VIRTUAL COMPTON SCATTERING ON THE PROTON
AT HIGH $s$ AND LOW $t$ $^a$

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Virtual Compton Scattering (VCS) at low transferred momenta to the proton ($t$) and sufficiently high c.m. energies ($s$) may be used to a) study $Q^2$-dependence of leading $t$-channel exchanges and b) look for onset of scaling behavior with increasing $Q^2$. I discuss the implications for perturbative and nonperturbative QCD and suggest possible experiments.

Introduction

Analysing various kinematic domains of Virtual Compton Scattering (VCS), one may obtain clues to various problems of strong interaction dynamics.

If $s \gg -t$, and $Q^2$ is fixed, the amplitude of VCS may be described (Fig.1a) by the sum of $t$-channel Reggeon ($R$) exchanges,$^{1}$

$$M = \sum_R s^{\alpha_R(t)} \beta_R(t, Q^2)e^{-\frac{i}{2}\pi \alpha_R(t)}, \quad (1)$$

where the sum is taken over all possible Reggeons with positive charge parity, i.e., Pomeron ($\alpha(0) \approx 1$), $f-a_2$ ($\alpha(0) \approx 1/2$) and pion ($\alpha(0) \approx 0$) trajectories. It implies that at asymptotically high energies ($s \rightarrow \infty$), the energy behavior of Compton amplitude is governed by the Pomeron exchange.

However, as I demonstrate below, if $s$ and $Q^2$ are in the range of a few (GeV/c)$^2$, and $t \approx -m^2$, where $m$ is a pion mass, a relative contribution from the $\pi^0$–exchange becomes large, exceeding even the diffractive (Pomeron) contribution at low $t$. Taking advantage of different quantum numbers and phases of $t$-channel exchanges, it is possible to separate these contributions$^2$ and measure the form factor of $\gamma^*\pi^0 \to \gamma$ transition ($F_{\gamma^*\gamma\pi^0}(Q^2)$) as a function of $Q^2$.

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Figure 1: a) VCS amplitude in terms of $t$-channel Reggeon exchanges, where $R$ stands for the Pomeron, $f - a_2$, and pion trajectories, and the dashed blob denotes the two-photon transition form factor; b) Quark triangle diagram for $\gamma^* M \rightarrow \gamma$ transition, where $M$ is a pseudoscalar meson.

Theoretically, the leading-twist QCD contribution to the $F_{\gamma^*\gamma^0\pi^0}$ transition form factor is given by the quark triangle diagram (Fig.1b) with no hard gluon exchange making this process 0th-order in QCD running coupling constant $\alpha_s$ with non-perturbative dynamics being contained in the pion distribution amplitude $\varphi_\pi(x).$ The function $\varphi_\pi(x)$ cannot be predicted by perturbative QCD (except for it asymptotic behavior) and it is crucial for understanding whether or not one may apply a perturbative QCD description to exclusive processes at given energies. The data on $F_{\gamma^*\gamma^0\pi^0}(Q^2)$ obtained at $e^+e^-$ colliders currently available for $Q^2$ up to 8 (GeV/c)$^2$ indicate that $\varphi_\pi(x)$ is close to its asymptotic form, and therefore ‘soft’, nonperturbative mechanisms are dominant in this energy range, in agreement with theoretical predictions based on QCD Sum Rules.

This conclusion is so important that it is desirable to have an independent measurement of $F_{\gamma^*\gamma^0\pi^0}(Q^2)$, and such a measurement via VCS was proposed earlier at Jefferson Lab$^6$ for transferred momenta $Q^2 = 1.0 \div 3.5$ (GeV/c)$^2$. Higher values of $Q^2$ are also possible for the Jefferson Lab energy upgrade.

As $Q^2$ increases, and $s$ stays large, in the low-$t$ limit one may observe scaling behavior of the VCS amplitude predicted recently by X. Ji$^8$ and A. Radyushkin.$^9$

**Transition Form Factors of Leading $t$-channel Exchanges**

Regge phenomenology proved very successful at describing energy dependence of hadronic total cross sections and differential cross-sections at low $t$ and high $s$. For VCS, it predicts the slope of the Regge trajectory $\alpha_R(t)$ to be independent of the photon virtuality $Q^2$. This is an important result of Regge theory which needs to be tested experimentally, as was indicated earlier (see,
As far as form factors of $\gamma^*\text{Reggeon} \rightarrow \gamma$ transitions are concerned, one needs a microscopic theory of Reggeon exchanges to be able to predict their $Q^2$ dependence. This task is challenging for the ‘soft’ Pomeron exchange limited to a few GeV energy scale, and this problem is still far from being solved in QCD.

