UNIVERSITY OF WISCONSIN
CENTER FOR PLASMA THEORY AND COMPUTATION
REPORT

A Study of Tearing Modes Via Electron Cyclotron
Emission From Tokamak Plasmas

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**A STUDY OF TEARING MODES VIA ELECTRON
CYCLOTRON EMISSION FROM TOKAMAK PLASMAS**

by

Chuang Ren

A dissertation submitted in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

(Physics)

at the

UNIVERSITY OF WISCONSIN - MADISON

1998

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Abstract

A STUDY OF TEARING MODES VIA ELECTRON CYCLOTRON EMISSION FROM TOKAMAK PLASMAS

Chuang Ren

Under the supervision of Professor James D. Callen
at the University of Wisconsin-Madison

The tearing mode is an important resistive magnetohydrodynamic (MHD) mode. The magnetic island structure which it nonlinearly produces can degrade the plasma confinement and reduce the number of fusion reactions. This thesis studies several tearing mode problems from both theoretical and experimental points of view.

A major part of this thesis is to demonstrate that Electron Cyclotron Emission (ECE) is an excellent diagnostic for studying an MHD mode structure and its properties in a tokamak plasma. It is shown that an MHD mode can be detected from the electron temperature fluctuations measured by ECE. The amplitude and phase profiles of the fluctuations contain detailed information about the mode structure.

The ECE fluctuation phase profile indicates the magnetic island deformation due to the combination of sheared flow and viscosity. A model is presented to relate qual-

itatively the observed phase gradient to the local magnetic field, flow velocity shear and viscosity in a 2D slab geometry, using an ideal Ohm's law and the plasma momentum equation including flow and viscosity. Numerical solution of the resultant Grad-Shafranov-like equation describing the deformed island shows that the experimentally observed value of the phase gradient can be obtained under realistic parameters for the shear in the flow velocity and viscosity.

The tearing mode stability parameter, Δ' , can be measured from the ECE fluctuation amplitude profile [Ren *et al.*, Phys. Plasmas **5**, 450 (1998)]. A fit function which has Δ' as one of the fitting parameters is derived for the ECE fluctuation amplitude profile. The theoretical function is then fitted to the experimental data from the Tokamak Test Fusion Reactor (TFTR). The obtained values of Δ' for 3/2 and 4/3 modes are clearly negative. These results represent the first direct determination of this key tearing mode parameter from internal plasma diagnostics, and support the previous findings that these modes are neoclassical tearing modes.

A new approach to the tearing mode stability boundary and saturation level is also presented. It uses the "almost ideal MHD" constraint, which is a relaxation of the ideal MHD frozen-flux constraint to allow changes in the magnetic field topology [Ren *et al.*, Phys. Plasmas **5**, 2574 (1998)]. The numerical results show that the obtained linear stability boundary is the same as that obtained from Δ' -theory [Furth *et al.*, Phys. Fluids **6**, 459 (1963)] and the saturation level is smaller than that of White *et al.* [Phys. Fluids **20**, 800 (1977)].

Acknowledgements

During my graduate study, I have been helped by many people. To express my deep gratitude to some of them here is the more enjoyable aspect of my thesis writing.

My deepest thanks go to my adviser Jim Callen. His knowledge and enthusiasm in plasma physics and fusion research are always sources of inspiration to me. The choice of my thesis topic is influenced by Jim's inclination of applying theoretical concepts to the experiments and I have really enjoyed working with experimentalists. He guides his students in a gentle but thoughtful way. Even with his busy schedule, he is still fully aware of their needs and gives effective help. He also provides the students opportunities to visit other fusion research centers and collaborate with the people there, thus allowing us a more comprehensive education. I was very lucky to be Jim's student.

I'd like to thank Chris Hegna, whom I always run to for help when Jim is not around. Chris taught me asymptotic analysis (among other things), the "bread and butter" of a plasma theorist.

The members of my oral exam committee—Prof. Cary Forest, Prof. Noah Herskovitz, Prof. Paul Terry, and Prof. Stewart Prager—have given good suggestions on how to improve this thesis, which I appreciate very much.

I'd like to thank my fellow graduate students at the University of Wisconsin: Kate Comer, Tom Gianakon, Eric Held, Scott Kruger, Yas Nishimura and Xin Wang in

Engineering Physics; and Jay Anderson, Ted Biewer, Brett Chapman, Darren Craig, Eduardo Fernandez, Paul Fontana, Kevin Mirus, Matt Stoneking and other students in Physics. They are the first friends I have made in this country. I can't count the number of things I learned from them, scientific or cultural. They definitely have made my new life in a foreign country rich and enjoyable.

My thesis work began from a summer visit to Princeton Plasma Physics Laboratory in 1994. I'd like to thank Zuoyang Chang, who spent a lot of his time, then and thereafter, in preparing the ECE and other data for my analysis. Without Zuoyang's help, I don't know that I could have finished this work. I also want to thank Eric Friedrickson and Mike Zarnstorff, without whose codes my work would have been a lot more difficult.

Since June, 1996 I have been working at the General Atomics (GA), where I have completed a significant part of this thesis and written the thesis. I have had the privilege of working with many nice people, among them Max Austin, Rob La Haye and Ted Strait on ECE signal analyzing; Ming Chu on island deformation due to sheared flow and viscosity; and Torkil Jensen on the nonlinear tearing mode study. Ming and Torkil also kindly encouraged me to write our collaborative research results into papers.

As a Chinese citizen, I have had to be escorted in and out of GA during the first one and a half years. Many people, especially Ming, Rob, Torkil and Alan Turnbull, took their time to escort me. Torkil, in his semi-retirement, sometimes rearranged his schedule just for my coming in and out of the company. During our many walks, Torkil

and I talk about everything and he often gives fatherly advice to me on many things. This almost makes me feel grateful for the escort policy. I also want to thank Tony Taylor for his continuous efforts and ultimate success in changing this policy.

I'd like thank our secretaries, Barb Griffith of Engineering Physics, Kay Shatrawka of Physics, and Sumie Kobayashi of GA for their kind help through all these years.

I also want to thank my friends at UCSD, Dezhe Jin and Bo Yang, for helping me borrow books from the library and other things while I have been away from my own campus.

Finally, I want to thank my family. Without the financial assistance from my brother and parents, I would not have been able to come to the U.S. to study. I am also ever grateful to my wife's understanding and strong support for my pursuing a career in science through all the years since we met.

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Chapter 1

Introduction

Magnetohydrodynamics (MHD) [1, 2] is a self-consistent fluid model for studying the macroscopic properties of a plasma, which is an ionized gas. Compared to the more complete kinetic model [3], MHD is simpler and more tractable. Nevertheless, it can still quite accurately describe the long-wavelength, low-frequency plasma behavior. It has wide applications in astrophysics, plasma processing and controlled fusion research.

The goal of magnetic fusion research is to find a magnetic field configuration that can confine a plasma of sufficiently high temperature and density long enough to produce useful energy from the fusion reaction. A magnetic confinement system usually consists of nested toroidal magnetic surfaces, on which the magnetic field lines lie. The magnetic confinement system must be in an MHD equilibrium state where the pressure force of the hot plasma is balanced by the magnetic force. In addition, a successful system must also be stable. Any small perturbation should not drive the system further away from the equilibrium. Among the various instabilities that a plasma can develop, MHD instabilities are the most dangerous ones. They grow rapidly and can degrade the confinement or even disrupt the whole plasma discharge.

