A PROBABILISTIC MODEL TO LIQUEFACTION ASSESSMENT OF DAMS

Nikolaos Simos, Carl J. Costantino and M. Reich

Department of Advanced Technology
Brookhaven National Laboratory
Upton, NY 11973

ABSTRACT

In an effort to evaluate earthquake liquefaction potential of soil media, several statistical models ranging from purely empirical to mathematically sophisticated have been devised. While deterministic methods define susceptibility of a soil structure to liquefaction, for a given seismic event, in the sense that the site does or does not liquefy, probabilistic approaches incorporate statistical properties associated with both the earthquake and site characterisation.

In this study a stochastic model is formulated to assess liquefaction potential of soil structures in general and earth dams in particular induced by earthquakes. Such earthquakes are realisations of a random process expressed in the form of a power spectral density. Uncertainties in the soil resistance to liquefaction are also introduced with probability density functions around in-situ measurements of parameters associated with the soil strength. The attempt of this study is to devise a procedure that will lead to a continuous probability of liquefaction at a given site. Monte Carlo simulations are employed for the probabilistic model. In addition a stochastic model is presented. The dynamic response of the two-phase medium is obtained with the help of the POROSLAM code and it is expressed in the form of a transfer function (Unit Response).

1.0 Introduction

During an earthquake event of considerable duration and intensity an earthen dam can experience partial or total failure that stems from either loss of soil strength due to liquefaction or reduction of the inherent resistance to sliding along a potential failure surface. In trying to assess the potential to failure in either mode, one has to essentially incorporate two types of probabilities. One that is associated with the occurrence of an earthquake in the proximity of the structure and the other with the state of the soil and its tendency to liquefy or fail in shear.

The coupling of the two essential components (excitation and soil state) with inherent statistical properties can only be achieved through response analyses that allow for the combined statistics to participate. A promising approach is the one involving the Unit Response of the domain that can eventually provide the system response resulting from a seismic event which in turn represents a stochastic process. Such procedure, however, implies that the domain exhibits linear behavior. While in typical soils this is only true in small strains, the benefits of the Unit Response approach compensate for the lack of nonlinear considerations.

The constitutive response of the granular soil skeleton and its coupling with the fluid phase is formulated around the Biot dynamic equations of motion. The finite element analysis utilised in the evaluation of the dynamic response is a linear in character but it treats the soil as a two-phase medium. While the drawback of linearity is somewhat compensated with the equivalent hysteretic damping, it is the two-dimensional pore water/soil skeleton interaction that provides a realistic description of the behavior of the soil in a dynamic mode. The solution takes place in the frequency domain.
and the resulting harmonic response, inverted with the use of Fast Fourier Transform techniques, provides the intergranular stress as well as the pore water pressure fluctuation during the seismic event.

The probability of liquefaction failure is evaluated by incorporating the in-situ strength conditions, which exhibit a random behavior around soil strength properties, and seismically induced random dynamic stresses at various potentially liquefiable locations within the cross section of the dam. The driving random forces in such process are the dynamic (cyclic) shear stress that is generated in the soil layers and the buildup of pore pressures. A limit state function that contains the failure criterion is established for the duration of the random earthquake and it is an indicator of the pore pressure build-up. It should be noted that while a linear analysis can only predict the buildup of pore pressures only heuristically, it can provide the level of shearing at low strains that the soil experiences during the seismic event. Since most probabilistic models of soil liquefaction provide deterministic answers (probability can only take the values of one for liquefaction occurrence or zero when no liquefaction occurs), the present model through Monte Carlo simulations both in the earthquake and in the soil strength, expressed as function of a single field parameter (SPT N-values or relative density $D_r$), results in a continuous probability between 0 and 1. Lastly, a purely stochastic model of liquefaction failure is presented based on relations linking the statistics of the random input (input power spectral densities) with the output counterpart along with the stochastic nature of the soil strength.

2.0 Seismic Response of a Dam

Dynamic analysis of the 2-phase medium

In assessing the dynamic response of the embankment the saturated state of the soil must be accounted for. The pore pressure of the water trapped in the soil skeleton will fluctuate during the earthquake and impact on the intergranular soil stresses. Since the strength of the soil is tied to the intergranular stresses, it is vital that the dynamic pore pressure be captured. The coupled behavior of pore water and soil skeleton requires that the medium must be treated as a two-phase one with governing equations that reflect the coupling.

