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THE MASSES OF IDENTIFIED POSITIVE HEAVY MESONS

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ABSTRACT

The charged secondary particles from all presently known decay modes of positive K mesons have been identified, and the masses of the primaries measured by the momentum-range method. More than 2000 K mesons were found in a rather large stack of nuclear emulsion after its exposure, behind the strong-focusing magnetic spectrometer at the Bevatron, to a target plunged into the 6.2-Bev circulating proton beam. The mean K-meson momentum at the stack was 360 Mev/c, and the proper time of flight was 1.4×10^{-8} sec. The masses of 263 systematically selected K mesons were measured, and the averages for the various decay modes are:

$$M_{\tau + \tau'} = 966.6 \pm 1.9 m_e,$$

$$M_{K\pi_2} = 966.7 \pm 2.0 m_e,$$

$$M_{K\mu_2} = 967.2 \pm 2.2 m_e,$$

$$M_{K\mu_3} = 967.6 \pm 6.3 m_e,$$

$$M_{K\beta_3} = 964 \pm 10 m_e.$$

The standard errors indicated apply to the relative masses; the uncertainties on the absolute masses are slightly larger. Various corrections are described. The results are discussed in the light of recent ideas concerning heavy-meson characteristics.

I. INTRODUCTION

The study of "strange" particles originated in 1947 when Rochester and Butler¹ reported the observation of two cloud-chamber events which they interpreted as the decay of a neutral particle and the decay of a charged particle of mass considerably greater than that of the pions. An important event in photographic emulsion occurred two years later with the discovery by Brown et al.² of a " τ " meson, which stopped in emulsion and decayed into three charged pions with a Q of about 70 Mev. The event firmly established the existence of unstable charged particles about a thousand times as heavy as the electron. In 1950 the group at California Institute of Technology³ confirmed the early cloud-chamber results, and by the following year⁴ it was recognized that these events could be divided into two different categories which are now known as K mesons (mass intermediate between the pion and nucleon) and hyperons (mass intermediate between the nucleon and the deuteron). We shall be concerned only with particles belonging to the former group.

In 1951 O'Ceallaigh⁵ observed, in emulsion, two stopping particles of mass roughly the same as that of the τ , but which decayed at rest into a muon and, apparently, two or more neutral particles. It appeared, then, that more than one mode of decay characterized the charged K mesons. Although the sign of the charge on these early mesons was not unambiguously determined, the absence of star prongs indicated that all particles yet seen had been either positively charged or extremely weakly interacting (or both). That the former was true has been borne out by the fact that in all unambiguous heavy-meson decays at rest yet found, whenever the charge was established by the behavior of the secondary endings it was found to be positive. Apparently all negative K mesons are absorbed before they can decay at rest, and thus their endings are usually characterized by stars. Experiments in emulsion and in cloud chambers, subsequent to those mentioned above, have shown that the K-meson class is occupied by negative as well as positive and neutral particles. The subclasses of particles of different charge have been subjected to intense investigation in the past few years,

and each is a study in itself, as also are the relationships that exist between them. This paper is restricted to consideration of only the positively charged group.

At the present time at least six different modes of decay are attributed to positive K mesons. These modes, and their accepted names, are listed in Table I, according to a phenomenological classification of type.

Table I

Modes of decay of positive K mesons ⁶		
Type	Name	Decay Mode
$K_{\pi 3}^+$	τ^+	$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$
	τ'^+	$\tau'^+ \rightarrow \pi^+ + \pi^0 + \pi^0$
$K_{\pi 2}^+$	θ^+, X^+	$K_{\pi 2}^+ \rightarrow \pi^+ + \pi^0$
$K_{\mu 2}^+$		$K_{\mu 2}^+ \rightarrow \mu^+ + \nu$
$K_{\mu 3}^+$		$K_{\mu 3}^+ \rightarrow \mu^+ + 2 \text{ or more neutral particles}^a$
$K_{\beta 3}^+$		$K_{\beta 3}^+ \rightarrow \beta^+ + 2 \text{ or more neutral particles}^b$

^a Yekutieli et al.⁷ have found an electron pair emitted from an otherwise typical $K_{\mu 3}$ ending, which suggests the presence of a π^0 in some cases.

^b The exact composition of the neutral particles here is not known.

(We shall henceforth drop the charge-indicating superscripts, since we shall be considering only the positive K mesons.)

This multiplicity of decay modes has provoked wide-spread interest in the properties of the parent particles. The primary goal of all K-meson research in the past few years has been to determine whether a

single particle is ultimately responsible for all modes of decay or if there is more than one distinct parent, and if several, how many different particles exist. Because the mass is more readily measurable than other intrinsic properties of this type of particle, mass measurements have continued to play an important role in K^+ -meson research. A difference in the masses of two parents is a sufficient (though not necessary) condition that the respective decay modes have come from different particles.

There are two general approaches to the determination of the mass of an unstable particle, and both have been employed in the investigation of K mesons.

One method obtains the mass of the immediate parent of the decay products from the total energy of the latter (known rest masses and the measured or deduced total kinetic energy Q of the secondaries). This mass (energy) is the same as that of the primary K meson if the latter has not undergone an undetected energy transition to a lower state (the immediate parent), which has then decayed according to the observed mode, as in the "cascade" scheme of Lee and Orear.⁸ The method can be used if all secondaries are charged as in the $K_{\pi 3}$ (τ) or in the cases of two-body decay such as the $K_{\pi 2}$ and $K_{\mu 2}$. It is of much less value in three- or more-body decays when only one secondary is charged, since the upper end of the energy spectrum of the latter must be well determined.

The second method measures directly the mass of the primary itself. In nuclear emulsions, direct mass determinations are commonly made by the methods of multiple scattering vs range, specific ionization vs range, and specific ionization vs multiple scattering. These methods are unfortunately subject to significant systematic errors and--often--low statistical weights. Statistical errors on one particle are generally in the 10%-20% region. On the other hand these methods depend on no experimental apparatus other than the microscopes used for the measurements, and are therefore the only methods of primary mass determination that it is possible to use in emulsions exposed to cosmic rays. The momentum-range method utilizes a magnetic field to determine the

momentum before the particle enters the stopping medium in which the range is measured. It has been successfully employed in cloud-chamber studies of K mesons by the École Polytechnique⁹ and the Princeton¹⁰ groups, using basically similar vertical arrangements of two Wilson Chambers, of which the top chamber has a momentum-analyzing magnetic field and the bottom, a multiplate chamber, affords the range measurement. The École Polytechnique group has been able to quote errors of 5%-10% on individual particle masses, about half as great as those typical of the scattering-range or ionization-range measurements mentioned above.

By the summer of 1954 a substantial amount of information on K^+ mesons had already been gathered.¹¹ All the presently known decay modes had been observed in emulsion, but their complete descriptions were not yet certain. A fairly accurate lifetime measurement had been made on the K_L group* as a whole, and progress had been made toward the analysis of the τ -decay characteristics in an effort to determine the spin-parity properties of the τ . However, nothing could yet have been said concerning a definite difference or equality between any of the parent mesons on the basis of the above data. In spite of the numerous mass measurements that had been made, only the τ mass was known accurately, and some of the other results were sources of confusion. The mass of the τ from a Q-value determination was known already in 1951¹² to be very close to 966 electron masses, which has been confirmed by subsequent data. Averaging the ionization-range and scattering-range measurements on primary particles of identified decay modes, the committee on K mesons at the Padua Conference¹³ found

$$M_{K_{\pi 2}} = 955 \pm 44 m_e.$$

$$M_{K_{\mu 3}} = 1035 \pm 25 m_e.$$

On the other hand, the seemingly best value for the primary mass determinations (momentum-range and Q value) of $K_{\mu 2}$ events had been calculated from cloud-chamber data⁹ to be $M_{K_{\mu 2}} = 920 \pm 30 m_e$. Primary

* The K_L group consists of a K meson decaying into but one fast charged secondary and one or more neutrals.

mass measurements on supposed $K_{\mu 2}$'s in emulsion gave values ranging from about 914 ± 30 to 1100 ± 60 electron masses, depending on the technique used, and although there was no real consistency among the measurements, the average did not seem compatible with a unique mass of $920 \pm 30 m_e$.¹⁴ There were also data indicating the existence of particles having a mass of about $1450 m_e$.¹⁵

In October 1954, in an effort to clarify the situation, the large "G" stack exposure to cosmic rays was made by several European laboratories in collaboration.¹⁶ The experiment was designed to obtain accurate Q-value measurements for the $K_{\mu 2}$ and $K_{\pi 2}$ by actually stopping the secondaries.