Basically, there are two approaches to the study of the stability problem using the MHD equations [1, 2]. One is the initial value approach. In this approach, a

particular small perturbation is introduced into the equilibrium to be studied and the MHD equations are used to advance the plasma state. If the perturbation grows larger and larger as time evolves, then the equilibrium is said to be unstable. Usually a numerical code has to be run to advance the MHD equations.

Another approach is the eigenvalue approach. Since the equilibrium is by definition time-independent, any perturbed quantity can be written in the form of $f(\mathbf{x}) \exp(i\omega t)$. Putting the perturbations of this form into the MHD equations and using the proper linearization, we obtain a set of eigenequations for the perturbations with their frequency, ω , as the eigenvalue. With the proper boundary conditions, the eigenequations can be solved, either numerically or analytically, and the eigenfrequency ω can be determined. If the imaginary part of a certain eigenfrequency is negative, then the eigenmode associated with this frequency will grow exponentially and is therefore an unstable mode.

When solving the eigenequations for nonideal MHD instabilities (and/or ideal MHD instabilities with small growth rate), a method of asymptotic matching is often used [2, 4]. The plasma is divided into two regions: inner and outer regions. In the outer region, which comprises most of the plasma, the effect of inertia is negligible. Furthermore, nonideal effects, such as resistivity and viscosity, and the nonlinear effect can also be neglected. The mode structure of these MHD instabilities in the outer region can be reasonably well described by the linearized, inertia-free ideal MHD equations. Generally, these equations become singular at the so-called rational surfaces, the equilibrium magnetic surfaces on which the magnetic field lines close on themselves after a finite

number of toroidal transits. In the inner region—the thin layers around these rational surfaces—the equations for the outer region are no longer appropriate and various previously neglected terms have to be taken into account for various modes, e.g. inertia for the ideal interchange mode [2], inertia and resistivity for the linear tearing mode [5], and resistivity and nonlinearity for the nonlinear tearing mode [6]. Eventually, the solutions of the outer and inner equations have to be matched asymptotically to determine the eigenfrequency and thus the stability of the mode.

In the asymptotic matching procedure, both the inner and outer solutions are expanded into series and the coefficients of the corresponding terms are matched. The coefficients of the inner solution are usually functions of ω . The coefficients of the outer solution, determined by the mode structure and the boundary conditions, are related to the energy available to the mode and generally determine whether the mode will grow or decay. Thus, it is very important to determine the behavior of the outer solution near the rational surface.

Because of its importance in stability studies, much effort has been devoted to the theoretical study of the outer solution, especially its behavior near the rational surface. The behavior of the outer solution depends on the equilibrium pressure and current profile. Given certain forms of the equilibrium pressure and current profiles, the outer equation can be solved analytically. The equilibrium pressure and current profile can also be expanded around the rational surface and the matching parameters—the asymptotic expansion of the coefficients of the outer solution near the rational surface—

can be obtained analytically [7, 8]. For more realistic pressure and current profiles and geometry, numerical codes can be used to obtain the outer solution [9, 10]. All these results provide useful insights into the various instability modes observed in the experiments.

While much work has been done on calculating the matching parameters from the outer solution, there is not much, if any, direct and quantitative comparison between theoretical and experimental results. This is due to the difficulty of accurately measuring the matching parameters experimentally. The matching parameters depend on the spatial variation of the perturbed magnetic field. It is extremely difficult to directly measure the local magnetic field with high spatial and temporal resolution. However, due to the fact that the particle diffusivity and the heat conductivity are far greater in the direction parallel to the magnetic field lines than in the direction perpendicular to them, the density and temperature of both the electrons and ions are practically constant within the magnetic surfaces. In other words, the density and temperature are surface functions to a high accuracy. This provides an opportunity to determine the shape of the magnetic surface by measuring the spatial distribution of the density and temperature. There are many diagnostics that can measure the density and temperature with high spatial and temporal resolution, e.g. Electron Cyclotron Emission (ECE) for electron temperature measurement [11] and Beam Emission Spectroscopy (BES) or reflectometry for density measurement [12]. To glean information about the matching parameters from these diagnostics, we can assume some general form param-

eterized by the matching parameters for the outer solution and thus obtain the shape of the resultant magnetic surfaces. Then, by assuming that the temperature or density is a surface function, we can obtain a fit function for the profile of the temperature or density which would be measured by the particular diagnostics. By fitting this function to the actual measured profile, we can determine the matching parameters. Then these experimentally determined matching parameters can be compared to the theoretically calculated ones.

This thesis describes some results in the study of tearing modes—an important class of resistive MHD modes [5]. It can be divided roughly into two parts. One part is about using the ECE diagnostic to study the mode structure. Specifically, a method [13] will be presented for measuring a critical matching parameter, Δ' , which is the stability parameter for the classical tearing mode [5] and also plays an important role in the growth and saturation of the neoclassical tearing mode [19, 21]. Also, the deformation of the mode structure due to sheared flow and viscosity and its manifestation in the ECE data will be presented [14]. The other part is about a new approach, using an “almost ideal MHD” constraint, to study numerically the stability and saturation of the tearing mode [15] in a simplified model.

The remainder of the thesis is organized as follows. Chapter 2 gives an introduction to the tearing modes. Chapter 3 describes the ECE diagnostic and the method used to analyze its signals to identify the tearing mode. Chapter 4 describes the deformation of the magnetic island due to sheared flow and viscosity as observed from the phase profile

of the ECE signals. Chapter 5 describes the derivation of a fit function that relates Δ' to the electron temperature fluctuations recorded by the ECE diagnostic. Chapter 6 shows the result of applying this fit function to the actual ECE data. Chapter 7 presents a new theoretical approach to obtain the stability condition and saturation of the tearing mode. Chapter 8 gives the summary.

Chapter 2

Tearing mode and Δ'

2.1 Classical tearing mode and Δ'

In 1963, Furth, Killeen and Rosenbluth (FKR) published a seminal paper [5] in which the existence of a new class of instabilities—the resistive MHD modes—was demonstrated. These modes are called resistive because they change the topology of the magnetic surfaces, which can only happen in a plasma with finite resistivity. They do not exist in the ideal MHD [1] regime. The most important of these modes is the tearing mode, which is driven by the current gradient. Near a resonant surface where the local magnetic field is perpendicular to its wave vector, the mode breaks the magnetic surfaces and reconnects them to form a so-called magnetic island structure. However, the reconnection only occurs in a narrow region around the resonant surface—the inner region. In the rest of the plasma—the outer region—the mode structure can be determined by the requirement that the plasma is in a force-balanced state even in the presence of the perturbed magnetic field. For a plasma where the pressure force is negligible, the linearized outer equation determining the mode structure can be written as

$$\mathbf{B}_0 \cdot \nabla \frac{j_{||1}}{B} + \mathbf{B}_1 \cdot \nabla \frac{j_{||0}}{B} = 0. \quad (2.1)$$

