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NUCLEAR MAGNETIC RESONANCE STUDY OF GRANULAR FLOWS

Work performed under PETC contract #DE-AC22-90PC90184

by

The Lovelace Institutes, Albuquerque, New Mexico

FINAL REPORT

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Introduction

This report describes the activities performed under Contract #DE-AC22-90PC90184 "Nuclear magnetic resonance study of granular flows" from October 1, 1990 to March 31, 1995, at The Lovelace Institutes (TLI).

Background

Nuclear Magnetic Resonance Imaging (NMRI) is a non-invasive method commonly used for diagnostic imaging in medicine. We extended its use to flowing granular materials and obtained concentration and velocity information inside the flow. We also investigated other parameters such as diffusion and energy dissipation that could lead to the determination of granular temperature.

Methods

Sample selection: NMRI studies of granular flows require particles with not only suitable mechanical properties such as size, shape, and coefficients of restitution and friction but also with sufficient number of mobile, NMRI-sensitive, nuclei. All of our early experiments were performed with 1.4 mm diameter mustard seeds which were approximately spherical and yielded good proton

The experimental arrangement for the rotating cylinder experiment. The vibration experiment used the same magnet, gradient coil, and probe but with a vibrating bar through the bore, instead of the cylinder.

NMRI signals from the oil in the seeds' cores. Some of the later experiments were performed with 1 and 4 mm diameter pharmaceutical pills containing liquid cores that also yielded excellent proton signals.

Apparatus: All the NMR experiments were done at TLI on an imager/spectrometer by Nalorac (Quest 4400) controlled by a DEC μ VAX-II computer, with an Oxford 1.89T/31cm superconducting magnet. The original unshielded Nalorac gradient coils were replaced by shielded coils by Magnex during this contract. All of the NMR probes we used were homemade, high-pass birdcage coils (Watkins and Fukushima, 1988). The data were processed in a Sun IPX workstation. The general arrangement is shown above.

The particles were usually in an acrylic container and rolled or shaken for the NMRI and other experiments. Specific geometries are described in the narratives that correspond to specific experiments.

Velocity: NMR velocity measurements can be made by two distinct methods. The first is by labeling the desired cross-section with a grid pattern some time before imaging (Icenogle, et al., 1992). The deformation of the labeling pattern in the delay interval gives information about velocity and concentration on a spatial scale of the order of the grid pattern. Although this gives a fairly coarse spatial resolution, the experiment is robust because it consists of adding a tagging preamble to a proven imaging sequence. Furthermore, minor imperfections in the signal intensity do not affect the measurement which depends only on the displacement of the tags. The figure below shows an example of such a tagging experiment for half-filled cylinder of

Grid-tag experiment of flowing mustard seeds at 18 rpm. The delays between tagging and imaging are 1, 10, 20, 50, and 100 ms, from left to right. The tagged cells translate and deform as a function of time. [No special efforts were made to reconcile the x and y scales in our 2-D images which lead to the out-of-round appearances of many of the images in this report. This is strictly a display problem and does not affect the quantitative information.]

mustard seeds being rotated at 18 rpm. The delays between the tagging and imaging vary from 1 to 100 ms and the evolution of the grid pattern clearly shows the velocity and shear patterns. Time sequence of such patterns are excellent visualization tools.

The second way to measure velocity is the phase method which yields high resolution velocity images but at a cost of more work, time, computer

memory, etc. An NMR image consists of amplitude and phase of the nuclear magnetization at every voxel with the amplitude related to the density while the phase to velocity. We can apply a magnetic field gradient in a sequence to make the phase shift proportional to the average velocity in the voxel with the proportionality factor an experimentally controllable parameter. It is quite straightforward to make a 256×256 velocity image in this way. The velocity resolution obtained so far is of the order of 0.1-1.0 mm/s with the maximum velocity in the range of 1-10 m/s. The duration of measurement in our experiments were of the order of tens of minutes so we looked only at steady state flows. However, it has become possible to make fast, low resolution (32×32) velocity image in tens of milliseconds (Kose, 1991), which can lead to direct determination of fluctuations.

Concentration corrected for diffusion: NMR measures the time average concentration of the particles in each spatial voxel by the intensity of the signal in that voxel. Thus, any confounding effects that might affect the NMR intensity will lead to errors in measured concentration. One such effect on the signal intensity in this experiment is that due to particulate collisions which, in effect, removes some signal intensity from the voxel. The crucial times in this consideration are those between collisions and the time required to obtain one spin-echo. The latter has been improved during this project, by improved gradient behavior described below, from 15 ms to some 4-5 ms. With this improvement, we have been able to make better extrapolations of the intensity to zero experimental echo time. Proving that there is a silver lining to every cloud, we use this effect of the signal intensity being attenuated by collisions, later, to study diffusion, collisional energy dissipation, etc.

Diffusion: Molecular diffusion on a microscopic scale has been measured by NMR for many decades. We extend this formalism to determine the macroscopic particulate diffusion coefficients of particles undergoing collisions. In short, random diffusion of particles during the NMRI sequence attenuates the signal intensity. As mentioned above, the signal intensity, obviously, depends on the concentration of the material, too, so the experiment must be designed carefully to separate various factors that affect the signal amplitude.

The procedure we used to get diffusion coefficient images is to insert diffusion sensitizing gradient pulses in the intervals before and after the π -pulse in a Hahn echo sub-sequence of a Fourier imaging sequence. The orientation of the gradients determines which component of diffusion coefficient D will be measured. In this way, anisotropy of the diffusion process in granular flows can be studied. We look at some anisotropic diffusion examples in the results section.

The diffusion coefficients are obtained with the assumption that the particles do not move appreciably during the diffusion sensitizing gradients which is the so-called short pulse approximation. We undertook the reexamination of this assumption and this is described under theoretical results.

Energy dissipation: Energy dissipation can be determined for flowing particles from the knowledge of velocity and acceleration. It is a quadratic function of a generalized shear so a knowledge of the velocity field can be used to calculate a dissipation image. It can also be calculated from conservation of energy. The energy dissipation as well as diffusion are related to granular temperature but the exact relationship is not known.