At about the same time were found the first K mesons produced in the Bevatron. Little new knowledge was gained, however, until the development, in early 1955, of the Kerth-Stork strong-focusing magnetic spectrometer.¹⁷ In addition to increasing the scanning efficiency in emulsion by a factor of 100, this system made possible the application of the momentum-range method to primary mass measurements in emulsion. An accurate determination of the relative masses of the K particles was now possible. By the time of the Pisa Conference in July 1955, the major inconsistencies of 1954 had been resolved. Both the "G" stack data¹⁶ (range of secondaries) and the results from the Richman group¹⁸ (momentum-range of primary) placed the mesons of the $K_{\mu 2}$ and $K_{\pi 2}$ equal to that of the τ to within $15 m_e$, and in addition the momentum-range measurements gave the $K_{\mu 3}$ - τ mass difference to be $12 \pm 16 m_e$. No evidence of a $1480 m_e$ particle was found in the "G" stack, and the original data were described as having been due to some unknown spurious effect.¹⁹

On the other hand, an analysis²⁰ of all existing data on τ decays indicated different spin-parity configurations for the $K_{\pi 2}$ and τ . Thus, the Pisa Conference pointed to a situation wherein there might exist a fundamental difference between the mesons parent to the $K_{\pi 2}$ and the τ decay product with no measurable mass difference attendant. It made manifest the need of further, more precise, measurements on all observable characteristics of the K^+ mesons.

The experiment described herein was designed to reduce the errors in the relative (and absolute) primary mass values of identified mesons, and is part of a general investigation of positive heavy mesons by the Richman Group. The significance of precise values from both the Q-value and primary mass measurements is discussed below (in Section IIIB). The data to be discussed were obtained from a rather large stack of nuclear emulsions exposed to positive particles by use of the strong-focusing magnetic spectrometer at the Bevatron. The stack consisted of ninety-five 600- μ Ilford G.5 pellicles, 9 by 17.5 inches.

II. THE EXPERIMENTAL PROCEDURE

A. The General Method

A momentum-range determination of the mass of a charged particle is based on well-established relationships that are known to describe the range of a particle of given charge, mass, and momentum.

In general, a measure of the momentum is achieved through a knowledge of the particle's charge and trajectory in a magnetic field through which the particle passes before entering the stopping medium or detector in which the range is measured. If the magnetic field is directed vertically, all particles entering any particular vertical section at the detector are characterized by a small band of momenta whose width is inversely proportional to the resolution of the system. Let us assign rectangular coordinates to the detector such that the z axis is in the direction of the magnetic field H and the x axis represents the average direction of the particles entering the detector (see Fig. 1). The y axis is in the direction of increasing particle momentum. The average momentum $p_0(y)$ of all particles entering the detector at a given value of y is determined by the value of the magnetic field and the radius of curvature of the corresponding average trajectory. An accurate determination of the momentum p at a particular y is obtained by measuring the ranges $R_P(p)$ of particles (in this case protons) of known mass M_P and charge Z_P that have entered the detector at y .

The range $R_K(p_K)$ of any particle of charge $Z_K (=1)$, mass M_K , and momentum p_K is given by

$$R_K(p_K) = \left(\frac{Z_P}{Z_K} \right)^2 \left(\frac{M_K}{M_P} \right) R_P \left(\frac{M_P}{M_K} p_K \right). \quad (1)$$

This equation forms the basis of the momentum-range method. The method thus depends on the knowledge of the mass M_P and the range-momentum relation $R_P(p)$ of the reference particle over the momentum regions p_K and $\frac{M_K}{M_P} p_K$. In this experiment $R_P(p)$ was taken from the proton range tables of Barkas and Young.²¹ The proton mass M_P was taken to be $1836.1 m_e$ (electron masses).

B. The Experimental Apparatus and Details of the Exposure

The essential elements of the strong-focusing magnetic spectrometer are shown in Fig. 2. A target was plunged into the 6.2-Bev proton beam in one of the straight sections of the Bevatron. Particles of proper charge and momentum coming from the target at 90° to the incident beam passed through the 0.090-inch aluminum wall of the Bevatron vacuum tank and through a compound lens formed by a series of quadrupole magnets, and were brought to a focus in a stack of nuclear emulsions placed behind a momentum-analyzing magnet. The flux of positively charged particles consisted mainly of protons, K mesons, and pions. The momentum selection of the analyzing magnet restricted the protons entering the stack to ranges typically in the vicinity of 1.5 cm and the K mesons to about 7 cm, while the pions passed completely through the stack. Brass was placed on the low-energy side of the magnet to shield the detector from neutral radiation from the target, and a considerable amount of lead and concrete was also used to shield from beam "spill-out" during the early part of the accelerating cycle and from associated secondary radiation. The copper target used was $3/8$ inch high and $5/8$ inch in the proton beam direction. The aperture of the lens was 2 inches and the analyzing magnet had a 2.5-inch gap.

In order to obtain sufficient strength in the lens it was necessary to use four quadrupole magnets. The first two formed a lens converging vertically and diverging horizontally, while the last two magnets were operated so as to diverge vertically and converge horizontally. The compound lens formed by these two elements was both vertically and horizontally convergent; however, it was astigmatic. Since the momentum resolution of the system was determined largely by the horizontal size of the target image at the stack (see Appendix D), there was an incentive to minimize the horizontal magnification to a value that was satisfactory in consideration of the concomitant increase in the vertical magnification. If the latter became too large, particles would be lost by striking the pole pieces of the analyzing magnet. The horizontal magnification used was about 0.6x, and the vertical magnification was about 1.5x. It turned

out that a significant number of particles did in fact strike the pole pieces, judging from the sharp fall-off of the particle density spectrum in the stack. This was apparently caused by a vertical misalignment of the lens "telescope" and perhaps by a poorer-than-expected vertical focus.

Proper current values for all the magnets were obtained from a series of wire-trajectory measurements made for particles of momenta in the vicinity of 360 mev/c. This method was found to be quite satisfactory in achieving proper focus and magnification conditions. The measurements were made considerably easier and more accurate after the construction by Mr. Leroy Kerth of a current regulator which compensated for currents induced in the wire by motion in the magnetic field.

The trajectories obtained from these wire measurements were used to position the stack when the exposure was made, so that the particles would enter approximately perpendicular to the front face of the stack. The stack was oriented with the plane of the emulsion lying in the x-z (vertical) plane in Fig. 1, and was placed so that the particles entered the middle of the long (17.5-inch) side, thus allowing enough emulsion above and below the "beam" to stop the muons from $K_{\mu 2}$'s that decayed in these directions. The bulk of the stack was thus shielded from the target by the pole pieces of the magnet.

The emulsions were clamped firmly between two sheets of bakelite 0.75 inch thick, and all four sides of the stack were milled flat. This allowed a fairly accurate determination of the volume of the stack and thus of its density, and in addition presented a smooth surface to the incident particles for an accurate range measurement. Fiducial marks for aligning the emulsions were provided at various points by exposing the milled sides to x-rays passed through a slit system. When developed, each emulsion thus contained a set of 18 black lines about $25\ \mu$ wide, extending about 0.5 inch into each emulsion, spaced at useful intervals about the periphery.

After processing, the glass-mounted emulsions were cut parallel to the x axis (short dimension) into three equal sections for ease in scanning. The center plates contained of the incident particles, while some of the K-meson secondaries went into the side sections.

The exposure, designed to place about 2000 K particles in the stack, was monitored by (a) a counter telescope located above the Bevatron tank that measured secondary-particle flux from the target; (b) a series of test exposures measuring the proton flux at the stack, using 1-by-3-inch glass-mounted 50- μ G.5 emulsions, which were exposed, developed, and scanned during the run; and (c) the Bevatron's permanently installed proton beam monitor ("induction electrodes"). The total proton flux required at the stack was estimated from the K-meson/proton ratio obtained previously,¹⁸ and was measured by correlating the sampled data from method (b) with the relative primary proton beam flux obtained from (a) and (c). The target was bombarded by 5×10^{12} protons.

C. Determination of the Proton Momenta

1. Measurement of the Projected Ranges

The stack was oriented so that the plane of the emulsions was parallel to the momentum-analyzing magnetic field; thus the average momentum of all particles entering a particular emulsion could be represented as a function of the index number of that emulsion, the numbers running from 1 to 95 in the direction of increasing momentum. The momentum variation across the stack was obtained from the projected ranges of the protons in eight plates. Since the absolute mass value obtained by this method is quite sensitive to the value of the estimated initial momentum, considerable effort was made to achieve an accurate representation of the proton range distribution in the stack. The average projected range was measured by determining the density of proton endings in strips 330 μ wide. These strips spanned the distribution of endings in a given plate. The range distributions were characterized by a root-mean-square deviation of about 6.5%, resulting from effects discussed in Appendix D. A curve was drawn to best fit each distribution, and the median range was taken as that corresponding to the peak value of the curve. A typical example of the distribution of proton endings in a single plate is shown in Fig. 3.