Here, $\mathbf{B}_0$ and $j_{||0}$ are the equilibrium magnetic field and the current component parallel to the field, respectively; and $\mathbf{B}_1$ and $j_{||1}$ are the corresponding perturbed components. The resonant surface is located where the operator $\mathbf{B}_0 \cdot \nabla$ equals zero. In a cylindrical coordinate system where $\mathbf{B}_0 = B_z \hat{z} + B_\theta(r) \hat{\theta}$ and $\mathbf{B}_1 = \nabla \psi_1 \times \hat{z}$ with $\psi_1 = \psi_1(r) \exp(im\theta - inz/R)$,

$$\mathbf{B}_0 \cdot \nabla = \frac{iB_\theta}{r}(m - nq)$$

with the safety factor $q \equiv rB_z/RB_\theta$. Thus, the resonant surface is at the rational surface, where $q = m/n$. At the resonant surface, Eq. (2.1) is singular and generally the radial derivative of ψ_1 is discontinuous. The singularity is resolved by using a full resistive inner equation that includes the effect of resistivity and inertia in a thin layer around the resonant surface. By matching the solutions of the inner equation and the outer equation, it is found [5] that the stability of the linear tearing mode depends on a parameter Δ' which can be determined from the outer equation (2.1),

$$\Delta' = \left[\left(\frac{d\psi_1}{dr} \right)_+ - \left(\frac{d\psi_1}{dr} \right)_- \right] / \psi_1(r_s). \quad (2.2)$$

Here, r_s is the radial location of the resonant surface and $(d\psi_1/dr)_+$ and $(d\psi_1/dr)_-$ are the derivatives of ψ_1 on the two sides of the rational surface. A positive Δ' means that the tearing mode is unstable.

In 1973, Rutherford published a theory about the growth of the tearing mode in the nonlinear regime where the magnetic island width exceeds the inner layer width [6]. Instead of the exponential growth of the linear regime, the mode grows algebraically

and the evolution equation of the amplitude can be written as

$$\frac{\partial \psi_1}{\partial t} = c \Delta' \sqrt{\psi_1},$$

where c is a positive quantity. This shows that Δ' also determines whether the amplitude would grow or decay in the nonlinear stage of the tearing mode.

White *et al.* [16] developed an analytical theory for the saturation of the tearing mode which uses the concept of nonlinear Δ' . The nonlinear Δ' is an extension of the linear Δ' defined in (2.2). Instead of using the derivatives of ψ_1 at the resonant surface, it uses the derivatives at the island width; i.e.,

$$\Delta'(w) = \left[\left(\frac{d\psi_1}{dr} \right) \right]_{r_s+w/2} - \left(\frac{d\psi_1}{dr} \right) \bigg|_{r_s-w/2} \bigg/ \psi_1(r_s).$$

Here, the island width is defined as $w \equiv 4r_s(B_{1r}L_q/mB_\theta r)^{1/2}$, where B_{1r} is the radial component of the perturbed field and $L_q \equiv q(dq/dr)^{-1}$ is the magnetic shear length. Later, Carreras *et al.* [17] compared this theory to experimental results. Since both the linear and nonlinear theory predict the mode would grow when Δ' is positive, we refer to them as the Δ' theory. In Chapter 7, a new theoretical method to obtain tearing mode stability and saturation using the "almost ideal MHD" constraint will be described.

2.2 Neoclassical tearing mode

While it was successful in interpreting the experiment results in the 1970's [6, 17], the Δ' theory has had difficulty in explaining the observed MHD instabilities in some of the large tokamaks built in the 1980's. In the neutral beam heated supershots in

the Tokamak Fusion Test Reactor (TFTR), tearing-type MHD modes are observed with low mode numbers ($m/n=3/2, 4/3$ and $5/4$) [18]. Theoretical interpretation and numerical simulation of these modes based on the Δ' theory do not yield satisfactory results [24, 25]. It is also found [19] that the Δ' 's for the low-mode-number tearing mode are usually negative, except for the $2/1$ mode which could have a positive Δ' but is less frequently seen in the experiment. Recently, Chang *et al.* [19] have shown that these modes can be quite successfully modeled by the nonlinear neoclassical tearing mode theory [21].

The neoclassical MHD theory takes into account the effect of the bootstrap current which is driven by the anisotropic part of the pressure tensor. The anisotropy of the pressure originates from the collisional viscosity between the trapped and passing particles, and is intrinsically a toroidal effect. The formation of the magnetic island reduces the pressure gradient and thus the bootstrap current. This in turn affects the nonlinear evolution of the island, which can now be expressed as

$$\frac{dw}{dt} = k_1 \frac{\eta}{\mu_0} (\Delta' + k_2 \frac{\sqrt{\epsilon} \beta_p L_q}{w L_p}). \quad (2.3)$$

Here, w is the island width, k_1 and k_2 are two positive numbers, η and μ_0 are the resistivity and vacuum permeability, respectively, ϵ is the inverse aspect ratio of the tokamak, $\beta_p \equiv p/(B_0^2/2\mu_0)$ is the ratio of plasma pressure to the energy density of the poloidal magnetic field, and $L_p \equiv -p(dp/dr)^{-1}$ is the pressure gradient length. This equation shows that the neoclassical term (the k_2 term in Eq. (2.3)) is a driving term for growth of the island in the normal magnetic shear case ($L_q > 0$). A neoclassical

MHD island can grow even when Δ' is negative and saturates when the two terms in Eq. (2.3) cancel each other. The agreement of this model and the experimental results in TFTR is very good [19].

2.3 Importance of determining Δ'

Since it reconnects the otherwise nested and separated magnetic surfaces, the tearing mode can significantly affect the confinement and the overall performance of the plasma. On TFTR, a low-mode-number tearing mode causes degradation in the neutron rate, stored energy and energy confinement time in the supersonic plasma—by tens of per cent [18]. On DIII-D, the $2/1$ tearing mode is usually found to set the "hard" β -limit in the sense that the mode frequently causes a plasma disruption; on the other hand, the $3/2$ mode sets a "soft" β -limit by reducing the plasma pressure without disrupting the plasma [20]. A tearing mode can also be induced by the external error field and reduce the plasma rotation [26]. How to suppress the tearing mode by means such as local current drive [22] or external correction coils [23] has also been an active research topic recently.

Because of the central role played by Δ' in the evolution of the island, how to determine Δ' is an important issue in the study of the tearing mode. In modeling of the neoclassical tearing mode in Ref. [19], the sign of Δ' for the observed mode indicates whether the mode is neoclassical or Δ' -driven and the value of the negative Δ' determines the saturation level of the mode. In Ref. [19], Δ' is obtained by solving

Eq. (2.1) numerically with the needed information about the equilibrium magnetic field coming from the experiment. There are also analytical formulae [7, 8] that can evaluate Δ' for high poloidal mode numbers when the equilibrium magnetic field configuration is given. The analytical result shows that it is the equilibrium current gradient near the resonant surface that determines Δ' .

The equilibrium magnetic field configuration can be represented by the safety factor profile $q(r)$. Since $j_{||0}$ is related to q' , q'' has to be used to solve Eq. (2.1). In Ref. [19], $q(r)$ is obtained from the transport analysis code TRANSP [27]. TRANSP can calculate the plasma parameters that are difficult to measure, like the current profile, using a few assumptions and fitting parameters that can be determined from the experimentally measured quantities like the electron density and temperature profiles. The q -profile thus obtained is not very accurate. It is even more unreliable to take the second derivative of it. A more direct and accurate way of determining q is to use the Motional Stark Effect (MSE) diagnostic [28]. MSE can accurately measure the local q at the discrete MSE channel positions. However, evaluating q'' near the resonant surface from a discrete q measurement still incurs large uncertainties. One of the aims of this thesis is to develop an independent, more reliable and direct way of determining Δ' experimentally from the Electron Cyclotron Emission (ECE) diagnostic.