Further, the ability of the soil to resist liquefaction is one hand dependent on its initial stress state (effective stress) and on the other hand on the intensity of the dynamic shear stress. The shear stress variation at different locations in the embankment and the foundation as well as the number of stress cycles during the earthquake event determine whether the soil is susceptible to such failure.

Therefore, to effectively analyse the system, the employed theoretical/computational model must enable;

a. The description of the domain as a two-phase medium.
b. The implementation of actual or representative earthquake input.
c. The evaluation of the time variation of stresses resulting from the seismic input.

In order to perform the dynamic analysis, which satisfies the above requirements, the POROSLAM code is employed. The code is a two-dimensional finite element representation of Biot's dynamic equations for both soil and fluid phases. Biot's equations are a linear description of the response of the soil skeleton and of the pore water in the form,

$$\frac{\partial \tau_{zz}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = \rho u_z + \rho_f \bar{w}_z$$

$$\frac{\partial \tau_{yz}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} = \rho u_y + \rho_f \bar{w}_y$$

and

$$-\frac{\partial p_f}{\partial x} = \rho_f \bar{u}_z + \frac{1}{f} \rho_f \bar{w}_z + \frac{\eta}{k} \bar{w}_z$$

$$-\frac{\partial p_f}{\partial y} = \rho_f \bar{u}_y + \frac{1}{f} \rho_f \bar{w}_y + \frac{\eta}{k} \bar{w}_y$$

(1)

where,

$$[u_z, u_y] = \text{components of displacement of the soil}$$

$$[w_z, w_y] = \text{components of displacement of the pore water}$$

$$\{\tau \} = \{ \sigma_{zz} - \alpha p_f, \sigma_{yy} - \alpha p_f, \sigma_{xy} \}^T$$

while, $f$ = porosity, $\rho$ = total mass density, $\rho_f$ = fluid mass density, $\alpha$ = compressibility of solid, $M$ = compressibility of the fluid, $\eta$ = fluid viscosity and $k$ = soil permeability.

The resultant equation that expresses the total stress vector in terms of the displacement vector while considering hysteretic damping, takes the form:

$$\{\tau \} = E_c [D_1] \left( [D_0] + [D_3] \frac{\partial}{\partial t} \right) \{u_z, u_y\}^T + \alpha^2 M [D_2] \{u_z, u_y\}^T + \alpha M [D_2] \{w_z, w_y\}^T$$

(2)
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
Explicit definition of the above equations can be found in [5].

### Harmonic Solution

The dynamic input can be either deterministic or stochastic. In the deterministic mode the given ground excitations or dynamic loads are expressed in the following form

\[
g(t) = \sum_{k=1}^{N} X(\omega_k) e^{i\omega_k t}
\]

where \( \omega_i \) are the frequencies of the harmonic analysis. In the stochastic mode the base excitation or the forcing function is expressed in terms of the complex input

\[
X(\omega_k) = e^{i\omega_k t}
\]

Evaluation of the response at the frequencies \( \omega_i \) leads to the complex frequency response \( H(\omega) \). The Fourier coefficients of a response quantity \( Y(\omega) \) (displacement or stress) from a synthetic input with coefficients \( X(\omega) \) can be obtained through the relation

\[
Y(\omega) = H(\omega) X(\omega)
\]

In probabilistic analysis that is based on a Monte Carlo scheme (multiple evaluation of the response at the same intensity level but with random distribution of peaks), the simulated earthquakes or dynamic loads belong to a certain family. Such family can be characterized by a response or a power spectrum. Earthquake (or load) records can be synthesized on the basis of these properties. For the case of an earthquake family described by the Kanai-Tajimi power spectrum

\[
S_x(\omega, \lambda) = S_0 \frac{(1 + 4\zeta^2 \alpha_0^2)}{(1 - \alpha_0^2)^2 + 4\zeta^2 \alpha_0^2}
\]

where, \( \alpha_0 = \frac{\omega}{\omega_p} \), \( \lambda = [\zeta, \omega_p, S_0]^T \).