Apparatus Improvements

Shielded gradient coil: Despite its many advantages, the usual MRI is relatively slow, with most applications limited to stationary objects or to objects undergoing periodic motion imaged synchronously with the motion. Attempts to speed up imaging have to confront the eddy currents induced in conducting surfaces by the pulsed magnetic fields associated with changing gradients (Lowe and Wysong, 1993; and references therein). Such eddy currents recover slowly and degrade not only imaging speeds but also image quality.

To estimate the effects of eddy current problems, consider that the usual anatomical medical image in a superconducting magnet takes several seconds to several minutes to make, depending on parameters chosen. Velocity images take even longer. On the other hand, the fastest images made, so far, have been in iron-core electromagnets not susceptible to eddy currents because of the lack of high electrical conductivity metals. Kose (1991) used an iron electromagnet to obtain a 32×32 two-dimensional velocity image in only 56 ms which is more than two orders of magnitude faster than it has been possible in uncompensated superconducting magnets, albeit in a small sample space. This clearly demonstrates the improvement possible with better eddy current remediation.

One method to combat the eddy current problems is to use shielded gradient coils (Turner, et al., 1988; Carlson, et al., 1992; and references therein). They are designed to cancel the magnetic field gradients at suitable locations outside the coil while maintaining reasonable gradients inside. The reduction of eddy current effects is achieved at the expense of available space because the extra coil windings take up considerable additional space. The problem is compounded because the shield coils partially cancel the desired field gradients inside the coil so that it takes more electrical current to generate the same magnetic field gradients. This, in turn, generates more heat in the coil assembly which leads to the need for water cooling which takes up even more space.

We had a gradient coil made commercially by Magnex Scientific. The design criterion was to get the maximum useful volume at a sacrifice of ultimate shielding performance which we would then try to rectify by analog compensation described below. The new coil has made possible the diffusion corrected concentration images of flowing seeds.

Analog compensation of gradient pulses:

In addition to the shielded gradient described above, we wished to further correct the eddy current problems by altering the shape of the driving waveform to the gradient coils so that the resulting "distorted" waveform is the desired one.

This, in principle, is a standard procedure in electrical engineering and only requires the determination of a transfer function for the system from a known input and a measured output, followed by a modified input corrected for the newly determined transfer function. [See, for example, Thomas (1969).] For eddy currents, the common compensation scheme consists of using suitable RC filters with the time constants matched to each eddy current decay times.

Unfortunately, the induced eddy currents do not have the same symmetry as the original inducing field because the eddy currents are generated on inhomogeneous, asymmetric, conducting surfaces. Thus, besides compensating the input waveform of a particular gradient component, we need to correct the other components of induced gradients, too. (Van Vaals and Bergman, 1990)

We model the eddy current, generated after the gradient $G_y(t)$ is turned off, as

$$B_y(t) = b_{oy}(t) + xg_{xy}(t) + yg_{yy}(t) + zg_{zy}(t) + \text{higher order terms in } x, y, z, \quad (1)$$

where $b_{oy}(t)$ is a spatially uniform induced field along B_o , $g_{xy}(t)$ is a linear induced field gradient in x-direction, and $g_{yy}(t)$ and $g_{zy}(t)$ are y and z gradients induced also by $G_y(t)$. The general interaction will, thus, be represented by a matrix with up to 12 elements relating the input and output gradients. Each nonzero term requires a hardware compensation circuit, each with an array of time constants and weights.

First, consider the eddy current correction for channel y with the cross terms neglected, as shown in Fig. 1(a). The transfer functions $G_{yy}(s)$ and $G_{oy}(s)$ model the magnetic field generated by current $i_y(t)$. This model with compensation is shown in Fig. 1(b). The output of $H_{yy}(s)$ goes to the y-gradient coil and through $H_{oy}(s)$ to the B_o coil. The induced field $B_y(t)$ is now given by

$$B_y(t) = b_{oy}(t) + yg_{yy}(t) + b_{oo}(t) + yg_{yo}(t). \quad (2)$$

Our goal is to design $H_{yy}(s)$ and $H_{oy}(s)$ so that $B_y(t) = 0$ after $i_y(t)$ is shut off. $H_{yy}(s)$ and $H_{oy}(s)$ are chosen to be

$$H_{yy}(s) = a_0 + \sum_{i=1}^{12} a_i \frac{s}{s + p_i}, \quad (3)$$

$$H_{oy}(s) = \sum_{i=1}^{12} b_i \frac{s}{s + p_i}, \quad (4)$$

where the gain constants a_i and b_i are chosen to cancel the eddy currents. In actual operation, we place a sample at the electrical center of the y-gradient coil ($y=0$) and null the eddy response by a suitable pulse from the B_o coil. Let $B_{yu}(t)$ be the uncompensated eddy current response and $B_{yi}(t)$ be the eddy current

Figure 1(a). The uncompensated system showing that the y-input $i_y(t)$ causes two responses to first order; $b_{oy}(t)$, a uniform field, and $g_{yy}(t)$, a y-gradient field. The higher order contributions are ignored for now.

Figure 1(b). The compensated system showing the added compensators and the extra current path through the B_o coil.

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response with $b_i = 1$, for $i = j$ and zero for other i values and $a_0 = 0$. Then the basis functions are

$$h_{oy}(i,t) = B_{yi}(t), \quad i = 1, 12 \quad (5)$$

where the b_i values are determined by minimizing the least squares error

$$E_o = \int (B_{yu}(t) + \sum_{i=1}^{12} b_i h_{oy}(i,t))^2 dt. \quad (6)$$

The sample probe can now be moved off-center to be sensitive to the gradient eddy current response. We measure the uncompensated response $B_{yu}(t)$ (with $a_i = 0$, $i = 1, 12$) and the basis functions $h_{yy}(i,t)$, $i = 1, 12$, with the previously calculated $H_{oy}(s)$. Then, a_i are determined by minimizing

$$E_1 = \int (B_{yu}(t) + \sum_{i=1}^{12} a_i h_{yy}(i,t))^2 dt. \quad (7)$$

These calculations can, of course, be iterated as necessary.

