TEMPERATURE, CHEMICAL POTENTIAL AND THE $\rho$-MESON

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1. Introduction. Models of QCD must confront nonperturbative phenomena such as confinement, dynamical chiral symmetry breaking (DCSB) and the formation of bound states. In addition, a unified approach should describe the deconfinement and chiral symmetry restoring phase transition exhibited by strongly-interacting matter under extreme conditions of temperature and density. Nonperturbative Dyson-Schwinger equation (DSE) models [1, 2] provide insight into a wide range of zero temperature hadronic phenomena; e.g., non-hadronic electroweak interactions of light- and heavy-mesons [3], and diverse meson-meson [4] and meson-nucleon [5] form factors. This is the foundation for their application at nonzero-$(T, \mu)$ [2],[6]-[8]. Herein we describe the calculation of the deconfinement and chiral symmetry restoring phase boundary, and the medium dependence of $\rho$-meson properties. We also introduce an extension to describe the time-evolution in the plasma of the quark’s scalar and vector self energies based on a Vlasov equation.

2. Dyson-Schwinger Equation at Nonzero-$(T, \mu)$. The dressed-quark DSE at nonzero-$(T, \mu)$ is

$$S^{-1}(\bar{p}_k) = i\gamma \cdot p A(\bar{p}_k) + i\gamma_4\omega_{k+} C(\bar{p}_k) + B(\bar{p}_k) = i\gamma \cdot p + i\gamma_4\omega_{k+} + \Sigma(\bar{p}_k),$$  

where $\bar{p}_k = (\bar{p}, \omega_{k+}), \omega_{k+} = \omega_k + i\mu,$ and $\omega_k = (2k+1)\pi T$ is the quark’s Matsubara frequency. The complex-valued scalar functions: $A(\bar{p}, \omega_{k+}), B(\bar{p}, \omega_{k+})$ and $C(\bar{p}, \omega_{k+})$, depend only on $(|\bar{p}|^2, \omega_{k+}^2)$. With a given dressed-gluon propagator the solutions are determined by

$$B(\bar{p}_k) - m_0 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} D(\bar{p}_k - \bar{q}_k) \frac{B(\bar{q}_k)}{\bar{q}_k C(\bar{q}_k) + B^2(\bar{q}_k)},$$  

$$C(\bar{p}_k) - 1 \bar{p}_k^2 = \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} D(\bar{p}_k - \bar{q}_k) \frac{\bar{p}_k \cdot \bar{q}_k C(\bar{q}_k)}{\bar{q}_k C(\bar{q}_k) + B^2(\bar{q}_k)},$$

where herein we only consider models where $A(p) = C(p)$. It is the interplay between the functions $B$ and $C$ that leads to confinement, realised via the absence of a Lehmann representation for the dressed-quark 2-point function [1, 2],[6]-[9]. $B \neq 0$ in the chiral limit signals DCSB.
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The phase transition is first order for any non-zero $\mu$ and second order for $\mu = 0$. The model has mean field critical exponents, which is a feature of the rainbow-ladder truncation [11]. The study of thermodynamic properties shows that it is essential to keep scalar and vector self-energies as well as their momentum dependence [8, 10].

Mesons are quark-antiquark bound states and their masses are obtained by solving the Bethe-Salpeter equation [12]. Here we focus on the vector channel and employing Eq. (4) the eigenvalue equation for the bound state mass is [13]

$$\eta^2 \Re \left\{ \sigma_B(\omega_{0+}^2 - 4M^2_{\rho\pm})^2 - \left[ \pm \omega_{0+}^2 - \frac{1}{4}M^2_{\rho\pm} \right] \sigma_C(\omega_{0+}^2 - 4M^2_{\rho\pm})^2 \right\} = 1, \quad (5)$$

where $\sigma_{B,C}(\vec{p}^2) = \{B(\vec{p}^2), C(\vec{p}^2)\}/[\vec{p}^2 C^2(\vec{p}^2) + B^2(\vec{p}^2)]$. The equation for the $\rho$-meson's transverse component is obtained with $[-\omega_{0+}^2 - 4M^2_{\rho\pm}]$ in Eq. (5) and in the chiral-limit component $M^2_{\rho\pm} = \frac{1}{2} \eta^2$, independent of $T$ and $\mu$. This is the $T = 0 = \mu$ result of Ref. [14]. Even for nonzero current-quark mass, $M_{\rho\pm}$ changes by less than 1% as $T$ and $\mu$ are increased from zero toward their critical values. Its insensitivity is consistent with the absence of a constant mass-shift in the transverse polarization tensor for a gauge-boson. For the longitudinal component one obtains in the chiral limit:

$$M^2_{\rho^+} = \frac{1}{2} \eta^2 - 4(\mu^2 - \pi^2 T^2). \quad (6)$$

The results for the medium-dependence of the $\rho$ meson are summarised in Fig. 2. As in the case of the dressed-quark mass function, the response to increasing $T$ and $\mu$ is anti-correlated: the $\rho$- mass decreases with increasing chemical potential and increases with temperature. This anti-correlation leads to an edge along which the $T$ and $\mu$ effects compensate and the mass remains unchanged up to the transition point.

