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The Fundamental Role of Solitons in Nonlinear Dispersive Partial Differential Equations

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A. Khare (Sachivalaya Marg) and Philip Rosenau (Tel Aviv University)

Abstract

This is the final report of a three-year Laboratory Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). Numerical simulations and mathematical analysis have proved crucial to understanding the fundamental role of solitons in the evolution of general initial data for quasilinear dispersive partial differential equations, such as the Korteweg-de Vries, nonlinear Schrödinger and the Kadomtsev-Petviashvili equations. These equations have linear dispersion and the solitons have infinite support. Recently, Philip Rosenau and Mac Hyman discovered a new class of solitons with compact support for similar equations with nonlinear dispersion. These 'compactons' display the same modal decompositions and structural stability observed in earlier integrable partial differential equations. They form from arbitrary initial data, are nonlinearly self stabilizing and maintain their coherence after multiple collisions, even though the equations are not integrable. In related joint research, Roberto Camassa and Darryl Holm, made the remarkable discovery that a similar nonlinear dispersive equation can be described by the evolution of solitons with a peaked solution. The equations are Hamiltonian and a subclass is biHamiltonian and, hence, possess an infinite number of conservation laws. This research is the opening for a far reaching and new understanding of the central role of solitons in nonlinear dispersion.

Background and Research Objectives

Our main interest is in the light that understanding the dynamics of the compactons will shed on the theory of solitons. The Korteweg-de Vries, nonlinear Schrödinger and other classical soliton equations are all integrable. This means, roughly, that their solutions satisfy infinitely many conservation laws, much as physical systems obey laws such as conservation of energy and momentum. Integrability helps explain solitons' extraordinary stability. The infinite number of conservation laws constrain the classical solitons so rigidly that they can hardly fall apart.

Compacton equations, however, are not integrable; they satisfy only a handful of conservation laws. So you would not expect the compactons to remain coherent when two of them collide. The unexpected and amazing result that the compactons emerge intact, just

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like classical, integrable solitons, indicate that the remarkable stability of solitary waves lies deeper than mere integrability.

Importance to LANL's Science and Technology Base and National R&D Needs

The robustness of these compactons and the inapplicability of the inverse scattering tools, that worked so well for integrable wave equations, makes it clear that the nonlinear mechanism that causes these structures is extremely robust. We have seen that elastic collision is accompanied by the birth of a compact oscillatory wave, which slowly decomposes into compacton-anticompactons pairs. This event has no counterpart in the conventional soliton theory. Naturally one would like to find a physical application for these compactons, say in nonlinear optics or developing a field theory with particles described by compactons.

Scientific Approach and Accomplishments

We have been investigating the role of compactons in nonlinear dispersive partial differential equations in pattern formation. The solitary wave solutions of these equations have several remarkable properties. The compactons collide elastically, but unlike the traditional nonlinear wave solitons, they have compact support. When two "compactons" collide, the interaction site is marked by the birth of a low-amplitude compacton-anticompacton pair. These equations seem to have only a finite number of local conservation laws. Nevertheless the behavior and the stability of the compact solitary waves is very similar to what is observed in completely integrable systems.

We have discovered a class of solitary waves with compact support (which we call compactons) that are solutions of a multi-parameter family of fully nonlinear dispersive partial differential equations. Compactons are solitary waves with the remarkable soliton property that after colliding with other compactons, they re-emerge with the same coherent shape. These particle-like waves exhibit elastic collisions that are similar to the soliton interactions associated with completely integrable PDEs supporting an infinite number of conservation laws. However, unlike the soliton collisions in an integrable system, the point where two compactons collide is marked by the creation of a low amplitude compacton-anticompacton pair.

Still more surprising is a brand new feature, not seen in classical solitons: when compactons collide, interact, and separate, they leave behind a wake of tiny ripples. We almost missed this in our first compacton calculations---and thought we saw only some numerical noise in the results, stemming from numerical errors in the computation. It was only when we decided to do an extra, high-resolution calculation to get rid of this numerical

noise, we realized that the ripples were real. These ripples are a real mystery. They seem to continue indefinitely, with tinier and tinier ripples arising, a kind of flotsam caused, perhaps, by the compacton equations' lack of integrability. There's no proof yet that the ripples do not finally die out, just numerical evidence that smaller and smaller ripples continue to arise.

We have also used numerical studies to demonstrate that, in addition to conventional solitons, the quintic Korteweg-de Vries equation supports multihumped solitary waves (doublets, triplets, quadruplets, etc.), referred to collectively as multiplets. Their peaks pulsate as they travel and undergo nearly elastic collisions with other multiplets. An N-humped multiplet can pulsate thousands of cycles before disassociating into an (N-1)-humped multiplet and a single peak solitary wave (singlet). Although multiplets are easily created from an initial wide compact pulse, they rarely are formed by fusing singlets or multiplets in collisions. We have discovered and investigated the emergence and evolution of multiplets, their nearly elastic collision dynamics and their eventual decomposition into singlets. The impact of cubic dispersion critically depends on the sign of cubic dispersion and its amplitude. For sufficiently large cubic dispersion, only a train of singlets emerge from an initial pulse with compact support. If the cubic dispersion is decreased, multiplets begin to emerge leading the train of singlets. The number of humps in the multiplet increases as the cubic dispersion is decreased, until below a critical point when the initial pulse decomposes into highly oscillatory waves.

Publications

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3. Hyman, J. and Rosenau, P. "Pulsating Multiplet Solutions of Quintic Wave Equations," to appear in *Physica D*, LA-UR-97-3704.