The linear least squares fitting and loading the appropriate gain constants to the 12 serial MDAC's (multiplying digital to analog converters) is done through a parallel port of an IBM PC compatible computer. The use of MDACs with serial inputs is the key to implementing this scheme. It greatly simplifies the interfacing, considering that 12 bits are required to specify the amplitude of each of twelve time constants. This scheme not only speeds up the minimization of the eddy current responses but its precision and reproducibility allow for the tailoring of the eddy current corrections to different experimental conditions such as different samples, rf coils, gradient coils, and magnets.

Our apparatus is a 1.9T/31cm Oxford superconducting magnet with a 24cm clear-bore shielded gradient coil made by Magnex Scientific. A 12mm diameter, 12mm long probe with a short T_1 water sample monitored the eddy current. The response $B_y(t)$ is measured by the method of Wyson and Lowe (1993). We used a 100 ms long, 1 G/cm gradient pulse $G_y(t)$ with a repetition time of 8s which is five times the largest time constant to be corrected.

In our system, there are eight terms that must be corrected.

	Output G_x	Output G_y	Output G_z	Output B_o
Input G_x	$g_{xx}(t)$	$g_{yx}(t)$	0	$b_{ox}(t)$
Input G_y	$g_{xy}(t)$	$g_{yy}(t)$	0	$b_{oy}(t)$
Input G_z	0	0	$g_{zz}(t)$	$b_{oz}(t)$

The compensation scheme with no cross-terms has been tested on the y gradient, the one that has the worst eddy current effects. The uncompensated and compensated responses, $b_{oy}(t)$ and $g_{yy}(t)$, are shown in Figs. 2 and 3. Both

Figure 2. The behavior of b_{oy} , the uniform component of the transient field, after turning off the y-gradient $G_y(t)$, as measured by NMR. The quantity plotted is the induced magnetic field intensity, in units of Hz, per change in the applied magnetic field intensity gradient, in units of Hz/cm, leading to the unit of Hz/Hz/cm. Both the raw and the first order corrected responses are shown.

Figure 3. The behavior of g_{yy} , the y-component of the transient magnetic field gradient caused by the eddy currents upon the turn-off of the magnetic field gradient, as measured by NMR. The quantity plotted is the change in the induced field gradient per unit change in the applied gradient, leading to the units of Hz/Hz. Both the raw and the first order corrected responses are shown.

compensations were successful and have prepared the way for corrections of x and z gradients and the cross-terms.

Experimental Results

Velocity and concentration in rotating cylinder (Nakagawa, et al., 1993): We fill a large-aspect ratio acrylic tube half full of mustard seeds and rotate it. Typically the cylinders are 30 to 60 cm long with inner diameters of up to 25% of the length. In separate experiments, we saw that the end effects usually do not propagate more than a fraction of the end plate diameter. Thus, our cylinder dimensions were chosen to eliminate end effects in the region of interest. Our MRI experiment for mustard seed flow selects a 8 mm thick slice near the center of a long cylinder, perpendicular to the axis.

A rotating, partially filled cylinder generates different kinds of flows depending on material property and rotation velocity. Most of our data are for spherical mustard seeds with 1.4 mm diameter, density 1.3 g/cm^3 , and with coefficient of restitution of 0.7 at room temperature. We usually chose rotation rates for the flow to be in the cascading region, i.e., the rotation is fast enough to avoid periodic slumping and slow enough so the free surface is fairly straight. Under these conditions, we obtained concentration images such as those in Fig. 4(a) for three different seeds with the yellowish colors being high concentration and the green being low.

At 5 rpm, the rotation is slow enough that the concentration is not affected very much and the free surface is flat for all three seeds. At 31 rpm, the concentration is still fairly uniform but the diffusion and collision of particles attenuate the signal as described in the next section, giving rise to an apparent decrease in the concentration. The free surface is no longer flat, even for the spherical mustard seeds. For the nonspherical seeds, the flow is much more chaotic in the first half of the flow before the flowing seeds order their orientations to achieve smoother flow and concomitant smaller angle of repose.

The corresponding transverse velocity components at 5 rpm for the three seeds are shown in Fig. 4(b). The color coding for the velocity is shown in the accompanying color bars with the background tan color being zero. For the X-velocity, red represents motion to the left and green to the right. For the Y-velocity, red and green represent down and up, respectively. Velocity vector and streamline displays, as shown below, can be made from such velocity component data. At this slow rotation velocity, most of the seeds are undergoing solid-body rotation while those that reached the region near the high point flow along the free surface in a lenticular region. The flow velocity is parallel to the free surface and is nearly symmetric about the middle of the free surface, i. e., the velocity maximum for any trajectory is at the center of the traverse, in agreement with de Gennes' simple derivation (quoted in Rajchenbach, 1990).

Fig. 4(a). Concentration images of mustard, poppy, and sesame seeds at 5 (top) and 31 rpm (bottom).

Fig. 4(b). Velocity component images of mustard, poppy, and sesame seeds at 5 rpm.

MUSTARD

POPPY

SESAME

5 RPM

31 RPM

Concentration Images

25 cm/s

X Velocity

-25 cm/s

25 cm/s

Y Velocity

-25 cm/s

Velocity Images (5 RPM)

Left: A velocity vector image for the mustard seeds flowing in a rotating cylinder. Each vector has a length proportional to the speed at that location and direction of the flow.
 Right: A streamline image of the same flow.

Figure 5(a) shows the X- and Y-velocities for the three seeds at a much higher rotation rate of 31 rpm. The free surface is no longer flat, as mentioned above and the flowing layer has become thicker and more complex.

The X- and Y- velocity fields can be used to obtain the speed and Fig. 5(b) shows the concentration as well as the flowing speed images of mustard seeds as a function of rotation speed. The development of the flowing layer as the rotation rate increases is clearly seen. Both quantities are quite symmetric about the mid-point of the free surface.