Chapter 3

ECE and MHD modes

3.1 Detecting MHD modes from ECE signals

In a magnetic field, the electrons gyrate around the magnetic field lines and emit radiation—the so-called Electron Cyclotron Emission (ECE)—at frequencies near harmonics of the electron cyclotron frequency $f_c = eB/2\pi m_e$, where m_e and e are the mass and charge of the electron, respectively, and B is the local magnetic field. In most modern fusion devices the electron density is high enough for the plasma to be optically thick to the radiation, except at the very edge of the plasma. Thus, the radiation achieves thermal equilibrium with the electrons and its intensity is a measure of the local electron temperature [11]. The dominant toroidal magnetic field in a tokamak has a $1/R$ dependence where R is the major radius of the device. This enables the ECE diagnostic to measure the electron temperature as a function of R with a horizontal viewing geometry (Fig. 3.1). The diagnostic usually consists of multiple channels set to receive signals of different frequencies originating from different major radii. The exact position for each channel depends on the total magnetic field strength in the presence of the plasma and can be determined from equilibrium codes like EFIT [29].

The plasma in a tokamak usually rotates toroidally due to direct input of toroidal momentum from sources like the neutral beam injection. The poloidal flow is damped

Figure 3.1: The viewing geometry of a typical ECE diagnostic

[30]. As the plasma rotates, an ECE diagnostic at a particular toroidal location actually can record the temperature from different toroidal parts of the plasma. If the magnetic surface is symmetric in the toroidal direction (axisymmetric), as it should be in the unperturbed equilibrium state, all the points with the same major radius R and vertical coordinate Z (but different toroidal angle ξ) belong to the same magnetic surface. Due to the free streaming of electrons along the magnetic field lines, the electron temperature is constant on a magnetic flux surface and is a surface function. Therefore, the temperature recorded by any ECE channel of a fixed receiving frequency would be constant for a rotating plasma without any MHD mode (Fig. 3.2(a)).

In the presence of an MHD mode, the toroidal symmetry of the magnetic surface is broken and the magnetic surface expands and shrinks periodically along the toroidal

direction (Fig. 3.3). The surface is also distorted periodically in the poloidal direction; however, since the plasma rotates mainly in the toroidal direction, it is the toroidal distortion that is significant for the ECE diagnostic. Because of the distortion, an ECE channel fixed to receive the signal from a particular R would only record the temperature from different magnetic surfaces as the plasma rotates by. Since the electron temperature differs from surface to surface, the ECE signal displays a periodic fluctuation in time (Fig. 3.2(b)). The amplitude of the fluctuation is proportional to the radial shift of the magnetic surface.

To determine whether an observed MHD mode is a tearing mode or not, it is important to know the relative phase between the temperature fluctuations on opposite sides of the mode resonant surface. Since the tearing mode reconnects the magnetic surfaces on the two sides of the resonant surface and the temperature decreases monotonically from the plasma core to edge, a distinctive feature of the tearing mode is the out-of-phase temperature fluctuations of the ECE channels on the two sides of the rational surface. This can be readily understood from Fig. 3.3. In Fig. 3.3, the effect of the plasma rotation can be represented by the movement of the ECE channels in the toroidal (ξ) direction. In going from A to B to C, the ECE channel inside (left of) the resonant surface would see the temperature first decrease and then increase. In the meantime, the ECE channel outside (right of) the resonant surface would see the temperature first increase and then decrease in going from A' to B' to C'. This so-called “ π -phase jump” is an important signature of the tearing mode (or really magnetic island

from reconnected flux surfaces) revealed in the ECE signal. In principle, an ideal MHD mode could have the odd symmetry to produce a "π-phase jump" in the ECE signal. However, without the field line reconnection, an ideal MHD mode would cause much smaller fluctuation amplitude than that of a resistive mode and is hard to observe in practice.

3.2 Data analysis technique

Due to the noise, not all the ECE signals have the clearly defined phase and amplitude indicated in Fig. 3.2(b). Fluctuations can also be caused by other MHD activities like ELMs (Edge Localized Modes), which are the narrow spikes in the ECE data (Fig. 3.4). Here we explain some of the techniques used to reduce the noise of the raw ECE data and get a more reliable measurement of the phase and amplitude of the fluctuations. We use the data from one DIII-D shot (92515) as an example.

The time window of the data to be analyzed is chosen between two consecutive ELMs so the raw data is ELM-free. In most of the cases the window is also chosen to be at a time when the mode saturates, so the amplitude and phase can be clearly defined. However, even during the nonlinear growing stage of the mode, the growth time is long compared to the typical time window width (about 5 ms) and the mode can be treated as being in steady state. In this particular shot, we choose a window from 1895 ms to 1900 ms. The raw data from several ECE channels are plotted in Fig 3.5. Even from the raw data, we can see the fluctuations on the two sides of Ch. 18

are out of phase. Therefore, Ch. 18 is probably very near to the rational surface. (The radial distance between 2 neighboring channels is about 2 cm.)

To reduce the statistical error, the entire time window is further divided into 5 bins; the amplitude and phase are measured individually for each bin and then are averaged over the bins. The amplitude and phase of the fluctuation in each bin is obtained by performing a Fast Fourier Transform (FFT) on the data. Suppose the raw data is denoted by a series of numbers T_i , $i = 1, \dots, N$. The time interval between the two consecutive data points is $1/f_s$, where f_s is the data sampling rate. The FFT coefficients C_k and S_k are defined as

$$\begin{aligned} C_k &= \sum_{i=1}^N T_i \cos\left(2\pi \frac{k-1}{N} \cdot i\right), \\ S_k &= \sum_{i=1}^N T_i \sin\left(2\pi \frac{k-1}{N} \cdot i\right), \\ k &= 1, \dots, N. \end{aligned}$$

The frequency for the k th FFT mode is $f_k = f_s(k-1)/N$ and the frequency resolution is $\Delta f = f_s/N$. For the k th FFT mode, the amplitude is

$$A_k = \frac{\sqrt{C_k^2 + S_k^2}}{N}$$

and the phase φ_k is determined from

$$\begin{cases} \sin \varphi_k = C_k / \sqrt{C_k^2 + S_k^2}, \\ \cos \varphi_k = S_k / \sqrt{C_k^2 + S_k^2}, \\ 0 \leq \varphi_k \leq 360. \end{cases} \quad (3.1)$$

To improve the statistics, the amplitude and phase are averaged between the different time bins for each channel. While averaging the amplitude is straightforward, averaging

the phase requires some extra precautions. Obviously, only the relative phases between the channels are relevant. By choosing an arbitrary channel as a reference channel, the relative phases of other channels in each bin can be obtained by taking the difference of the absolute phases (defined in Eq. (3.1)) of those channels and the reference channel. For each channel, the possible values of the relative phase from different bins lie between -360 degrees to +360 degrees. Certain adjustments have to be done before these numbers can be averaged. Due to the possible experimental and statistical errors, it is not proper to convert every negative angle to a positive angle by adding 360 degrees to it. For example, suppose the actual relative phase for a channel is 130 degrees and the measurement from a certain bin gives a result of -30 degrees due to a 160 degree error. If the result is converted to 330 degrees, the error is 200 degrees and this would certainly skew the final average. Assuming the error is always small, we adjust the relative phases from each bin by adding or subtracting 360 degrees to make their deviation from the actual value no more than 180 degrees. Of course, the actual value is not known beforehand. This can be done in an iterative way by using the bin-averaged value as an estimate of the actual value.