A synthetic time history \( g(t) \) generated from the form

\[
g(t) = 2\zeta \sum_{i=1}^{N} \sqrt{S_x(\omega_i)} \Delta \omega \cos(\omega_i t + \phi_i)
\]

is a realization of the process described by the power spectrum. In order to accommodate the nonstationary part of the ground excitation, the simulated acceleration or force is with the nonstationary function \( \zeta(t) \). In the above expression

\[
w_i = i\Delta \omega \quad \Delta \omega = \frac{\omega_u}{N}
\]

where \( \omega_u \) is a cutoff frequency and \( \phi_i \) is a vector of random phase angles uniformly distributed between 0 and \( 2\pi \). Different choices of the vector of random phase angles will lead to a different simulated dynamic inputs. The synthetic process \( g(t) \) is periodic with a period \( T_0 = \frac{2\pi}{\Delta \omega} \).

### 3.0 Probabilistic Liquefaction

As it has been previously mentioned, the assessment of the probability of liquefaction failure at a given site given that an earthquake has occurred is typically deterministic or in other words the probability takes the value of 1 for liquefaction occurrence or the value 0 for no liquefaction. In this study a procedure through which a continuous probability between 0 and 1 can be established will be adopted.

In assessing the likelihood of liquefaction one has to keep in mind that several sources of uncertainty exist. Uncertainties emanate from the definition of the earthquake and from the characterization of the particular site. There are various vectors of parameters that can characterize both the earthquake and the site. A typical set of parameters for an earthquake are the peak ground acceleration and the duration or the magnitude. The equivalent vector for the site indicating resistance to liquefaction could consist of the corrected standard penetration resistance or the relative density and the percent fines content. The two independent uncertainty sources will be addressed separately but the limit state function that reflects the failure criterion will incorporate both uncertainties.

Consider the following limit state function,

\[
g(\chi, \psi) = (SF)_{LIQ} - 1
\]

\[
= \frac{\text{Strength}(\chi)}{\text{Induced Stress}(\psi)} - 1 = 0
\]

where \( \chi \) = the parameter vector of the site and \( \psi = \psi(pga, T) \) is the earthquake load vector. The
probability that \( g(\chi, \psi) < 0 \) conditional to the occurrence of an earthquake and a given vector of site soil parameters \( \chi = \bar{\chi} \) is

\[
P [ g(\chi, \psi) < 0 | \text{quake} ] = \int_{\Psi} P [ g(\chi, \psi) < 0 ] \ d\Psi
\]

where \( \Psi(\psi) \) the probability distribution of the parameters in the earthquake characterization vector. Quantities with a bar over them are considered to be known. To reduce the complexities, the parameter indicating the earthquake duration is no longer considered random (tendency to only incorporate the strong motion portion of the record which in turn can be defined by an average value). Hence, the conditional probability can be written

\[
P [ g(\chi, \psi) < 0 | \text{quake} ] = \int_{\alpha} P [ g(\chi, \alpha) < 0 ] \ d\Psi \tag{8}
\]

where \( \alpha \) is the random peak acceleration of the earthquake. If one estimates the earthquake occurrence rate \( \lambda \) per year for the particular site from appropriate hazard curves then the probability of liquefaction failure per year at the site and at a specific location characterised by the vector \( \chi = \bar{\chi}(N, D_r, etc) \) can be deduced from

\[
P[ g(\chi, \alpha) < 0 | \text{year} ] = 1 - \exp\{ -\lambda \int_{\alpha} P [ g(\chi, \alpha) < 0 ] \ d\Psi(\alpha) \} \tag{9}
\]

In order to incorporate the uncertainties of the site characterisation into the probability measure, the randomness of the vector parameters in \( \chi = \bar{\chi}(N, D_r, etc) \) must be established. The elements of the above vector are indicative of the soil liquefaction resistance. If, for the sake of simplicity, one can deduce a single parameter that can fairly represent the soil, then the randomness of the selected parameter will characterise the uncertainties of the site. The SPT blow count N values can play that role.