It was necessary to make certain modifications to these values before the incident-proton momentum could be obtained.

2. Corrections to the Measured Projected Ranges

(a) Range shortening due to scattering. The measured projected range was shorter than the true range along the track because of the multiple scattering incurred while the protons were slowing down in the emulsion. The average shortening for protons of a particular range was calculated by a method described in Appendix A, and this quantity was then added to the measured projected range. The effect amounted to about 0.5% of the total range and varied slightly across the stack.

(b) Effect of the entrance angle. It follows from the geometry of the apparatus that an "average" particle would enter perpendicular to the face of the stack at only one lateral (y) position, the remainder either climbing or diving in the emulsion when viewed on the microscope. This small but quite significant effect both shortened the projected range and created a difference between the entrance plate and the plate in which the protons actually came to rest. The correction is described in Appendix B.

(c) Effect of the air path. The above corrections afforded an accurate description of the true average proton range $R_{0P}(y)$ and momentum $p_{0P}(y)$ as a function of the entrance position y as shown in Figs. 4 and 5, respectively. Another modification of these values was still required, however, before it was possible to define the average incident momentum $p_{0K}(y)$ of the K mesons for use in Eq. (1). It was necessary to correct for the difference between p_{0K} and p_{0P} resulting from the different rates of energy loss experienced by the mesons and protons over the air path following the entrance to the analyzing magnet. As a result of the correction, a modified proton range curve was constructed to represent the ranges $R_{0P}(p_{0K})$ for protons of momentum equal to the average K-meson momentum as a function of entrance position. This curve has been included in Fig. 4. The correction itself is described further in Appendix C.

D. Scanning for the K Mesons

K mesons entered the emulsion with a relative ionization about 1.8 times minimum. A swath perpendicular to the incident-particle direction (parallel to the z axis) and about 1 cm behind the stopping protons was scanned for tracks whose grain densities were from 1.5 to 3 times minimum and whose directions lay within 15° of that of the x axis. Such tracks were followed to their endings. The average K-meson track was thus followed for about 4 cm. This procedure allowed a reasonably uniform efficiency for the observation of particles of mass 800 to 1150 electron masses. Pions were at minimum ionization and were not followed.

E. The Identification of the Modes of Decay

Almost all τ mesons, with their striking three-prong decay, were identified at once by the scanners. Furthermore, the scanners followed to their endings all other secondaries recognized as being of low energy. In this manner most of the low-energy decay τ 's and some $K_{\mu 3}$'s were identified. Systematic identification of other events was accomplished by two methods:

(a) Following the Secondaries. All secondaries were followed if their initial direction would have allowed them to stop in the stack if they were muons from $K_{\mu 2}$'s. This afforded the identification of some particles at the ends of their ranges and others by their behavior at great distances from the decay point. Pions interacting in flight were recognized as being from $K_{\pi 2}$'s after a blob-count showed that their velocities were too high for them to have come from a τ '. A secondary leaving the stack at a distance greater than 12 cm from the decay point was said to have been from a $K_{\mu 2}$; the possibility of high-energy secondary from $K_{\mu 3}$ was ignored because of the very low frequency of occurrence of such particles. If for some reason a particle went out of the stack or was lost less than 12 cm from the K ending, it was identified by blob-count where last seen. In all, a total of 59 $K_{\mu 2}$'s, 31 $K_{\pi 2}$'s, 2 $K_{\mu 3}$'s, 2 τ 's, and 2 $K_{\beta 3}$'s were found in this selection of events.

(b) Blob Count Near the Decay Point. One hundred and eighty K_L secondaries throughout the stack were selected for blob-counting near the decay point. The only criterion in the selection was that the tracks should not have exceeded a dip of one plate in 1.5 mm projected length, so as to avoid the serious errors incurred in blob-counting steep tracks. An average of 800 blobs were counted per track, and all counts were normalized to those obtained from known $K_{\mu 2}$ secondaries in the same plate, so as to allow for variation in development. The results are shown in Fig. 6. Shaded rectangles correspond to secondaries, identified by the first method, that also fell into the second group. Events lying in a 4% overlap region between the two distributions ($K_{\mu 2}$ and $K_{\pi 2}$) were eliminated in the selection of events for mass measurement. Two additional $K_{\mu 3}$'s were identified in this group when a heavier blob count than normal was noticed, and one $K_{\beta 3}$ secondary was noticed by abnormal scattering and confirmed by measurement. It was thought that the small number of $K_{\mu 3}$ and $K_{\beta 3}$ modes that might appear in the main $K_{\mu 2}$ and $K_{\pi 2}$ blob count distributions would not be sufficient to significantly affect the mass value obtained with their inclusion. Forty-six $K_{\mu 2}$'s and twenty-four $K_{\pi 2}$'s identified in this manner were included in the mass measurements.

F. The Measurement of the K Ranges

After the decay mode of a meson had been identified, the range of the primary was measured. The following procedure was selected as the most accurate method of determining the true ranges of the particles, considering the general character of their tracks. Most of the tracks were within 10° of being parallel to the x axis in the emulsion, which was one of the two directions of motion of the microscope stage. Each track was followed backward from the point of decay. Coordinates (x_i, y_i, z_i) were recorded for each point where there had accrued a change in direction (from that of the previous point) greater than about 3° . The range was then obtained essentially from the sum of the chord lengths that were computed from these sets of coordinates. This method avoided

errors due to random misalignment of the emulsions (which was less than 25μ), air gaps, * and "rub-off" or other similar losses of parts of the tracks near the surfaces of the emulsion. Occasional systematic shifts in alignment were taken into account. Any systematic error in the computed total range was negligible. Statistical errors were very small (less than 100 microns) compared to the range straggling.

Each track was followed to as near the edge of the emulsion as possible in order to establish its entrance position for the momentum determination. When a track was lost in the densely populated region of the protons, the entrance point was estimated by assuming that the track went half the remaining distance to the edge maintaining the direction when last seen, and then followed the average particle direction for that plate as determined from the protons. An error of one plate in the entrance position would have resulted in an individual mass error of $1.5 m_e$. Most estimates should have erred randomly by less than half a plate, or $0.8 m_e$.

G. The Evaluation of the Masses

By means of Eq. (1) and the proton range-momentum relations,²¹ it was possible to construct a family of isomass curves by plotting, for various masses, the meson range $R_K(p_K)$ as a function of the proton range for the same momentum, $R_P(p_K)$. A small-scale reproduction is shown in Fig. 7. Each event was recorded as a point on the graph with coordinates (R_K, R_P) , using the measured value of R_K and the value $R_P = R_{0P}(p_{0K})$ appropriate to the meson's entrance position as described in Section II C2. The corresponding mass was obtained by interpolating between the two nearest isomass lines. The masses were estimable to $0.5 m_e$, but actually the nearest integral value was used.

* The effect of air gaps was taken into account in the density correction in Appendix E.

III. THE EXPERIMENTAL RESULTS

A. The Characteristics of the Mass Distributions

In Table II are given the averages of the masses obtained from the measurements, the errors, and certain small corrections that are discussed in Appendix E along with the systematic uncertainties. Also listed are the values and standard errors of the mass differences $M_K - M_\tau$ and $M_K - M_{(\tau + \tau')}$. The standard errors are based on the standard deviations of the individual distributions, except for the $K_{\mu 3}$ and $K_{\rho 3}$, whose small number of events made it more reasonable to use the average standard deviation of the other distributions.