The amplitude of the first FFT mode A_1 of each ECE channel is the average electron temperature observed by that channel and a plot of A_1 versus channel position (which is determined by EFIT) gives the equilibrium electron temperature profile (Fig. 3.6). It should be pointed out that the bump at the edge (from $R=2.22$ m out) is not real. These edge channels are fixed at the frequencies that correspond to positions outside the

last closed magnetic surfaces where the plasma density is quite low. The radiation they observe is actually coming from the core region but with relativistically "slide-down" frequencies [31]. Relativistically, the cyclotron frequency is $f_c = eB/2\pi\gamma m$ with the relativistic factor $\gamma = [1 - (v/c)^2]^{-1/2}$. With an electron temperature on the order of 10 keV, the frequencies for the majority of the electrons do not differ significantly from the nonrelativistic case. However, there are some high energy electrons in the tail of the Maxwellian distribution whose frequencies down-shift to that of the edge channels. This can also be confirmed by the fact that the phases of the edge channel fluctuations are closely correlated to that of certain core channels. However, the signal amplitudes of the edge channels no longer reflect the temperature of the electrons since the radiation is not coming from the local thermal electrons.

A coherent MHD mode usually can be identified from the narrow, well-defined peak in the frequency spectrum (Fig. 3.7). For most of the channels, the frequencies of the peaks are the same. The frequency of the peak from the BCE channels usually agrees well with that from other diagnostics like Mirnov coils and Beam Emission Spectroscopy (BES) [12]. For this particular example, the magnetic coils give a frequency of 24.5 kHz, which agrees with the 24 kHz from Fig 3.7. Comparing with other diagnostics can also resolve the issue of aliasing of the FFT. Once the MHD mode frequency is determined, we define the amplitude and phase of the fluctuations corresponding to the MHD mode as the amplitude and phase of the FFT mode with that MHD mode frequency.

From Fig 3.7 we can also see that the fluctuations have second harmonics, as predicted by Fitzpatrick [32]. The fluctuations depend on the equilibrium temperature profile and the mode structure (see the next chapter). They are not pure sinusoidal waves. In particular, due to the magnetic field line reconnection, the temperature inside the island is flat (Fig 3.3). The channel near the rational surface almost does not see any fluctuation. That is why the fundamental mode of Ch. 18 is significantly suppressed (Fig 3.7).

Shown in Fig. 3.8(a) are the bin-averaged phases and amplitudes of the fluctuations at 24 kHz. From the phase plot, we can see a phase jump of about 160 degree between $R = 2.02$ m and 2.06 m. This indicates that the observed mode is a tearing mode and the resonant surface is around $R = 2.04$ m. Indeed, the q -profile from the Motional Stark Effect (MSE) diagnostic and EFIT shows this is where the $3/2$ surface is located (Fig. 3.8(b)). This also agrees with the mode number $m/n=3/2$ obtained from the poloidal and toroidal arrays of external magnetic probes (mimov coils). We also notice that the phases on the two sides of the resonant surface vary with radial position, instead of being constant. We believe this is due to the deformation of the magnetic islands by the sheared flow and viscosity, the details of which will be described in Chapter 4.

This particular shot is a Negative Central Shear (NCS) plasma [33]. It has a negative magnetic shear region from the plasma core to about $R = 2.02$ m, where the shear reverses (Fig. 3.8(b)). We notice there is no magnetic island at the $3/2$ surface in the negative shear region ($R = 1.98$ m). This is an indication that the observed mode is

neoclassically driven, because the neoclassical term in Eq. 2.3 becomes positive only in the positive shear region for a normal pressure profile ($dp/dr < 0$).

The amplitude plot in Fig. 3.8(a) shows a double peak structure. As will be shown in the Chapter 5, the amplitude peak corresponds to the separatrix of the island and the detailed amplitude profile can give us the information we need to determine Δ' .

Figure 3.2: (a) Electron temperature from ECE when no MHD mode is present. (b) Electron temperature from ECE in the presence of an MHD mode. (c) Magnetic field fluctuation measured by the external magnetic coils. The mode starts at around 2400 ms and saturates around 2700 ms. Indicated also are the times when the data in (a) and (b) are taken.

Figure 3.3: Flux surfaces of a magnetic island. The rational surface is at $R = R_r$. Also indicated are 2 possible trajectories swept by ECE channels. The electron temperature from the 2 ECE channels should be out of phase.

Figure 3.4: Edge Localized Mode (ELM) as observed from ECE data.

Figure 3.5: Raw ECE data from several channels. Notice that the fluctuations on the two sides of ECE18 are out of phase.

Figure 3.6: The average electron temperature profile from ECE. The sharp negative spikes are due to an inaccurate baseline calibration of individual channels. The last closed flux surface is at 2.2 m. The bump at 2.3 m is due to relativistic frequency slide-down.

Figure 3.7: Fourier spectrum of 3 neighboring channels.

Figure 3.8: (a) The bin-averaged phase and amplitude profiles at 24 kHz. (b) The q -profile from MSE and EFIT.

Chapter 4

Magnetic island structure with sheared flow and viscosity

In the previous chapter, it was shown (Fig. 3.8(a)) that the phase profile of the ECE electron temperature fluctuations is not the expected step function with constant phase on both sides of the rational surface. It has a phase gradient of about 5 degree/cm outside the magnetic island. This phase gradient could indicate that the magnetic island is deformed by the viscous drag of the sheared flow in the vicinity of the island. In this chapter, we present a model to calculate this phase gradient [14].

4.1 Basic equations

We assume that the magnetic island reaches saturation when the ECE data is taken. In a coordinate system moving along with the island, the island is in a steady state. Therefore, the momentum equation and Ohm's law can be written as,

$$\rho \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{V} + \mathbf{S}_V, \quad (4.1)$$

$$-\nabla \Phi + \mathbf{V} \times \mathbf{B} = 0. \quad (4.2)$$

Here, $\mathbf{V}$, $\mathbf{j}$ and $\mathbf{B}$ are velocity, current density and magnetic field, respectively, and ρ , p and Φ are the mass density, pressure and electric potential, respectively. A constant

viscosity is represented by ν . Finally, $\mathbf{S}_V$ represents a momentum source, which is necessary to achieve steady state when viscous dissipation is present.

We assume a constant density and an incompressible flow. In a two-dimensional slab geometry with the third dimension being the symmetry direction ($\partial/\partial z = 0$), $\mathbf{B}$ and $\mathbf{V}$ can be written as,

$$\mathbf{B} = B_z \mathbf{z} + \mathbf{z} \times \nabla \Psi, \quad (4.3)$$

$$\mathbf{V} = \mathbf{z} \times \nabla U. \quad (4.4)$$

Therefore, the ideal Ohm's law becomes

$$-\nabla \Phi + B_z \nabla U + \mathbf{z}(\mathbf{z} \cdot \nabla U \times \nabla \Psi) = 0.$$

The symmetry in z -direction requires that U is a function of the magnetic flux Ψ . It follows then that Φ and B_z are all functions of Ψ . Thus, we have $\Phi = \Phi(\Psi)$, $B_z = B_z(\Psi)$ and $U = U(\Psi)$.