Figure 6(a) shows the X- and Y-components of acceleration for flowing mustard seeds. These components are computed from the velocity fields as $a_i = u_i \partial u_i / \partial x_i + u_j \partial u_i / \partial x_j$, where the indices i and j correspond to X and Y. The scale of $\pm 75 \text{ cm/s}^2$ for the color bars is a typographical error and should be $\pm 750 \text{ cm/s}^2$. Note that there is a small flow-independent acceleration due to the rotation of the cylinder which can be subtracted from the images.

The velocity profile along a perpendicular bisector for a rotation rate of 17 rpm is shown below, together with a schematic of the flow geometry. The profile along A-B consists of $r-r'$, the solid-body rotation portion with its linear velocity, and F, the sliding layer. The departure of the velocity from the linear region at r' , the beginning of the sliding layer, is consistent with a quadratic dependence. The velocity becomes more linear as the free surface is approached. The variation of the velocity with distance is never abrupt, lending credence to the notion that the velocity field is determined by minimizing energy dissipation which is proportional to the square of shear.

Fig. 5(a). Velocity component images of mustard, poppy, and sesame seeds at 31 rpm.

Fig. 5(b). Concentration and speed images of flowing mustard seeds at 5, 20, and 31 rpm.

MUSTARD

POPPY

SESAME

25 cm/s

X Velocity

-25 cm/s

25 cm/s

Y Velocity

-25 cm/s

Velocity Images (31 RPM)

5 RPM

20 RPM

31 RPM

Concentration

40 cm/s

Speed

MUSTARD SEEDS

Schematic of flow geometry (left) and the velocity profile along A-B (right).

The depth of the sliding layer, F in the previous figure, normalized to the cylinder radius, increases approximately as the rotating rate Ω to the power of $1/2$ up to a rotation rate of 24 rpm as shown below. This corresponds to the peak velocity increasing approximately as $\sqrt{\Omega}$ as well, from the discussion in the theory section later. Clearly, the $\sqrt{\Omega}$ dependence can be maintained only for small Ω because of the finite thickness of the flow geometry.

The depth of the flowing layer as a function of rotation speed.

Diffusion and energy dissipation in rotating cylinder: In addition to the velocity and concentration distributions and profiles, we have measured diffusion coefficients and energy dissipation associated with collisions. Figure 6(b) shows diffusion images for mustard seeds at 5, 20, and 31 rpm rotation rates. Here, the plotted diffusion coefficient is the trace of the diffusion tensor; i.e., $(D_x + D_y + D_z)/3$,

Fig. 6(a). X and Y acceleration component images of mustard seeds at 5, 20, and 31 rpm. Positive acceleration, represented by the reddish colors, are to the right for the X component and down for the Y component. The extremes of the color bars should be marked $\pm 750 \text{ cm/s}^2$ instead of $\pm 75 \text{ cm/s}^2$.

Fig. 6(b). Diffusion and viscous loss images of flowing mustard seeds at 5, 20, and 31 rpm.

5 RPM

20 RPM

31 RPM

75 cm/s/s

X Acceleration

-75 cm/s/s

75 cm/s/s

Y Acceleration

-75 cm/s/s

Diffusion

Viscous Loss

MUSTARD SEEDS

where the individual components are measured in separate experiments. Unlike the velocity, the diffusion is not symmetric; it becomes larger in the lower half of the flow where the seeds decelerate more chaotically than how they accelerated in the upper half. Besides the inhomogeneity, the diffusion coefficient for this flow also exhibits anisotropy, i.e., the x, y, and z components of D are different.

Figure 6(b) also shows the viscous dissipation $e_{11}^2 + e_{22}^2 + e_{12}^2 - (e_{11} + e_{22})^2/3$ where $e_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2$ is a generalized shear that measures the deformation of a fluid element. The general viscous dissipation is $2\mu(e_{ij}e_{ij} - \Delta^2/3)$ where μ is the fluid viscosity (Batchelor, 1970) so the plot assumes that μ is constant and dissipation in the axial direction is ignored.

The energy dissipation can be calculated from conservation of energy, as well. In the absence of any dissipation, the rate of change of total energy for flows in a rotating cylinder should be zero. Thus, if the rate of change of kinetic and potential energies do not add to zero, the difference is the energy being dissipated which is also equal to that being put into the system. Images of energy dissipation of mustard seeds in a rotating cylinder at four rotation speeds is shown below. [These images show rotational contributions in the solid-body rotation region but they add to zero over the entire cylinder.] The diffusion image and the dissipation images calculated from shear and from energy conservation look remarkably similar. As already mentioned, they peak near the bottom of the flow as the seeds decelerate chaotically.

Images of energy dissipation calculated from energy conservation for different rotation speeds.

The energy dissipation over the entire cylinder can be summed to yield the power required to rotate the cylinder and this can be verified by experiment. The global measurement of the power was performed by rolling the half-filled cylinder on an incline plane (door). The cylinder accelerates for a short time

before attaining a uniform speed. The rate of change of kinetic energy is zero for uniform rotation so the rate of change of potential energy equals the power. The time for the cylinder to roll a significant fraction of the door was recorded as a function of the inclination. The power is proportional to the rotation frequency, as shown below, and exceeds the NMRI result by 10%. This is reasonable because the NMRI measurement ignored seeds at the top of the free surface which contribute significant dissipation.

The power required to rotate the cylinder (which equals the energy dissipation of the particles) as a function of the rotation speed.

Flow and diffusion of particles are expected to behave differently for different particle shapes. The figures below show the diffusion coefficients for sesame seeds which have a prominent short axis, i.e., it is flattened. The

Images of horizontal (left) and vertical (right) diffusion coefficient components for flowing nonspherical (sesame) seeds.

horizontal component of D increases monotonically as it did for the spherical mustard seeds but the vertical component reaches a maximum somewhere near the center of the cylinder and decreases thereafter. This arises because the seeds tumble chaotically as they are released at the top of their travel but organize themselves to flow with their flat sides parallel to each other part way down. Thus, the diffusion coefficient becomes decidedly anisotropic, i.e., the vertical component of velocity variation becomes smaller than the horizontal component. The determination of such collisional anisotropies represents a breakthrough in granular flow measurements.