Histograms showing the actual distributions of the $K_{\pi 3} = \tau + \pi$, $K_{\pi 2}$, and $K_{\mu 2}$ masses are presented in Fig. 8. The results of an examination of these distributions are found in Table III. In each group were a few abnormally high mass values, which probably resulted from collisions with the magnet pole piece or from inelastic scatters in the emulsion. In order to obtain the most valid average of the masses, these events were eliminated on the basis of Chauvenet's Criterion.* It is of some interest that the relative number of events in this category was about the same for all decay modes. As distributions were nearly gaussian, no great error should have resulted from the assignment of the mass cutoff values on the basis of the normal probability distribution and the standard deviations of the individual distributions. As a check, the averages and standard deviations were computed for the mass distributions modified by the inclusion of the excluded value nearest the cutoff. These results are also given in Table III. As a further check on the internal consistencies of the distributions, averages were taken on events falling in the lower, middle and upper mass regions of each group, and the third and fourth moments, m_3 and m_4 , were also computed. These partial averages and the values m_3/σ^3 (skewness) and m_4/σ^4

* Chauvenet's Criterion eliminates events from a sample of size n , where the probability is less than $1/2n$ that the event in question is a part of the distribution. See, for instance, Beers.²²

Table II

Particle Type	Values of the average masses and corrections (All quantities pertaining to the mass are in units of the electron mass m_e .)							All
	π	π'	$\pi + \pi'$	K_{e2}	$K_{\mu 2}$	$K_{\mu 3}$	K_{p3}	
Mean measured mass	966.0	967.4	966.1	966.4	966.9	967.3	961	966.5
Standard error (m_e)	± 2.1	± 4	± 1.9	± 2.0	± 2.2	± 6	± 10	± 1.1
Estimated systematic error	± 1.6							
Number of events used	77	21	98	54	96	11	4	263
Averaging correction ^a	-0.3 ± 0.1							
Range-momentum correction ^a	$+0.7 \pm 0.5$							
Emulsion density correction ^a	$+0.4 \pm 0.1$							
Inelastic events correction ^a	$+0.5 \pm 0.25$							
Corrected absolute mass	966.3	967.7	966.6	966.7	967.2	967.6	964	966.8
Combined statistical and systematic uncertainty on absolute mass	± 2.7	± 4.1	± 2.6	± 2.7	± 2.8	± 6.3	± 10	± 2.1
$M_K - M_\pi$ (standard errors)		1.4 ± 4.5		0.4 ± 1.9	0.9 ± 3.0	1.3 ± 6.3	-2.7 ± 10	
$M_K - M_{\pi + \pi'}$ (standard errors)				0.1 ± 2.8	0.6 ± 3.0	1.0 ± 6.3	-1.0 ± 10	

^a See Appendix E.

Table III

Characteristics of the main mass distributions (All mass quantities are in units of the electron mass m_e .)			
Particle	$K_{\pi 3}$	$K_{\pi 2}$	$K_{\mu 2}$
Total no. identified	104	59	100
Number accepted	98	54	96
Mean mass	966.3	966.4	966.9
Standard error	± 1.9	± 2.0	± 2.2
Standard deviation σ	18.7	14.9	21.3
Chauvenet's $\frac{\Delta M_{\max}}{\sigma}$	2.79	2.60	2.79
Chauvenet's cutoff	1017.7	1005.2	1026.5
Nearest excluded mass M_x	1021	1017	1027
$\frac{\Delta M_x}{\sigma}$	2.90	3.40	2.82
Alternate mean mass ^a	966.8	967.4	967.5
Alternate std. error ^a	± 1.9	± 2.2	± 2.2
Alternate std. dev. σ^a	19.3	16.3	21.8
Alternate $\frac{\Delta' M_x}{\sigma^a}$	2.80	3.05	2.73
Mean of lower half	952	957	953
Mean of center half	964	965	961
Mean of upper half	981	976	984
Skewness $\frac{m_3}{\sigma^3}$	0.20	0.49	9.53
Peakedness $\frac{m_4}{\sigma^4}$	2.83	3.15	3.15

^a The alternate values include excluded mass M_x nearest the cutoff.

(peakedness) are included in Table III. All three distributions are nearly the same in these characteristics as well as in average value. They all display a slight skewness as expected; however, the peakedness is nearly the same as that of a gaussian (3.0). The skewness is mentioned below.

One property of the three distributions that is significantly different in the three cases is the standard deviation.* If a series of random samplings, each of size n_1 , is taken from a normal universe whose standard deviation is σ_T , then the standard deviations σ_{ij} (j denotes a specific sample) measured on each sample of size n_1 will themselves vary about σ_T , and their distribution will have a standard deviation $S\sigma_i = \frac{\sigma_T}{\sqrt{2n_1}}$. Using these relations and assuming that the standard deviations measured in this experiment are random samples taken from normal distributions, we find the probability less than about 1/400 that the $\sigma_{K_{\pi 2}} = 14.9 \pm 1.4$ and $\sigma_{K_{\mu 2}} = 21.3 \pm 1.6$ are random samples of the same distribution. The difference is 6.4 ± 2.1 , which corresponds to 3.0 standard deviations, and has the probability 1/400. The probability that the difference $\sigma_T - \sigma_{K_{\pi 2}} = 3.8 \pm 2.0$ (1.9 standard deviations) or greater would occur from a common distribution is about 1/17.

There are two explanations for this discrepancy other than that of an improbable statistical fluctuation, and neither of these is very satisfactory. Both assume that the true standard deviation σ_T should be close to that of the $K_{\pi 2}$, and that the $K_{\mu 2}$ and perhaps the $K_{\pi 3}$ distributions are broader than the $K_{\pi 2}$ as a result of some process not occurring to the $K_{\pi 2}$. If the $K_{\mu 2}$ and $K_{\pi 3}$ mesons are actually the daughters of a heavier, short-lived parent meson that had decayed, say, in about 5×10^{-9} second after creation (when the parent was about midway between the target and the lens), their momentum spreads could have been broadened because the recoil momentum of the decay

* Actually the third and fourth moments are also different in each case, but when they are normalized to the standard deviations (i.e., to become the "skewness" and "peakedness"), the resulting quantities are consistent.

had effectively broadened the target. Although this explanation cannot be completely excluded, angular deflections from such decays might have been detected both in the following of $K_{\mu 2}$ and τ meson tracks from cosmic-ray stars in emulsion and in cloud-chamber data, and there apparently has been no evidence in support of the scheme.*

An alternative to the above is the possibility that the τ and $K_{\mu 2}$ are capable of undergoing exotic small inelastic processes in matter, the effect of which would be a shorter-than-normal range, as well as a broader range distribution than is provided by conventional straggling, so that the actual masses would be lower than those obtained from measurements using the known range-momentum relations. This idea is more tenuous than the first. Any such processes should be very much less important than the Coulomb interaction. To a certain extent, the effect is refuted by the fact that the relative density of abnormally high mass values (shorter ranges) is not significantly different from that for the $K_{\pi 2}$. If such a difference existed between the interactions of various particle types, it would also undoubtedly have been noticed in the many scattering measurements that have been made on the primaries of K mesons. In the absence of any data supporting the above explanations, we are apparently left to digest a rather unpalatable abnormally large statistical fluctuation.

The Predicted Mass Distribution

For a test of the reasonableness of the actual mass distribution, it was possible to construct a predicted distribution curve from the expected behavior of the variables influencing the measurement. We must consider the effects of range straggling and the momentum distribution characteristic of each entrance point.

* Note added later: It is of interest to note that Stork⁴⁰ has calculated the mean life of the $K_{\beta 3}$ to be $(0.49^{+0.20}_{-0.12}) \times 10^{-8}$ sec by comparing the relative abundances of the $K_{\beta 3}$ mode to those of the other modes (τ , $K_{\mu 2}$ and $K_{\pi 2}$) of known lifetimes found in emulsion stacks in which the mesons had different average times of flight. The probability is 1/500 that the lifetime of the $K_{\beta 3}$ is the same as that of the other modes (1.2×10^{-8} sec).

In the velocity region of the K mesons, the mean range R' of a particle of mass M_0 and momentum p behaves as $R'/M_0 = C_1(p/M_0)^{3.0}$, where C_1 is constant. A measure M of the mass M_0 based on the known momentum p and the actual range R will not in general yield the true mass $M = M_0$ because of the range straggling of R about R' . In fact, the measured value M will follow the relation $M = C_2 R^{-0.5} p^{1.5}$. We define R_0 by the equation

$$M_0 = C_2 R_0^{-0.5} p_0^{1.5},$$

where $p_0(y)$ is the average momentum of all particles of the same entrance position y . The true momentum p varies about p_0 because of the finite target width (Appendix D). From these relations, it follows that the mass value M obtained by using the measured range R and the mean momentum $p_0(y)$ can be expressed as

$$\frac{M}{M_0} = \left(\frac{R}{R_0(y)} \right)^{-0.5} \left(\frac{p}{p_0(y)} \right)^{1.5}. \quad (2)$$

We make the reasonable assumption that the relative rms range straggle $\frac{\sigma_R(y)}{R_0(y)}$ was constant across the stack, as was the relative momentum standard deviation $\frac{\sigma_p(y)}{p(y)}$. From the average standard deviation $\sigma_M = 19.1 m_e$ actually obtained from the mass distributions, we find in Appendix D that $\frac{\sigma_p}{p} = 1.2\%$ for the calculated value of $\sigma_R/R = 1.6\%$.