Now let's look at each of the three components of the momentum equation (4.1). Since the z components of the velocity and the momentum source $\mathbf{S}_V$ are assumed to be zero (they are decoupled from the rest of the equations to a certain degree), the z component of the equation is trivially satisfied:

$$\nabla B_z \cdot \mathbf{z} \times \nabla \Psi = 0.$$

The "poloidal" component of the equation is:

$$-B_z \cdot \nabla(p + \rho V^2/2) + \nu \nabla \Psi \cdot \nabla(\nabla^2 U) + B_z \cdot \mathbf{S}_V = 0,$$

with $\mathbf{B}_p = \mathbf{z} \times \nabla \Psi$. Integrating this equation, we obtain a Bernoulli relation:

$$p + \rho V^2/2 = H(\Psi) + \int_0^{\Psi} \frac{dL_p}{B_p} [\nu \nabla \Psi \cdot \nabla(\nabla^2 U) + \mathbf{B}_p \cdot \mathbf{S}_V], \quad (4.5)$$

with L_p being the length along a magnetic field line and H an arbitrary surface function.

This requires the solubility condition $\oint \frac{dL_p}{B_p} [\nu \nabla \Psi \cdot \nabla(\nabla^2 U) + \mathbf{B}_p \cdot \mathbf{S}_V] = 0$.

To simplify the "radial" ($\nabla \Psi$) component of the momentum equation, we assume that the momentum source has no "radial" part, $\nabla \Psi \cdot \mathbf{S}_V = 0$. We further assume that the part of the enthalpy ($p + \rho V^2/2$) that varies within magnetic flux surfaces has no "radial" variation, $\nabla \Psi \cdot \nabla \int_0^{\Psi} \frac{dL_p}{B_p} [\nu \nabla \Psi \cdot \nabla(\nabla^2 U) + \mathbf{B}_p \cdot \mathbf{S}_V] = 0$. With these two assumptions, the "radial" component of the momentum equation is:

$$\nabla^2 \Psi + F(\Psi) - \mu_0 \rho U \nabla^2 U = \mu_0 \nu \frac{\nabla \Psi \cdot \mathbf{z} \times \nabla(\nabla^2 U)}{|\nabla \Psi|^2}, \quad (4.6)$$

with $F(\Psi) = B_z B'_z + \mu_0 H'$ and μ_0 the vacuum permeability.

Eq. (4.6) is a generalized equation for an MHD equilibrium with flow and viscosity. In particular, it can be used to describe a steady state magnetic island with deformation caused by sheared flow. This can be shown most easily when the viscous term, the right hand of the equation, is treated as a perturbation. We assume that without the perturbation the island is a normal one—one with left-right (about the rational surface) and up-down (about the O -point) symmetry. Using $\nabla^2 U = U''(\nabla \Psi)^2 + U' \nabla^2 \Psi$ and for the unperturbed island, we can see that the viscous term is odd both about the rational surface and the O -point. A perturbation with this symmetry property is able to cause the deformation which is consistent with the experimental data.

An analytically solvable example using particular functions F and U has been found by M. S. Chu [14]. The result gives an expression for the scaling of the phase gradient:

$$\frac{d\theta}{d\omega} \propto k \frac{\rho V_D V_p}{B_p^2 / \mu_0}.$$

Here, k is the wavenumber of the island, V_p and B_p are the flow velocity and magnetic field evaluated at the separatrix, respectively. The viscous diffusion velocity across the island V_D is defined as $V_D \equiv \nu / \rho \omega$.

4.2 Numerical results

In this section, we solve Eq.(4.6) numerically using more realistic B_z , U , etc. We treat the viscous term as a perturbation. The total flux function can be divided into two parts: $\Psi = \Psi_0 + \Psi_1$, with $\Psi_1 \ll \Psi_0$. The unperturbed part Ψ_0 represents a normal symmetric island, for which the equation is (de-dimensionalizing all the variables and neglecting viscosity):

$$\nabla^2 \Psi_0 + [F(\Psi_0) - \epsilon U' U'' |\nabla \Psi_0|^2] / (1 - \epsilon U'^2) = 0. \quad (4.7)$$

Here, the dimensionless number $\epsilon \equiv \rho V^2 / (B^2 / \mu_0)$ represents the flow magnitude relative to the magnetic field (V and B are the typical "poloidal" flow velocity and magnetic field). The flow is assumed small enough that no Alfvén resonance ($1 - \epsilon U'^2 = 0$) is present.

The equation for Ψ_1 , the perturbed part, can be obtained by linearizing Eq. (4.6):

$$\nabla^2 \Psi_1 + R(\Psi_0) \Psi_1 + T(\Psi_0) \nabla \Psi_0 \cdot \nabla \Psi_1 = \nu_s S(\Psi_0), \quad (4.8)$$

with R , S and T calculated from Ψ_0 ,

$$R(\Psi_0) = \frac{2\epsilon U''U''(F(\Psi_0) - \epsilon U''U''|\nabla\Psi_0|^2)}{(1 - \epsilon U'^2)^2} + \frac{[F' - \epsilon|\nabla\Psi_0|^2(U'U''' + U'^2)]}{(1 - \epsilon U'^2)},$$

$$T(\Psi_0) = -\frac{2\epsilon U''U''}{(1 - \epsilon U'^2)},$$

$$S(\Psi_0) = \frac{2U''}{(1 - \epsilon U'^2)^2} \frac{\nabla\Psi_0 \cdot \mathbf{z} \times \nabla(|\nabla\Psi_0|)}{|\nabla\Psi_0|}. \quad (4.9)$$

Here, ν_a is the dimensionless viscosity, $\nu_a \equiv (\nu V/w)/(B^2/\mu_0)$. The inhomogeneous part S acts as the source for the perturbation. As stated before, it has the proper symmetry. It is also proportional to U'' , the flow shear.

The functions of F (related to B_z and enthalpy) and U' (related to flow velocity) are two arbitrary flux surface functions whose forms have to be specified before Ψ can be solved. We use the following forms for F and U' :

$$F = a + b\Psi,$$

and

$$U' = \tanh[\beta(\Psi_s - \Psi)],$$

with a , b , β and Ψ_s being 4 constant parameters. We usually choose Ψ_s to be near the separatrix of the island so that the flow shear concentrates mostly near the separatrix.

The equations are solved in a rectangular region. The dimension in the y -direction is one wavelength of the island and the dimension in the x -direction is about 10 island half-

widths. The rational surface is located at the left boundary. Therefore, we only solve for one side of the island. The other side can be easily reconstructed by symmetry.

To introduce the initial island configuration, we use the following boundary condition for Ψ_0 : $\Psi_{0R} = 0$ at the right boundary and $\Psi_{0L} = 1/2 - w^2/4 \cos(ky)$, where k and w are the wavenumber and half-width of the island. The boundary condition for Ψ_1 at the left boundary is $\Psi_{1L} = 0$ (no perturbation at the rational surface). The boundary condition for Ψ_1 at the right boundary is related to the physical reality. If there is another rational surface nearby and the modes are coupled, then Ψ_{1R} should be equal to a prescribed value. If there is no mode-coupling and the right boundary is sufficiently far away from the rational surface, the proper boundary condition should be $\Psi_{1R} = 0$. Here we use the latter. The boundary conditions in the y -direction are periodic for both Ψ_0 and Ψ_1 .