To establish its statistics for a particular site one has to consider the following: It is not realistic to accept a distribution of N-values for a site as a function merely of the depth and the type of soil unless one considers the soil medium perfectly homogeneous. If such is the case, then for a given site and type of soil the N-values can be estimated based on the effective overburden stress (not random) and the relative density of the soil which in turn can be considered as the random variable. In most applications, however, one has available a profile of the SPT N-value with depth from actual in-situ testing. It is expected that variability of the N-value at any depth exists along with the variability with depth. Rather than attempting to establish a distribution for the entire medium, randomness to the value of N at a given location can be assumed. If a good size sample is available (very rare) for every location, the statistical properties can then be established (mean, variance etc.) as well as the distribution type.

If (as in most cases) a single value of N is available for the location, a distribution can be assumed about the value which can represent the mean. The random nature of the members of the assumed distribution will provide the uncertainty in the liquefaction resistance.

Denoting the probability distribution about the SPT N-value at a location \( h \) as \( \Xi(N) \) \( h \) then the probability of a liquefaction failure at the specified site and this particular location within a time period \( T \) years can be estimated through the expression

\[
P[ g < 0 | T ] = \int_{\Xi} \left[ 1 - \exp\{ -\lambda T \int_{\alpha} P [ g(\chi, \alpha) < 0 ] \ d\Psi \} \right] \ d\Xi \tag{10}
\]

The integrals \( \int_{\Xi} \) and \( \int_{\alpha} \) are evaluated between the range of expected minima and maxima for the site (i.e. \( \int_{\alpha} = \int_{\alpha_{\min}}^{\alpha_{\max}} \) and \( \int_{\Xi} = \int_{-\sigma_0}^{+\sigma_0} \)).

For the process to be complete an explicit definition of the limit state function \( g(\chi, \psi) - 1 = 0 \) must be established. There are several ways in defining soil strength or resistance to liquefaction and seismically induced action. Specifically, the vulnerability of the soil to liquefaction is determined by the level of stress or strain that develops during a dynamic event or by the build-up of pore pressure in the saturated soil. In addition, one must keep in mind that the failure criterion it is not an instantaneous event but rather the result of an accumulated effect for the span of the earthquake and can be estimated based on the induced cycles of the soil straining. For a shear stress or strain failure type the developed stress or strain in the soil must exceed a threshold value for failure to occur. Typically, an equivalent stress ratio \( \tau_{\text{equiv}} = \tau_{\text{equiv}} \) and an equivalent number of cycles \( N_{\text{equiv}} = \sigma_0 \) estimated from the shear stress response at a particular location and the resistance to liquefaction is determined as the ratio of \( \tau_{\text{equiv}} \) from appropriate \( \frac{\tau}{\sigma_0} \) v.s. \( N \) curves. For complex stress state, however, where the impinging seismic waves cause more than shear action, it is suggested that strength curves be developed (from...
laboratory tests) reflecting the strength of the soil as a function of the stress invariant $\sqrt{J_2}$.

An alternative definition of the limit state function is the evaluation of the resistance to liquefaction based on the generated pore pressures. With this approach liquefaction occurs at zero effective stress. Since the state of failure is reached progressively, a cumulative damage index is evaluated as the earthquake passes by and it has the form

$$D(t) = \sum_{\text{induced cycles}} \frac{1}{N_{\text{liq}}}$$

(11)

$N_{\text{liq}}$ represents the number of uniform cycles required to induce liquefaction at the shear stress level of one of the actual stress cycles the soil is subjected to.

Fan. 1 Relative Density $D_r$ vs. $(N_1)_{60}$

The shear stress ratio $\tau_r$ can be related to $N_{\text{liq}}$ through empirical formulae of the type

$$N_{\text{liq}} = \xi \tan \left[ \frac{\pi (r_0(D_r) - r_r)}{2(r_0(D_r) - r_\infty)} \right]$$

(12)

In the above expression $\tau_r$ is the shear stress ratio for an induced cycle, $r_0(D_r)$ and $r_\infty$ are shear stress ratios associated with the strength of the soil with variation in the relative density $D_r$ (see Figure 2) and $\xi$ is a curve fitting parameter. The last expression actually ties both the induced effect and the soil resistance. Thus, given the shear stress history at a particular location (generated randomly) and for a randomly selected SPT $N$-value, the corresponding relative density $D_r$ can be deduced from the appropriate for the soil relation of Figure 1.