3. Nonequilibrium Application. The time evolution of the self energies can be studied using Vlasov's equation

$$\partial_t f(p, x) + \partial_p E(p, x) \partial_x f(p, x) - \partial_x E(p, x) \partial_p f(p, x) = 0. \quad (7)$$

Solving this equation is complicated for two reasons. (i) The energy is a functional of the scalar and vector self energies, which in general are nonzero and momentum-dependent. While the scalar self energy is small in the plasma phase due to chiral symmetry restoration, the vector self energy remains significant [10]. (ii) The absence of a Lehmann representation for the dressed-quark
As a test whether this simplification still yields necessary and qualitatively important features, such as $C \neq 1, B \neq m_0$, in Fig. 3 we compare the momentum dependence obtained in the models specified by Eqs. (4,8) in the vicinity of $T_c$. Both functions are well reproduced and hence Eq. (8) can be used to model the persistence of non-perturbative effects in the deconfined domain. The solution of Eqs. (7,9-10) provide the time-evolution of the quark self-energy and distribution function.

As in the case of thermal equilibrium, the vector self energy plays an important role. Neglecting $\Sigma^C$ and the momentum dependence of $\Sigma^B$ a simpler equation is obtained

$$\partial_t f(p, x) + \frac{p}{E(p, x)} \partial_x f(p, x) - m(x) \partial_x m(x) \partial_p f(p, x) = 0,$$

with $m(x)$ the quark mass obtained as a solution of the gap equation in models without confinement. This equation has been widely studied; e.g. Refs. [15]. However, we anticipate that the numerical solution of Eq. (7) will yield significantly different results because of the presence and persistence of the vector self energy in the deconfined domain.
\( \Upsilon \) and \( J/\psi \) Production from 800-GeV Protons Incident on \( D_2 \) and \( H_2 \) Targets (collaboration list)

Fermilab experiment E866 is designed to detect pairs of oppositely charged muons produced in collisions between 800 GeV protons and variety of fixed targets. The primary motivation for E866 was to detect dimuon events generated in Drell-Yan reactions, but the detector is sensitive to any process which produces dimuon pairs. One such process is heavy quark vector meson production. \( \Upsilon \) and \( J/\psi \) particles are produced when partons from the beam and target annihilate and form a virtual gluon which then hadronizes into a heavy resonance state. These states have several percent branching ratios into dimuon pairs and are detected during normal data acquisition. The virtual gluon which produce the resonance can be generated by the annihilation of either a quark/antiquark pair or a pair of gluons. Resonance production is therefore sensitive to both the quark and gluon distributions within the beam and target. \( J/\psi \) production is believed to occur dominantly via gluon-gluon fusion, while \( \Upsilon \) production is thought to have contributions from both gluon-gluon fusion and quark-antiquark annihilation. Because the gluon distribution for the proton and neutron are thought be the same, the per nucleon \( J/\psi \) production cross section is expected to be the same for hydrogen and deuterium. The \( \Upsilon \) production ratio, on the other hand, is expected to be larger than unity since \( \bar{u}^n(x) > \bar{u}^p(x) \) for the Bjorken-\( x \) values probed by E866 in \( \Upsilon \) production.

The E866 hydrogen and deuterium data samples contain approximately 30000 \( \Upsilon \) and 1 million \( J/\psi \) events. The production cross sections are determined by fitting the measured mass distribution to the simulated mass distributions of the relevant vector mesons and the underlying Drell-Yan continuum. This separation is performed for several bins in X-Feynman, to determine the kinematic dependence of the cross section. The absolute \( \Upsilon \) and \( J/\psi \) production cross sections for hydrogen and deuterium are obtained from the extracted results by normalizing the yields to the integrated luminosity and correcting the results by the acceptance of the detector. The cross sections values can then be compared to predictions based various models using the various parton distributions available in the literature. Color evaporation model (CEM) calculations have been performed for both \( J/\psi \) and \( \Upsilon \) production from \( H_2 \) and \( D_2 \). Although the CEM model is extremely simple and cannot predict the absolute value of the cross sections, the X-Feynman shape is reproduced by the CEM calculation. Further, the unknown scale factor in the CEM calculations cancels when the \( D_2/H_2 \) per-nucleon ratio is calculated. The extracted cross section ratio, \( D/2H \), for \( J/\psi \) and \( \Upsilon \) production are shown in figure ???. The \( J/\psi \) ratio is close to unity, which is consistent with expectations assuming gluon fusion production. The \( \Upsilon \) ratio is also consistent with unity, which is in disagreement with the CEM calculation when either the MRST or CTEQ5M parton distribution functions (PDFs) are used. The deviation from the prediction may indicate that the PDFs underestimate the hard gluon \( (x \sim 0.25) \) distribution in the proton.