However, QCD predictions for the two–photon transition form factors related to the $t$–channel of VCS are available for the case of $\pi^0$ exchange. For the review of theoretical approaches, see Ref. 1. I would also like to mention here effective quark models for $F_{\gamma^*\gamma\pi^0}$ based on the extended NJL model $^{11}$ and the model of dynamical dressing of propagators and vertices $^{12}$, the latter also addressed the off-shell behavior of $F_{\gamma^*\gamma\pi^0}$ for the proposed Jefferson Lab experiment. $^{3}$

The lightest meson which can be exchanged in the $t$–channel of VCS is $\pi^0$. It may be possible to extract the corresponding form factor doing a Chew–Low extrapolation like for the charged pion form factor measurements $^{4,5}$ scheduled at Jefferson Lab.

**Exchange of $\pi^0$**

In the real Compton scattering on the proton at high $s$ and low $t$, the main contribution to the cross section is known to be diffractive, due to the Pomeron exchange. I will demonstrate here that the situation is different for the case of virtual photons, because of strong enhancement of the $\pi^0$–exchange term.

At small $t$, the $\pi$–trajectory is determined by the pion Born term. The matrix element of the corresponding transition is

$$\mathcal{M}^{(\pi^0)}_{\gamma p \rightarrow \gamma p} = e^2 F_{\gamma^*\gamma\pi^0}(Q^2) g_{\pi NN} F_{\pi NN}(t) D_{\pi}(t) \epsilon_{\mu \nu \alpha} \bar{\epsilon}_\alpha \epsilon_\nu q_\alpha q'_\beta \bar{u}' \gamma_5 u,$$  \hspace{1cm} (2)

where $\epsilon(\epsilon')$ is the polarization 4–vector of initial (final) photon, and $q(q')$ is its momentum ($Q^2 = -q^2$), and $u(u')$ is a bispinor of the initial (final) proton. The pion propagator has the form $iD_{\pi}(t) = (t - m_{\pi}^2)^{-1}$, and I also assumed a conventional monopole form for the cut–off form factor $F_{\pi NN}(t) = \Lambda^2/(\Lambda^2 - t)$.

Define four coincidence structure functions (SF) for the (unpolarized) $p(e,e'\gamma)p$ cross section as

$$\frac{d^5\sigma}{dE'd\Omega_e d\Omega_p} = \frac{\alpha^3}{16\pi^3} \frac{E'}{E} \frac{|p|_{c.m.}}{mW} \frac{1}{Q^2} \frac{1}{1-\epsilon} \left[ \sigma_T + \epsilon \sigma_L + \epsilon \cos(2\varphi)\sigma_{LT} \right] \frac{1}{\cos^2 \theta_e \tan \theta_e},$$  \hspace{1cm} (3)

and

$$\epsilon^{-1} = 1 - 2 \frac{q_\alpha q'_\beta}{q^2} \tan \theta_e.$$  \hspace{1cm} (4)
where $E(E')$ is the initial (final) electron energy, $\theta_e$ is the electron scattering angle, $\varphi$ is the azimuthal angle, and $m$ is the proton mass. The matrix element given by eq.(2) yields the following contributions to SF:

\[
\sigma_T^{(\pi^0)} = [ (|q| - q_0 \cos \theta)^2 + (|q| \cos \theta - q_0)^2 ] X, \\
\sigma_L^{(\pi^0)} = 2Q^2 \sin^2(\theta)X, \\
\sigma_{LT}^{(\pi^0)} = 2\sqrt{Q^2} (|q| - q_0 \cos \theta) \sin(\theta)X, \\
\sigma_{TT}^{(\pi^0)} = Q^2 \sin^2(\theta)X, \\
X = \frac{-t}{(t - m^2)^2}[q_0' F_{\pi NN}(t) g_{\pi NN} F_{\gamma^* \gamma^0}(Q^2)]^2, 
\]

where the c.m. energy of the (initial) final photon is given by $q_0 = \frac{E^2 + W^2 - m^2}{2W}$ ($q_0' = \frac{W^2 - m^2}{2W}$), and $\theta$ is the c.m. angle of outgoing photon.

