The Superconducting Super Collider

Beam-Beam Stability in the SSC A Preliminary Report

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SSC Central Design Group

January 1988

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**BEAM-BEAM STABILITY IN THE SSC
A PRELIMINARY REPORT**

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I. Introduction

The stability of proton orbits in the SSC is threatened by two principle elements in the beam system: magnet errors and the beam-beam interaction. Of these, magnet errors are considered to be of greater importance because they are more tenacious (CDR [1] pp.63-65,124-137). Beam-beam effects can be reduced if necessary by widening the beam crossing angle (although such widening inevitably results in loss of luminosity). Magnet errors, however, cannot be removed and their effects are difficult to compensate for.

The stability of the beam system is characterized by a curve in the two-dimensional amplitude plane which separates dynamically stable trajectories, at small amplitudes, from dynamically unstable ("chaotic") trajectories, at large amplitudes. The area of stable motion is referred to here as the "stability aperture". Protons outside the stability aperture exhibit chaotic motion and are lost rapidly from the beam. Those inside the aperture, in the absence of single particle scattering, remain confined indefinitely. Although the edge of the stability aperture is not a "fine line", as it would be for a one-degree-of-freedom system with time dependence, the amplitudes representing the transition from stable to unstable motion are fairly well defined in the SSC.

The primary purpose of this investigation is to roughly determine the size and shape of the stability aperture induced by the beam-beam interaction alone, and its dependence on the principle machine parameters. This information, together with a determination of the magnet error aperture [2-4] and the combined total aperture, is needed to select the nominal beam crossing angle.

The term "stability aperture" is used here in lieu "dynamic aperture" because the two quantities are measured slightly differently and therefore represent different properties. The simulation runs used here represent

200 times as many machine revolutions as the runs used previously to establish the dynamic aperture for magnet errors (150,000 revolutions compared to 400). Furthermore, a particle is deemed to be unstable here when it exhibits erratic motion in phase space; it doesn't have to diffuse all the way to the hard aperture, as in the dynamic aperture studies. The stability aperture is therefore expected to be somewhat smaller than the dynamic aperture as it has been previously measured.

The beam-beam stability aperture is mapped for three different working tunes and with varying combinations of the three principle elements in the beam-beam interaction: the "short-range" force, the "long-range" force, and tune modulation. Also investigated are the sensitivities of the stability aperture to variations of the principle machine parameters (e.g. chromaticity, synchrotron frequency, tunes, tune shifts, and beam crossing angle).

The results indicate that the most significant nonlinear element in the SSC beam system is the "long range" force (CDR [1] p.195-202). This force is a direct consequence of the small-angle crossing at the interaction points. The long range force causes particle motion to be unstable when amplitudes are large enough to bring them close to the opposing beam while approaching and departing the interaction point.

The width of the stability aperture is determined primarily by the crossing angle of the beams at the interaction point. This width (in radians) is roughly the beam separation less 15 μ radians. The nominal width of the beam is 10 μ radians and the nominal beam separation is 75 μ radians. This gives a stability aperture of about 60 μ radians or 6σ .

Other parameters also effect the width of the stability aperture, but less so. The aperture width for net linear tune shifts of $\xi_x = \xi_z = .0007$ is about $\Delta a_z = 7\sigma$. When these are quadrupled to $\xi_x = \xi_z = .003$, the width shrinks to about $\Delta a_z = 5.5\sigma$.

When tune modulation is added to the model, the aperture width shrinks by about 8% at the nominal parameter values (the deleterious effects of tune modulation have been previously investigated by Piwinski [5]). Doubling or halving either the modulation frequency or depth has little effect on the aperture width.

Based on previous experiences with simulation, this study is believed to underestimate instability. The simulation neglects many sources of resonance excitation, the most significant of which are magnet errors and other lattice imperfections. While a more careful study would probably show stability aperture widths smaller than those found here, it's possible that they would not be very much smaller since the motion inside the aperture appears to be very stable, i.e. stability doesn't begin to seriously suffer from beam-beam effects until amplitudes get very close to exceeding the stability aperture.

A parallel investigation of beam-beam stability has been conducted by John Irwin [6]. His simulation is more sophisticated than the simulation used here: his beam-beam interactions are represented individually, he assumes alternating crossing planes, and he includes the effect of magnet imperfections in the IR quadrupole triplets. Irwin's estimation of the size of the stability aperture agrees well with the finding presented here.

II. Simulation Model

The simulation model used here is quite primitive in comparison to the sophisticated tracking codes that have recently been developed to evaluate the performance of the machine lattice. However, the simplicity of the simulation, together with the fact that it's performed on a CRAY-2, gives it a very short run time and allows for the investigation of the beam's long-term behavior (typical simulation runs in this study correspond to 150,000 revolutions or about 44 seconds of actual run time)

It should be emphasized that because the simulation is so simple, it is also quite conservative. The dynamical elements in the model are included at the strengths specified in the design report. No attempt is made to compensate for the effects of dynamical elements not included in the model: either those present in the design or those that will inevitably be present due to imperfections in the actual machine. It is believed that these additional effects, if included, could only result in a further loss of dynamical stability. Thus, actual machine performance is expected to be worse than that observed in the simulation.

The simulation focuses entirely on the role of the beam-beam interaction. The model machine has a perfectly linear lattice and a single beam-beam kick which consists of a "short range" part and a "long-range" part. These are the simulation's only elements. The system is two-dimensional (includes both horizontal and vertical oscillations) and both tunes are modulated (the depth and phase of modulation can be different for the two oscillators, but their frequencies must be the same).

In the simulation model, the short range kicks to the horizontal and vertical momenta are precisely those that would be received were the beams of perfectly Gaussian, round, cross-section and colliding head on. They are thus inaccurate in several respects. If the amplitude of the synchrotron oscillation is non-zero, the actual short range force oscillates at

twice the synchrotron frequency while the center of the opposing beam sways (as it would were there a non-zero dispersion at the interaction point). Even when the amplitude of synchrotron oscillation is zero, the apparent vertical width of the opposing beam depends on the vertical velocity of the particle. A more accurate short-range model would excite more resonances and thus yield poorer performance than the present model.

The short-range kick used in the simulation model will be referred to in subsequent discussions as the "head-on" kick, to distinguish it from the actual short-range kick received with a non-zero crossing angle.

The model of the long-range kick is more accurate than that for the short-range. It is divided into two parts: one that comes before the head-on kick and one that comes after. It is assumed that the betatron phase advance between the first and second long-range kicks is exactly 180° in both planes and that the head-on kick comes half-way between them (90° after the first long-range kick). The crossing is assumed to take place in the vertical plane. Both the first and second long-range kick represent the accumulated phase displacements of 30 identical bunch passings (and are therefore strictly representative of only particles near the middle of a collision train). While the 30 passings are taking place, it is assumed that the two beams are separated by a constant distance of 7.5σ .

The single beam-beam interaction in the simulation is given nominal linear head-on tune shifts of $\xi_x = \xi_z = .0015$. This is intended to approximate the combined tunes shifts of the four interaction points in the actual machine.

