table II (cont.)

31. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,9/2,\pm 1/2\rangle$	.6244	.8840	.9701	1	.9714	.9014	.7579
$\|1,4,7/2,\pm 1/2\rangle$	∓ .1150	∓ .0077	± .0055	0	∓ .0160	∓ .0471	∓ .1243
$\|2,2,5/2,\pm 1/2\rangle$	- .5744	- .4295	- .2379	0	.2336	.4099	.5708
$\|2,2,3/2,\pm 1/2\rangle$	± .1818	± .0229	∓ .0022	0	∓ .0146	∓ .0612	∓ .1763
$\|3,0,1/2,\pm 1/2\rangle$	.4836	.1828	.0475	0	.0377	.1159	.2466

32. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,9/2,\pm 1/2\rangle$	∓ .4721	∓ .2930	∓ .1306	0	∓ .0892	∓ .2595	∓ .4330
$\|1,4,7/2,\pm 1/2\rangle$	.6218	.6774	.7960	1	.8494	.6732	.5786
$\|2,2,5/2,\pm 1/2\rangle$	∓ .2536	∓ .4560	∓ .4762	0	± .4326	± .6031	± .5498
$\|2,2,3/2,\pm 1/2\rangle$	- .2048	- .2773	- .3022	0	.2753	.2080	.0930
$\|3,0,1/2,\pm 1/2\rangle$	± .5332	± .4129	± .1770	0	± .0867	± .2692	± .4098

Table II (cont.)

33. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,1/2,\pm 1/2\rangle$	.5002	.2882	.3837	0	- .2132	- .3216	- .4410
$\|1,4,3/2,\pm 1/2\rangle$	± .5214	± .5006.	± .4375	0	∓ .3676	∓ .5424	∓ .5931
$\|2,2,5/2,\pm 1/2\rangle$	- .0829	.3050	.6580	1	.7884	.4626	.1360
$\|2,2,3/2,\pm 1/2\rangle$	∓ .5469	∓ .4686	∓ .2682	0	∓ .2820	∓ .1882	∓ .5769
$\|3,0,1/2,\pm 1/2\rangle$	- .4147	- .5947	- .5196	0	.3438	.3872	.3201

34. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,9/2,\pm 3/2\rangle$	∓ .1724	∓ .0912	∓ .0356	0	± .0216	± .0509	± .0914
$\|1,4,7/2,\pm 3/2\rangle$	.3096	.3072	.2948	0	- .1784	- .2128	- .2261
$\|2,2,5/2,\pm 3/2\rangle$	∓ .4931	∓ .3804	∓ .2498	0	∓ .1846	∓ .2692	∓ .1422
$\|2,2,3/2,\pm 3/2\rangle$	.6038	.6907	.7779	1	.6119	.5502	.5003
$\|3,0,1/2,\pm 3/2\rangle$	∓ .5164	∓ .5249	∓ .5260	0	± .7478	± .7595	± .7577

Table II (cont.)

35. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,4,9/2,\pm 1/2\rangle$	.3276	.2030	.0825	0	.0504	.1183	.2070
$\|1,4,7/2,\pm 1/2\rangle$	$\pm$ .4821	$\pm$ .4428	$\pm$ .3317	0	$\pm$ .3335	$\pm$ .4528	$\pm$ .4979
$\|2,2,5/2,\pm 1/2\rangle$	.5964	.6082	.4705	0	- .3203	- .4263	- .4872
$\|2,2,3/2,\pm 1/2\rangle$	$\pm$ .5112	$\pm$ .4753	$\pm$ .4812	0	$\mp$ .6853	$\mp$ .6418	$\mp$ .6441
$\|3,0,1/2,\pm 1/2\rangle$	.2078	.4085	.6559	1	.5601	.4328	.3078

36. $\Omega = \pm 11/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,11/2,\pm 11/2\rangle$	1	1	1	1	1	1	1

37. $\Omega = \pm 9/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,11/2,\pm 9/2\rangle$	.9116	.9922	.9987	1	.9994	.9980	.9950
$\|1,5,9/2,\pm 9/2\rangle$	$\pm$ .4112	$\pm$ .1245	$\pm$ .0502	0	$\mp$ .0357	$\mp$ .0624	$\mp$ .1001

Table II (cont.)

38. $\Omega = \pm 9/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,5,11/2,\pm 9/2\rangle$	$\mp$ .4112	$\mp$ .1245	$\mp$ .0502	0	$\pm$ .0357	$\pm$ .0624	$\pm$ .1001
$\lvert 1,5,9/2,\pm 9/2\rangle$	.9116	.9922	.9987	1	.9994	.9980	.9950

39. $\Omega = \pm 7/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,5,11/2,\pm 7/2\rangle$	.6160	.8964	.9840	1	.9942	.9845	.9664
$\lvert 1,5,9/2,\pm 7/2\rangle$	$\mp$ .0661	$\pm$ .0319	$\pm$ .0366	0	$\mp$ .0432	$\mp$ .0808	$\mp$ .1371
$\lvert 2,3,7/2,\pm 7/2\rangle$	- .7850	- .4420	- .1742	0	.0980	.1559	.2176

40. $\Omega = \pm 7/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,5,11/2,\pm 7/2\rangle$	$\mp$ .4766	$\mp$ .2002	$\mp$ .0818	0	$\mp$ .0776	$\mp$ .1294	$\mp$ .1844
$\lvert 1,5,9/2,\pm 7/2\rangle$	.7621	.9190	.9622	1	.3403	.2661	.2204
$\lvert 2,3,7/2,\pm 7/2\rangle$	$\mp$ .4382	$\mp$ .3396	$\mp$ .2599	0	$\pm$ .9371	$\pm$ .9552	$\pm$ .9578

Table II (cont.)

41. $\Omega = \pm 7/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1, 5, 11/2, \pm 7/2\rangle$	.6272	.3954	.1581	0	- .0738	- .1187	- .1793
$\|1, 5, 9/2, \pm 7/2\rangle$	$\pm$ .6441	$\pm$ .3929	$\pm$ .2700	0	$\mp$ .9393	$\mp$ .9605	$\mp$ .9657
$\|2, 3, 7/2, \pm 7/2\rangle$	.4379	.8302	.9498	1	.3350	.2515	.1877

42. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1, 5, 11/2, \pm 5/2\rangle$	.6916	.9011	.9754	1	.9856	.9594	.9116
$\|1, 5, 9/2, \pm 5/2\rangle$	$\pm$ .1453	$\pm$ .0371	$\pm$ .0277	0	$\mp$ .0410	$\mp$ .0834	$\mp$ .1519
$\|2, 3, 7/2, \pm 5/2\rangle$	- .5998	- .4238	- .2178	0	.1634	.2679	.3763
$\|2, 3, 5/2, \pm 5/2\rangle$	$\mp$ .3752	$\mp$ .0841	$\mp$ .0189	0	$\mp$ .0103	$\mp$ .0294	$\mp$ .0650

Table II (cont.)

43. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,5,11/2,\pm 5/2\rangle$	$\mp$ .5119	$\mp$ .3435	$\mp$ .1121	0	$\mp$ .1164	$\mp$ .2140	$\mp$ .3146
$\lvert 1,5,9/2,\pm 5/2\rangle$	- .5921	- .5048	.8086	1	.5353	.4050	.3379
$\lvert 2,3,7/2,\pm 5/2\rangle$	$\mp$ .5384	$\mp$ .7888	$\mp$ .3602	0	$\pm$ .8366	$\pm$ .8879	$\pm$ .8815
$\lvert 2,3,5/2,\pm 5/2\rangle$	- .3122	+ .0713	- .4516	0	+ .0046	- .0426	- .0993

44. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,5,11/2,\pm 5/2\rangle$	.3500	.2257	.1853	0	- .1217	- .1817	- .2590
$\lvert 1,5,9/2,\pm 5/2\rangle$	$\mp$ .5232	$\mp$ .4762	$\pm$ .4312	0	$\mp$ .8294	$\mp$ .8848	$\mp$ .8937
$\lvert 2,3,7/2,\pm 5/2\rangle$	- .1937	.2790	.8828	1	.5147	.3477	.2153
$\lvert 2,3,5/2,\pm 5/2\rangle$	$\pm$ .7524	$\pm$ .8028	$\pm$ .0219	0	$\mp$ .1798	$\mp$ .2514	$\mp$ .3024

Table II (cont.)

45. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,11/2,\pm 5/2\rangle$	$\mp$ .3702	$\mp$ .1381	$\mp$ .0407	0	$\mp$ .0114	$\mp$ .0275	$\mp$ .0532
$\|1,5,9/2,\pm 5/2\rangle$	.5994	.7191	.3995	0	- .1546	- .2148	- .2601
$\|2,3,7/2,\pm 5/2\rangle$	$\mp$ .5593	$\mp$ .3471	$\mp$ .2087	0	$\pm$ .0918	$\pm$ .1377	$\pm$ .1873
$\|2,3,5/2,\pm 5/2\rangle$	.4423	.5860	.8917	1	.9836	.9665	.9457

46. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,11/2,\pm 3/2\rangle$	.5714	.8721	.9686	1	.9758	.9256	.8311
$\|1,5,9/2,\pm 3/2\rangle$	$\mp$ .0734	$\pm$ .0113	$\pm$ .0159	0	$\mp$ .0301	$\mp$ .0687	$\mp$ .1104
$\|2,3,7/2,\pm 3/2\rangle$	- .5870	- .4504	- .2442	0	.2149	.3627	.5068
$\|2,3,5/2,\pm 3/2\rangle$	$\pm$ .1030	$\mp$ .0113	$\mp$ .0098	0	$\mp$ .0136	$\mp$ .0463	$\mp$ .1171
$\|3,1,3/2,\pm 3/2\rangle$	.5594	.1908	.0434	0	.0250	.0690	.1377

Table II (cont.)

47. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,\tfrac{7}{2},\pm\tfrac{3}{2}\rangle$	$\mp$ .4628	$\mp$ .2606	$\mp$ .1214	0	$\mp$ .1243	$\mp$ .2223	$\mp$ .4160
$\|1,5,\tfrac{9}{2},\pm\tfrac{3}{2}\rangle$	.5948	.7141	.8350	1	.7173	.5281	.4401
$\|2,3,\tfrac{7}{2},\pm\tfrac{3}{2}\rangle$	$\mp$ .0806	$\mp$ .3114	$\mp$ .3817	0	$\pm$ .6542	$\pm$ .7473	$\pm$ .6929
$\|2,3,\tfrac{5}{2},\pm\tfrac{3}{2}\rangle$	- .3734	- .4169	- .3314	0	.1258	.0611	.0222
$\|3,1,\tfrac{3}{2},\pm\tfrac{3}{2}\rangle$	$\pm$ .5349	$\pm$ .3890	$\pm$ .2800	0	$\pm$ .1618	$\pm$ .2913	$\pm$ .3907

48. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,\tfrac{7}{2},\pm\tfrac{3}{2}\rangle$	.4966	.3061	.1906	0	- .1760	- .2472	- .3312
$\|1,5,\tfrac{9}{2},\pm\tfrac{3}{2}\rangle$	$\pm$ .4653	$\pm$ .4164	$\pm$ .3955	0	$\mp$ .6411	$\mp$ .7547	$\mp$ .7618
$\|2,3,\tfrac{7}{2},\pm\tfrac{3}{2}\rangle$	- .1282	.3135	.6907	1	.6694	.3552	.1681
$\|2,3,\tfrac{5}{2},\pm\tfrac{3}{2}\rangle$	$\mp$ .5369	$\mp$ .3703	$\mp$ .1675	0	$\mp$ .2612	$\mp$ .4133	$\mp$ .4996
$\|3,1,\tfrac{3}{2},\pm\tfrac{3}{2}\rangle$	- .4818	- .7053	- .5497	0	.2041	.2095	.1790

Table II (cont.)

49. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,7/2,\pm 3/2\rangle$	∓ .2436	∓ .1118	∓ .0356	0	± .0128	± .0381	± .0743
$\|1,5,3/2,\pm 3/2\rangle$	.4213	.4198	.3113	0	- .1622	- .1752	- .1760
$\|2,3,7/2,\pm 3/2\rangle$	∓ .5259	∓ .3750	∓ .2126	0	∓ .1258	∓ .2218	∓ .3015
$\|2,3,5/2,\pm 3/2\rangle$	.5931	.7380	.8684	1	.6633	.5016	.4172
$\|3,1,3/2,\pm 3/2\rangle$	∓ .3555	∓ .3568	∓ .3200	0	± .7375	± .8168	± .8358

50. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,7/2,\pm 3/2\rangle$	.3916	.2557	.0978	0	.0357	.0881	.1446
$\|1,5,3/2,\pm 3/2\rangle$	± .6882	± .3745	± .2216	0	± .2171	± .3407	± .4186
$\|2,3,7/2,\pm 3/2\rangle$	.5966	.6792	.5219	0	- .2490	- .3234	- .3794
$\|2,3,5/2,\pm 3/2\rangle$	± .4582	± .3797	± .3283	0	∓ .7085	∓ .7572	∓ .7497
$\|3,1,3/2,\pm 3/2\rangle$	.2058	.4346	.7491	1	.6226	.4465	.3177

Table II (cont.)

51. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,11/2,\pm 1/2\rangle$	.5124	.8333	.9608	1	.9634	.8588	.6318
$\|1,5,9/2,\pm 1/2\rangle$	± .2432	± .1553	± .0746	0	∓ .0736	∓ .1428	∓ .2342
$\|2,3,7/2,\pm 1/2\rangle$	- .5443	- .4629	- .2586	0	.2500	.4413	.5753
$\|2,3,5/2,\pm 1/2\rangle$	∓ .3609	∓ .1557	∓ .0406	0	∓ .0392	∓ .1380	∓ .3056
$\|3,1,3/2,\pm 1/2\rangle$	.3529	.1784	.0504	0	.0468	.1535	.3082
$\|3,1,1/2,\pm 1/2\rangle$	± .3567	± .1058	± .0143	0	∓ .0104	∓ .0574	∓ .1618

52. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,11/2,\pm 1/2\rangle$	± .2126	± .1282	± .0350	0	∓ .0298	∓ .1025	∓ .2073
$\|1,5,9/2,\pm 1/2\rangle$	.6842	.7412	.8615	1	.8802	.7928	.7252
$\|2,3,7/2,\pm 1/2\rangle$	± .2411	± .3891	± .3758	0	± .3778	± .4262	± .3783
$\|2,3,5/2,\pm 1/2\rangle$	- .3278	- .2890	- .2522	0	.2258	.2562	.2453
$\|3,1,3/2,\pm 1/2\rangle$	∓ .5229	∓ .4391	∓ .2257	0	± .1739	± .3369	± .4694
$\|3,1,1/2,\pm 1/2\rangle$	- .2182	- .0803	+ .0262	0	+ .0213	+ .0134	- .0865

Table II (cont.)

53. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,5,\tfrac{11}{2},\pm\tfrac{1}{2}\rangle$	.6506	.4492	.2631	0	- .2515	- .4588	- .6646
$\vert 1,5,\tfrac{9}{2},\pm\tfrac{1}{2}\rangle$	$\mp$ .2197	$\mp$ .3245	$\mp$ .3170	0	$\mp$ .3163	$\mp$ .3392	$\mp$ .3137
$\vert 2,3,\tfrac{7}{2},\pm\tfrac{1}{2}\rangle$	- .2028	.2826	.7036	1	.7460	.4500	.0748
$\vert 2,3,\tfrac{5}{2},\pm\tfrac{1}{2}\rangle$	$\pm$ .3999	$\pm$ .4620	$\pm$ .3335	0	$\mp$ .3296	$\mp$ .4614	$\mp$ .4890
$\vert 3,1,\tfrac{3}{2},\pm\tfrac{1}{2}\rangle$	- .1352	- .3775	- .4020	0	.3842	.4295	.3255
$\vert 3,1,\tfrac{1}{2},\pm\tfrac{1}{2}\rangle$	$\mp$ .5560	$\mp$ .5071	$\mp$ .2600	0	$\mp$ .1548	$\mp$ .2731	$\mp$ .3305

54. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,5,\tfrac{11}{2},\pm\tfrac{1}{2}\rangle$	$\pm$ .1585	$\pm$ .0653	$\pm$ .0128	0	$\mp$ .0060	$\pm$ .0150	$\pm$ .0711
$\vert 1,5,\tfrac{9}{2},\pm\tfrac{1}{2}\rangle$	.4881	.4167	.2940	0	- .2869	- .3812	- .4264
$\vert 2,3,\tfrac{7}{2},\pm\tfrac{1}{2}\rangle$	$\pm$ .4158	$\pm$ .3197	$\pm$ .1612	0	$\mp$ .0435	$\mp$ .1698	$\mp$ .3008
$\vert 2,3,\tfrac{5}{2},\pm\tfrac{1}{2}\rangle$	.5321	.5473	.6054	1	.7293	.6238	.5538
$\vert 3,1,\tfrac{3}{2},\pm\tfrac{1}{2}\rangle$	$\pm$ .5208	$\pm$ .6482	$\pm$ .6924	0	$\pm$ .5689	$\pm$ .6473	$\pm$ .6448
$\vert 3,1,\tfrac{1}{2},\pm\tfrac{1}{2}\rangle$	+ .0970	- .0145	- .2037	0	.2454	.1324	.0093

Table II (cont.)

55. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,3/2,\pm 1/2\rangle$	.4675	.2740	.1001	0	.0850	.1975	.3215
$\|1,5,1/2,\pm 1/2\rangle$	∓ .3532	∓ .3129	∓ .1911	0	± .1641	± .2595	± .3262
$\|2,3,7/2,\pm 1/2\rangle$	.4734	.5663	.4624	0	- .4664	- .5863	- .5841
$\|2,3,5/2,\pm 1/2\rangle$	∓ .0002	∓ .0772	∓ .2031	0	∓ .2954	∓ .2679	∓ .2404
$\|3,1,3/2,\pm 1/2\rangle$	- .2982	- .0248	.3698	1	.6419	.3718	.1226
$\|3,1,1/2,\pm 1/2\rangle$	± .5862	± .2068	± .2494	0	∓ .4991	∓ .5830	∓ .6121

56. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,5,3/2,\pm 1/2\rangle$	∓ .2599	∓ .0904	∓ .0373	0	± .0190	± .0476	± .0875
$\|1,5,1/2,\pm 1/2\rangle$	.2479	.2228	.1694	0	.1027	.1481	.1796
$\|2,3,7/2,\pm 1/2\rangle$	∓ .4556	∓ .3625	∓ .2388	0	∓ .1373	∓ .2201	∓ .2952
$\|2,3,5/2,\pm 1/2\rangle$	.3650	.4110	.6447	0	- .4688	- .4915	- .4927
$\|3,1,3/2,\pm 1/2\rangle$	∓ .4739	∓ .4606	∓ .4108	0	± .2904	± .3482	± .3845
$\|3,1,1/2,\pm 1/2\rangle$	.4036	.4747	.5731	1	.8162	.7513	.6945

Table II (cont.)

57. $\Omega = \pm 13/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 13/2\rangle$	1	1	1	1	1	1	1

58. $\Omega = \pm 11/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 11/2\rangle$	.9221	.9935	.9990	1	.9995	.9984	.9959
$\|1,6,11/2,\pm 11/2\rangle$	$\pm$ .3870	$\pm$ .1141	$\pm$ .0457	0	$\mp$ .0324	$\mp$ .0566	$\mp$ .0907

59. $\Omega = \pm 11/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,11/2,\pm 11/2\rangle$	$\mp$ .3870	$\mp$ .1141	$\mp$ .0457	0	$\pm$ .0324	$\pm$ .0566	$\pm$ .0907
$\|1,6,11/2,\pm 11/2\rangle$	.9221	.9935	.9990	1	.9995	.9984	.9959

Table II (cont.)