We can now obtain a plot of the relative mass distribution function $P(M/M_0)$ by graphical methods. We first construct a curve defining the distribution $P(R'/R_0)$ by using the relation $R'/R_0 = (p/p_0)^{3.0}$ and a plot of the gaussian distribution $P(p/p_0)$ of standard deviation 0.012, weighted according to $\frac{d(R'/R_0)}{d(p/p_0)}$. Next, the distribution $P(R/R_0)$ is obtained by folding into $P(R'/R_0)$ the straggling distribution $P(R/R')$, a gaussian of the calculated standard deviation 0.016. Finally, the function $P(M/M_0)$ can be plotted by using the relation $M/M_0 = (R/R_0)^{-0.5}$ and the distribution $P(R/R_0)$ weighted according to $\frac{d(M/M_0)}{d(R/R_0)}$. From

$P\left(\frac{M}{M_0}\right)$ the distribution function $P(M)$ was obtained by using $M_0 = 966.5 m_e$, and a normalized curve of this predicted distribution was plotted along with a histogram of all events combined; the result is shown in Fig. 9. The fit is surprisingly good. Deviations from a gaussian are greatest in the high-mass tail, but can be noticed in the peak as well. The total effect of these deviations is not large; nevertheless the skewness is apparent and its result on the mass average is discussed below in Appendix E. The curve also demonstrates the validity of the mass cutoff used in the averaging.

B. The Average Masses

The average mass values and standard errors are such that any mass differences (see Table II) existing among the τ , $K_{\mu 2}$, and $K_{\pi 2}$ are less than $3 m_e$ with a probability of 70%, and that the $K_{\mu 3}$ and $K_{\beta 3}$ masses are similarly equal to the others to within $6 m_e$ and $10 m_e$ respectively. The average values and standard errors of the masses are shown graphically in Fig. 10. The results are in agreement with all recent relative mass measurements,^{18, 23, 24} and the standard errors here are at least twice as small as have been obtained heretofore. In addition, the errors on the absolute mass are small enough that the close agreement between the primary τ mass and that from the measurement of the Q strengthens the validity of the accepted π meson mass values. The internal consistency of the proton range-energy relation over regions apropos of this experiment, and its absolute behavior over the regions relating to the ranges of pions from the τ decay, are also supported.

The agreement among the average values is closer than expected for the given standard errors. Normally one-third of the data should fall outside the standard error, and in fact all values agree to well within their errors. Presumably the small variance in the averages is fortuitous.

C. Significance of the Results

The results indicate at least a very near degeneracy among the K-meson masses regardless of the associated decay modes. Less than two years ago this fact would have been exciting if not startling, however, in the past year, there has been a general trend toward an acceptance of the idea, and in recent months some theoretical attempts have been made to reconcile this degeneracy with the increasingly apparent difference between the spin-parity configurations of the τ (0, -)^{25, 26} and $K_{\pi 2}$ (0, +).³² It has also been established that, at least for the particles that come to rest in the order of 10^{-8} sec,* the τ , $K_{\mu 2}$, and $K_{\pi 2}$ all have very nearly the same mean life.^{27, 28}

Lee and Orear⁸ propose a scheme wherein there are two primary heavy mesons, the θ and the τ , the heavier of which is capable of decaying via γ decay into the lighter, which has a different spin-parity configuration. The lifetime of the heavier corresponds to the measured value of about 10^{-8} sec, and the lighter (daughter) decays in a time short compared to 10^{-8} sec, so that its measured lifetime is the same as the parent's if the measurement is made on stopped particles. Either or both the τ and θ may contribute to the $K_{\mu 2}$, $K_{\mu 3}$, and $K_{\pi 3}$. Thus, if this scheme is correct, the primaries of all decay modes in this experiment are the same particle (hence no primary mass differences). However, the masses based on the Q-value determinations should differ between the θ and τ , unless the transition energy between the two states is too small to be detectable as a mass difference. Several variations of the scheme are possible. Accurate Q-value determinations of the $K_{\pi 2}$ and $K_{\mu 2}$ masses as well as of the τ mass are obviously important as a means of evaluating the validity of this cascade concept of K-meson behavior. It might be mentioned that the recoil from the decay of a K particle at rest into a γ -ray and another K particle would have been too short for identification as such unless the energy transition were greater than about 15 Mev (recoil of 3 microns). Q-value

* The proper time of flight of the K mesons in this experiment was 1.4×10^{-8} sec.

mass measurements²⁹ dictate that any transition must be an order of magnitude smaller, and hence the recoil would not have been detected.

Lee and Yang³⁰ and Gell-Mann³¹ have recently suggested that the apparent θ - τ mass degeneracy actually follows from an invariance law. They assume that the τ and θ have identical spin (0)³² but opposite parity, and propose a new "parity conjugation" operator that changes the τ of minus parity into a θ of plus parity, and vice versa. The identity or near identity of the τ and θ masses results from conservation of parity conjugation by strong interactions. Gell-Mann believes that parity conjugation is conserved by electromagnetic as well as strong interactions, and that the conservation fails only in weak (leptonic) interactions, hence the mass difference should be only a small fraction of an electron mass. One would expect rather long ($\sim 10^{-8}$ sec) but not identical lifetimes. On the other hand, Lee and Yang admit the possibility of a violation of the conservation of parity conjugation in electromagnetic interactions, and thus the existence of a mass difference between the θ^+ and τ^+ of the same order of magnitude as that between the π^0 and $\pi^\pm$,³⁰ or smaller, which could be compatible with the Lee-Orear scheme. A spin 0-0 transition³² requires the emission of at least two γ 's, and a mass difference should be of the order of 10 Mev⁸ (thus measurable) for a lifetime of about 10^{-8} sec.

From the results of this experiment alone one can make no choice between these ideas; the data simply indicate that the particles passing through the system, with lifetimes of at least 10^{-8} sec, have close to the same mass. A genetic relationship as per Lee and Orear is, in principle, allowed by these data; however, the Q-value measurements require θ , τ mass difference in this scheme to be less than about 2 or 3 Mev.²⁹ The "parity conjugation" ideas are agreeable with existing data, with the stipulation that the measured stopped-particle lifetimes are the same^{27, 28} and that, from this experiment, if the actual lifetimes are the same, any mass difference is no greater than about 1 Mev, for if the lifetimes are equal we have indeed measured the masses of the actual parents to the decay products.

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APPENDICES

A. Shortening of the Projected Proton Range Due to Multiple Scattering in the Emulsion

The range-energy relation deals with the average range $\langle R \rangle_{av.}$ along the track and includes the effect of range straggling.

If an accurate value is to be obtained for the energy of a group of monoenergetic particles by measuring the average projected range $\langle R_x \rangle_{av.}$, deviations from the original (x) direction must be taken into account. The greatest cause of such deviations is small-angle multiple scattering, and we confine the correction to this effect. The very infrequent large-angle Coulomb and nuclear scatters are of minor importance in the average shortening of proton ranges.

In general the difference between an increment dR and its projection dR_x is $dR - dR_x = dR (1 - \cos \theta)$, where θ is the angle between the actual path and the direction along which R_x is measured. Over the entire range R_0 the total difference is

$$R_0 - R_{0x} = \int_0^{R_0} (1 - \cos \theta) dR. \quad (3)$$

The protons with which we are concerned have an average absolute angular deviation θ of less than 20° to within ten microns of the end of their range, thus we are justified in employing the small-angle approximation $\cos \theta = 1 - \frac{\theta^2}{2}$, with which Eq. (3) becomes

$$R_0 - R_{0x} = \int_0^{R_0} \frac{\theta^2(R)}{2} dR.$$

The average shortening of a large number of tracks is

$$\langle R_0 - R_{0x} \rangle_{av.} = \int_0^{R_0} \langle \frac{\theta^2(R)}{2} \rangle_{av.} dR,$$

and, since the distribution in θ is gaussian, we have

$$\langle R_0 - R_{0x} \rangle_{av.} = \int_0^{R_0} \frac{\theta_{rms}^2(R)}{2} dR; \quad (4)$$

the range shortening is calculated in terms of the root-mean-square multiple-scattering angle as a function of the range.