The horizontal and vertical tunes $\nu_x(t)$ and $\nu_z(t)$ are modulated at the synchrotron tune ν_s , at depths $\Delta\nu_x = M_x$ and $\Delta\nu_z = M_z$,

$$\begin{aligned}\nu_x(t) &= \nu_{x0} + M_x \cos 2\pi\nu_s t \\ \nu_z(t) &= \nu_{z0} + M_z \cos 2\pi\nu_s t\end{aligned}$$

where v_{x0} and v_{z0} are the working point tunes. The time t is normalized to the period of revolution. Modulation in the simulation represents only chromaticity (at the predicted magnitude $M_x = M_z = .0015$); other modulation effects, such as those due to dispersion or magnet current ripple, are not included in the simulation. The simulation uses normalized variables x, x', z, z' for the transverse displacements and velocities. The normalized horizontal displacement x and velocity x' are related to the actual displacement and velocity X and X' by

$$x = \eta_x \sqrt{\beta_x^*} / \sigma = (X / \sigma) \sqrt{\beta_x^* / \beta_x(s)}$$

$$x' = \eta'_x \sqrt{\beta_x^*} / \sigma = (X' / \sigma) \sqrt{\beta_x^* \beta_x(s)}$$

The variables z and z' are defined by a similar expression. The constants β_x^* and β_z^* are the values of the horizontal and vertical betafunctions $\beta_x(s)$ and $\beta_z(s)$ at the interaction point. The constant σ is the rms beam half-width at the interaction point.

The expressions defining the long and short-range kicks are discussed in detail in the following three sections. The actual map used in the simulation is summarized below.

Linear oscillation between interaction regions

$$x_1 = x' \sin 2\pi(v_x(t) - .5) + x \cos 2\pi(v_x(t) - .5)$$

$$x'_1 = x' \cos 2\pi(v_x(t) - .5) - x \sin 2\pi(v_x(t) - .5)$$

$$z_1 = z' \sin 2\pi(v_z(t) - .5) + z \cos 2\pi(v_z(t) - .5)$$

$$z'_1 = z' \cos 2\pi(v_z(t) - .5) - z \sin 2\pi(v_z(t) - .5)$$

First long-range kick $R^2 \equiv x_1^2 + (z_1 + d)^2$

$$x'_2 = x'_1 + 30 \cdot 8\pi\xi_x \cdot x_1 \cdot (1 - \exp[-R^2/2]) / R^2$$

$$z'_2 = z'_1 + 30 \cdot 8\pi\xi_x \cdot (z_1+d) \cdot (1 - \exp[-R^2/2]) / R^2$$

Linear oscillation between long-range and head-on kicks

$$x_3 = x'_2$$

$$x'_3 = -x_2$$

$$z_3 = z'_2$$

$$z'_3 = -z_2$$

Head-on kick $R^2 \equiv x_3^2 + z_3^2$

$$x'_4 = x'_3 + 4\pi\xi_x \cdot x_3 \cdot (1 - \exp[-R^2/2]) / R^2$$

$$z'_4 = z'_3 + 4\pi\xi_x \cdot z_3 \cdot (1 - \exp[-R^2/2]) / R^2$$

Linear oscillation between head-on and long-range kicks

$$x_5 = x'_4$$

$$x'_5 = -x_4$$

$$z_5 = z'_4$$

$$z'_5 = -z_4$$

Second long-range kick $R^2 \equiv x_5^2 + (z_5-d)^2$

$$x'_6 = x'_5 + 30 \cdot 8\pi\xi_x \cdot x_5 \cdot (1 - \exp[-R^2/2]) / R^2$$

$$z'_6 = z'_5 + 30 \cdot 8\pi\xi_x \cdot (z_5-d) \cdot (1 - \exp[-R^2/2]) / R^2$$

The simulation model underestimates instability because it ignores many sources of nonlinear resonance excitation. The principle additional sources of resonance excitation are lattice imperfections. These include but are not limited to magnet field errors.

II.A Head-on Force

The beam can be represented by a charge distribution that is translationally invariant in the Y direction and axially symmetric about the Y axis with a Gaussian cross-section

$$\rho(R) \propto \exp[-R^2/2\sigma^2] \quad (1)$$

where

$$R^2 = X^2 + Z^2$$

and σ is a constant.

The horizontal and vertical forces $g_X(X,Z)$ and $g_Z(X,Z)$ on a charged particle at X,Z are then proportional to

$$\begin{aligned} g_X(X,Z) &\propto \frac{X}{R^2} (1 - \exp[-R^2/2\sigma^2]) \\ g_Z(X,Z) &\propto \frac{Z}{R^2} (1 - \exp[-R^2/2\sigma^2]) \end{aligned} \quad (2)$$

The transverse impulses $\Delta X'$ and $\Delta Z'$ received by a particle that collides "head-on" (the two colliding beams are moving along the same design orbit) with a bunch of the opposing round beam are proportional to the forces (2) and given by

$$\begin{aligned} \Delta X'(X,Z) &= \frac{-8\pi \xi_X \sigma^2 X}{R^2 \beta_X^*} (1 - \exp[-R^2/2\sigma^2]) \\ \Delta Z'(X,Z) &= \frac{-8\pi \xi_Z \sigma^2 Z}{R^2 \beta_Z^*} (1 - \exp[-R^2/2\sigma^2]) \end{aligned} \quad (3)$$

where

$$X' \equiv dX/ds$$

$$Z' \equiv dZ/ds$$

The constants ξ_x and ξ_z are the horizontal and vertical tune shifts, β^*_x and β^*_z are the horizontal and vertical betafunctions at the interaction point, and σ is the beam width. The expressions (3) assume that the betafunctions are approximately constant over one bunch length about the interaction point and that the betatron phase advance through this region is negligible.

II.B The Short-Range Force with a Non-Zero Crossing Angle

In the SSC, the beams cross at a small angle $\alpha = 75$ μ radians rather than colliding head-on. The head-on force given in the preceding section must therefore be modified to take into account this non-zero crossing angle.

Each time a particle passes through an interaction region, it also passes through a bunch of the opposing beam. The transverse impulse that it receives from that bunch depends on the angle α at which it crosses the bunch. It is assumed that the crossing takes place in the vertical plane and that

1. Beam bunches have a round, Gaussian, transverse section with RMS width $\sigma = 5 \times 10^{-6}$ m, and a Gaussian longitudinal distribution with RMS width $\sigma_s = 7.5 \times 10^{-2}$ m. The bunches are separated by a distance $S_B = 4.8$ m (see fig.1).

2. The force felt by a particle in the interaction region is transverse and can be determined entirely by the transverse density profile of the opposing beam at the precise position s of the particle.

A particle in a beam, because it can only "see" transversally, doesn't know how fast the opposing beam is moving; it only knows the frequency at which it is passing bunches of the opposing beam. If the particle assumes that the opposing beam is at rest in the lab frame, then the bunch lengths and spacings of the opposing beam appear to be half of (what we know to be) their actual lab frame values.