60. $\Omega = \pm 9/2$

δ	−.4	−.2	−.1	0	.1	.2	.4
$\lvert 1,6,13/2,\pm 9/2\rangle$	.6004	.8987	.9852	1	.9948	.9810	.9696
$\lvert 1,6,11/2,\pm 9/2\rangle$	$\pm$ .0402	$\pm$ .0443	$\pm$ .0371	0	$\mp$ .0402	$\mp$ .0743	$\mp$ .1248
$\lvert 2,4,9/2,\pm 9/2\rangle$	− .7986	− .4364	− .1671	0	.0930	.1489	.2106

61. $\Omega = \pm 9/2$

δ	−.4	−.2	−.1	0	.1	.2	.4
$\lvert 1,6,13/2,\pm 9/2\rangle$	$\mp$ .4601	$\mp$ .1736	$\mp$ .0632	0	$\mp$ .0328	$\mp$ .1091	$\mp$ .1727
$\lvert 1,6,11/2,\pm 9/2\rangle$	.7995	.9496	.9861	1	.7412	.3871	.2612
$\lvert 2,4,9/2,\pm 9/2\rangle$	$\mp$ .3861	$\mp$ .2610	$\mp$ .1536	0	$\pm$ .6704	$\pm$ .9156	$\pm$ .9497

62. $\Omega = \pm 9/2$

δ	−.4	−.2	−.1	0	.1	.2	.4
$\lvert 1,6,13/2,\pm 9/2\rangle$	.6540	.4028	.1590	0	− .0959	− .1257	− .1736
$\lvert 1,6,11/2,\pm 9/2\rangle$	$\pm$ .5993	$\pm$ .3103	$\pm$ .1620	0	$\mp$ .6700	$\mp$ .9190	$\mp$ .9572
$\lvert 2,4,9/2,\pm 9/2\rangle$	.4616	.8611	.9739	1	.7361	.3736	.2317

Table II (cont.)

63. $\Omega = \pm 7/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 7/2\rangle$	.5894	.8750	.9739	1	.9861	.9592	.9032
$\|1,6,11/2,\pm 7/2\rangle$	± .0612	± .0160	± .0216	0	∓ .0470	∓ .0908	∓ .1857
$\|2,4,9/2,\pm 7/2\rangle$	- .6620	- .4673	- .2243	0	.1576	.2617	.3772
$\|2,4,7/2,\pm 7/2\rangle$	∓ .4560	∓ .1252	∓ .0255	0	∓ .0132	∓ .0388	∓ .0856

64. $\Omega = \pm 7/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 7/2\rangle$	± .6184	± .0224	∓ .0650	0	∓ .0587	∓ .1431	∓ .2254
$\|1,6,11/2,\pm 7/2\rangle$	.6016	.7868	.8844	1	.7928	.6374	.5599
$\|2,4,9/2,\pm 7/2\rangle$	± .4598	± .2244	∓ .1473	0	± .6061	± .7368	± .7886
$\|2,4,7/2,\pm 7/2\rangle$	+ .2104	- .5745	- .4380	0	+ .0259	- .0313	- .1177

Table II (cont.)

65. $\Omega = \pm 7/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,7/2,\pm 7/2\rangle$	.7941	.8566	.2126	0	-.1536	-.2637	-.3597
$\|1,6,9/2,\pm 7/2\rangle$	±.5073	±.1383	±.2541	0	±.5038	±.7413	±.7819
$\|2,4,7/2,\pm 7/2\rangle$	-.2043	.7284	.0292	1	.7692	.5683	.4066
$\|2,4,9/2,\pm 7/2\rangle$	±.7387	±.4919	±.1603	0	±.1784	±.2622	±.3042

66. $\Omega = \pm 7/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,7/2,\pm 7/2\rangle$	±.3391	±.1590	±.0449	0	±.0131	±.0120	±.0631
$\|1,6,9/2,\pm 7/2\rangle$	.6120	.4012	.1908	0	-.1223	-.1850	-.2015
$\|2,4,7/2,\pm 7/2\rangle$	±.5533	±.4481	±.2577	0	±.1257	±.1897	±.2455
$\|2,4,9/2,\pm 7/2\rangle$	.4496	.6422	.9825	1	.3035	.0637	.0157

Table II (cont.)

67. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1, 6, 13/2, \pm 5/2\rangle$	.5281	.8545	.9654	1	.9781	.9375	.8660
$\|1, 6, 11/2, \pm 5/2\rangle$	∓ .0598	± .0046	± .0116	0	∓ .0261	∓ .0581	∓ .1160
$\|2, 4, 9/2, \pm 5/2\rangle$	- .5960	- .4800	- .2571	0	.2055	.3777	.4604
$\|2, 4, 7/2, \pm 5/2\rangle$	± .0636	∓ .0201	∓ .0102	0	∓ .0103	∓ .0129	∓ .0808
$\|3, 2, 5/2, \pm 5/2\rangle$	.5985	.1976	.0412	0	.0192	.0504	.0984

68. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1, 6, 11/2, \pm 5/2\rangle$	∓ .4032	∓ .1884	∓ .0706	0	∓ .0704	∓ .2431	∓ .3921
$\|1, 6, 11/2, \pm 5/2\rangle$	.5888	.7528	.8973	1	.8603	.5619	.3986
$\|2, 4, 9/2, \pm 5/2\rangle$	± .0117	∓ .2054	∓ .2072	0	± .4460	± .7437	± .7453
$\|2, 4, 7/2, \pm 5/2\rangle$	- .5097	- .5446	- .3831	0	.1789	.0959	- .0024
$\|3, 2, 5/2, \pm 5/2\rangle$	± .4804	± .2427	± .0123	0	± .0918	± .2506	± .3632

Table II (cont.)

69. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 5/2\rangle$	.5152	.3532	.2237	0	- .1937	- .2386	- .2813
$\|1,6,11/2,\pm 5/2\rangle$	$\pm$ .3706	$\pm$ .2529	$\pm$ .1764	0	$\mp$ .4312	$\mp$ .7506	$\mp$ .8077
$\|2,4,9/2,\pm 5/2\rangle$	- .1899	.3089	.7562	1	.8392	.4710	.2113
$\|2,4,7/2,\pm 5/2\rangle$	$\mp$ .5052	$\mp$ .2494	$\mp$ .0558	0	$\mp$ .1767	$\mp$ .3502	$\mp$ .4498
$\|3,2,5/2,\pm 5/2\rangle$	- .5531	- .8084	- .5864	0	.2028	.1877	.1463

70. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 5/2\rangle$	$\mp$ .2646	$\mp$ .0999	$\mp$ .0266	0	$\mp$ .0007	$\pm$ .0186	$\pm$ .0590
$\|1,6,11/2,\pm 5/2\rangle$	.5413	.5180	.3526	0	- .2176	- .2419	- .2132
$\|2,4,9/2,\pm 5/2\rangle$	$\mp$ .4804	$\mp$ .2997	$\mp$ .1491	0	$\mp$ .0147	$\mp$ .1250	$\mp$ .2456
$\|2,4,7/2,\pm 5/2\rangle$	.5849	.7607	.9052	1	.8987	.6945	.4925
$\|3,2,5/2,\pm 5/2\rangle$	$\mp$ .2531	$\mp$ .2308	$\mp$ .1824	0	$\pm$ .3805	$\pm$ .6657	$\pm$ .8051

Table II (cont.)

71. $\Omega = \pm 5/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 5/2\rangle$	.4723	.3156	.1110	0	.0303	.0682	.1169
$\|1,6,11/2,\pm 5/2\rangle$	± .4685	± .3177	± .1982	0	± .1014	± .2428	± .3601
$\|2,4,9/2,\pm 5/2\rangle$	.6146	.7363	.5448	0	- .2334	- .3088	- .3453
$\|2,4,7/2,\pm 5/2\rangle$	± .3725	± .2492	± .1746	0	∓ .3592	∓ .6203	∓ .7406
$\|3,2,5/2,\pm 5/2\rangle$	.2625	.4418	.7881	1	.8974	.6755	.4346

72. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 3/2\rangle$	.6073	.8486	.9593	1	.9690	.9062	.7892
$\|1,6,11/2,\pm 3/2\rangle$	± .0467	± .0172	± .0126	0	∓ .0238	∓ .0542	∓ .1142
$\|2,4,9/2,\pm 3/2\rangle$	- .6195	- .4891	- .2767	0	.2428	.4038	.5495
$\|2,4,7/2,\pm 3/2\rangle$	∓ .1045	∓ .0259	∓ .0079	0	∓ .0106	∓ .0385	∓ .1101
$\|3,2,5/2,\pm 3/2\rangle$	.4280	.1940	.0341	0	.0372	.1048	.2155
$\|3,2,3/2,\pm 3/2\rangle$	± .2262	± .0457	± .0059	0	∓ .0033	∓ .0163	∓ .0597

Table II (cont.)

73. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 3/2\rangle$	$\pm$ .5347	$\pm$ .3048	$\mp$ .0342	0	$\mp$ .0533	$\mp$ .2577	$\mp$ .4259
$\|1,6,11/2,\pm 3/2\rangle$	.4111	.5783	.8824	1	.9015	.6284	.4801
$\|2,4,9/2,\pm 3/2\rangle$	$\pm$ .0630	$\pm$ .3745	$\mp$ .0552	0	$\pm$ .2993	$\pm$ .5880	$\pm$ .5063
$\|2,4,7/2,\pm 3/2\rangle$	- .2629	- .4104	- .4259	0	.2888	.2330	.1182
$\|3,2,5/2,\pm 3/2\rangle$	$\mp$ .5965	$\mp$ .5099	$\pm$ .0367	0	$\pm$ .0978	$\pm$ .3723	$\pm$ .5494
$\|3,2,3/2,\pm 3/2\rangle$	- .3407	+ .0625	+ .1857	0	+ .0441	- .0022	- .1262

74. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 3/2\rangle$	.2351	.3174	.2598	0	- .2333	- .3012	- .3443
$\|1,6,11/2,\pm 3/2\rangle$	$\mp$ .4517	$\mp$ .4012	$\pm$ .1010	0	$\mp$ .2757	$\mp$ .6073	$\mp$ .6560
$\|2,4,9/2,\pm 3/2\rangle$	- .1433	.2833	.8020	1	.8403	.4540	.1165
$\|2,4,7/2,\pm 3/2\rangle$	$\pm$ .5872	$\pm$ .4210	$\mp$ .0052	0	$\mp$ .1655	$\mp$ .2197	$\mp$ .5572
$\|3,2,5/2,\pm 3/2\rangle$	- .0249	- .4596	- .5181	0	.3623	.3613	.2641
$\|3,2,3/2,\pm 3/2\rangle$	$\mp$ .6122	$\mp$ .5192	$\mp$ .1030	0	$\mp$ .0690	$\mp$ .1660	$\mp$ .2393

Table II (cont.)