For the results presented here, the values of the two parameters ϵ and ν_a are chosen to be $\epsilon = 0.8$ and $\nu_a = 0.05$. With a deuterium plasma density of $5 \times 10^{19} \text{ m}^{-3}$, island half-width of $w = 2 \text{ cm}$ and "poloidal" magnetic field at the separatrix of $B = 15 \text{ Gauss}$, this corresponds to a flow velocity $V = 3 \text{ km/s}$ and viscosity $\nu/\rho = 3.8 \text{ m}^2/\text{s}$. The anomalous viscosity observed in tokamaks is about $1 \text{ m}^2/\text{s}$. Notice that the so-called "poloidal" (y -) direction here corresponds in reality to the direction that is perpendicular both to the total magnetic field at the rational surface and the radial direction.

We solve Eqs. (4.7) and (4.8) on a 51×51 grid. The lengths in the x - and y -directions are 1.0 (with $w=0.12$) and 4.189, respectively. The numerical values used to represent

F and U' are: $a = -42356$, $b = 2.01$, $\beta = 20$ and $\Psi_s = 0.495$. The flow shear is quite high and also localized. Shown in Fig. 4.1 is a contour plot of Ψ_0 (solution to Eq. (4.7)) in the xy -plane. This is an island without the viscous drag.

In Fig. 4.2 we plot the viscous term (Eq. (4.9)). This term shows the expected odd symmetry about the mid-plane. It becomes small away from the separatrix, where the field lines are hardly bent and there is almost no flow shear.

The contour plots of the perturbation Ψ_1 and the total flux magnetic flux function $\Psi = \Psi_0 + \Psi_1$ are plotted in Figs. 4.3 and 4.4. We can see the deformed island structure in Fig. 4.4. To better quantify the deformation and relate it to the observed phase gradient, we plot the y coordinate of the points where $d\Psi/dy = 0$ in Fig. 4.5. In an unperturbed island (Fig. 4.1), these points are all in the mid-plane ($n_y = 25$).

We can see from Fig. 4.5 that there are two regions that have a phase gradient. One is inside the separatrix and the other is outside. The phase gradient observed from ECE data is probably coming from the part outside the separatrix. Since the resolution of a single ECE channel is about the island half-width, any feature within the separatrix cannot be resolved. The average phase gradient determined from Fig. 4.5 is about 12 degree/half-width, which agrees with the experimental value. We have to point out again here the viscosity used is about three times greater than the "usual" anomalous viscosity.

Figure 4.1: Contour plot of Ψ_0 : the unperturbed island.

Figure 4.2: Contour plot of the viscous term (S/ν_n). Notice the odd symmetry about the mid-plane.

Figure 4.3: Contour plot of ψ_1 , the perturbation.

Figure 4.4: Contour plot of $\Psi_0 + \Psi_1$. This shows an island deformed by the shear flow in the presence of viscosity.

Figure 4.5: The position of the points where $d\Psi/dy = 0$ in the deformed island (Fig. (4.1)). For an unperturbed island, those points should be at the mid-plane ($n_y = 25$.)

Chapter 5

Fit function for ECE electron temperature fluctuations

In Chapter 3, we have seen that in the presence of the tearing mode the ECE diagnostic sees an electron temperature fluctuation. The electron temperature fluctuation profile, $\tilde{T}_e(\mathbf{x})$, depends on the magnetic field perturbation. This provides us an opportunity to measure Δ' , a property of the magnetic field perturbation, from $\tilde{T}_e(\mathbf{x})$. In this chapter, we derive a fit function, with Δ' as one of the fitting parameters, for the $\tilde{T}_e(\mathbf{x})$ profile seen by the ECE diagnostic.

We only consider a single mode here. The coupling between the modes at different rational surfaces [34], with the same toroidal mode number but different poloidal numbers, will be neglected. This is because the coupling is small in a high aspect ratio device and also it is often weakened by the different plasma flow velocities at the different rational surfaces.

Cylindrical coordinates (r, θ, z) are used, where θ and z represent the poloidal and toroidal coordinates, respectively, and r is the radial coordinate. The equilibrium magnetic field can be written as

$$\mathbf{B} = B_z \hat{z} + B_\theta(r) \hat{\theta}.$$

For a thin magnetic island, i.e., $w/r_s \ll 1$, where w is the island width and r_s is the

radius of the rational surface, the equilibrium field near the rational surface can be expanded as:

$$\begin{aligned} \mathbf{B} &= B_z \hat{z} + B_\theta(r_s) \hat{\theta} + \nabla \psi_0 \times \hat{z}, \\ \psi_0 &\equiv - \left[\frac{1}{2} B_\theta'(r_s) x^2 + \frac{1}{6} B_\theta''(r_s) x^3 \right], \\ x &\equiv r - r_s. \end{aligned} \quad (5.1)$$

It is necessary to retain the x^3 term in ψ_0 because, as we will show later, it is of the same order as the Δ' effect.

The island can be modeled by superimposing a perturbed field $\tilde{\mathbf{B}}$ on the equilibrium field:

$$\begin{aligned} \tilde{\mathbf{B}} &= \nabla \psi_1 \times \hat{z}, \\ \psi_1 &\equiv - \frac{r}{m} B_* \left(1 + \frac{x}{\alpha_\pm} \right) \cos \theta, \\ \theta &\equiv m\theta - nZ/R_0. \end{aligned} \quad (5.2)$$

Here, m and n are the poloidal and toroidal mode numbers, respectively, R_0 is the major radius of the torus, and the subscripts "+" and "-" of α are for $\alpha > 0$ and $\alpha < 0$, respectively. Therefore, Δ' can be parameterized by the difference between the reciprocals of α_+ and α_- :

$$\begin{aligned} \Delta' &\equiv [(\partial \psi_1 / \partial r)_+ - (\partial \psi_1 / \partial r)_-] / \psi_1(0) \\ &= 1/\alpha_+ - 1/\alpha_-. \end{aligned} \quad (5.3)$$

In principle, there could be another term, proportional to $x \ln x$, in the expression

of ψ_1 [35]. However, this term will not contribute to Δ' and is also not found to be significant in fitting the experimental data. Therefore, we neglect this term.

Figure 5.1: (a) Contour plot of lines of constant y (constant ψ) for Eq. (5.4) in the x, ϕ plane. Dashed lines represent trajectories scanned by two ECE channels located at x_1 and x_2 . The amplitude of the electron temperature fluctuations recorded by the channel at x_1 is proportional to the difference of T_e between A and B. Since T_e is constant along the contour, it is also equal to the difference of T_e between $d(x_1)$ and x_1 . (b) The electron temperature profile across the island O - point ($\phi = \pi$). The T_e is assumed to be flat within the separatrix (from x_- to x_+).