For calculations of the net change in transverse momenta due to bunch crossing the longitudinal variable s will be the distance (in lab frame units) of the particle from the nominal interaction point (the place where the exact centers of the two bunches cross in the lab frame). The value of s is negative before the interaction point, positive after. The variable Z_0 is

the vertical distance of the particle from the design orbit of the opposing bunch; the variable Z_S is the vertical distance of the particle from the design orbit of its own bunch. The particle is assumed to be above the opposing beam, with Z_0 positive, before the crossing (see fig.2).

The displacement Z_0 can be written as a function of the effective opening angle α_{eff} , the vertical displacement of the particle from its own design orbit when it passes the interaction point $Z_S(s=0)$, and the longitudinal displacement s :

$$Z_0 = Z_S(s=0) - s\alpha_{\text{eff}} \quad (4)$$

The effective opening angle α_{eff} is the sum of the actual opening angle α and an additional term due to the betatron phase advance. If dZ_S/ds can be assumed constant during the crossing, the effective opening angle is given by

$$\alpha_{\text{eff}} = \alpha + \Delta Z_S / \Delta s \quad (5)$$

where ΔZ_S and Δs are the changes in Z_S and s that occur during the crossing. The particle is inside the crossed bunch for a distance $\Delta s = .07$ m, and the betafunction is approximately constant at $\beta^* = .5$ m in both planes. So the betatron phase advance during this time is

$$\begin{aligned} \Delta\phi &= \Delta s / \beta^* = .07 / .5 \\ &= .14 \text{ radians} \\ &\approx 8 \text{ degrees} \end{aligned}$$

For a particle with a vertical betatron amplitude of $A_Z = \sigma$, this corresponds to a maximum possible displacement of

$$\begin{aligned} \max(\Delta Z_S) &= \Delta\phi A_Z = .14 \sigma \\ &= .65 \times 10^{-6} \text{ m} \end{aligned}$$

This, in turn, makes the effective crossing angle

$$\begin{aligned}\alpha_{\text{eff}} &= \alpha + \Delta Z_s / \Delta s \\ &= 75. \times 10^{-6} + 10. \times 10^{-6} \text{ radians}\end{aligned}$$

This 12% adjustment to the crossing angle can probably be neglected, but since ΔZ_s is proportional to amplitude, this error reaches 100% at $A_z = 8.3\sigma$. Therefore, it's possible to make the approximation that α_{eff} is equal to the actual crossing angle α only when the Z amplitude is small ($A_z < 2\sigma$). A careful study would take this effect into account by setting the effective opening angle equal to

$$\begin{aligned}\alpha_{\text{eff}} &= \alpha + (A_z / \beta_z^*) \sin \varphi \\ &= \alpha + (A_z^2 - Z^2)^{1/2} / \beta_z^*\end{aligned}$$

or, using the fact that $\beta_z^* = .5 \text{ m}$

$$\alpha_{\text{eff}} = \alpha + 2\sigma (a_z^2 - z^2)^{1/2} \quad (6)$$

where a_z and z are the vertical amplitude and displacement normalized to the nominal beam width σ .

The beams are assumed to be highly relativistic, so that only the density of the opposing beam at the point exactly transverse to the particle has an effect on the particle at a given time. The density of the opposing bunch transverse to the particle is given by

$$\rho(s) \propto \exp [-(2s - \delta s)^2 / 2\sigma_s^2] \quad (7)$$

where δs is the longitudinal displacement of the particle from the center of its own bunch. The factor of 4 in the numerator of the exponent comes from the fact that the opposing bunches appear to have half their lab

frame lengths.

The instantaneous force felt by a proton, given the two assumptions stated above, is

$$g(X_0, Z_0, s) \propto \rho(s) Z_0 (1 - \exp[-R^2/2\sigma^2]) / R^2 \quad (8)$$

where $R^2 = X_0^2 + Z_0^2$. It's convenient to change the longitudinal displacement variable s in (7) to

$$s' \equiv 2s - \delta s$$

so that

$$s = (s' + \delta s) / 2 \quad (9)$$

The impulses $\Delta X'$ and $\Delta Z'$ to the horizontal and vertical velocities of the particle are given by the s' integrals of the force:

$$\Delta Z' \propto \int_{-\infty}^{+\infty} ds' \exp[-s'^2/2\sigma_s^2] Z_0(s') (1 - \exp[-R^2/2\sigma^2]) / R^2 \quad (10)$$

with, from (4) and (9)

$$R^2 = X(s')^2 + Z(s')^2$$

$$X_0(s') = X_s(s=0)$$

$$Z_0(s') = Z_s(s=0) - \alpha_{\text{eff}} (s' + \delta s) / 2 \quad (11)$$

The constant $X_s(s=0)$ is the horizontal displacement of the particle from the center of its own beam. It is assumed here to be constant, though the horizontal betatron phase, like the vertical phase, actually changes by about 8° during the crossing. A similar expression gives the horizontal impulse

$$\Delta X' \propto \int_{-\infty}^{+\infty} ds \exp [-s'^2/2\sigma_s^2] X_0(s)/R^2 (1 - \exp [-R^2/2\sigma^2]) \quad (12)$$

The integrands of (10) and (12) are illustrated in figs. 3 and 4 as the product of two component functions: a Gaussian representing the projected density of the opposing bunch onto its design orbit, and the force function that would apply were the opposing beam unbunched. The case shown is for $X_0 = X_S = 0$.

The integrals (10) and (12) depend entirely on three quantities; the horizontal displacement X_0 and two others which will be designated p_1 and p_2 . The first p_1 is the ratio of the Gaussian half-width to the half-width of the force function (see figs. 3 and 4),

$$p_1 = \sigma_s / [2\sigma/\alpha] \quad (13)$$

The second p_2 is the ratio of the distance between the centers of the Gaussian and force function to the Gaussian half-width,

$$p_2 = |2 Z_S(s=0)/\alpha_{\text{eff}} - \delta s| / [2\sigma/\alpha] \quad (14)$$

The numerators of both p_1 and p_2 are shown explicitly in figs. 3 and 4. The first parameter p_1 will be called the "opening parameter". It's also equal to the ratio of the change in the vertical displacement Z_0 of a particle as it passes through the opposing beam, to the the beam full width. For the SSC,

$$\alpha = 75 \times 10^{-6} \text{ radians}$$

$$\sigma_s = .075 \text{ m}$$

$$\sigma = 5 \times 10^{-6} \text{ m}$$

so

$$p_1 = \sigma_s \alpha / 2\sigma = .56$$

The integrals (10) and (12) can be easily approximated in two limiting cases: when $p_1 \ll 1$ and when $p_1 \gg 1$. The former corresponds to a small crossing angle and in the limit $\alpha \rightarrow 0$, gives the case of head-on collisions. In this case, the force function widths in figs. 3 and 4 are infinite compared to the Gaussian width so the integrals collapse to the value of the force functions at $X_S(s=0)$, $Z_S(s=0)$.