Segregation in rotating cylinder: Particles having different properties tend to segregate into their own species when they are in relative motion. In many industrial particle handling situations such as transporting coals and mixing heterogeneous components, segregation is undesirable. Rotating cylinders (drums) are commonly used to mix, dry, or mill different particles and happens to have a geometry that is well suited to NMRI studies. Therefore, we have studied the axial and radial segregation of heterogeneous mixtures in a partially filled horizontal cylinder (Nakagawa, 1994), the same geometry used in the velocity, concentration, diffusion, etc., studies described above.

When a horizontal, acrylic cylinder containing heterogeneous sand was rotated by an electric motor, 13 to 21 distinct bands corresponding to particles with differing coarseness formed in a few minutes. Even within a band, there was a distribution of particles with different coarseness, i.e., the coarsest particles were in the centers of the bands containing the coarser particles. The initial number of bands seem not to follow any rules as a function of rotation speeds between 9 and 17 rpm.

With a bimodal distribution of particles, there were fewer initial bands at the higher rotation speeds and more at the lower speeds. If the rotation continued, we found that the number of bands always decreased. For example, at 30.7 rev/min, the bands decreased from the initial 9 to 3 in less than 400 minutes. The bands decreased in number regardless of the rotation speed and the rate of change seemed to increase with faster rotation speeds.

Cross-sectional images at six locations shown in the sketch below the images after 200 rotations. The small, 1 mm, beads have the appearance of a continuum after the averaging over the slice thickness, while the large, 4 mm, beads can be resolved.

NMR can be used to study segregation phenomena if the different components can be distinguished. We use pharmaceutical pills that are two different sizes but otherwise identical. They can be distinguished because the fill factor of the liquid center is different for the two sizes and because the imaging resolution can be chosen so the different size particles will appear with different coarseness in the image. The set of images above show cross-sectional images after 200 rotations of an equal mixture of 1 and 4 mm diameter pills. The radially segregated beads can be identified easily and the size of the core containing the smaller pills is uneven through the cylinder.

After 3000 rotations, we obtained cross-sectional images as shown below. Now, the large beads have moved to the left and the small beads to the right. The third slice from the left is the only one showing both phases. A few large particles remain attached to the right end-plate which shows up in the rightmost slice.

Similar images after 3000 rotations with the six slice locations shown (in addition to the two end plates).

Segregation and flow during shaking: Segregation can also be induced by relative motions of particles in the absence of flow. Therefore, in order to separate the effects of flow (and shear) from vibrational displacements, we have studied diffusion in particles whose motions are predominantly shaking. Because it is difficult to create significant shaking without some accompanying flow, we will compare situations with vastly different fractions of the two contributions as in rotating cylinder generating primarily flow accompanied by diffusion versus the sample shaking experiment in which the flow is only secondary.

Shaking was accomplished by a flexible, 2-1/2 meter, wooden beam with one end attached to a 12" loud speaker. The beam is placed in the magnet bore so that the antinode of its vibration mode is in the center of the magnet. The sample chambers were boxes of various shapes fixed to the beam at the center of the magnet. The loudspeaker was driven by a sinusoidal signal from a function generator amplified by an audio amplifier. The shaking experiments are not steady state experiments on the timescale of NMR measurements, unlike the steady flow experiments. Therefore, we synchronized the data acquisition to

the vibration with an adjustable delay so that we could measure flow data at any part of the shaking cycle.

Small changes in the shaking geometry induce dramatic changes in the convection patterns and these are easily measured. When the shaking mode is adjusted so that most of the motion is vibrational, interesting variations in the instantaneous concentration and velocity, as a function of depth, can be seen. These variations are sharply dependent on the frequency and phase of the vibration cycle, as might be expected.

Direct Numerical Simulation Results

A 3-dimensional, direct numerical simulation developed by Walton (Walton and Braun, 1993) was used to study mustard seed flows in the rotating drum. Parameters such as the mean rotation of the seeds and the mean time between collisions are difficult to measure directly but can be obtained from these simulations. The output of the simulation may also be used as input to numerical simulation of the NMR experiments.

Rotation rates of 15, 30, 45, and 60 rpm have been used to study flows which are geometrically similar to the mustard seed experiments. The simulations use 2000 1.89 mm spheres in a 12.6 cm ID rotating cylinder. A one cm length of the cylinder was simulated and periodic boundary conditions were used along the axis of the cylinder. The output was processed with PV-WAVE [Visual Numerics, Boulder, CO] to produce images of concentrations, velocities, and angular velocities. The variance of the linear velocity components was also displayed as an image. Each simulation was observed every 0.5 s for the first 3

The left image shows the horizontal X component of velocity in a half-filled cylinder of mustard seeds measured by NMR. The right hand image shows the vertical Y component of velocity for the same flow. Cylinder rotation rate was 24 rpm.

s and the flow reached steady state after 2 s. Therefore, tables of coordinates and linear and angular velocity components were written out every 0.05s after the first 3 s. NMR measurements of V_x and V_y for mustard seeds in a cylinder rotated clockwise at 24 rpm is shown above as gray scale images. X-velocity, at left, is positive to the right as indicated by darker gray values. The positive

direction for Y-velocity, at right, is upward. This rotation rate is between the two we saw in Figs. 4(b) and 5(a).

Similar results are obtained from the numerical simulation in this range of rotation rates, an example of which is given below. The velocities are approximately twice those in the figure above due to geometric scale-up.

The left image shows the horizontal component of velocity in a numerical simulation of flow in a half-filled cylinder of mustard seeds. The right image shows the simulated vertical component of velocity for the same flow. Cylinder rotation rate was 15 rpm.

The dynamic angle of repose increases linearly with rotation speed according to NMRI experiments and direct numerical simulations. The latter underestimates the measured angle of repose, however.

Figure 7 shows concentration and velocity components from NMRI and simulation at 30 rpm. The agreement is good. The thickness of the flowing layer

Figure 7. Concentration, X-velocity, and Y-velocity images from NMRI (left) and direct numerical simulation (right) for rotations at 30 rpm. The agreement is excellent for the velocities but not as good for the concentration because of the collision losses that were not fully compensated. The NMR images are also out-of-round, giving the false impression that the free surface angle is not as steep as they were in Figs. 4-6 at comparable speeds.