75. $\Omega = \pm 3/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,6,1/2,\pm 3/2\rangle$	± .3566	± .2056	± .0262	0	± .0105	± .0610	± .1287
$\vert 1,6,1/2,\mp 3/2\rangle$	.3762	.3154	.3595	0	- .2796	- .3132	- .3326
$\vert 2,4,3/2,\pm 3/2\rangle$	± .6178	± .5883	± .1711	0	∓ .0986	∓ .2853	∓ .4037
$\vert 2,4,3/2,\pm 3/2\rangle$	.4271	.3622	.4912	1	.7754	.5838	.5226
$\vert 3,2,5/2,\pm 3/2\rangle$	± .3942	± .6153	± .4661	0	± .4417	± .6573	± .6596
$\vert 3,2,3/2,\pm 3/2\rangle$	+ .1043	- .0478	- .6182	0	.3401	.2096	.0410

76. $\Omega = \pm 3/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\vert 1,6,3/2,\pm 3/2\rangle$	.3647	.1987	.1002	0	.0602	.1328	.2426
$\vert 1,6,1/2,\pm 3/2\rangle$	∓ .5672	∓ .5249	∓ .1271	0	± .1530	± .3367	± .4305
$\vert 2,4,3/2,\pm 3/2\rangle$	.2438	.3741	.4851	0	- .3659	- .4443	- .4926
$\vert 2,4,3/2,\pm 3/2\rangle$	± .0585	∓ .0638	∓ .0428	0	∓ .3107	∓ .4451	∓ .4210
$\vert 3,2,5/2,\pm 3/2\rangle$	- .3916	- .0605	.6703	1	.8019	.5010	.2347
$\vert 3,2,3/2,\pm 3/2\rangle$	± .5736	± .7335	± .5260	0	∓ .3153	∓ .4717	∓ .5299

Table II (cont.)

77. $\Omega = \pm 3/2$

ε	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,\tfrac{1}{2},\pm\tfrac{3}{2}\rangle$	∓ .1725	∓ .0670	∓ .0187	0	± .0055	± .0166	± .0428
$\|1,6,\tfrac{3}{2},\pm\tfrac{3}{2}\rangle$	.4018	.3598	.2562	0	.0958	.1478	.1731
$\|2,4,\tfrac{1}{2},\pm\tfrac{3}{2}\rangle$	∓ .3880	∓ .2325	∓ .1118	0	∓ .0411	∓ .0791	∓ .1498
$\|2,4,\tfrac{3}{2},\pm\tfrac{3}{2}\rangle$	.6240	.7198	.7585	0	- .4374	- .4787	- .1616
$\|3,2,\tfrac{5}{2},\pm\tfrac{3}{2}\rangle$	∓ .3894	∓ .3301	∓ .2464	0	± .1400	± .1917	± .3032
$\|2,2,\tfrac{3}{2},\pm\tfrac{3}{2}\rangle$	.3475	.4300	.5342	1	.8822	.8401	.8005

78. $\Omega = \pm 1/2$

ε	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,\tfrac{1}{2},\pm\tfrac{1}{2}\rangle$	.5278	.8383	.9561	1	.9631	.8829	.7210
$\|1,6,\tfrac{3}{2},\mp\tfrac{1}{2}\rangle$	∓ .0640	± .0028	± .0043	0	∓ .0080	∓ .0235	∓ .0703
$\|2,4,\tfrac{3}{2},\mp\tfrac{1}{2}\rangle$	- .5714	- .4957	- .2863	0	.2640	.4414	.5862
$\|2,4,\tfrac{3}{2},\pm\tfrac{1}{2}\rangle$	± .0987	∓ .0017	∓ .0027	0	∓ .0048	∓ .0221	∓ .0884
$\|3,2,\tfrac{5}{2},\mp\tfrac{1}{2}\rangle$	.4677	.2116	.0609	0	.0503	.1512	.3206
$\|3,2,\tfrac{3}{2},\pm\tfrac{1}{2}\rangle$	∓ .2378	∓ .0060	± .0010	0	∓ .0027	∓ .0193	∓ .0865
$\|4,0,\tfrac{1}{2},\pm\tfrac{1}{2}\rangle$	- .3787	- .0816	- .0109	0	.0071	.0361	.1165

Table II (cont.)

79. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 1/2\rangle$	∓ .4533	∓ .3097	∓ .0411	0	∓ .0241	∓ .2963	∓ .4736
$\|1,6,11/2,\pm 1/2\rangle$	.4725	.5683	.8562	1	.8875	.5694	.4153
$\|2,4,9/2,\pm 1/2\rangle$	± .0878	∓ .2804	∓ .1146	0	± .1731	± .4685	± .2596
$\|2,4,7/2,\pm 1/2\rangle$	- .3979	- .4331	- .4598	0	.4032	.3432	.2348
$\|3,2,5/2,\pm 1/2\rangle$	± .3542	± .4494	± .0342	0	± .0630	± .4691	± .5964
$\|3,2,3/2,\pm 1/2\rangle$	.0453	.1593	.1874	0	+ .1240	- .0566	- .0976
$\|4,0,1/2,\pm 1/2\rangle$	∓ .5269	∓ .2959	± .0662	0	∓ .0113	± .1655	± .3197

80. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 1/2\rangle$	.3656	.2653	.2625	0	- .2669	- .2804	- .2922
$\|1,6,11/2,\pm 1/2\rangle$	± .4129	± .4247	± .0808	0	∓ .1462	∓ .5364	∓ .5651
$\|2,4,9/2,\pm 1/2\rangle$	- .2296	.1860	.7548	1	.8043	.1925	- .0150
$\|2,4,7/2,\pm 1/2\rangle$	∓ .5159	∓ .4089	∓ .0474	0	∓ .1012	∓ .4508	∓ .5891
$\|3,2,5/2,\pm 1/2\rangle$	- .0672	- .4196	- .5361	0	.4737	.4249	.2139
$\|3,2,3/2,\pm 1/2\rangle$	± .4596	± .3398	± .0427	0	∓ .0830	∓ .2921	∓ .3964
$\|4,0,1/2,\pm 1/2\rangle$	.4014	.5058	.2517	0	.1378	.2323	.2127

Table II (cont.)

81. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,6,13/2,\pm 1/2\rangle$	± .2249	± .1072	± .0286	0	∓ .0275	∓ .0500	∓ .1050
$\lvert 1,6,11/2,\pm 1/2\rangle$	.3320	.3067	.2761	0	- .1905	- .2650	- .3144
$\lvert 2,4,9/2,\pm 1/2\rangle$	± .5080	± .3772	± .1838	0	± .1147	± .2165	± .3132
$\lvert 2,4,7/2,\pm 1/2\rangle$	.4350	.4532	.5397	1	.5231	.5266	.5206
$\lvert 3,2,5/2,\pm 1/2\rangle$	± .5598	± .6166	± .5208	0	± .3972	± .4715	± .4904
$\lvert 3,2,3/2,\pm 1/2\rangle$	+ .1303	+ .0268	- .1119	0	- .2605	- .2804	- .2991
$\lvert 4,0,1/2,\pm 1/2\rangle$	± .2479	± .4073	± .5604	0	± .6712	± .5496	± .4280

82. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\lvert 1,6,13/2,\pm 1/2\rangle$	.4612	.2926	.1039	0	.0412	.1970	.3248
$\lvert 1,6,11/2,\pm 1/2\rangle$	± .4337	± .3682	± .2164	0	± .1707	± .3648	± .4533
$\lvert 2,4,9/2,\pm 1/2\rangle$	.2984	.5143	.4501	0	- .4662	- .5166	- .4566
$\lvert 2,4,7/2,\pm 1/2\rangle$	∓ .0413	∓ .0035	± .0783	0	∓ .1595	∓ .1570	∓ .1090
$\lvert 3,2,5/2,\pm 1/2\rangle$	- .4491	- .3805	.3479	1	.6088	.2132	- .1085
$\lvert 3,2,3/2,\pm 1/2\rangle$	∓ .4833	∓ .4165	∓ .3077	0	∓ .3659	∓ .5272	∓ .5808
$\lvert 4,0,1/2,\pm 1/2\rangle$	- .2704	- .5413	- .7197	0	.4710	.4617	.3460

Table II (cont.)

83. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 1/2\rangle$	$\mp$ .0827	$\mp$ .0323	$\mp$ .0091	0	$\mp$ .0016	$\mp$ .0127	$\mp$ .0321
$\|1,6,11/2,\pm 1/2\rangle$	.1139	.2659	.1918	0	.1266	.1717	.2019
$\|2,4,9/2,\pm 1/2\rangle$	$\mp$ .2318	$\mp$ .1349	$\mp$ .0634	0	$\pm$ .0308	$\pm$ .0712	$\pm$ .1236
$\|2,4,7/2,\pm 1/2\rangle$	.5691	.5963	.5919	0	- .4555	-.4519	- .4368
$\|3,2,5/2,\pm 1/2\rangle$	$\mp$ .3676	$\mp$ .2947	$\mp$ .2091	0	$\mp$ .1554	$\mp$ .2369	$\mp$ .3041
$\|3,2,3/2,\pm 1/2\rangle$	.3302	.6023	.6838	1	.6755	.5987	.5413
$\|4,0,1/2,\pm 1/2\rangle$	$\mp$ .3174	$\mp$ .3238	$\mp$ .3123	0	$\pm$ .5432	$\pm$ .5886	$\pm$ .6054

84. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,6,13/2,\pm 1/2\rangle$	.3349	.1808	.0593	0	- .0406	- .1116	- .2301
$\|1,6,11/2,\pm 1/2\rangle$	$\mp$ .4530	$\mp$ .4255	$\mp$ .3170	0	$\pm$ .3303	$\pm$ .3934	$\pm$ .3776
$\|2,4,9/2,\pm 1/2\rangle$	.4606	.4648	.3079	0	.1814	.3954	.5166
$\|2,4,7/2,\pm 1/2\rangle$	$\mp$ .2257	$\mp$ .2903	$\mp$ .3723	0	$\mp$ .5634	$\mp$ .4152	$\mp$ .3414
$\|3,2,5/2,\pm 1/2\rangle$	- .0324	+ .2785	+ .5212	0	- .4593	- .5026	- .3783
$\|3,2,3/2,\pm 1/2\rangle$	$\pm$ .4858	$\pm$ .5676	$\pm$ .6232	0	$\mp$ .5726	$\mp$ .4428	$\mp$ .3258
$\|4,0,1/2,\pm 1/2\rangle$	- .4272	- .2935	+ .0508	1	+ .0087	+ .2155	- .3953

Table II (cont.)

B5. $\Omega = \pm 15/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 15/2\rangle$	1	1	1	1	1	1	1

B6. $\Omega = \pm 13/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 13/2\rangle$	.9350	.9950	.9992	1	.9996	.9987	.9967
$\|1,7,13/2,\pm 13/2\rangle$	$\pm$ .3547	$\pm$ .1000	$\pm$ .0403	0	$\mp$ .0288	$\mp$ .0505	$\mp$ .0813

B7. $\Omega = \pm 13/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 13/2\rangle$	$\mp$ .3547	$\mp$ .1000	$\mp$ .0403	0	$\pm$ .0288	$\pm$ .0505	$\pm$ .0813
$\|1,7,13/2,\pm 13/2\rangle$	.9350	.9950	.9992	1	.9996	.9987	.9967

Table II (cont.)

88. $\Omega = \pm 11/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,5/2,\pm 1/2\rangle$	.9715	.9902	.9963	1	.9954	.9814	.9722
$\|1,7,3/2,\pm 1/2\rangle$	$\pm$.0219	$\pm$.0903	$\pm$.0373	0	$\pm$.0363	$\pm$.0668	$\pm$.1119
$\|2,5,3/2,\pm 1/2\rangle$	-.8125	-.4363	-.2668	0	.0890	.3433	.7057

89. $\Omega = \pm 11/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,5/2,\pm 1/2\rangle$	$\pm$.4963	$\pm$.1472	$\pm$.0542	0	$\pm$.0008	$\pm$.0134	$\pm$.1135
$\|1,7,3/2,\pm 1/2\rangle$	.8650	.9728	.9933	1	.9852	.8678	.4565
$\|2,5,3/2,\pm 1/2\rangle$	$\pm$.3728	$\pm$.1829	$\pm$.1019	0	$\pm$.1708	$\pm$.4907	$\pm$.8756

90. $\Omega = \pm 11/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,5/2,\pm 1/2\rangle$	.6978	.4115	.1559	0	-.0979	-.1575	-.1973
$\|1,7,3/2,\pm 1/2\rangle$	$\pm$.5343	$\pm$.2262	$\pm$.1097	0	$\pm$.1482	$\pm$.4974	$\pm$.8827
$\|2,5,3/2,\pm 1/2\rangle$	.4770	.8829	.9837	1	.9813	.8560	.4988

Table II (cont.)

31. $\Omega = \pm 3/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
[illegible]⟩	.9674	.9941	.9740	1	.9983	.9680	.9318
[illegible]⟩	± .0939	± .0464	± .0316	0	± .0393	± .0729	± .1192
[illegible]⟩	- .4876	- .4613	- .2229	0	.1478	.2391	.3198
[illegible]⟩	± .7108	± .0576	± .0118	0	± .0058	± .0217	± .0391

32. $\Omega = \pm 3/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
[illegible]⟩	± .6249	± .1062	± .0329	0	± .0082	± .0759	± .2535
[illegible]⟩	.4915	.6678	.9771	1	.9721	.8152	.4358
[illegible]⟩	± .5671	± .0444	± .0909	0	± .2001	± .5622	± .8436
[illegible]⟩	- .2473	- .7354	- .3627	0	.3790	.1172	.0064

Table II (cont.)

93. $\Omega = \pm 9/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,{}^{15}/_2,\pm{}^9/_2\rangle$	-.2135	.4402	.2179	0	-.3505	-.2381	-.2574
$\|1,7,{}^{13}/_2,\pm{}^9/_2\rangle$	$\mp$.1665	$\mp$.2506	$\mp$.3212	0	$\mp$.3888	$\mp$.5432	$\mp$.8163
$\|2,5,{}^{9}/_2,\pm{}^9/_2\rangle$	.1853	.9548	.9664	1	.9075	.7883	.3586
$\|2,5,{}^{11}/_2,\pm{}^9/_2\rangle$	$\mp$.9381	$\mp$.1126	$\mp$.0025	0	$\mp$.0717	$\mp$.1700	$\mp$.3662

94. $\Omega = \pm 9/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,{}^{15}/_2,\pm{}^9/_2\rangle$	$\mp$.3618	$\mp$.1152	$\mp$.0729	0	$\mp$.0062	$\mp$.0158	$\mp$.0331
$\|1,7,{}^{13}/_2,\pm{}^9/_2\rangle$	.7796	.6993	.3534	0	-.1337	-.1930	-.2665
$\|2,5,{}^{9}/_2,\pm{}^9/_2\rangle$	$\mp$.4139	$\mp$.7333	$\mp$.0993	0	$\mp$.0663	$\mp$.0737	$\mp$.1075
$\|2,5,{}^{11}/_2,\pm{}^9/_2\rangle$	.3068	.6658	.9299	1	.9899	.9783	.9626

Table II (cont.)

5. $\Omega = \pm 7/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,5/2,\pm 3/2\rangle$	.4963	.8462	.9634	1	.9795	.9636	.8810
$\|1,2,3/2,\pm 3/2\rangle$	± .0238	± .0093	± .0246	0	± .0357	± .0706	± .2276
$\|2,2,5/2,\pm 3/2\rangle$	- .6137	- .5002	- .2662	0	.1974	.3700	.6462
$\|2,2,3/2,\pm 3/2\rangle$	± .0273	± .0319	± .0116	0	± .0092	± .0280	± .0071
$\|3,3,5/2,\pm 3/2\rangle$	.6137	.1792	.0317	0	.0248	.0389	.0745

6. $\Omega = \pm 7/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,5/2,\pm 3/2\rangle$	± .3737	± .2027	± .0729	0	± .0037	± .1314	± .3573
$\|1,2,3/2,\pm 3/2\rangle$	.5601	.7182	.8563	1	.9573	.7791	.4166
$\|2,2,5/2,\pm 3/2\rangle$	± .0765	± .2140	± .1974	0	± .1973	± .5013	± .1455
$\|2,2,3/2,\pm 3/2\rangle$	- .5986	- .6175	- .4154	0	.2101	.2077	.0371
$\|3,3,5/2,\pm 3/2\rangle$	± .4272	± .3280	± .2733	0	± .0711	± .1331	± .2607

Table II (cont.)

[illegible]. Ω = ± 1/2

ξ	-.4	-.2	-.1	0	.1	.2	.4
|1, 2, ½, ± ½⟩	.5377	.3759	.2502	0	- .2502	- .2996	- .[illegible]
|1, 2, ½, ± ½⟩	± .3952	± .2044	± .1167	0	± .1171	± .1326	± .[illegible]
|2, 2, ½, ± ½⟩	- .2131	.5735	.8482	1	.9634	.7352	.2463
|2, 2, ½, ± ½⟩	± .4729	± .3813	± .0000	0	± .0974	± .2640	± .4783
|3, 1, ½, ± ½⟩	- .[illegible]	- .5026	- .1129	0	.1518	.2031	.3209

[illegible]. Ω = ± 3/2

ξ	-.4	-.2	-.1	0	.1	.2	.4
|1, 2, ½, ± ½⟩	± .2925	± .0948	± .[illegible]	0	± .0006	± .0096	± .0444
|1, 2, ½, ± ½⟩	.6206	.3787	.3735	0	- .2032	- .2275	- .1681
|2, 2, ½, ± ½⟩	± .4257	± .2626	± .1167	0	± .0113	± .0987	± .2189
|2, 2, ½, ± ½⟩	.5608	.7513	.9082	1	.9710	.9969	.4188
|3, 3, ½, ± ½⟩	± .3195	± .3505	± .1180	0	± .3922	± .6728	± .8643

Table II (cont.)

9. [illegible]

i	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]

10. [illegible]

i	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]

Table [illegible] (cont.)

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]

Table II (cont.)

10a. $\beta = \pm 1/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
|1,1,½,±½⟩	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
|1,1,½,±½⟩	[illegible]	[illegible]	[illegible]	0	− [illegible]	− [illegible]	− [illegible]
|2,2,½,±½⟩	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
|2,2,½,±½⟩	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
|3,3,½,±½⟩	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
|3,3,½,±½⟩	− [illegible]	− [illegible]	− [illegible]	0	[illegible]	[illegible]	[illegible]

10b. $\beta = \pm 3/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
|1,1,½,±½⟩	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
|1,1,½,±½⟩	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
|2,2,½,±½⟩	[illegible]	[illegible]	[illegible]	0	− [illegible]	− [illegible]	− [illegible]
|2,2,½,±½⟩	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
|3,3,½,±½⟩	− [illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
|3,3,½,±½⟩	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]

Table II (cont.)