The latter case, $p_1 \gg 1$, corresponds to large-angle crossing. In this limit, the width of the force functions in figs. 3 and 4 are much smaller than the widths of the Gaussians. So the vertical kick $\Delta Z'$ goes to zero (it's an integral over an odd function) while the horizontal kick reduces to the integral of the horizontal force function,

$$\Delta X' \propto \exp [-(2 Z_S(s=0) - \delta s)^2 / 2 (\alpha_{\text{eff}} \sigma_S)^2] \times \\ X_S(s=0) \int_{-\infty}^{+\infty} ds' (1 - \exp [-R^2/2\sigma^2]) / R^2 \quad (15)$$

The SSC cannot be reliably represented by either limiting case since its parameter $p_1 = .56$ is neither large nor very small. Figures 3 and 4 depict fairly accurately the relative widths of the Gaussian and force functions.

The parameter p_1 is independent of the interaction point coordinates $X_S(s=0)$ and $Z_S(s=0)$, and is thus a simple constant characterizing the machine. If the values of p_1 and the horizontal interaction point displacement $X_0(s=0)$ are known, the shapes of the force functions in figs. 3 and 4 are completely determined.

The second parameter p_2 gives the remaining information necessary to find the value of the impulse. These values are shown in figs. 5 and 6 as a function of

$$p_2 = \frac{1}{\sigma} \left(Z_S(s=0) - \frac{\alpha \delta s}{2} \right) \quad (16)$$

for several different values of α (and thus of p_1). In figure 5, $\Delta Z'$ is plotted for $X_S(s=0)=0$, while in figure 6, $\Delta X'$ is plotted for $X_S(s=0)=\sigma$.

The head-on case is given by $\alpha = 0$, so that the actual crossing angle of $\alpha=75$ μ radians gives a tune shift (proportional to the slope of the impulse function at $p_2=0$) that is some 10% less than the head-on tune shift.

Figures 5 and 6 also show the way in which a slow synchrotron modulation couples into the beam-beam force. As a proton's longitudinal energy oscillates, its position δs inside its bunch also oscillates. This results in a slow oscillation of the 2nd term in (16). Since this term corresponds to the center of betatron oscillation in figs. 5 and 6, it is apparent that the modulation affects the vertical and horizontal motion in different ways. From fig. 5, a slow oscillation of the betatron oscillation center about $p_2=0$ with a typical excursion of $\Delta p_2 = p_1$ (corresponding to $\delta s = \pm \sigma_S$) gives an effect very similar to vertical dispersion at the interaction point. It corresponds to a dispersion-induced excursion of the design orbit of between .25 and .5 beam widths.

In the horizontal direction, fig. 6, a longitudinal displacement δs does not create a dispersion-like effect, but it does serve to attenuate the magnitude of the horizontal impulse. Synchrotron oscillation thus results in a modulation of the horizontal beam-beam force at twice the synchrotron frequency. Here, as in fig. 5, a typical excursion of p_2 due to synchrotron modulation is $\Delta p_2 = p_1$.

Although this information about the nature of the synchro-betatron coupling due to a non-zero crossing angle in the short range beam-beam interaction will not be used in the simulations that appear in the remainder of this report (which uses the head-on force in place of the short range

force), it could be used to improve the validity of future simulation models. It is not altogether clear whether a careful treatment of the short range force would improve or further degrade the performance of the simulation model. One might expect that the head-on force's 10% over-estimation of the actual tune shift would more than offset whatever deleterious effects might arise from the synchrobetatron coupling mentioned above. Furthermore, it appears from the studies presented in the following sections that the long range beam-beam force is the dominant cause of dynamical instability in the SSC and that a precise treatment of the short range force is unnecessary.

II.C Long-Range Forces

In addition to the "short-range" kick received by a particle as it passes through a bunch of the opposing beam at an interaction point, there are several "long-range" kicks that are received immediately preceding and following the short-range kick. These auxiliary kicks are due to other bunches in the opposing beam that are not actually crossed by the particle, but are nonetheless passed by it in the interaction region. The length of the low beta interaction region in the SSC is about $2L^* = 150$ m, with the interaction point at its center. Since bunches are 4.8 meters apart, the particle passes about $2 \times 75 / 4.8 = 30$ bunches before it gets to the interaction point and about 30 bunches after it passes the interaction point. Each time it passes one of these bunches, it receives a kick.

To construct a model for these kicks, it is first necessary to determine the phase advance between them. The betafunction in the interaction region is given by

$$\beta = \beta^* + s^2/\beta^* \quad (17)$$

Since the beam width σ is proportional to $\beta^{1/2}$, the transverse envelope of each beam is hyperbolic with respect to s . The asymptotes come out at an angle of

$$\alpha_1 \approx \sigma/s = (\beta/\beta^*)^{1/2} (\sigma^*/s) \quad (18)$$

where σ^* is the beam width at the interaction point (see fig.7). Since from (17),

$$s = (\beta\beta^*)^{1/2} \quad s \gg \beta^*$$

equation (18) gives an opening angle of

$$\alpha_1 = \sigma^*/\beta^* = 5 \times 10^{-6} / .5 = 10 \text{ } \mu\text{radians}$$

Since the crossing angle α is 75 μ radians, the ratio α/α_1 of beam separation to beam half-width is approximately a constant 7.5 when $s \gg \beta^* = .5$ m. This ratio can be changed by varying α , σ^* or β^* .

The phase advance in the interaction region is given by the integral

$$\Delta\phi = \int_x^y ds/\beta(s) = \arctan (s/\beta^*) \Big|_x^y \quad (19)$$

Thus, if the phase is zero when the particle passes the interaction point, it changes very quickly to $+90^\circ$ just after the crossing kick and to -90° just before the crossing kick. Evaluating (19) gives the precise relative phase values for the few long-range kicks on either side of the head-on kick.

Kick	-3	-2	-1	0	+1	+2	+3
$\Delta\phi$	-86°	-84°	-78°	0°	$+78^\circ$	$+84^\circ$	$+86^\circ$

Since the particle receives many kicks on either side of the interaction point, most of these kicks are received at betatron phases very close to 90° before the interaction point or 90° after the interaction point.

All long-range kicks on the same side of the interaction point change both the emittance and phase by exactly the same amount. This is because, although the beam separation increases linearly with the distance from the interaction point (so that the transverse force falls off as $1/s$), the betafunction is increasing as s^2 so that the sensitivity of the emittance and phase also increases linearly with s .

Furthermore, the 30 long-range kicks before the interaction point change the emittance and phase by the same amount as the 30 kicks after the interaction point. This is because the phase advance between these two

sequences of long range kicks is 180° .

Like the short-range beam-beam interaction, the long-range kicks affect both resonance excitation and nonlinearity in the betatron oscillation. The nonlinearity induced by the long-range kicks is small for small betatron amplitudes because the kicks are in the same direction on both sides of the beam. However, particles that happen to enter the interaction region with vertical displacements away from the opposing beam receive weaker kicks than those with vertical displacements towards the opposing beam. This differential results in a net tune shift which has been studied in detail by Neuffer and Peggs [7]. Their results are reviewed below.