With a given $D_r$ at the location (conditional to the value $N$) the $N_{\text{liq}}$ vs. $\tau_r$ relationship is established. At the same time, by continuously tracking the time history of the randomly induced stress at the same location for the entirety of the random earthquake, the limit state function is established and can be implemented in the probability integrals. For any synthetic earthquake the damage index is evaluated and related to the pore pressure ratio $\tau_{pf} = \frac{P_f}{a_0}$ with the simple relation $\tau_{pf} = D(t)$. Liquefaction occurs when the pore pressure and the effective stress are equal or $\tau_{pf} = 1$. Thus the limit state function is defined as

$$g(\chi, \psi) = \tau_{pf} - 1 = 0$$

Fan. 2 Cyclic Test Data for Sand

Probability Distribution Functions

Equations 6 through 12 incorporate the probability distribution functions $\Psi$ and $\Xi$ of the two parameter vectors $\psi$ and $\chi$. It has been assumed that the earthquake vector $\psi = \psi(\alpha)$ has uncertainty only in the peak ground acceleration. If for a given site one accepts that there exists a low PGA value $a_0$ below which the earthquakes for the site are insignificant and that the occurring earthquakes constitute a Poisson process with an average rate $\lambda_0$ per year, for a time interval $(0, T)$ (where $T$ could be the lifetime of the
structure) the significant earthquakes can be deduced from

\[ P(N_T = n) = \frac{(\lambda T)^n}{n!} e^{-\lambda T} \quad (13) \]

The distribution of the peak accelerations of Eqn (13) can take the following exponential form

\[ \Psi_{PGA}(\alpha) = 1 - e^{-\beta(\alpha - \alpha_0)} \quad \alpha \geq \alpha_0 \]

\[ = 0 \quad \alpha < \alpha_0 \]

The parameter \( \beta \) is site specific and it ranges between 1.5 and 2.3. The smaller the value of \( \beta \) the higher the frequency of larger earthquakes.

The uncertainty in the soil strength vector is summarized in the SPT N-value for the specific depth where liquefaction susceptibility is sought. On the basis of in-situ SPT testing certain distribution functions can be deduced. For a given site where \( n \) borings have been performed with variability of blow counts in the depth as well as variability between their individual profiles the following assumptions can be made: (a) there exists a normal distribution with mean and variance approaching that of all the data in the n-borings and it represents the entire site. (b) at different depths \( h \) a normal distribution about the mean value of \( \bar{N}(h) \) also exists along with a standard deviation \( \sigma_N(h) \). Given that there exists great variability of the soil profile in actual sites, it will be a hopeless task to attempt and deduce a continuous probability distribution as a function of depth. Instead, the distribution and its properties mentioned above (formed from actual boring test data) will represent the particular depth. If one is interested in the probability of liquefaction failure at a specific depth where no data are available, the distribution of the entire site can be used. Thus, the random N value at the selected location will be a member of the normal distribution with an expected mean and standard deviation.

Monte Carlo Simulations

Through this approach a continuous probability of liquefaction failure will be deduced by relying on deterministic assessment for any individual realization of the involved stochastic processes (randomness of the induced ground acceleration at any level of peak acceleration and randomness of the soil strength parameter N).

The methodology that will lead to the evaluation of the probability integrals (Eqns 6,10) is presented below. The ultimate goal is to determine the probability that the site will liquefy at any depth during its lifetime. In the process, however, the annual probability of liquefaction at any depth conditional to the occurrence of an earthquake as well as the probability of liquefaction at various depths as a function of peak ground acceleration can be estimated.

Prior to entering a double loop in which the soil strength parameter and the earthquake temporal variation are randomly selected, the effective overburden stress \( \sigma' \), the unit response of the soil domain \( H(\omega) \) and the distribution parameters \( \bar{N}(h_i) \) and \( \sigma_N(h_i) \) are evaluated. With this information at hand the flow chart of the process will be as follows:

**External Loop**

- The range of depth \( h_i \) of interest starting from \( h_0 \) and extending to \( h_{max} \) such that \( h_i = i\Delta h \) is selected.
- At each increment characterized by \( \bar{N}(h_i) \) and \( \sigma(h_i) \) \( n \) values of \( N \) are randomly selected from the governing normal distribution. This is achieved by generating random normal (Gaussian) deviates from a distribution with zero mean and unit variance. When the deviate (multiplied by \( \sigma_N(h_i) \)) is superimposed on the mean \( \bar{N}(h_i) \) a random soil strength parameter is \( N^* \) deduced and with this value the inner loop is entered.