[illegible] [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	- [illegible]	- [illegible]	- [illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]

[illegible] [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	- [illegible]	- [illegible]	- [illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
[illegible]	± [illegible]	± [illegible]	± [illegible]	0	± [illegible]	± [illegible]	± [illegible]
[illegible]	- [illegible]	- [illegible]	- [illegible]	0	[illegible]	[illegible]	[illegible]

Table II (cont.)

10F. $\Omega = \pm 3/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,15/2,\pm 1/2\rangle$	± .4080	± .2996	± .0538	0	± .0708	± .2791	± .4564
$\|1,2,13/2,\pm 1/2\rangle$	.4456	.6134	.8776	1	.9122	.7464	.4426
$\|2,5,9/2,\pm 1/2\rangle$	± .1396	± .3213	± .0662	0	± .1204	± .3989	± .4768
$\|2,5,7/2,\pm 1/2\rangle$	- .5033	- .5710	- .4450	0	.3748	.4154	.2097
$\|3,3,7/2,\pm 1/2\rangle$	± .2952	± .2603	± .0030	0	± .0026	± .2554	± .5753
$\|3,3,5/2,\pm 1/2\rangle$	.2766	.3290	.1546	0	.0884	.1064	- .0410
$\|4,1,3/2,\pm 1/2\rangle$	± .4569	± .0468	± .0490	0	± .0052	± .0467	± .1923

10F. $\Omega = \pm 3/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,15/2,\pm 3/2\rangle$	.3858	.3890	.2978	0	- .2129	- .3822	- .1312
$\|1,2,13/2,\pm 3/2\rangle$	± .2093	± .2277	± .0626	0	± .0699	± .3763	± .6067
$\|2,5,9/2,\pm 3/2\rangle$	- .2461	.3096	.9289	1	.8937	.5943	.1001
$\|2,5,7/2,\pm 3/2\rangle$	± .4398	± .2302	± .0085	0	± .0839	± .3151	± .5748
$\|3,3,7/2,\pm 3/2\rangle$	- .1267	- .5625	- .4490	0	.3727	.4764	.7748
$\|3,3,5/2,\pm 3/2\rangle$	± .4452	± .1779	± .0115	0	± .0448	± .2654	± .2995
$\|4,1,3/2,\pm 3/2\rangle$	.5270	.5482	.1999	0	.0584	.2316	.1291

Table II (cont.)

109. $\Omega = \pm 3/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 3/2\rangle$	∓.3755	∓.1769	∓.0406	0	∓.0361	±.0656	±.1821
$\|1,7,13/2,\pm 3/2\rangle$	.5025	.4836	.3435	0	-.3125	-.3915	-.3611
$\|2,5,11/2,\pm 3/2\rangle$	∓.4002	∓.4138	∓.2040	0	∓.1109	∓.2839	∓.4828
$\|2,5,9/2,\pm 3/2\rangle$	.1102	.2331	.4688	1	.7631	.5637	.4111
$\|3,3,7/2,\pm 3/2\rangle$	±.1501	∓.2536	∓.4074	0	±.3954	±.5389	±.5203
$\|3,3,5/2,\pm 3/2\rangle$	-.5246	-.6358	-.4462	0	.3794	.3537	.2038
$\|4,1,3/2,\pm 3/2\rangle$	±.3693	±.2016	∓.1882	0	±.0778	±.1730	±.3141

110. $\Omega = \pm 3/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 3/2\rangle$	.4683	.2818	.0813	0	.2664	.1470	.2192
$\|1,7,13/2,\pm 3/2\rangle$	±.4640	±.3950	±.1510	0	±.1484	±.3328	±.4824
$\|2,5,11/2,\pm 3/2\rangle$	.2466	.5037	.3894	0	-.3522	-.4658	-.4474
$\|2,5,9/2,\pm 3/2\rangle$	∓.1249	∓.0513	±.1056	0	∓.2612	∓.3157	∓.3084
$\|3,3,7/2,\pm 3/2\rangle$	-.4437	-.0693	.5918	1	.7845	.5006	.3441
$\|3,3,5/2,\pm 3/2\rangle$	∓.4464	∓.3006	∓.1524	0	∓.2397	∓.4487	∓.5680
$\|4,1,3/2,\pm 3/2\rangle$	-.3046	-.6426	-.6591	0	.7044	.3344	.2579

Table II (cont.)

111. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 1/2\rangle$	$\mp$ .1258	$\mp$ .0432	$\mp$ .0104	0	$\mp$ .0061	$\mp$ .0091	$\mp$ .0224
$\|1,7,13/2,\pm 1/2\rangle$	.3159	.2695	.1531	0	.0349	.0570	.0736
$\|2,5,11/2,\pm 3/2\rangle$	$\mp$ .3030	$\mp$ .1638	$\mp$ .0694	0	$\pm$ .0260	$\pm$ .0597	$\pm$ .1010
$\|2,5,9/2,\pm 3/2\rangle$	.5702	.6028	.5364	0	- .1865	- .2092	- .2162
$\|3,3,7/2,\pm 3/2\rangle$	$\mp$ .4153	$\mp$ .3340	$\mp$ .2377	0	$\mp$ .1760	$\mp$ .2156	$\mp$ .3199
$\|3,3,5/2,\pm 3/2\rangle$	.4707	.5920	.7315	1	.4907	.4273	.3855
$\|4,1,3/2,\pm 3/2\rangle$	$\mp$ .2453	$\mp$ .2706	$\mp$ .3097	0	$\pm$ .8374	$\pm$ .8326	$\pm$ .8303

112. $\Omega = \pm 3/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 1/2\rangle$	.2603	.1017	.0175	0	- .1227	- .0231	- .0561
$\|1,7,13/2,\pm 1/2\rangle$	$\pm$ .3684	$\pm$ .3304	$\pm$ .2479	0	$\mp$ .0646	$\mp$ .1793	$\mp$ .2451
$\|2,5,11/2,\pm 3/2\rangle$	.5113	.3536	.1266	0	.0224	.1095	.1822
$\|2,5,9/2,\pm 3/2\rangle$	$\pm$ .4454	$\pm$ .4477	$\pm$ .5322	0	$\pm$ .3469	$\pm$ .5158	$\pm$ .5543
$\|3,3,7/2,\pm 3/2\rangle$	.5206	.6234	.4721	0	- .1871	- .2937	- .3393
$\|3,3,5/2,\pm 3/2\rangle$	$\pm$ .1599	$\pm$ .0364	$\pm$ .0087	0	$\mp$ .7039	$\mp$ .6734	$\mp$ .6248
$\|4,1,3/2,\pm 3/2\rangle$	.2002	.4058	.4450	1	.5739	.3870	.3030

Table II (cont.)

III. $\lambda = \pm 1/2$

ξ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,2,\frac{1}{2},\pm\frac{1}{2}\rangle$	.5139	.8142	.9666	1	.9585	.8746	.7052
$\|1,2,\frac{3}{2},\pm\frac{1}{2}\rangle$	$\pm$ .0294	$\pm$ .0045	$\pm$ .0035	0	$\mp$ .0064	$\mp$ .0147	$\mp$ .0362
$\|2,2,\frac{1}{2},\pm\frac{1}{2}\rangle$	- .6038	- .5325	- .3153	0	.2790	.4568	.5964
$\|2,2,\frac{3}{2},\pm\frac{1}{2}\rangle$	$\mp$ .0636	$\mp$ .0070	$\mp$ .0023	0	$\mp$ .0031	$\mp$ .0128	$\mp$ .0502
$\|3,1,\frac{1}{2},\pm\frac{1}{2}\rangle$	.1692	.7195	.1659	0	.0577	.1549	.2343
$\|3,3,\frac{3}{2},\pm\frac{1}{2}\rangle$	$\pm$ .1158	$\pm$ .0136	$\pm$ .0017	0	$\mp$ .0014	$\mp$ .0107	$\mp$ .0231
$\|4,1,\frac{1}{2},\pm\frac{1}{2}\rangle$	- .2950	- .0480	- .0111	0	.0028	.0618	.1464
$\|4,1,\frac{3}{2},\pm\frac{1}{2}\rangle$	$\mp$ .2143	$\pm$ .0277	$\mp$ .0017	0	$\mp$ .0010	$\mp$ .0092	$\mp$ .0515

Table II (cont.)

114. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/4,\pm 1/2\rangle$	± .4468	± .2014	∓ .0069	0	∓ .0080	∓ .1180	∓ .3885
$\|1,7,9/4,\pm 1/2\rangle$	.1067	.5926	.8841	1	.9146	.7466	.5117
$\|2,5,11/4,\pm 1/2\rangle$	∓ .0920	± .2200	∓ .0054	0	± .0528	± .2167	± .1943
$\|2,5,9/4,\pm 1/2\rangle$	- .4564	- .5601	- .4338	0	.3607	.5296	.4344
$\|3,3,9/4,\pm 1/2\rangle$	∓ .3631	∓ .2549	± .0072	0	± .0039	± .1590	± .4782
$\|3,3,5/4,\pm 1/2\rangle$	.2021	.3726	.1627	0	.1207	.2636	.1655
$\|4,1,3/4,\pm 1/2\rangle$	± .4476	± .0831	∓ .0098	0	∓ .0113	± .0214	± .3202
$\|4,1,5/4,\pm 1/2\rangle$	+ .2162	- .1884	- .0595	0	+ .0288	+ .0771	- .0479

Table II (cont.)