Multiple scattering has been treated theoretically by Williams³³ and others,³⁴ who have obtained expressions for the mean-square projected angle $\langle \Delta \theta_x^2 \rangle_{av.}$ between tangents to the path at each end of a small length interval as a function of various properties of the scattering medium and the scattered particle. In order to most satisfactorily describe the scattering in photographic emulsion, we choose to use an expression basically derived from theory but fitted to experimental data on multiple scattering in emulsion. This function evaluates the mean absolute projected angle between two successive chords connecting points at intervals Δt along the path. We shall refer to this angle as $\langle \Delta \theta_{x \text{ chord}} \rangle$, and when expressed in radians it has the value³⁵

$$\langle \Delta \theta_{x \text{ chord}} \rangle_{av.} = \frac{\pi}{180} \frac{K}{pv} (\Delta t/100)^{1/2} \text{ radians}, \quad (5)$$

for a singly charged particle of momentum p (Mev/c) and velocity v (cm/sec), when measured over a length Δt (microns). The "scattering constant" K ($\frac{\text{degrees} \times \text{Mev}}{100 \text{ microns}}$) is determined empirically for a particular type of emulsion, and is a slowly varying function of velocity and the cell length Δt .

Equation (5) must now be modified to deal with the spatial (nonprojected) angle between tangents used in Eq. (4). A calculation by Fermi³⁶ shows that the average absolute projected angle $\langle \Delta \theta_x \rangle_{av.}$ between tangents to the path at small intervals Δt can be written in terms of the average chord angle as

$$\langle \Delta \theta_x \rangle_{av.} = \sqrt{3/2} \langle \Delta \theta_{x \text{ chord}} \rangle_{av.}$$

and the average of the actual angles $\Delta \theta$ (small) in space is related to the average of their projections θ_x by $\langle \Delta \theta \rangle_{av.} = \sqrt{2} \langle \Delta \theta_x \rangle_{av.}$. Further, since the distribution in $\Delta \theta$ is gaussian, the root-mean-square value $\Delta \theta_{rms}$ is $\sqrt{\pi/2}$ times the arithmetic mean, whence

$$\Delta \theta_{rms} = \sqrt{3\pi/2} \langle \Delta \theta_{x \text{ chord}} \rangle_{av.}$$

With the use of the nonrelativistic relation $p v = 2 T$ and the approximation

$$T = AR^\eta (m)^{1-\eta}, \quad (6)$$

we obtain

$$\Delta \theta_{rms}^2 = \frac{3\pi}{8A^2} \left(\frac{\pi}{180} \right)^2 K R^{-2\eta} m^{-2(1-\eta)} \left(\frac{\Delta t}{100} \right) \quad (7)$$

for a particle of residual range R (microns), mass m in units of the proton mass, and kinetic energy T (Mev). $\Delta \theta$ is in radians.

A large number of particles, initially traveling in the same direction with residual range R_0 , will, after reaching a point with residual range R , exhibit a mean-square angular deflection that may be expressed as

$$\theta_{rms}^2 = \int_{R_0}^R \frac{d\theta_{rms}^2}{dR} dR. \quad (8)$$

We let $\Delta t \rightarrow -dR$ in Eq. (7) (the minus sign derives from the fact that R is measured backwards from the end of the range), and $\Delta \theta_{rms}^2 \rightarrow d\theta_{rms}^2$, then Eq. (8) becomes, upon integration:

$$\theta_{rms}^2 = 10^{-2} \left(\frac{\pi}{180} \right)^2 \frac{K^2 m^{-2(1-\eta)}}{A^2 (1-2\eta)} (R_0^{1-2\eta} - R^{1-2\eta}). \quad (9)$$

The evaluation of Eq. (4), with the use of Eqs. (9) and (6), yields the desired expression for the average range shortening for protons of initial range R_0 and kinetic energy T_0 :

$$\langle R_0 - R_{0x} \rangle_{av.} = 10^{-2} \left(\frac{\pi}{180} \right)^2 \left(\frac{3\pi}{16} \right) \frac{K^2}{2(1-\eta)} \left(\frac{R_0}{T_0} \right)^2. \quad (10)$$

Taking the value of $K = 27 \frac{\text{degrees} \times \text{Mev}}{100 \text{ microns}}$ from Menon's data³⁵ as a suitable average over the ranges considered, and an average $\eta = 0.57$ from Barkas,²¹ we obtain the final result:

$$\langle R_0 - R_{0x} \rangle_{av.} = 1.53 \times 10^{-3} \left(\frac{R_0}{T_0} \right)^2. \quad (11)$$

These corrections amounted to roughly 0.5% of the proton ranges. If the small-angle approximation is valid, Eq. (2) is exact. The approximation $p v = 2 T$ holds for 65-Mev protons, but the assumed constant values of η and K may have caused an error, probably less than 15% of the total correction, or 0.075% of the range. The result would have been an error in momentum of 0.022% or 0.03% in meson mass. Any systematic mass error incurred in this correction, then, was probably less than $0.3 m_e$.

B. The Correction for Nonnormal Angles of Incidence

As described briefly in Section IIC, the average incident angle varied according to the entrance position. The resulting dip of the track caused a shortening of the projected proton ranges and also meant that entrance plate was not that in which the ranges were measured. The average tangent of the dip angle ϕ_D that a track makes with the emulsion plane was determined in four different plates across the stack. These averages were plotted as shown in Fig. 11, and a straight line was drawn to approximate a fit. The measured proton ranges were then modified and the appropriate entrance plate was chosen on the basis of the values of $\tan \phi$, taken from the curve.

The average of the tangent was taken from the measurements on a group of individual proton tracks selected in a systematic manner to avoid any bias. In each emulsion, these tracks were followed from about 3 mm from the edge of the emulsion until they either left the plate or had traveled about 2 mm. The depth of the track in the emulsion was recorded at each end point and the difference was increased by the factor 2.5 to compensate for shrinkage during development. It was also necessary to correct the measured distance between the end points on all diving tracks because of a distortion of the original y - z plane (parallel to the face of the stack) in each emulsion. This distortion resulted from an inward shift of the bulk of the emulsion during development, while the bottom (glass) surface remained fixed, as shown in Fig. 12.

It can be seen that the uppermost 40μ of the emulsion was left relatively undistorted, the shift having been equal for all depths in this region. The effect was greatest at the edge of the emulsion, decreasing to zero at about 10 mm from the edge. The character and magnitude of the shift appropriate to the dip angle measurements were determined by examining steeply diving tracks in the region. The original direction of a track was established in the undistorted region and the relative distortion of the emulsion below was ascertained from the measured displacement of the actual track from the projection of the undistorted track at various depths. A curve similar to Fig. 12 was constructed for each plate in which the dip angles were measured. It can be seen that for diving tracks this distortion would effectively increase the dip that had characterized the track in the unprocessed emulsion.

The error in mass, ΔM , resulting from an error in the dip angle, $\Delta \phi_D$ ($\approx \Delta \tan \phi_D$), can be shown to follow the relation

$$\frac{\Delta M}{M} = \left(\frac{\phi_D}{3.4} + \frac{R_p}{p} \frac{dp}{dy} \right) \Delta \phi_D. \quad (12)$$

When the values $R_p = 1.5$ cm, $P = 360$ Mev/c, and the averages $\langle \phi_D \rangle_{av.} = 0.04$ radians and $\langle \frac{1}{p} \frac{dp}{dy} \rangle_{av.} = 0.0185$ are used, Eq. (12) shows the error in meson mass to have been less than $0.4 m_e$ for an error $\Delta \phi_D$ less than 0.01 radian.

C. The Correction for Momentum Loss in Air

Protons and mesons of equal momentum have different rates of loss of energy and momentum in matter. In the experiment reported herein this fact assumed significance in the air path following the onset of the momentum analysis, and resulted in a difference in the momenta of the mesons and protons of the same entrance position. A correction for the effect was made by computing this difference and shifting the proton range curve (Fig. 4) laterally in the y direction at distance dictated by the momentum dispersion dp/dy so that the new range corresponded to a proton momentum identical to that of the K 's for the same entrance y . Several values of dp/dy were obtained by graphically differentiating the (proton) momentum curve (Fig. 5) and from these values a curve (Fig. 13) was constructed to represent dp/dy as a function of the entrance plate.

In the correction, it was necessary to consider the paths between the entrance to the magnet and the emulsion stack (see Fig. 2). An exact analytical treatment would have been very difficult because of complexities arising from the action of the magnetic field and the geometry of the system. The help of several justifiable approximations, however, made possible a relatively simple yet quite accurate semianalytic procedure.

Since the total change in momentum, and hence in radius of curvature along the air path in the magnet, was only about 0.7% for the protons and 0.2% for the mesons, we can restrict the approximation to the first order in small quantities and to zeroth order in large quantities. The rate of change of momentum, dp/dx , can be treated as a constant over the air path, its magnitude depending on the particle type and initial momentum P_0 :

$$dp/dx = -K_1(p_0). \quad (14)$$

While in the magnetic field B , the particle trajectory has a radius of curvature

$$\rho = \frac{3.336 \text{ pc}}{BZ} \text{ cm}, \quad (15)$$

where pc is in Mev, B is in kilogauss, and the net charge Z on the particle is in electronic charge units ($Z = 1$ here). Assuming B is constant, we have p equal to a constant times the momentum:

$$p = k_2 p_0 \quad (16)$$

and hence $dp/dx = -k_1 k_2$.