The long range impulses can be represented by the head-on force described in section II.A. If $D(s)$ is the vertical displacement of the opposing beam from the particle's beam at the point s where the bunch is passed, and $\sigma(s)$ the beam width at s , the vertical impulse $\Delta z'$ to the particle is given by

$$\Delta z' = \frac{-8\pi\xi_z z_0}{R^2} (1 - \exp[-R^2/2]) \quad (20)$$

$$\text{where } R^2 = x^2 + z_0^2 \quad (21)$$

$$z_0 = z + D(s)/\sigma \quad (22)$$

with ξ_z the head-on vertical linear tune shift and N the total number of long range kicks occurring in the interaction region. The vertical long-range tune shift is given by

$$\xi_{LRZ} = \frac{N}{4\pi} \frac{\partial}{\partial z} \Delta z' (x=0, z=0) \quad (23)$$

Where N is the total number of long range kicks (nominally $N=60$).

Substituting (20) into (23) gives

$$\xi_{LRZ} = \frac{N}{4\pi} \left. \frac{\partial}{\partial Z} \right|_{Z=0} \frac{-8\pi \xi_Z Z_0}{R^2} (1 - \exp[-R^2/2]) \quad (24)$$

Evaluation of (24) results in

$$\xi_{LRZ} = -2N\xi_Z \left\{ \frac{\sigma^2}{D^2} (1 - \exp[-D^2/2\sigma^2]) - \exp[-D^2/2\sigma^2] \right\} \quad (25)$$

In the limit where $D \gg \sigma$,

$$\xi_{LRZ} = \frac{-2N\xi_Z\sigma^2}{D^2} \quad (26)$$

In the SSC, $D/\sigma = 7.5$ and $N = 60$ so the long-range linear tune shift at $X=Z=0$ is expected to be

$$\xi_{LRZ} = \frac{-2N\xi_Z\sigma^2}{D^2} = -2.13 \xi_Z$$

Since small amplitude protons are outside the opposing beam when the long-range kicks take place, the horizontal tune shift is equal in magnitude but opposite in sign to the vertical tune shift.

$$\xi_{LRX} = 2.13 \xi_X$$

The long-range tune shifts, like the their short-range counterparts, depend on the amplitudes of the betatron oscillations. The dependence of the tune shifts on the amplitudes, due to the long range interaction alone, is illustrated in fig. 8. The tune shift values $\Delta\nu_X(a_X, a_Z)$ and $\Delta\nu_Z(a_X, a_Z)$ in fig. 7 were computed using numerical integration of the expression

$$\Delta\nu_Z(a_X, a_Z) = \frac{60}{(2\pi)^3 a_Z} \iint d\theta_X d\theta_Z \Delta Z'(a_X, a_Z, \theta_X, \theta_Z) \cos \theta_Z$$

where $\Delta Z'$ is defined by (20) and a_Z is the interaction point amplitude

normalized to σ .

The same expressions hold for the horizontal nonlinear tune shift, but with "x" substituted for "z". Amplitude contours from $(a_x, a_z) = (0,0)$ to $(9,9)$ are shown with a grid spacing of $\Delta a = \sigma$ for both degrees of freedom. The crossing is assumed to occur in the vertical plane.

The long-range tune shifts are compared to the corresponding short-range head-on in fig. 9. Here, amplitude contours are only shown up to $(a_x, a_z) = (5,5)$. The long-range force is clearly stronger than the head-on, especially at large amplitudes. Furthermore, the natures of the two forces are very different. For example, with the long range force, the signs of the x and z tune shifts are opposite, while for the short range force, they are the same.

If both long and short-range forces are present, the tune spread folds over, as shown in fig. 10.

Figure 11 shows the tune spread that would result if the crossing-plane were vertical at one low-beta interaction point and horizontal at the other. The alternate-crossing tune spread is included here for comparison but is not considered in the simulation study described in the following section.

III. Simulation Results

The simulation tracks the trajectory of a particle in the model system by iteratively evaluating a symplectic map. The trajectory is embedded in a five dimensional extended phase space, and is thus impossible to fully represent in a 2-dimensional plot. Partial representations of trajectories may be obtained, however, by looking at 2-D sections and projections of the full phase space. There are two types of 2-D plots that are particularly useful with the beam system.

In the first type, points are plotted in the z, z' plane (or x, x' plane) at certain prescribed intervals of time. This plot is useful because it can demonstrate the integrity (or lack thereof) of KAM curves at certain betatron amplitudes, and shows explicitly the resonance characteristics of any chosen resonance.

The second type of plot is a projection of the trajectory onto the 2-D amplitude space (a_x, a_z). This projection is useful because it shows the amplitude diffusion characteristics of the particle as a function of amplitude. The location of the stability aperture appears clearly in these plots.

III.A Phase Plane Plots: z vs z'

To get clean pictures of KAM curves in the beam system, it is usually necessary to take a codimension-two section of the 5-dimensional phase space and project out one of the remaining dimensions. Since the KAM surface is 3-dimensional to begin with, the result will be a 1-dimensional curve in a 2-dimensional plane.

In the beam system, the five phase space dimensions are x, x', z, z' and t . To obtain clean KAM curves in the z, z' plane, one would normally take two

sections by plotting the z, z' position of the trajectory only at those times when t is an integer (or some multiple of the synchrotron period if synchrotron modulation is present), and only when x or x' is equal to some given value. But because the trajectory is calculated in steps using a map, x and x' never land precisely on the section and this method, if strictly adopted, would result in no points at all being plotted. The usual solution to this problem is to take a slice of x (or x') that has a finite width; that is, plot the point when x (or x') falls within some narrow range about the given value. The wider this range, the shorter the time necessary to generate the plot, but the more "smeared" the KAM curve that results. If this selection criterion eliminates 49 out of every 50 points that would be plotted at a synchrotron period of 1000 revolutions, then only one point is plotted for every 50,000 iterations calculated. To obtain a mere 100 points would then require 5 million iterations of the map.

Sometimes, it's desirable to know the width of a single resonance which dominates the motion in some region of amplitude space. It's usually possible to plot relatively clear KAM curves in the near vicinity of a known resonance at rates substantially higher than those associated with the double-section technique described in the preceding paragraph. This is especially true with one dimensional resonances. Consider a resonance defined by the equation

$$m_x \omega_x + m_z \omega_z = n$$

If $m_x = 0$, then the resonance does not couple the horizontal and vertical motion (it couples the vertical to the longitudinal only). Therefore, it is not necessary to take a section of the x, x' motion to obtain a clear portrait of the resonance in the z, z' plane. This is analogous to projecting a straight 2-dimensional tube in 3-dimensions onto a 2-dimensional surface. The resulting projection is two dimensional unless the tube is perpendicular to the projection surface, in which case it's 1-dimensional. Thus, plots of such "one-dimensional" or "parametric" resonances can be obtained fairly

quickly. This is also true of synchro-betatron sidebands of parametric resonances (which should be plotted at the synchrotron frequency).