**Inner Loop**

- The loop is performed over the range of peak acceleration anticipated for the site \( a_0 \) to \( a_{max} \) with increments of \( \Delta a \). At each increment the probability distribution \( \Psi(a_j) \) is computed \( a_j = j\Delta a \). The peak acceleration \( a_j \) will reflect on the power spectral density function of Eqn (4).
- With a power spectrum \( S(\omega, \lambda) \) but for \( M \) realizations of the random phase angle \( \phi_i \), \( M \) synthetic histories of the form in Eqn (5) are deduced and through Eqn (3) the response of the domain is calculated. For each synthetic history of peak acceleration \( a_j \) the temporal variation of the induced shear stress and the liquefaction damage index are evaluated. The criterion for failure during any such simulated event is that the damage index is greater than 1.0.
- The probability of failure at any \( a_i \), conditional to a strength value \( N^*(h_i) \) can be computed in the following sense

\[ P [SF(h_i) < 0 || N = N^* || pga = a] = \frac{M_f}{M} \quad (14) \]
where, \( M_f \) are the number of simulated earthquakes at \( PGA = \alpha \) that induced failure and \( M \) is the total number of simulations.

- Repetition of the process for the selected PGA range \((\alpha_0, ..., \alpha_{max})\) a fragility curve can be developed reflecting the susceptibility of the soil to liquefaction provided that the strength of the soil is \( \eta(h_i) \). The probability that \( SF_{LQF}(h_i, ||N^*||) < 0 \), conditional to the occurrence of an earthquake is

\[
P[SF_{LQF}(h_i, ||N^*||) < 0||\text{quake}] = \int_{\alpha_0}^{\alpha_{max}} P[SF_{LQF}(h_i, N^*) < 0||\alpha]d\Psi(\alpha)
\]

The probability of failure for the lifetime \( T \) of the dam

\[
P_T(h_i, N^*) = 1 - \exp(-\lambda_0 T P[SF_{LQF}(h_i, N^*) < 0||\text{quake}])
\]

- The probability of liquefaction failure at depth \( h_i \) at which the strength parameter is normally distributed about the mean \( \eta(h_i) \) and with \( \sigma_N(h_i) \) can be written

\[
P[g < 0||h = h_i] = \sum_n P[g(N, \alpha) < 0||N = N^*] \Xi(N^*, \eta, \sigma)
\]

where \( \Xi \) is the normal distribution density function of \( N \) at depth \( h_i \).

- Since we have discretized the distribution function of the soil strength parameter with the depth and since, for this stage, no other strength parameter that has a depth-dependent distribution is involved, the probability of liquefaction failure in the soil under consideration is simply the maximum of the probabilities calculated at each depth, hence

\[
P[g < 0] = \max (P[g < 0||h = h_i])
\]

The above steps have been incorporated in the post-processing features of the POROSLM code. The stress temporal variation is computed according to the procedures of section 2 and utilized in the statistical evaluation.

**Stochastic Approach**

In an effort to avoid Monte Carlo simulations, which are very time consuming, a purely stochastic methodology has been devised to address the probability of liquefaction at a site where some actual soil strength data are available. As in the previous approach it will be assumed that the induced stresses from either random or deterministic earthquakes and the soil strength are deterministically or statistically independent.