If ϕ represents the angle through which the trajectory turns in a path length x , and if $\phi = 0$ at the entrance to the magnet, then at any value of ϕ , we have,

$$\begin{aligned} dx(\phi) &= p d\phi \\ &= [p_0 + (dp/d\phi)\phi] d\phi \end{aligned}$$

Since $(dp/d\phi)\phi_{\text{total}}$ is less than $0.01 p_0$,

we obtain $dx \approx p_0 d\phi$, (17)

and thus $dp = -k_1 k_2 p_0 d\phi = -k_3 d\phi$. (18)

The trajectory of a particle in the magnet, then, can be well approximated by the involute of an inextensible string of initial length p_0 winding around a cylinder of radius $k_3 = k_1 k_2 p_0$, as shown in Fig. 14, with k_3 exaggerated relative to the other dimensions ($k_3 \approx 10^{-2} p_0$). The locus of the center of curvature is actually a spiral, but the error caused by the circle approximation is very small ($\sim 3\%$ of the correction).

Referring to Fig. 14, we note that, at the exit from the magnet, the lateral separation Δy_m between a truly circular trajectory and the "actual" trajectory can be expressed as

$$\begin{aligned} \Delta y_m &= y'_m - y_m = p_0 + \Delta p + k_3 \sin \phi - p_0 \\ &= \Delta p + k_3 \sin \phi, \end{aligned}$$

and using Eq. (18) we have $\Delta y_m = -k_3 (\phi - \sin \phi)$. (19)

The angle $\Delta \phi$ between the two trajectories is

$$\Delta \phi = \phi' - \phi \approx \frac{k_3}{p_0} (1 - \cos \phi). \quad (20)$$

If the path length between the magnet and the stack is $x_A(p_0)$, the difference in y at the stack is

$$\begin{aligned}\Delta y_s &= y_s^i - y_s = y_m^i - y_m - x_A \sin \Delta \phi = \Delta y_m - x_A \Delta \phi \\ &= -k_3 \left[(\phi - \sin \phi) + \frac{x_A}{\rho_0} (1 - \cos \phi) \right] \\ &= \frac{dp}{dp} \frac{dp}{dx} \left[\rho_0 (\phi - \sin \phi) + x_A (1 - \cos \phi) \right].\end{aligned}\quad (21)$$

We designate the path length in the magnet as $x_m(\rho_0)$. The total path in air, x_T , after momentum analysis begins, is $x_T = x_m + x_A$, and we assume that these distances are the same for both mesons and protons of the same momentum. We now assign the subscripts K and P to all functions representing mesons and protons when these functions are substantially different for the two cases. If the momentum dispersion at the magnet is dp/dy (slowly varying function of y), then the difference Δp_s in the momentum between the protons and mesons at the entrance to the stack for any value of y is

$$\begin{aligned}\Delta p_s(y) &= (p_K - p_P)_s = \left(\frac{dp_K}{dx} - \frac{dp_P}{dx} \right) x_T + \frac{dp}{dy} \left[-(y_s^i - y_s)_K + (y_s^i - y_s)_P \right] \\ &= \left(\frac{dp_K}{dx} - \frac{dp_P}{dx} \right) \left\{ x_T - \frac{dp_0}{dp} \frac{dp}{dy} \left[\rho_0 (\phi - \sin \phi) + x_A (1 - \cos \phi) \right] \right\}.\end{aligned}\quad (22)$$

The value of Δp was computed for three different values of y . The proton momenta appropriate to these points were obtained from Fig. 5, and $\frac{dp}{dy}$ from Fig. 14. The remaining quantities in Eq. (22) were determined for each point as follows:

1. $\frac{dp}{dx} = \frac{1}{\beta c} \frac{dT}{dx}$: values of $\frac{dT}{dx}$ in air for both the protons and mesons were obtained from the range-energy tables (for protons) of Rich and Madey.³⁷ Appropriate numbers in the table were modified to hold for the mesons. The air density was taken equal to 1.19×10^{-3} g/cm³ (23°C).
2. ρ_0 and $d\rho_0/dp$ were determined by using the measured values of p and B (=9.25 kilogauss) in Eq. (15).
3. $\phi(\rho_0)$ was extracted from the original wire-trajectory data.
4. In order to include the effect of the fringe field, x_m was taken to be equal to $\rho_0 \phi$ rather than $x_{m(\text{geom})}$ the length predicted by ρ_0 , by the angle of incidence at the magnet, and by the magnet dimensions.

5. x_A , the magnetic-field-free path length was taken to be the physical distance from the stack to the magnet for a particular p_0 , less the effective extent of the fringe field as estimated from $\frac{x_m - x_m(\text{geom.})}{2}$ (factor 1/2 so as to include only the exit edge).

Knowing the momentum difference Δp between the mesons and the protons, one could calculate the distance Δy by which the proton range curve should be shifted so as to accurately represent the true meson momentum. The expression

$$\Delta y = - \Delta p / (dp/dy) \quad (\text{function of } y)$$

was used in computing the shift for each of the three points. A smooth curve $\Delta y(y)$ was then constructed to fit the points, and the proton range curve was changed accordingly. It was then possible to compute the masses as described in section II G.

This correction effectively increased the K mass values by an amount varying from $6.3 m_e$ at the high-momentum side to $8.5 m_e$ for the low momenta. The calculation should have been good to about 20%, or $1.5 m_e$.

D. Discussion of the Statistical Errors

Deviations of the individual mass values from the means were due largely to the spread in momentum characterizing any point at the face of the stack, and the range straggling of the K mesons. To a much lesser extent the precision of measurement was also affected by the uncertainty in determining the entrance position of a particular meson and by errors in measuring the true ranges of the mesons and the protons.

The incident K mesons were in a velocity region such that the range-momentum relationship could be expressed by $\frac{R}{M} = C_1 \left(\frac{p}{M} \right)^{3.0}$, (23)

$$\text{hence} \quad M = C_2 (R)^{-0.5} (p)^{1.5}, \quad (24)$$

where C_1 and C_2 are constants.

The percentage straggling can be considered independent of momentum over the small momentum interval existing at each entrance position. Thus we can consider, as contributing to the variations of M about the actual mass M_0 , the independent variations of R , due to straggling, about $R_0(y)$ and p about $p_0(y)$. R_0 is defined (Sec. III A 1.) by $M_0 = C_2 R_0^{-0.5} p_0^{1.5}$ where $p_0(y)$ is the mean momentum of particles entering the stack at a given position y . We assume that the relative standard deviations $\frac{\sigma_R}{R_0}$ and $\frac{\sigma_p}{p_0}$ were constant across the stack. The root-mean-square deviations are related according to

$$\frac{\sigma_M}{M_0} = \sqrt{(0.5 \frac{\sigma_R}{R_0})^2 + (1.5 \frac{\sigma_p}{p_0})^2} \quad (25)$$

The dominant contribution to $\frac{\sigma_R}{R_0}$ was the K-meson range straggling of 1.6%. The errors in measurement of the meson range were less than 0.2% and could be neglected in σ_R compared to the straggling, since each type of error enters the total in quadrature.

The total $\sigma_p/p_0 = 1.2\%$ was obtained from Eq. (25) by use of the observed value of $\sigma_M/M_0 = 1.9\%$ and the calculated $\sigma_R/R_0 = 1.6\%$. The errors in momentum deriving from inaccuracies in estimating the entrance position (0.1%) and in ascertaining the true mean proton range (0.2%) were an order of magnitude smaller than the total, and thus were negligible compared to the spread in incident momentum at each point on the face of the emulsion stack. This momentum spread was due mainly to the multiple scattering in the 0.09-inch Al window in the Bevatron vacuum tank and to a smaller extent to the finite size of the target and the multiple scattering in the air. The effect of the scattering in such a system is to give an apparently finite width to a line target, resulting in an apparent target that varies in density according to a gaussian distribution.

In this experiment the scattering was such as to produce a 1.2-inch width for a line target for the protons and a 0.63-inch width for the mesons, where the width represents twice the standard deviation on the gaussian. The actual target was 0.625 inch wide, and when the

effect of the scattering was folded into this square distribution, there resulted apparent targets that were in both cases almost gaussian and had widths of 1.2 inches (protons) and 0.70 inch (mesons).