It's also possible to obtain individual portraits of coupling resonances quickly (one point per synchrotron period), but the plotting plane must be modified. The ordinate of the new plane may be either amplitude, but the abscissa must be the resonance phase ψ defined by

$$\psi = m_x \theta_x + m_z \theta_z$$

where θ_x and θ_z are the horizontal and vertical betatron phases.

The following figures show particle motion in the z, z' plane. They were obtained by plotting one point per synchrotron period. KAM curves that are fairly circular are far from any influential resonance. Curves that appear smeared out are either chaotic bands or stable trajectories under the influence of coupling resonances (sometimes these cases can be distinguished since trajectories of the latter tend to have a braided appearance). Clearly defined islands or waving KAM curves near islands are always associated with parametric resonances $m_x=0$.

Only one quadrant of the z, z' plane is plotted. In the absence of the long-range force, the plots are symmetric with respect to reflection across either the z or z' axes (note that to achieve this symmetry, the points are plotted half-way through the head-on kick). In the presence of the long-range force, symmetry exists only with respect to reflection across the z axis. The z' axis symmetry is broken by the strong dipole moment in the long-range force.

The phase-plane plots are first used to verify that no glaring mistakes were made in the simulation coding. The amplitude-dependent tune shifts of the vertical betatron oscillation at $a_x=0$, as measured in the simulation, are compared to the numerically derived values used to plot fig.8. The

comparison is shown in fig. 12 and is good enough to validate at least the coding for the vertical motion. The small but systematic discrepancy shown in this figure appears to be due not to inaccuracies in the simulation, but to an overestimation by the integration algorithm used to calculate the tune spreads shown in figs. 8-11.

Figures 14 through 19 show a series of plots illustrating the relative effects of various beam-beam elements on the integrity of KAM surfaces close to a 10th order resonance. Phase trajectories starting at $a_x = 2$, and 20 different values of vertical amplitude are plotted in the z, z' plane. The points are plotted at the synchrotron frequency (only one point is plotted every 527 iterations of the map). The three elements examined are 1) the short-range force, 2) the long-range force, and 3) tune modulation. The working points and beam spreads for these six plots are shown in figs. 13a-13c.

In the absence of all three elements, the motion is perfectly linear and integrable. Modulation alone cannot destroy integrability and thus leaves the KAM curves completely intact.

When only the head-on force is present (without modulation), the motion remains largely stable, as shown in fig. 14. The initial conditions are close to a 10th order parametric resonance which appears clearly in the plot at about $a_z = 5$. Although there must be some breakdown of KAM surfaces in this case (thin filaments of chaos are known to permeate the phase space in a tenuous but dense and wholly interconnected web), the breakdown is so small that it doesn't appear in the figure and has no practical significance. A vast majority of the phase space remains regular.

When tune modulation is added to the head-on force, the primary resonance splits into a multiplet of sidebands, of which only one is visible in fig. 15. Although the unstable areas are known to be much larger here than in the unmodulated case, they are still too small to be observed and remain inconsequential to beam lifetime.

Beam stability suffers far more from the long-range interaction than from the head-on force. This is apparent in fig. 16 where only the long-range force is present. Although the motion remains very stable at amplitudes below $a_z = 6.4$, stability rapidly deteriorates at higher values. The stability aperture for this case approximately intersects the point $(a_x, a_z) = (2, 5.4)$. Particles that are initiated outside of the stability aperture show a rapid diffusion. Since these runs correspond to a real machine time of 44 minutes, the rate of amplitude diffusion above the stability aperture is excessive; particles in this region are lost to the hard aperture in a matter of minutes.

When modulation is added to the long-range force, the stability aperture moves in slightly, to about $a_z = 5.8$, as shown in fig. 17. The rate at which particles beyond the stability aperture are lost appears to be much higher when modulation is present.

Addition of the head-on force to the long-range force (in the absence of modulation) has very little impact, as shown in fig. 18. In fact, although the head-on force seems to have removed the 10th order resonance apparent in fig. 16, the stability apertures in figs. 16 and 18 are nearly identical. Figure 19 shows the case where all three effects are present. Again, comparison with fig. 17 shows little impact from the head-on force.

As mentioned in section II.B, the head-on approximation to the short range force used in the simulation is approximately 10% larger than the actual short range force associated with the non-zero crossing angle. Thus the significance of the short range force is even less than it appears in these plots, making the issues raised in section II.B concerning the proper representation of the short-range force unimportant; the short-range force could probably be omitted altogether without affecting the veracity of the model.

III.B Amplitude Space Plots: a_x vs a_z

To see the instability aperture explicitly, it is necessary to plot the trajectories of many particles with different initial conditions in the 2-dimensional amplitude space a_x, a_z . In this section, a series of such plots are shown, as well as a number of simple graphs in amplitude space which show, in one figure, the stability apertures produced by several different simulation cases.

The amplitudes are defined by

$$a_x = x^2 + x'^2$$

$$a_z = x^2 + (x' - \gamma)^2$$

where $\gamma = .14$ is an empirically derived constant designed to approximately re-center the z, z' orbits which are shifted up in the phase plane due to the strong dipole moment of the long-range force.

In each case, the parameters are given their design values unless otherwise specified. The design values are:

Tune shifts	$\xi_x = \xi_z = .0015$ (combined)
Modulation Depths	$M_x = M_z = .0015$
Modulation Period	$P_s = 527$ revolutions
Crossing Angle	$\alpha = 75$ μ radians
Long-range Separation	$D/\sigma = 7.5$
Crossing plane	Vertical
Number of LR kicks	$N = 60$
Horizontal Tune	$\nu_x = .27$
Vertical Tune	$\nu_z = .28$

In each case, the stability aperture is fairly apparent. The deviations of a

trajectory from its initial conditions (its "smear") increase as the initial conditions approach higher amplitudes. However, they are strictly confined as long as the initial conditions are inside the stability aperture (running the simulation longer would not significantly enlarge the "smear" of a stable orbit about its initial conditions).

Three different values for the tunes are used in these runs. The three working points are shown in fig. 20 and labeled "a" through "c". Point "a" is chosen to be close to a tenth order resonance $(m_x, m_z) = (0, 10)$ and corresponds approximately to the tunes of figs. 14-19. The positions of the tune spreads in the region close to point "a" are shown in fig. 13. The tune spreads for points "b" and "c" are shown in figs. 21 and 22. Point "b" corresponds to the nominal design tunes, and point "c" is in an area practically devoid of resonances below 16th order but close to the powerful main coupling resonance $m_x, m_z = 1, -1$.

Figures 23 through 26 show the four cases examined in the previous section. These runs are identical to those of figs. 14-19, except that here there are more particles, they are distributed in both a_x and a_z , and their trajectories are plotted in the amplitude plane.