115. $\Omega = \pm 1/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 1/2\rangle$	.3161	.4395	.3074	0	-.2739	-.4086	-.3779
$\|1,7,13/2,\pm 1/2\rangle$	± .2552	± .1515	± .0184	0	∓ .0333	∓ .1604	∓ .3585
$\|2,5,11/2,\pm 1/2\rangle$	-.2719	.3688	.8229	1	.8548	.5373	.0339
$\|2,5,9/2,\pm 1/2\rangle$	± .4730	± .1964	± .0013	0	∓ .0333	∓ .1743	∓ .4572
$\|3,3,7/2,\pm 1/2\rangle$	-.0391	-.5702	-.4548	0	.4214	.5906	.3561
$\|3,3,5/2,\pm 1/2\rangle$	∓ .5045	∓ .2309	∓ .0140	0	∓ .0283	∓ .1651	∓ .4305
$\|4,1,3/2,\pm 1/2\rangle$	.2348	.3629	.1399	0	.1149	.3105	.3643
$\|4,1,1/2,\pm 1/2\rangle$	± .5434	± .3148	± .0369	0	∓ .0235	∓ .1252	∓ .2712

Table II (cont.)

11b. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,1,1/2,\pm 1/2\rangle$	± .3968	± .2045	± .0238	0	± .0174	± .1168	± .2698
$\|1,1,3/2,\pm 1/2\rangle$	.4953	.4257	.3779	0	- .3429	- .4217	- .4347
$\|2,2,3/2,\pm 1/2\rangle$	± .4209	± .4944	± .1331	0	∓ .1156	∓ .3829	∓ .5113
$\|2,2,5/2,\pm 1/2\rangle$	.1508	.2362	.5009	1	.6446	.3901	.2830
$\|3,3,5/2,\pm 1/2\rangle$	∓ .1274	± .3165	± .2986	0	± .3104	± .4126	± .1853
$\|3,3,7/2,\pm 1/2\rangle$	- .4232	- .4060	- .5920	0	.5631	.4725	.3673
$\|4,4,7/2,\pm 1/2\rangle$	∓ .4316	∓ .4601	∓ .0774	0	± .0295	± .3251	± .4749
$\|4,4,9/2,\pm 1/2\rangle$	- .1535	- .0014	+ .3773	0	+ .1973	+ .1024	- .0440

Table II (cont.)

117. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,5/2,\mp 1/2\rangle$	.4799	.2161	.0900	0	.0746	.1909	.3983
$\|1,7,[illegible]/2,\mp 1/2\rangle$	∓ .5066	∓ .5019	∓ .0857	0	± .1152	± .3827	± .5190
$\|2,1,7/2,\pm 1/2\rangle$	.1541	.3632	.4737	0	- .4007	- .4850	- .3902
$\|2,5,3/2,\pm 1/2\rangle$	± .2570	± .1104	∓ .0480	0	∓ .1250	∓ .3117	± .0137
$\|1,3,5/2,\pm 1/2\rangle$	- .4229	- .0682	.6288	1	.7065	.2031	- .1942
$\|3,3,5/2,\pm 1/2\rangle$	± .2447	± .1396	± .1744	0	∓ .2206	∓ .4290	∓ .4623
$\|4,1,3/2,\mp 1/2\rangle$	- .0960	- .2660	- .5163	0	.4723	.4671	.2573
$\|4,1,1/2,\pm 1/2\rangle$	∓ .4699	∓ .6094	∓ .3348	0	∓ .1849	∓ .3560	∓ .1909

11B. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 1/2\rangle$	± .3346	± .0394	± .0300	0	∓ .0059	∓ .0237	∓ .0628
$\|1,7,13/2,\pm 1/2\rangle$	.2794	.2202	.1323	0	.0893	.1524	.1933
$\|2,5,11/2,\pm 1/2\rangle$	± .3300	± .1632	± .0757	0	± .0519	± .1231	± .2179
$\|2,5,9/2,\pm 1/2\rangle$	.4985	.5168	.4645	0	- .4040	- .4509	- .4515
$\|3,3,7/2,\pm 1/2\rangle$	± .4862	± .4012	± .3186	0	∓ .2567	∓ .3710	∓ .4543
$\|3,3,5/2,\pm 1/2\rangle$	.4105	.4787	.5094	1	.6074	.5239	.4608
$\|4,1,3/2,\pm 1/2\rangle$	± .3720	± .5154	± .6242	0	± .6184	± .5858	± .5292
$\|4,1,1/2,\pm 1/2\rangle$	.0547	.0206	- .1014	0	.0903	.0501	- .0110

Table II (cont.)

119. $\Omega = \pm 1/2$

ζ	-.4	-.2	-.1	0	.1	.2	.4
$\|1, 7, 15/2, \pm 1/2\rangle$	.2862	.1135	.0237	0	- .0147	- .0583	- .1422
$\|1, 7, 13/2, \pm 1/2\rangle$	∓ .4170	∓ .3710	∓ .2193	0	∓ .1495	∓ .2610	∓ .3301
$\|2, 5, 11/2, \pm 1/2\rangle$	.4874	.3470	.1444	0	.1067	.2390	.3731
$\|2, 5, 9/2, \pm 1/2\rangle$	∓ .4045	∓ .4863	∓ .5332	0	± .4929	± .5379	± .5218
$\|3, 3, 7/2, \pm 1/2\rangle$	.2766	.4576	.4131	0	- .3962	- .4792	- .4596
$\|3, 3, 5/2, \pm 1/2\rangle$	± .1449	± .1621	± .0012	0	∓ .2818	∓ .1592	∓ .1730
$\|4, 1, 3/2, \pm 1/2\rangle$	- .2633	.0006	.2134	1	.5208	.3094	.1001
$\|4, 1, 1/2, \pm 1/2\rangle$	± .4211	± .5068	± .6142	0	∓ .4725	∓ .4809	∓ .4727

Table II (cont.)

120. $\Omega = \pm 1/2$

δ	-.4	-.2	-.1	0	.1	.2	.4
$\|1,7,15/2,\pm 1/2\rangle$	$\mp$ .0552	$\mp$ .0185	$\mp$ .0043	0	$\mp$ .0036	$\mp$ .0063	$\mp$ .0167
$\|1,7,13/2,\pm 1/2\rangle$	.1279	.0814	.0485	0	- .0207	- .0409	- .0605
$\|2,5,11/2,\pm 1/2\rangle$	$\mp$ .1738	$\mp$ .0922	$\mp$ .0354	0	$\pm$ .0159	$\pm$ .0386	$\pm$ .0716
$\|2,5,9/2,\pm 1/2\rangle$	.3279	.2753	.2418	0	.1287	.1781	.2082
$\|3,3,7/2,\pm 1/2\rangle$	$\mp$ .3722	$\mp$ .2946	$\mp$ .1789	0	$\mp$ .0961	$\mp$ .1523	$\mp$ .2076
$\|3,3,5/2,\pm 1/2\rangle$	.5126	.5219	.5770	0	- .4130	- .4428	- .4466
$\|4,1,3/2,\pm 1/2\rangle$	$\mp$ .5042	$\mp$ .5555	$\mp$ .4688	0	$\pm$ .3300	$\pm$ .3722	$\pm$ .4018
$\|4,1,1/2,\pm 1/2\rangle$	.4328	.4908	.5942	1	.8331	.7792	.7373

Table III. Radial components of the isotropic harmonic oscillator wave functions.

$$R_{1,\ell}(\alpha r) = \left[\frac{\alpha^3}{\pi^{\frac{1}{2}}} \, \frac{(\ell+1)\,!\, 2^{2\ell+3}}{(2\ell+2)\,!} \right]^{\frac{1}{2}} (\alpha r)^{\ell} \; e^{-\frac{1}{2}\alpha^2 r^2}$$

$$R_{2,\ell}(\alpha r) = \left[\frac{\alpha^3}{\pi^{\frac{1}{2}}} \, \frac{(\ell+1)\,!\, 2^{2\ell+2}}{(2\ell+3)\,!} \right]^{\frac{1}{2}} \left[2(\alpha r)^2 - (2\ell+3) \right] (\alpha r)^{\ell} \; e^{-\frac{1}{2}\alpha^2 r^2}$$

$$R_{3,\ell}(\alpha r) = \left[\frac{\alpha^3}{\pi^{\frac{1}{2}}} \, \frac{(\ell+2)\,!\, 2^{2\ell+1}}{(2\ell+5)\,!} \right]^{\frac{1}{2}} \left[4(\alpha r)^4 - 4(2\ell+5)(\alpha r)^2 + (2\ell+5)(2\ell+3) \right]$$

$$\cdot \, (\alpha r)^{\ell} \; e^{-\frac{1}{2}\alpha^2 r^2}$$

$$R_{4,\ell}(\alpha r) = \left[\frac{\alpha^3}{3\pi^{\frac{1}{2}}} \, \frac{(\ell+3)\,!\, 2^{2\ell+1}}{(2\ell+7)\,!} \right]^{\frac{1}{2}} \left[8(\alpha r)^6 - 12(2\ell+7)(\alpha r)^4 \right.$$

$$\left. + \, 6(2\ell+7)(2\ell+5)(\alpha r)^2 - (2\ell+7)(2\ell+5)(2\ell+3) \right]$$

$$\cdot \, (\alpha r)^{\ell} \; e^{-\frac{1}{2}\alpha^2 r^2}$$

e IV. Decoupling Constants.

tal ber	δ =.1	δ =.2	δ =.4
1	4.8817	4.5637	3.7485
2	-2.4288	-.3989	+.6523
3	+1.5107	-.3445	-.3423
4	-.2124	+.0207	+.2657
5	-.7500	-.2414	-.7248
1	-5.7914	-5.0819	-3.3366
2	3.3904	2.3231	1.5466
3	-2.0507	-1.1534	-1.5658
4	1.4123	.9568	.6055
5	-1.0920	-.9936	-.9348
6	1.1320	.9490	.6617
[illegible]	6.8486	6.4946	5.6531
[illegible]	-5.2418	-.0359	+1.8151
[illegible]	4.2423	-1.0111	-2.8372
1	-.4530	-.4669	-.3600
2	1.8880	.5026	-.0183
3	-1.4665	-1.1694	-.8663
[illegible]	-1.7710	-.3281	+.6636
[illegible]	-7.8305	-7.4679	-6.6006
[illegible]	6.6070	5.0235	.3076
[illegible]	-5.7047	-4.2262	+.6524

Table V. Energy Level Ordering.