The resulting magnitude of the relative spread in momentum may be estimated by the equation

$$\frac{dp}{p} = \frac{1}{p} \frac{dp}{dy} M_H L_T$$

where dp/dy is the change in momentum per cm across the stack, M_H the horizontal magnification of the lens system, and L_T the effective width of the target. The quantities p and dp/dy were found (from the proton ranges) to be about 360 Mev/c and 6.6 Mev/c/cm. The magnification was estimated from the wire measurements with an accuracy of about 30% to be about 0.6x. These values and those above for the estimated effective target size yield momentum spreads of 0.9% for the mesons, to be compared with the value 1.2% deduced from the mass distribution, and 1.6% for the protons, compared to 1.9% deduced from the proton range distribution. The magnitude of the increase in the momentum uncertainty due to scattering in the air was not calculated. However, a simple estimate indicated that the effect was no more than one-third as large as that caused by the scattering in the aluminum window. It is probable, then, that the slight disagreement between the predicted and deduced momentum spreads is due to the air-scattering. In general the agreement is very good, considering the uncertainty in the horizontal magnification.

E. Discussion of Possible Systematic Errors and some Small Corrections Affecting the Absolute Mass Values

Certain systematic errors may have affected the results. If a particular deviation of a measured value from the true value is calculable, a correction can be made to compensate for the error. Such cases are treated in Appendices A, B, and C. Some of the effects indicated below may be more properly classed as corrections than errors, and some possess the properties of both. For the sake of simplicity--albeit at some expense of logical order--we shall ignore this semantical problem. It is to be noticed, however, that in any case these errors apply to the absolute mass values alone; the values of the masses relative to each other are affected by only the nonsystematic errors discussed above unless there exist differences in energy-loss processes among the various types. The effects treated in the appendices above would have given slightly greater spreads to the mass distributions, as well as systematic deviations, had they not been considered.

Systematic errors may have resulted from:

1. Errors in the various corrections considered in appendices above. The magnitudes of these errors are estimated at the end of each Appendix and they affect the absolute mass as follows:

Error in range-shortening-correction $\sim 0.3 m_e$

Error in entrance-angle correction $\sim 0.4 m_e$

Error in effect-of-air correction $\sim 1.5 m_e$

We shall assume that these are randomly distributed and may be added in quadrature. Then the total uncertainty of these corrections should have been less than $1.6 m_e$.

2. Errors in the range-momentum relations.

The masses were determined on the basis of Eq. (1) rewritten as

$$M_K = M_P \frac{R_K(p_K)}{R_P\left(\frac{M_K}{M_P} p_K\right)}, \quad (26)$$

where $R_K(p_K)$ is the measured range of a meson of momentum p_K , and R_P is the proton range for momentum $\frac{M_P}{M_K} p_K$. The momentum p_K

was obtained from the tables of Barkas and Young,²¹ for the proton range $R_p(p_K)$ from Fig. 5. Thus, the mass determination involves the two different momentum regions p_K and $(M_p/M_K)p_K$. In the region of p_K , the proton range R_1 follows the approximation $R_1 = K_1 p_K^{1.35}$, and in the region of $p_2 = (M_p/M_K)p_K$ it obeys Eq. (23): $R_2 = K_2 p_2^{3.0}$. It can be shown that if the ranges R_1 and R_2 should be increased by ΔR_1 and ΔR_2 , respectively, in order to correct errors in the range-momentum relations, then the mass values obtained in the experiment should be corrected by adding

$$\Delta M = \left(\frac{\Delta R_2}{R_2} - \frac{3.0}{1.35} \frac{\Delta R_1}{R_1} \right) M \quad (27)$$

to the measured values. The range curves used were compared with the range tables recently recalculated by Barkas.³⁸ The curve expressing $R_1(p_1)$ was found to agree almost exactly with the new values, but the values for $R_2(p_2)$ were slightly lower than the revised points in the upper third of the momentum region concerned, the maximum error being about 0.55% in R_2 . It was estimated, by examining the particles' range-distribution, that one third of the mass values should be increased by $2m_e$, thus a total correction of $0.7 m_e$ on all mass values should be made if the new range values are correct.

An error in the value of the mean ionization potential used³⁸ (323 ± 11 ev) is reflected in a constant percentage error in the momentum deduced from any range, according to Barkas.³⁹ Equation (27) may be rewritten as

$$\Delta M = 3.0 (\Delta p_2/p_2 - \Delta p_1/p_1) M.$$

Thus if the percentage error is constant owing to an error in the mean ionization potential, the mass should not be affected by the latter error.

In summary, we find that all mass values in this experiment should be increased by about $0.7 m_e$ to correct for small errors in the range-momentum relations²¹ that were used. This correction appears in Table II.

3. Deviation of the average mass from the true mass.

Equation (24) indicates the behavior of the mass values determined as a function of the meson range R and momentum p . Symmetric distributions in R (straggling) and p_K (finite target thickness) thus lead to a small asymmetry in the mass distribution, and hence, to a deviation of the average value of the mass from the true mass. A calculation based on Eq. (24) and good to the second order in small variations of R and p yielded $\langle M \rangle - M_0 = +0.2 m_e$, where $\langle M \rangle$ is the average obtained by our method and M_0 the true mass.

The average mass given by the predicted mass distribution (Fig. 9) was computed to be $0.3 m_e$ higher than M_0 , in good agreement with the above calculated results. Choosing the value obtained from the predicted distribution as the more reliable, we then must lower all average mass values by $0.3 m_e$, as indicated in Table II.

4. Correction for emulsion density.

The measured density of the emulsion was $3.80 \pm 0.02 \text{ g/cm}^3$. Barkas and Young's tables²¹ were based on an emulsion density of 3.815 g/cm^3 . In order to be valid for the measured density, all ranges in the tables should be increased by $0.04 \pm .05\%$. From Eq. (27) and $M = 965 m_e$ it follows that all masses should be increased by $0.4 \pm 0.5 m_e$, as has been done in Table II.

5. Acceptance of inelastically scattered particles.

An examination of the entrance angles and positions of all τ 's and K_{μ}^{2} 's indicated that a significant fraction of the abnormally high mass values may have resulted from particles' having scattered off the pole pieces; others may have experienced inelastic collisions in the emulsion. If this was true, none of these events should have been included in the mass averaging, and the mass cutoff was justified. There may have been cases similar to these, whose mass values still lay within the acceptance limits, thus affecting the average mass in spite of the cutoff procedure. In an attempt to determine the result of their inclusion, it was assumed that the distribution of these events was of constant density from the average mass value to the point at $1100 m_e$. The density was taken from the points lying outside the curve in Fig. 9. Such a distribution would have increased the average value by $0.5 m_e$. A correction of $-0.5 \pm 0.25 m_e$ has been included in Table II to account for this effect.

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Fig. 1. Sketch of the basic experimental arrangement, indicating the coordinate system assigned to the emulsion stack.

MU-9161

Fig. 2. The strong-focusing spectrometer.

Fig. 3. A typical proton-ending distribution (plate 75).

Fig. 4. The corrected proton range curves $R_p(p_p)$ and $R_p(p_K)$ vs plate number. The upper curve was used to represent the K-meson momentum p_K in the mass determination as indicated in Fig. 7. (All measured K-particles entered in Plates 10 to 87.)

Fig. 5. Incident proton momentum p_p plotted as $\beta\gamma = \frac{1}{c} \frac{p_p}{M_p}$ vs plate number.

Fig. 6. Normalized results of blob counts made on K_L secondaries near the point of decay. Average number of blobs counted per track: 790. Curve $\sigma = 3.75\%$.

Fig. 7. Curves giving $R_K (M_K, p_K)$ vs $R_P (M_P, p_K)$ for various values of M_K .

W-11218

Fig. 8. Histograms showing the $\tau + \tau'$, K_{τ^2} , and K_{μ^2} mass distributions. The shaded events were eliminated from the mass averages by Chauvenet's criterion.

MASS DISTRIBUTION OF ALL MODES AND THE
PREDICTED DISTRIBUTION
SHADED EVENTS OMITTED FROM $\bar{M}$

WU-11212

Fig. 9. Histogram of the masses of all decay modes combined, and the predicted mass distribution (normalized). The shaded events were eliminated from the mass average.

Fig. 10. The average masses of the indicated K-meson type and the statistical standard errors.

MD-11251

Fig. 11. The tangent of the incident dip angle ϕ_D vs plate number.

Fig. 12. The typical lateral (inward) shift of the emulsion after development, measured about 3 mm from the leading edge of one of the plates. Direction of the incident particles in the emulsion is indicated.

HU-11253

Fig. 13. The momentum dispersion $\frac{dp}{dy}$ according to the entrance plate number.

MU-11254

Fig. 14. Diagram used in obtaining the correction for the momentum loss in air.