The $\pi^0$ pole contributes the most to transverse photoabsorption in VCS; the corresponding SF is shown in Fig. 2. Contributions to the other structure functions are suppressed at small $t$ by the factor of $t$ for $\sigma_{LT}$ and $t^2$ for $\sigma_L$ and $\sigma_{TT}$. The overall factor $-t/(t - m^2)^2$ from Eq.(9) has a pole in the unphysical region, $t = m^2$, turns to zero at $t = 0$, and has a maximum in the physical region at $t = -m^2$. As can be seen from Fig. 2, dependence of the $\pi^0$ contribution on $\theta$ changes dramatically when going from real to virtual photons. When $Q^2 = 0$, it is suppressed at forward angles (i.e., low $t$); in contrast, for $Q^2 \neq 0$, it is peaked at forward angles. This result is due to the Lorentz structure of the $\gamma^*\gamma^0$-vertex defined by Eq.(2): $\epsilon_{\mu\nu\alpha\beta} \epsilon_{\mu'\nu'\alpha'\beta'} q_0 q_0'$.

Enhancement of $\pi^0$ exchange makes it large enough to reach, and even exceed, the magnitude of diffractive term. For instance, assuming the VMD–model for the $\gamma^*Pomeron \rightarrow \gamma$ transition form factor, $\pi^0$ contribution to $\sigma_T$ is evaluated to be 25% higher than from the Pomeron at $Q^2 = 2$ (GeV/c)$^2$ and, respectively, three times higher at $Q^2 = 3$ (GeV/c)$^2$. (This result is obtained at $W = 2.5$ GeV, $t = -m^2$.)

It creates favorable conditions for extracting the form factor $F_{\gamma^* \gamma^0}$ from VCS experiments.

**Separation of the $t$–channel exchanges**

One can attempt to disentangle various $t$–channel exchanges in VCS.

Indeed, the Pomeron a) has quantum numbers of vacuum (except for its spin), b) contributes almost purely to the imaginary part of VCS amplitude at low $t$, and c) does not flip the nucleon spin. On the other hand, the pion a)
Figure 2: Contribution of the $\pi^0$-exchange Born diagram to the (dimensionless) structure function $\sigma_T$ of transverse virtual photoabsorption; the invariant mass of final $\gamma p$ state is taken $W = 2.5$ GeV.

is isovector and pseudoscalar b) contributes to the real part of VCS amplitude at low $t$, and c) flips the nucleon spin.

These circumstances may be used to design the experiments in order to separate these mechanisms, especially if nuclear targets are used. For instance, coherent VCS on $^4$He excludes pseudoscalar $t$-channel exchanges ($\pi^0, \eta, \eta'$), thus providing useful information on $Q^2$ evolution of the diffractive (Pomeron) term. Coherent VCS on deuterium target would rule out the $\pi^0$-exchange, but keep exchange of other pseudoscalars with zero isospin. On the other hand, if the hadronic target undergoes an isovector transition of any kind (e.g. threshold deuteron dissociation), it would be completely due to the $\pi^0$ exchange.

In addition, both diffractive and pseudoscalar $t$-channel exchanges are strongly suppressed in interference between VCS and the Bethe–Heitler amplitudes for the case of unpolarized particles. If, however, spin effects are included, the asymmetry and/or recoil polarization due to the proton polarized normal to the reaction plane would be caused by interference between diffractive and $\pi^0$ terms (in which case the electron beam polarization is not required), while the sideways (in-plane) asymmetry/polarization with longitudinally polarized electrons would be mainly caused by interference between $\pi^0$-exchange and the Bethe–Heitler amplitudes.

Summary

- Exchanges of $\pi^0$ and the Pomeron in $t$-channel provide the largest contributions to the amplitude of VCS on the proton at small $t$ and $s$ in the region of
a few GeV$^2$.

- It is possible to separate these contributions and study $Q^2$–dependence of the corresponding form factors.
- For the form factor $F_{\gamma\gamma\pi^0}$, it gives information about the pion distribution amplitude and QCD corrections. It also discriminates between predictions of effective quark models.
- Further increasing energies and momentum transfers, and keeping $t$ small, one may observe a transition to the scaling limit of VCS predicted and studied theoretically by X. Ji$^8$ and A. Radyushkin$^9$.

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References