Concentration

X-Velocity

Y-Velocity

NMRI

Simulation

and the steeper inclinations at higher rotation rates are also captured by the simulation. The distribution of velocity variance predicted by the simulation corresponds moderately well to that from NMRI diffusion images.

The figure above shows the dynamic angle of repose as a function of Ω , from the simulation and from NMRI. Although the dynamic angle of repose increases as Ω for both, the simulation consistently underestimates the NMRI values. This could be due to a slight nonsphericity of the real particles as demonstrated by Walton and Braun (1993) and Yamane, et al. (1995).

Theoretical Results

Free surface shape and angle of repose: Professor V. M. Kenkre of our subcontractor, University of New Mexico, has performed analyses of free surface shape based, in part, on de Gennes' formalism (quoted in Rajchenbach, 1990) that the flow be confined to the surface and the flux through any volume containing the free surface be conserved. Mass and momentum balance equations were written with gravity, dry friction, and collisional damping force as parameters to be varied. The resulting differential equations to be satisfied by the flows along the free surface were solved analytically and numerically to yield free surface profiles. The dependence of the free surface shape and the dynamic angle of repose, so obtained, were studied as a function of rotation rate Ω , filling fraction, and the damping constant.

The derivation predicts the development of an S-shape for the free surface for any reasonable rotation speed, friction, and damping. It also predicts that the angle of repose at the center of a half-filled cylinder be proportional to Ω for small Ω but increase more rapidly for large Ω , in general agreement with the experiments with spherical mustard seeds. The extension of this derivation to a flow with some depth has not been carried out.

Granular flow formalism: Dr. John Anderson of Rochester Institute of Technology, a visitor to Professor Kenkre's group, has performed some comparisons of the granular hydrodynamics due to Jenkins and Haff.

Velocity depth profile: The velocity field of flowing materials may be determined from the principle that the flow will arrange itself to minimize the total energy dissipation (Rouse, 1978). For granular flows in a partially filled rotating drum, we simplify the dissipation minimization condition to apply only to the line R which is the perpendicular bisector to the flow, thus dealing only with a one-dimensional integral

$$\frac{dE}{dt} \propto \int_R \left(\frac{du}{dr} \right)^2 dr \quad (1)$$

to minimize where r is measured along R. Here u is the velocity in the frame rotating with the cylinder. This simplification is made plausible by the observation that the velocity profile along R for granular flow seems independent

of perturbations away from R in contrast, for example, to typical liquid flow (Jeong, et al., 1993) in which minor disturbance propagates throughout the sample. In addition, we require that the net flux across the radius be zero and that the velocity and $\mu(dv/dr)$ be continuous everywhere and that the velocity be V at $r = 0$, i.e., at the free surface. Equation (1) can be minimized by Euler-Lagrange variational methods (Morse and Feshbach, 1953) with zero net flux across R, with the solution

$$u = ar^2 + br + c. \quad (2)$$

The minimization of Eq. (1) implies that the excursions of du/dr be minimized at all costs and the quadratic form is the smoothest variation possible. This prediction is borne out in our experiments as shown in the velocity profile on page 9.

For a half-full cylinder, the velocities are perpendicular to R and the concentration is close to the packed concentration, i.e., the free surface is at $r = 0$ and the mass flux depends only on velocity. The bulk of the seeds rotate as a solid body and flow occurs only near the free surface.

We now fit the quadratic solution and its derivative at r_0 to a linear velocity of the solid body rotation in order to derive a relation between r_0 and the velocity V at the free surface. Let $v_1(r)$ be the velocity in the flowing layer and $v_2(r)$ the velocity of the packed layer:

$$v_1(r) = a(r_0 - r)^2 + b(r_0 - r) + c, \quad 0 \leq r \leq r_0 \quad (3)$$

and

$$v_2(r) = -\Omega r, \quad r_0 \leq r \leq R. \quad (4)$$

The continuity of v and dv/dr at r_0 and the condition $v(0)=V$ lead to

$$v_1 = v \left(1 - \frac{r}{r_0} \right)^2 - \Omega r, \quad 0 \leq r \leq r_0 \quad (5)$$

as the velocity of the flowing layer.

The conservation of mass flux across R, the assumption of constant packing density, and the linear velocity in the lower part lead to

$$r_0 V = (3/2) \Omega r^2 \quad (6)$$

a not surprising result. That the increase of mass flow with Ω might be partitioned equally in the two parameters, leading to $\sqrt{\Omega}$ dependence for r_0 and for V, is reasonable. Both r_0 and V, as measured, agree with this $\sqrt{\Omega}$ dependence; $r_0(\Omega)$ was shown in Fig. RR.

General theory of diffusion measurements by NMR: We have re-examined the standard NMR diffusion technique to see under what conditions the usually assumed short pulse condition holds. This was done from a general expression for the diffusion attenuation and evaluating the fractional contribution to the signal attenuation during the diffusion sensitizing gradient. The somewhat surprising result is that the narrow pulse approximation is much more restrictive than it had been thought; the diffusion sensitizing gradient length must be of the order of 1%, or less, of the duration required for the nuclei to traverse across a bounded region.

Conclusions and Summary

We have demonstrated the efficacy of NMRI to granular flow studies of objects that yield NMR signals (seeds, pharmaceutical pills) in confined spaces (rotating cylinders, shaking boxes). Excellent results for velocity and concentration of flowing and colliding particles have been obtained non-invasively. We studied flows in geometries that are otherwise impossible to study because of the optical opacity of the materials. In addition, we obtained data for diffusion and collisional losses which must be related to granular temperature. We also measured the global collisional and frictional energy loss for a rotating cylinder containing granular material and compared it to that obtained from the NMRI velocity and acceleration data. We used the same NMR technique to study the interplay between radial and axial segregation of heterogeneous particles in a partially filled rotating cylinder. We also measured flow and diffusion of vibrating particles in the NMRI apparatus to ascertain that such experiments are, indeed, possible. Some theoretical studies were initiated to explain some of the velocity profiles and free surface shapes of rotating drum flow. A direct numerical simulation of the drum flow successfully predicted a number of features, including velocity, concentration, and dissipation profiles in rotating cylinders.