Consider the case when the excitation is a earthquake that belongs to a weakly stationary process \( a(t) \) of duration \( T \), zero mean and autocorrelation and power spectral density functions given by the relations,

\[
E[a(t)] = 0 \quad E[a(t + \tau)a(t)] = R_{aa}(\tau)
\]

\[
\Phi_a = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{aa}(\tau)e^{-i\omega \tau}d\tau
\]

\[
R_{aa}(\tau) = \int_{-\infty}^{\infty} \Phi_a(\omega)e^{i\omega \tau}d\omega
\]

The synthesized time history

\[
g(t) = 2 \sum_{i=1}^{N} \sqrt{\Phi_a(\omega_i)} \Delta \omega \cos(\omega_i t + \phi_i)
\]

can be assumed to represent \( a(t) \) as \( N \rightarrow \infty \) and has both the mean and the autocorrelation of the stochastic process \( a(t) \).

With Complex Unit Response Function of the multi-degree of freedom system \( H(\omega) \) available for either unit base excitation or unit harmonic load, the following powerful relation holds,

\[
\Phi_{out}(\omega) = H(\omega) \Phi_{inp}(\omega) H^T(\omega)
\]  

where \( \Phi_{inp} \) and \( \Phi_{out} \) are the Power Spectral Densities of the input and output respectively. Provided that the excitation is stationary, the response is also stationary and all the mathematical relations pertaining to a stochastic process, which satisfies that criterion, hold. These statistical properties of the response process are listed below

\[
\sigma_Y^2 = \int_0^\infty \Phi_Y(\omega) d\omega
\]

\[
\lambda_Y^i = \int_0^\infty \omega^i \Phi_Y(\omega) d\omega, \quad i = 1, 2, ...
\]

By utilizing the statistical properties of Equation (15) the probability density function of the induced shear stress at a location can be evaluated. If the
random stress $\tau$ is a narrow-band (lightly damped) Gaussian stationary random process with zero mean, the distribution of the peak magnitude in the time history realization is of the Rayleigh type,

$$p_{\tau}(\tau) = \frac{\tau}{\sigma_{\tau}^2} \exp\left(-\frac{\tau^2}{2\sigma_{\tau}^2}\right) \quad 0 \leq \tau < \infty$$

The expected equivalent cycles per unit time

$$N_{\text{eqv}} = \frac{\omega_{\text{eqv}}}{2\pi} \quad \omega_{\text{eqv}} = \sqrt{\frac{\lambda^2_{\tau}}{\sigma_{\tau}^2}}$$

(17)

so the total number of cycles for the duration $T$ of the event are equivalently

$$N = \frac{\omega_{\text{eqv}} T}{2\pi}$$

and the number of cycles at a shear stress level $\tau_i$

$$N(\tau_i) = \frac{\omega_{\text{eqv}} T}{2\pi} p(\tau_i)$$

(18)

The random accumulated damage per unit time caused by a random earthquake with peak acceleration $pga = \alpha$ is

$$\langle D(t) \|_{pga=\alpha} \rangle = \int_0^\infty \frac{N(\tau) \|_{pga=\alpha}}{N_{\text{req}}(\tau)} d\tau$$

(19)

where $N_{\text{req}}(\tau_i)$ is the number of stress cycles that can lead to failure at the stress level $\tau$.

In order to incorporate the uncertainties in the soil strength the randomness of the strength parameter $N_{\text{req}}$ is considered. If the soil strength against liquefaction is considered to be solely a function of the random relative density $D_r$ with a probability density $p_{D_r}$ (in the previous section it has been assumed Gaussian), then the required cycles to liquefaction failure will also be random and can be written as $N_{\text{req}} = N_{\text{req}}(\tau, D_r)$. Consequently, the damage conditional to the occurrence of an earthquake of peak acceleration $\alpha$

$$\langle D(t) \|_{pga=\alpha} \rangle = \int_0^\infty \left[ \int_{D_r} N(\tau) \|_{pga=\alpha} \frac{1}{N_{\text{req}}(\tau, D_r)} p_{D_r} dD_r \right] d\tau$$

(20)

The random accumulated damage for any earthquake anticipated in the site can be expressed as follows,

$$\langle D(t) \|_{\text{quake}} \rangle = \int_0^{\sigma_{\text{max}}} (D(t) \|_{pga=\alpha}) d\Psi$$

(21)

4. References


6. POROSLAM. Two-Dimensional Dynamic Solution of Elastic Saturated Porous Media, N. Simos, C.J. Costantino, C. Miller, Earthquake Research center, City Univ. of New York.


Acknowledgments

This work was performed under the auspices of DOE