Occupation number \ δ	-.4	-.2	-.1	.1	.2	.4
1 2	1 1/2+	1 1/2+	1 1/2+	1 1/2+	1 1/2+	1 1/2+
3 4	2 3/2-	2 3/2-	2 3/2-	3 1/2-	3 1/2-	3 1/2-
5 6	3 1/2-	3 1/2-	3 1/2-	2 3/2+	2 3/2-	2 3/2-
7 8	4 1/2-	4 1/2-	4 1/2-	4 1/2-	4 1/2-	4 1/2-
9 10	8 1/2+	5 5/2+	5 5/2+	8 1/2+	8 1/2+	8 1/2+
11 12	5 5/2+	6 3/2+	6 3/2+	6 3/2+	6 3/2+	6 3/2+
13 14	6 3/2+	8 1/2+	8 1/2+	5 5/2+	5 5/2+	5 5/2+
15 16	10 1/2+	10 1/2+	10 1/2+	10 1/2+	10 1/2+	10 1/2+
17 18	7 3/2+	7 3/2+	7 3/2+	9 1/2+	9 1/2+	9 1/2+
19 20	9 1/2+	9 1/2+	9 1/2+	7 3/2+	7 3/2+	7 3/2+
21 22	11 7/2-	11 7/2-	11 7/2-	17 1/2-	17 1/2-	17 1/2-
23 24	14 3/2-	12 5/2-	12 5/2-	14 3/2-	14 3/2-	14 3/2-
25 26	17 1/2-	14 3/2-	14 3/2-	12 5/2-	12 5/2-	12 5/2-
27 28	12 5/2-	17 1/2-	17 1/2-	11 7/2-	11 7/2-	19 1/2-
29 30	13 5/2-	16 3/2-	16 3/2-	19 1/2-	19 1/2-	[illegible] 1/2-

Table V (cont.)

[illegible]	−.4	−.2	−.1	.1	.2	.4
31 32	16 3/2−	13 3/2−	15 1/2−	16 3/2−	18 1/2−	17 1/2−
33 34	19 1/2−	19 1/2+	13 5/2−	18 1/2−	16 3/2−	16 3/2−
35 36	21 9/2+	18 1/2−	18 1/2−	15 3/2−	15 3/2−	31 1/2+
37 38	24 5/2+	15 3/2−	15 3/2−	13 5/2−	13 5/2−	27 3/2+
39 40	15 3/2−	21 9/2+	20 1/2−	20 1/2−	20 1/2−	13 5/2−
41 42	18 1/2−	20 1/2−	21 9/2+	31 1/2+	31 1/2+	15 3/2−
43 44	22 7/2+	22 7/2+	22 7/2+	27 3/2+	27 3/2+	20 1/2−
45 46	31 1/2+	24 5/2+	24 5/2+	24 5/2+	24 5/2+	24 5/2+
47 48	27 3/2+	27 3/2+	27 3/2+	22 7/2+	22 7/2+	33 1/2+
49 50	24 7/2+	31 1/2+	31 1/2+	21 9/2+	21 9/2+	22 7/2+
51 52	29 3/2+	26 5/2+	26 5/2+	33 1/2+	33 1/2+	51 1/2−
53 54	33 1/2+	23 7/2+	23 7/2+	32 1/2+	32 1/2+	29 3/2+
55 56	20 1/2−	33 1/2+	33 1/2+	29 3/2+	29 3/2+	32 1/2+
57 58	26 5/2+	29 3/2+	29 3/2+	28 3/2+	28 3/2+	21 9/2+
59 60	36 11/2−	25 5/2+	28 3/2+	26 5/2+	26 5/2+	46 3/2−

Table V (cont.)

[illegible]	-.4	-.2	-.1	.1	.2	.4
61–62	39 7/2-	28 3/2+	25 5/2+	25 5/2+	51 1/2-	26 5/2+
63–64	52 1/2-	32 1/2+	32 1/2+	23 7/2+	45 3/2-	28 3/2+
65–66	37 9/2-	36 11/2-	35 1/2+	35 1/2+	35 1/2+	42 5/2-
67–68	25 5/2+	37 9/2-	30 3/2+	51 1/2-	23 7/2+	35 1/2+
69–70	32 1/2+	39 7/2-	36 11/2-	34 1/2+	25 5/2+	53 1/2-
71–72	46 3/2-	30 3/2+	37 9/2-	46 3/2-	42 5/2-	23 7/2+
73–74	42 5/2-	35 1/2+	39 7/2-	30 3/2+	39 7/2-	39 7/2-
75–76	28 3/2+	42 5/2-	34 1/2+	42 5/2-	34 1/2+	25 5/2+
77–78	38 9/2-	51 1/2-	42 5/2-	39 7/2-	30 3/2+	47 3/2-
79–80	57 13/2+	46 3/2-	46 3/2-	37 9/2-	37 9/2-	52 1/2-
81–82	44 5/2-	41 7/2-	51 1/2-	36 11/2-	53 1/2-	76 1/2+
83–84	41 7/2-	34 1/2+	41 7/2-	53 1/2-	36 11/2-	30 3/2+
85–86	48 3/2-	38 9/2-	38 9/2-	48 3/2-	48 3/2-	34 1/2+
87–88	30 3/2+	48 3/2-	44 5/2-	52 1/2-	52 1/2-	37 9/2-
89–90	53 1/2-	53 1/2-	40 7/2-	47 3/2-	47 3/2-	72 3/2+

Table V (cont.)

[illegible]	-.4	-.2	-.1	.1	.2	.4
91 92	35 1/2+	44 5/2−	48 3/2−	44 5/2−	44 5/2−	44 5/2−
93 94	60 9/2+	40 7/2−	43 5/2−	43 5/2−	78 1/2+	47 3/2−
95 96	58 11/2+	57 11/2+	53 1/2−	41 7/2−	72 3/2+	36 11/2−
97 98	63 7/2+	43 5/2−	47 3/2−	40 7/2−	41 7/2−	67 5/2+
99 100	40 7/2−	47 3/2−	52 1/2−	38 9/2−	43 5/2−	80 1/2+
101 102	47 3/2−	52 1/2−	57 11/2+	55 1/2−	55 1/2−	41 7/2−
103 104	67 5/2+	58 11/2+	50 3/2−	78 1/2+	67 5/2+	63 7/2+
105 106	52 1/2−	60 9/2+	45 5/2−	72 3/2+	38 9/2−	55 1/2−
107 108	43 5/2−	50 3/2−	58 11/2+	54 1/2−	54 1/2−	43 5/2−
109 110	78 1/2+	63 5/2+	55 1/2−	67 5/2+	63 7/2+	74 3/2+
111 112	72 3/2+	45 5/2−	60 9/2+	50 3/2−	40 7/2−	79 1/2+
113 114	59 11/2+	67 5/2+	63 7/2+	63 7/2+	50 3/2−	54 1/2−
115 116	85 15/2−	55 1/2−	54 1/2−	60 9/2+	60 9/2+	113 1/2−
117 118	34 1/2+	72 3/2+	67 5/2+	49 3/2−	80 1/2+	38 9/2−
119 120	62 9/2+	*78 1/2+	72 3/2+	45 5/2−	49 3/2−	60 9/2+

Table V (cont.)

[illegible]	-.4	-.2	-.1	.1	.2	.4
121 122	65 7/2+	54 1/2-	49 3/2-	58 11/2+	58 11/2+	50 3/2-
123 124	80 1/2+	62 9/2+	78 1/2+	56 1/2-	45 5/2-	47 7/2-
125 126	69 5/2+	59 7/2+	56 1/2-	57 11/2+	79 1/2+	108 3/2-
127 128	74 3/2+	49 3/2-	62 9/2+	80 1/2+	74 3/2+	73 3/2+
129 130	45 5/2-	69 5/2+	59 7/2+	74 3/2+	56 1/2-	69 5/2+
131 132	84 13/2-	65 7/2+	65 7/2+	77 1/2+	57 13/2+	100 5/2-
133 134	88 11/2-	85 15/2-	69 5/2+	69 5/2+	73 3/2+	58 11/2+
135 136	50 3/2-	62 7/2+	61 9/2+	73 3/2+	69 5/2+	82 1/2+
137 138	55 1/2-	80 1/2+	78 3/2+	65 7/2+	65 7/2+	49 3/2-
139 140	61 9/2+	61 9/2+	80 1/2+	68 5/2+	113 1/2-	45 5/2-
141 142	68 5/2+	74 3/2+	64 7/2+	64 7/2+	68 5/2+	65 7/2+
143 144	79 1/2+	73 3/2+	68 5/2+	82 1/2+	82 1/2+	95 7/2-
145 146	92 9/2-	79 1/2+	79 1/2+	62 9/2+	106 3/2-	67 5/2+
147 148	95 7/2-	68 5/2+	73 3/2+	62 9/2+	100 5/2-	58 1/2-
149 150	62 7/2+	86 13/2-	85 15/2-	113 1/2-	64 7/2+	115 1/2-

Table V (cont.)

resonance number \ E	-.4	-.2	-.1	.1	.2	.4
151 152	73 3/2+	56 1/2-	71 5/2+	106 3/2-	62 9/2+	57 13/2+
153 154	87 13/2-	88 11/2-	86 13/2-	76 3/2+	76 3/2+	71 3/2+
155 156	100 5/2-	71 5/2+	82 1/2+	59 7/2+	95 7/2-	114 1/2-
157 158	54 1/2-	[illegible] 1/2+	66 7/2+	100 5/2-	84 1/2+	84 1/2+
159 160	106 3/2-	91 9/2-	76 3/2+	84 1/2+	61 9/2+	108 3/2-
161 162	113 1/2-	76 3/2+	88 11/2-	95 7/2-	91 9/2-	91 9/2-
163 164	90 11/2-	66 7/2+	91 9/2-	71 5/2+	59 7/2+	62 9/2+

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