What follows is a partial list of useful granular flow information gathered from our work, mostly, on flows of spherical particles in a partially filled rotating drum.

1. Thickness of flowing layer increases as square-root of angular velocity of the rotating drum for slow rotations.
2. Free surface angle is proportional to the angular rotation rate for slow flows.
3. Flow velocity changes smoothly as a function of depth from the free surface.
4. Flow velocity changes nearly quadratically along the free surface.
5. Energy dissipation is proportional to angular velocity.
6. Energy dissipation profile is closely related to diffusion coefficient profile.
7. Diffusion anisotropy and heterogeneity depend on particle shapes; anisotropic particles exhibit the most anisotropy in diffusion.
8. Axial segregation evolves with the help of radial segregation.
9. Simulation of drum flows agrees with experiment.

Future Prospects

This project has opened up a new technology for the non-invasive study of granular materials. Additional useful capabilities on the horizon include

- more sophisticated measurements and interpretation of diffusion;
- making the connection between diffusion, collisional losses, and granular temperature;
- more complete studies of vibrating particles; and
- combination of NMRI and chute flow geometry.

References

- G. K. Batchelor, An Introduction to Fluid Dynamics (Cambridge, London, 1970); p. 153.
- J. W. Carlson, K. A. Derby, K. C. Hawryszko, and M. Weidman, *Magn. Reson. Med.* 26, 191 (1992).
- D. Freeman and R. Hurd, in NMR Basic Principles and Progress, vol. 27 (Springer-Verlag, Berlin, 1992); pp. 199-222.
- M. V. Icenogle, A. Caprihan, and E. Fukushima, *J. Magn. Reson.* 100, 376-381 (1992).
- E.-K. Jeong, S. A. Altobelli, and E. Fukushima, *Physics of Fluids*, 6, 2901-2906 (1994).
- K. Kose, *J. Magn. Reson.* 92, 631 (1991).
- K. Kose, *J. Magn. Reson.* 98, 599 (1992).
- I. J. Lowe and R. E. Wysong, *J. Magn. Reson. B* 101, 106 (1993).
- P. M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw-Hill, New York, 1953); pp. 275-280.
- M. Nakagawa, *Chem. Engng. Sci.* 49, 2540-2544 (1994).
- M. Nakagawa, S. A. Altobelli, A. Caprihan, E. Fukushima, and E.-K. Jeong, *Experiments in Fluids* 16, 54-60 (1993).
- J. Rajchenbach, *Phys. Rev. Letters* 65, 2221 (1990).
- H. Rouse, Elementary Mechanics of Fluids (Dover, New York, 1978); Section 28.
- J. B. Thomas, An Introduction to Statistical Communication Theory (John Wiley & Sons, New York, 1969); Chapter 4.
- R. Turner, *J. Phys. E: Sci. Instrum.* 21, 948 (1988).
- J. Van Vaals and A. Bergman, *J. Magn. Reson.* 90, 52 (1990).
- O. R. Walton and R. L. Braun, "Simulation of rotary-drum and repose tests for frictional spheres and rigid sphere clusters," in *Proceedings, Joint DOE/NSF Workshop on Flow of Particulates and Fluids*, September 29 - October 1, 1993, Ithaca, NY; pp. 131-148.
- J. C. Watkins and E. Fukushima, *Rev. Sci. Instrum.* 59, 926-929 (1988).
- R. E. Wysong and I. J. Lowe, *Magn. Reson. Med.* 29, 119 (1993).
- R. E. Wysong, D. P. Madio, and I. J. Lowe, *Magn. Reson. Med.* 31, 572 (1994).
- K. Yamane, T. Tanaka, Y. Tsuji, M. Nakagawa, and S. A. Altobelli, "DEM and MRI Studies of Particulate Flows in a Rotating Cylinder," ASME, South Carolina, August, 1995.

APPENDIX

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Publications

M. Nakagawa, S. A. Altobelli, A. Caprihan, E. Fukushima, and E.-K. Jeong, "Non-invasive Measurements of Granular Flows by Magnetic Resonance Imaging," *Experiments in Fluids*, 1993, 16:54-60.

M. Nakagawa, "Axial segregation of granular flows in a horizontal rotating cylinder," *Chem. Engng. Sci.*, 1994; 49:2540-2544.

M. Nakagawa, S. A. Altobelli, A. Caprihan, and E. Fukushima, "Evolving phenomena of granular mixture in a horizontal rotating cylinder," *Nature*, submitted, 1995.

Presentations and Abstracts

S. A. Altobelli, A. Caprihan, E. Fukushima, and M. Nakagawa, "Multiphase Flow Studies by NMR with Application to Granular Flows," Joint DOE/NSF Workshop on Flow of Particulates and Fluids, Worcester, Massachusetts, October, 1991 (National Technical Information Service, Springfield, VA 22161); pp. 34-40.

S. A. Altobelli, A. Caprihan, E. Fukushima, E.-K. Jeong, and M. Nakagawa, "NMR Imaging of Flowing Liquids and Solids," 33rd ENC, Asilomar, California, March 29-April 2, 1992.

M. Nakagawa and E.-K. Jeong, "Application of NMR to Rotating Granular Flow," ASCE Ninth Engineering Mechanics Conference, College Station, Texas, May 25-27, 1992.