Fig. 23	Head-on force only
Fig. 24	Long-range force only
Fig. 25	Long-range force and modulation
Fig. 26	Long-range, head-on, and modulation

Figure 23 shows again that the head-on force, acting alone, cannot significantly affect the stability of the motion. The $m_z=10$ resonance appears clearly as a vertical smear about the position of the resonance (shown with a dashed line).

The far greater effect of the long-range force is clearly seen in figure 24. Diagonal smears on the right side of the figure indicate the presence of an

unidentified coupling resonance. The location of the stability aperture is clearly defined and appears as a (hand-drawn) curve.

When modulation is added to the long-range force, the stability aperture shrinks slightly and unstable orbits appear to diffuse more rapidly (fig. 25).

Addition of the head-on force doesn't significantly alter further either the the stability aperture or the diffusion rate (fig. 26). In fact, by removing the resonance which appears on the right side of fig. 25, the head-on force seems to enlarge the stability aperture slightly.

Figure 27 shows a graph in which the stability apertures seen in figs. 23 through 26 can be compared to one another. While the presence of modulation seems to significantly reduce the stability aperture, the short-range force seems to have little over-all effect.

Figure 28 shows the same stability apertures shown in fig. 27, but with tunes corresponding to working point "b" of fig. 20 (the nominal SSC design values). The aperture here is slightly larger than those at point "a", but the effects of modulation and the head-on force seem to be approximately the same. At both points, the stability aperture at small vertical amplitudes is between $a_z=5.5$ and $a_z=6.0$.

The conditions of fig. 29 are identical to those of figs. 27 and 28 but with tunes at the working point "c" in fig. 20. This point is heavily influenced by the $1, -1$ coupling resonance but by relatively few others. Because the strong coupling resonance causes very long, sweeping smears in the amplitude space (fig. 30), it is difficult to accurately place the stability aperture in this case. However, it appears to be significantly larger here (between 6.0 and 6.5) than at points "a" or "b". Comparison of fig. 29 to figs. 27 and 28 seem to indicate that "smear" in the amplitude plane is a poor indicator of motion stability.

The locations of the stability apertures in figs. 27-29 are nearly the same. This suggests that the loss of stability at vertical amplitudes near $a_z = 5.5$ occurs very rapidly. It also implies that the location of the stability aperture in the SSC will not be terribly sensitive to the tunes.

The figures that follow explore the sensitivity of the stability aperture to parameters other than the tunes. Both long-range and head-on forces, as well as modulation, are present in all the remaining figures.

Figure 31 shows the sensitivity of the stability aperture to the modulation frequency. Three synchrotron periods are represented:

$$P_s = 265$$

$$P_s = 527 \text{ (the design value)}$$

$$P_s = 1052$$

The plot shows, surprisingly, that frequency variations in this range have an almost negligible effect on the stability aperture. Since modulation was shown in fig. 28 to be significant, it appears that the range investigated here is just not wide enough to demonstrate the variation.

Figure 32 shows the sensitivity of the stability aperture to the modulation depth. Four modulation depths are represented:

$$(\Delta v_x, \Delta v_z) = (.007, .007)$$

$$(\Delta v_x, \Delta v_z) = (.0015, .0015) \text{ (the design value)}$$

$$(\Delta v_x, \Delta v_z) = (.0030, .000)$$

$$(\Delta v_x, \Delta v_z) = (.0015, -.0015)$$

Again, the differences between these cases is too small to be significant. The stability aperture seems to independent of modulation depth variations in this range.

Figures 33 and 34 show the sensitivity of the stability aperture to tune shift. In both figures, three tune shift values are represented:

$$\xi_x = \xi_z = .0007$$

$$\xi_x = \xi_z = .0015 \text{ (the design value)}$$

$$\xi_x = \xi_z = .0030$$

The two figures differ only in the tune values: the fig. 33 tunes correspond to point "b" in fig. 20, while the fig. 34 tunes correspond to point "a". The stability aperture clearly shrinks as the tune shift increases, though not as much as one might guess. A quadrupling of the tune shifts from .0007 to .0030 moves the aperture in from about 7.0 to about 5.5 in fig. 33, and from about 7.5 to 4.0 in fig. 34.

Figure 35 shows the sensitivity of the stability aperture to the crossing angle, and thus to the ratio D/σ of beam separation to beam width in the interaction region. Three cases are shown:

$$\alpha = 50 \text{ } \mu\text{radians}$$

$$D/\sigma = 5.0$$

$$\alpha = 75 \text{ } \mu\text{radians}$$

$$D/\sigma = 7.5 \text{ (design value)}$$

$$\alpha = 100 \text{ } \mu\text{radians}$$

$$D/\sigma = 10.0$$

The effect here is very large, as might be expected. It is clear that the single most important factor in determining the location of the stability aperture is the crossing angle. A very rough rule, at least for this simple simulation, seems to be that the vertical opening angle of the stability aperture is approximately 15 μ radians less than the beam separation distance α . With a nominal beam width angle of 10 μ radians, this corresponds to an aperture width in normalized amplitude that is about 1.5σ smaller than the beam separation (nominally 7.5σ).

IV. Discussion and Summary

The results presented in this report describe the principle features of the beam-beam stability aperture in the SSC.

1) The small but non-zero crossing angle creates a "long-range" beam-beam force which causes loss of dynamical stability at amplitudes above $.80 \times \alpha\beta^*/\sigma^*$ in the crossing plane. In the SSC, with $\sigma^*=5 \times 10^{-6}$ m, $\alpha=75$ μ rad, and $\beta^*=.5$ m, the stability aperture has a width of, at most, $\Delta A = 5.5\sigma$ in the crossing plane.

2) This estimate of the width of the stability aperture is optimistic. The actual width is less, though how much less is uncertain. Since the motion observed in the simulation runs performed in this study appear to be very stable at small amplitudes, it is unlikely that the actual beam-beam stability aperture is very much smaller than the above value.

3) Parameters other than the crossing angle also effect the width of the stability aperture, but less so. The aperture width for net linear tune shifts of $\xi_x = \xi_z = .0007$ is about $\Delta a_z = 7\sigma$. When these are quadrupled to $\xi_x = \xi_z = .003$, the width shrinks to about $\Delta a_z = 5\sigma$.

4) When tune modulation is added to the model, the aperture width shrinks by about 8%. Doubling or halving either the modulation frequency or depth has little effect on the aperture width.

A complete study of the SSC's stability aperture must also include a determination of 1) The magnet error stability aperture and 2) The stability aperture when both magnet errors and the beam-beam interaction are present. Previous studies of the short term stability when only magnet errors are present [2-4] should be extended to examine the corresponding long term stability in the presence of the beam-beam interaction. A preliminary study of the combined effect of magnet errors and the beam-beam interaction [6] suggests that at nominal parameter values, magnet errors do not significantly reduce the size of the stability aperture.

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Figure 1

Bunch dimensions and separation.

Figure 2

Short-range geometry and definition of variables

Figure 3

The beam-beam force in the crossing plane Z is the integral of the product of the two functions shown over s' .

Figure 4

The beam-beam force in the non-crossing plane X is the integral of the product of the two functions shown over s' .

Figure 5

The vertical impulse (the integral of the two functions graphed in fig. 3). The impulse is plotted as a function of $p2$, the vertical offset of the opposing beam relative to the particle's actual trajectory. Four different opening angles are shown. The SSC design tune shift is about 90% of the head-on tune shift. The horizontal displacement X is zero.

Figure 6

The horizontal impulse (the integral of the two functions graphed in fig. 4). The impulse is plotted as a function of p_2 , the vertical offset of the opposing beam relative to the particle's actual trajectory. Four different opening angles are shown. The SSC design impulse is about 93% of the head-on impulse. The value of the horizontal displacement is $X = \sigma$.

Figure 7

Long-range geometry and definition of variables

Figure 8

Tune spread on the long range force alone, in units of the head-on linear tune shift. Amplitudes run from 0 to 9σ in both the vertical and horizontal directions.

Figure 9

Tune spreads for the head-on and long-range forces, showing the relation between them and their orientations relative to the working point (marked with a cross). Amplitudes up to 5 sigma in both the vertical and horizontal directions are shown. The crossing is in the vertical plane .

Figure 10

Tune spread for head-on force and long range force together when the crossing planes are vertical at both low beta interaction points. Tune shift is measured in units of the linear head-on linear tune shift ξ . The working point is marked with cross.

Figure 11

Tune spread for head-on force and long range force together when the crossing planes are different in the two low beta interaction points. Tune shift is measured in units of the linear head-on linear tune shift ξ .

Figure 12

Comparison of the theoretical long-range tune shift (curve) with simulation (crosses). The horizontal amplitude is zero. The tune shift values are measured in units of the head-on linear tune shift.

Figure 13a

Tune spread for the head-on force relative to a 10th order resonance. The tunes are $\nu_x = .2632$ $\nu_z = .3002$ and correspond to the simulation runs shown in figs. 14 & 15. Thick lines show resonances of 10th order and lower; thin lines are 16th order and lower.

Figure 13b

Tune spread for the long-range force relative to a 10th order resonance. The tunes are $\nu_x = .2682$ $\nu_z = .2959$ and correspond to the simulation runs shown in figs. 16 and 18. Thick lines show resonances of 10th order and lower; thin lines are 16th order and lower.

Figure 13c

Tune spread for head-on and long-range forces together relative to a 10th order resonance. The tunes are $\nu_x = .2682$ $\nu_z = .2959$ and correspond to the simulation runs shown in figs. 17 and 19. Thick lines show resonances of 10th order and lower; thin lines are 16th order and lower.

Figure 14

Vertical phase plane plot for the head-on force only. The long-range force and modulation are turned off. The tunes and tune spread for this run are shown in figure 13a. The resonance which appears at about 5 sigma is the $\nu_z = .3$ parametric resonance.

Figure 15

Vertical phase plane plot for the head-on force with modulation. The long-range force is turned off. The tunes and tune spread for this run are shown in figure 13a.

Figure 16

Vertical phase plane plot for the long-range force only. The head-on force and modulation are turned off. The tunes and tune spread for this run are shown in figure 13b.

Figure 17

Vertical phase plane plot for the long-range force with modulation. The head-on force is turned off. The tunes and tune spread for this run are shown in figure 13b.

Figure 18

Vertical phase plane plot for the head-on and long-range forces combined. The modulation is turned off. The tunes and tune spread for this run are shown in figure 13c.

Figure 19

Vertical phase plane plot for the head-on and long-range forces with modulation. The tunes and tune spread for this run are shown in figure 13a.

Figure 20

The full tune plane showing the locations of the working points "a", "b", and "c". All resonances through 10th order, including all odd order resonances, are shown.

Fig. 21

Tune spread with all resonances through 16th order (labels on resonances indicate their order). Tunes are $\nu_x = .27$ $\nu_z = .28$ corresponding to the point "b" in fig. 20.

Fig. 22

Tune spread with all resonances through 16th order (labels on resonances give their orders). Tunes are $\nu_x = 0.1970$ $\nu_z = 0.1868$ corresponding to the point "c" in fig. 20.

Figure 23

Stability aperture for the head-on force alone, close to a 10th order resonance. This case corresponds to the tunespread shown in fig. 13a and the phase plot shown in fig. 14. The trajectories of 112 particles initially dispersed on a grid are projected into the amplitude plane. The run time corresponds to 158100 revolutions or 44 seconds of machine operation.

Figure 24

Stability aperture for the long-range force alone, close to a 10th order resonance. This case corresponds to the tunespread shown in fig. 13b and the phase plot shown in fig. 16. The trajectories of 112 particles initially dispersed on a grid are projected into the amplitude plane. The run time corresponds to 158100 revolutions or 44 seconds of machine operation.

Figure 25

Stability aperture for the long-range force with modulation, close to a 10th order resonance. This case corresponds to the tunespread shown in fig. 13b and the phase plot shown in fig. 17. The trajectories of 112 particles initially dispersed on a grid are projected into the amplitude plane. The run time corresponds to 158100 revolutions or 44 seconds of machine operation.

Figure 26

Stability aperture for the head-on and long-range forces together, with modulation alone, close to a 10th order resonance. This case corresponds to the tunespread shown in fig. 13c and the phase plot shown in fig. 19. The trajectories of 112 particles initially dispersed on a grid are projected into the amplitude plane. The run time corresponds to 158100 revolutions or 44 seconds of machine operation.

Figure 27

Comparison of the stability apertures shown in figures 23-26.
 LR = Long-range, HO = Head-on, M = Modulation.

Figure 28

Comparison of stability apertures at working point "b" (the nominal SSC design tunes). LR = Long-range, HO = Head-on, M = Modulation.

Figure 29

Comparison of stability apertures at working point "c" .
LR = Long-range force, HO = Head-on, M = Modulation.

Figure 30

Trajectories in the amplitude plane corresponding to the line labeled "LR" in fig.29. Only particles at very large and very small horizontal amplitudes are plotted to show the sweeping smear produced by the main coupling resonance.

Figure 31

Sensitivity of stability aperture to variations in modulation frequency. The working tunes here are the design values (point "b" in fig. 20).

Figure 32

Sensitivity of stability aperture to variations in modulation depth. The tunes here are the design values (point "b" in fig. 20). Curve A corresponds to $M_x = M_z = .0007$, B corresponds to $M_x = M_z = .0015$, C to $M_x = M_z = .003$, and D to $M_x = -M_z = .0015$.

Figure 33

Sensitivity of stability aperture to variations in tune shift. The tunes here are the design values (point "b" in fig. 20).

Figure 34

Sensitivity of stability aperture to variations in tune shift. The tunes here are those corresponding to point "a" in fig. 20.

Figure 35

Sensitivity of stability aperture to variations in crossing angle. The tunes here are the design values (point "b" in fig. 20).