- M. Nakagawa, A. Caprihan, and E. Fukushima, "MRI of Granular Flows in a Rotating Cylinder," 1992 Cavitation and Multiphase Flow Forum of The Fluids Engineering Division of ASME, Los Angeles, June 22-25, 1992.
- S. A. Altobelli, A. Caprihan, E. Fukushima, E.-K. Jeong, and M. Nakagawa, "NMR Imaging of Flowing Systems," 26th Congress Ampere on Magnetic Resonance, Extended Abstracts, Athens, Greece, September, 1992; pp. 16-17.
- S. A. Altobelli, A. Caprihan, H. Cheng, E. Fukushima, E.-K. Jeong, and M. Nakagawa, "Granular Flow Studies by NMR," DOE/NSF Workshop on Flow of Particulates and Fluids, Gaithersburg, Maryland, September 17-18, 1992; pp. 48-54.
- M. Nakagawa, A. Caprihan, E. Fukushima, S. A. Altobelli, and E.-K. Jeong, "Granular Flows in a Rotating Cylinder: Non-Invasive Measurement by MRI," 47th meeting of Society for Soil Mechanics, Tohoku University, Sendai, Japan, September, 1992.
- L. Z. Wang, "Diffusion Image of Granular Flow," New Mexico Regional NMR Meeting, Albuquerque, February 27, 1993.
- S. A. Altobelli, "Velocity Measurements of Suspensions, Slurries, and Seeds," 34th ENC, St. Louis, March, 1993.
- M. Nakagawa, "Emergence of Patterns in Particulate Flow," Seminar, Department of Physics, University of New Mexico, April 28, 1993.
- A. Caprihan and M. Nakagawa, "NMRI of Multiphase Flows," Porous Media Expo, Albuquerque, May 6-8, 1993.
- M. Nakagawa, S. A. Altobelli, A. Caprihan, and E. Fukushima, "NMR Imaging of Granular Flows," ASME/ASCE/Society of Engineering Science Meeting, Charlottesville, Virginia, June 6-9, 1993
- L. Z. Wang, A. Caprihan, and E. Fukushima, "A general formulation of diffusion measurements by NMR," Gordon Conference, Wolfeboro, NH, July 12-16, 1993.
- M. Nakagawa, S. A. Altobelli, A. Caprihan, and E. Fukushima, "Non-invasive measurements of granular flows by magnetic resonance imaging," Powders and Grains 93, Proceedings of the 2nd International Conference on Micromechanics of Granular Media, Birmingham, England, July 12-16, 1993; edited by C. Thornton (A. A. Balkema, Rotterdam, 1993), pp. 383-387.

L. Z. Wang, A. Caprihan, and E. Fukushima, "General Treatment of Diffusion in the Presence of Boundaries and Short or Long Gradient Pulses," Second International Conference on NMR Microscopy, Heidelberg, September 6-9, 1993.

E. Fukushima, "Strategies and Techniques in Flow, Diffusion, Imaging, and Combinations Thereof," Lectures (4) at Ampere Summer School and Symposium, Protoroz, Slovenia, September, 1993.

S. A. Altobelli, A. Caprihan, H. Cheng, E. Fukushima, M. Nakagawa, and L. Z. Wang, "Progress Report: Granular Flow Studies by NMR," Joint DOE/NSF Workshop on Flow of Particulates and Fluids, Ithaca, September 29 - October 1, 1993; Proceedings, pp. 233-241.

M. R. Endicott, V. M. Kenkre, A. Caprihan, and M. Nakagawa, "Free Surface Profiles of Granular Material in a Rotating Cylinder: A Simple Model," American Physical Society, Pittsburgh, March 21-25, 1994.

A. Caprihan, S. A. Altobelli, E. Fukushima, M. Nakagawa, and L. Z. Wang, "Nmr of Velocity Fluctuations and Energy Dissipation in Granular Flows," 35th ENC, Asilomar, April 10-15, 1994.

S. A. Altobelli, A. Caprihan, E. Fukushima, M. Nakagawa, and H. A. Cheng, "Granular Flow Studies By NMR," 12th U.S. National Congress of Applied Mechanics, Seattle, June 28, 1994; p.151.

S. A. Altobelli, A. Caprihan, E. Fukushima, M. Nakagawa, and L. Z. Wang, "Particle Flow Studies by Nuclear Magnetic Resonance," Seminar on Flows of Granular Materials in Complex Geometries, International Energy Agency, Albuquerque, New Mexico, August 15-16, 1994; in S. L. Passman, E. Fukushima, and R. E. Evans, editors, "Meeting on Flows of Granular Materials in Complex Geometries," Sandia Report, SAND94-2732, Nov., 1994..

E. Fukushima, S. A. Altobelli, A. Caprihan, E.-K. Jeong, and M. Nakagawa, "Studies of Multiphase Flow by Nuclear Magnetic Resonance Imaging," The First International Particle Technology Forum, Denver, August 17-19, 1994.

S. A. Altobelli, A. Caprihan, H. Cheng, E. Fukushima, M. Nakagawa, and L. Z. Wang, "Progress Report: Granular Flow Studies by NMR - 1994," Joint DOE/NSF Workshop on Flow of Particulates and Fluids, Philadelphia, October 5-7, 1994; Proceedings, pp. 69-77.

S. L. Passman, E. Fukushima, and R. E. Evans, editors, "Meeting on Flows of Granular Materials in Complex Geometries," Sandia Report, SAND94-2732, Nov., 1994.

I. J. Lowe, book review of Principles of Nuclear Magnetic Resonance Microscopy, Paul T. Callaghan. Oxford University Press, Oxford, 1991; in J. Magn. Reson. A, 1994; 110:121-122.

M. Nakagawa, S. A. Altobelli, A. Caprihan, and E. Fukushima, "Segregation Phenomena in a horizontal rotating cylinder," Mechanics of Granular Materials, 10th ASCE Engineering Mechanics Speciality Conference, Boulder, Colorado, May 21-24, 1995.

S. A. Altobelli, A. Caprihan, E. Fukushima, M. Nakagawa, and L. Z. Wang, "NMRI Studies of Granular Flows in a Rotating Horizontal Cylinder," IUTAM Symposium on Hydrodynamic Diffusion of Suspended Particles, Estes Park, Colorado, July 22-25, 1995.

M. Nakagawa, S. A. Altobelli, A. Caprihan, and E. Fukushima, "Unexpected Behavior of Particles in Horizontal Rotating Cylinder," IUTAM Symposium on Hydrodynamic Diffusion of Suspended Particles, Estes Park, Colorado, July 22-25, 1995.

K. Yamane, T. Tanaka, Y. Tsuji, M. Nakagawa, and S. A. Altobelli, "DEM and MRI Studies of Particulate Flows in a Rotating Cylinder," ASME, South Carolina, August, 1995.