The equation for a given magnetic field line, as illustrated in Fig. 5.1a in the $x - \phi$

plane, is $\psi_0 + \psi_1 = \text{const.}$, and becomes

$$\frac{1}{2}x^2(1 + \epsilon_1 x) + \left(\frac{w}{4}\right)^2(1 + \epsilon_2 x)\cos\phi = y, \quad (5.4)$$

where $\epsilon_1 \equiv B_\theta''(r_s)/3B_\theta'(r_s)$, $\epsilon_2 \equiv (1/\alpha_\pm + 1/r_s)$, $(w/4)^2 \equiv r_s B_\theta / m B_\theta'$. Here, y is a constant that labels different magnetic field lines. The value of y on the separatrix is $y = (w/4)^2$. Compared to the conventional magnetic island equation, the new terms $\epsilon_1 x$ and $\epsilon_2 x$ describe both the effect of the cylindrical geometry and Δ' . This equation illustrates that these effects are of higher order in w/r_s than the other terms. Therefore, what determines Δ' are the fine details of the island shape—the small difference between the two sides of the magnetic island.

Because of the large parallel electron heat conductivity, the electron temperature T_e can be assumed to be constant along all magnetic field lines. Also, T_e is assumed to have the same value ("flat") within the separatrix due to the magnetic field line reconnection [36]. The T_e profile at the O-point of the island (where $\phi = -\pi$) is illustrated in Fig. 5.1b. The equilibrium temperature gradient on the two sides of the island is denoted by $(dT/dx)_\pm$. From Fig. 5.1a it can be seen that the amplitude of the temperature fluctuation, $\tilde{T}_e(x_1)$, as seen by an ECE channel located at a radial position x_1 outside the separatrix, is proportional to the temperature difference between points A and B, which can be written as $(dT/dx)_- [d(x_1) - x_1]$. For a point x_2 inside the separatrix, we have $\tilde{T}_e(x_2) = (dT/dx)_+ [d(x_2) - x_+]$. Here $d(x)$ is the radial coordinate at the O-point of a field line whose radial coordinate at the X-point (where $\phi = 0$) is

x . In general, we have

$$\tilde{T}_s(x) = \begin{cases} |dT/dx|_{\pm} |d(x) - x_{\pm}| & \text{for } |x| \leq |x_{\pm}| \\ |dT/dx|_{\pm} |d(x) - x| & \text{for } |x| > |x_{\pm}| \end{cases} \quad (5.5)$$

Here, $x_{\pm}$ are the radial coordinates of the separatrix at the O -point, i.e., they are the half widths of the island on the two sides of the rational surface.

Both $x_{\pm}$ and $d(x)$ can be calculated from Eq. (5.4). To the lowest order in w/τ , where the Δ' effect can be considered, the results are

$$x_{\pm} = \pm \frac{w}{2} - \left(\frac{w}{4}\right)^2 (2\epsilon_1 - \epsilon_2), \quad (5.6)$$

$$d(x) = \pm \sqrt{(w/2)^2 + x^2} \pm [\epsilon_1 x^3/2 + (w/4)^2 \epsilon_2 x] / \sqrt{(w/2)^2 + x^2} - \epsilon_1 [(x^2/2)^2 + 2(w/4)^2] + \epsilon_2 (w/4)^2. \quad (5.7)$$

Now, to determine $(dT/dx)_{\pm}$, we first write

$$(dT/dx)_{\pm} = (dT/dy) \Big|_{y=(w/4)^2} \times (\partial y/\partial x) \Big|_{\phi=-\pi, x=x_{\pm}}. \quad (5.8)$$

From Eq. (5.4), we obtain

$$(\partial y/\partial x) \Big|_{\phi=-\pi, x=x_{\pm}} = \pm w/2 + \epsilon_1 (w/2)^2. \quad (5.9)$$

To find dT/dy , we assume that no sources or sinks of heat exist within the narrow region near the rational surface so the heat flux across the field line is continuous [36]. Therefore, we obtain

$$\int \nabla T \cdot d\mathbf{S} = \text{const.},$$

or,

$$\frac{dT}{dy} \cdot \int |\nabla y| dl = \text{const.} \quad (5.10)$$

Both $|\nabla y|$ and dl (the differential line length along a field line) can be found from Eq. (5.4):

$$|\nabla y| = \left| x + \frac{3}{2} \epsilon_1 x^2 + \left(\frac{w}{4}\right)^2 \epsilon_2 \cos \phi \right|, \quad (5.11)$$

$$dl = (r_s + x) d\phi. \quad (5.12)$$

Substituting Eqs. (5.11) and (5.12) into Eq. (5.10) and expressing x in terms of (y, ϕ) from Eq. (5.4), the integral in Eq. (5.10) can be performed over ϕ (from $-\pi$ to π). Then, setting $y = (w/4)^2$ and using Eqs. (5.8) and (5.9), $(dT/dx)_{\pm}$ can be determined to within a constant:

$$\left(\frac{dT}{dx}\right)_{\pm} = A \left\{ 1 \pm \frac{w}{2} \left[\left(1 - \frac{\pi}{4}\right) \epsilon_1 - \frac{\pi}{4r_s} \right] \right\}. \quad (5.13)$$

Finally, an analytical fit function for the electron temperature fluctuation profile $\tilde{T}_e$ is obtained from Eqs. (5.5-5.7) and (5.13):

$$\begin{aligned} \tilde{T}_e(x) = & \left\{ A \left\{ 1 \pm \frac{w}{2} \left[\left(1 - \frac{\pi}{4}\right) \epsilon_1 - \frac{\pi}{4r_s} \right] \right\} \right\} \\ & \times \left| \sqrt{\left(\frac{w}{2}\right)^2 + x^2} - \frac{w}{2} + \frac{\epsilon_1 x^3/2 + (w/4)^2 \epsilon_2 x}{\sqrt{(w/2)^2 + x^2}} \mp \epsilon_1 \frac{x^2}{2} \right| \end{aligned} \quad (5.14)$$

when $|x| \leq |x_{\pm}|$, and

$$\begin{aligned} \tilde{T}_e(x) = & \left| A \left\{ 1 \pm \frac{w}{2} \left[\left(1 - \frac{\pi}{4} \right) \epsilon_1 - \frac{\pi}{4r_e} \right] \right\} \right| \\ & \times \sqrt{\left(\frac{w}{2} \right)^2 + x^2 - \frac{w}{2} + \frac{\epsilon_1 x^2/2 + (w/4)^2 \epsilon_2 x}{\sqrt{(w/2)^2 + x^2}}} \mp \epsilon_1 \left[\frac{x^2}{2} + 2 \left(\frac{w}{4} \right)^2 \right] \\ & \pm \left[\epsilon_2 \left(\frac{w}{4} \right)^2 - x \right] \end{aligned} \quad (5.15)$$

when $|x| > |x_{\pm}|$. The five parameters, $A, w, \epsilon_1, 1/\alpha_+$ and $1/\alpha_-$ (in ϵ_2), are to be found by fitting the fit function to the experimental data. The value of Δ' is then given by Eq. (5.3).

Chapter 6

Δ' measured from ECE data

6.1 "Jogging" the plasma

In order to determine Δ' by fitting Eqs. (5.14) and (5.15) to the experimental data, the electron temperature fluctuation profile $\tilde{T}_e(x)$ must be measured with a high spatial resolution near the rational surface. Under normal conditions, this is difficult to do using a finite number of ECE channels at fixed radial positions. For example, on the Tokamak Fusion Test Reactor (TFTR), the distance between two neighboring ECE channels is about 5 cm, which is greater than the width of a typical island. To achieve a high resolution ECE measurement, Zarnstorff *et al.* [37] did the so-called "jog" experiment on TFTR. In a "jog" experiment, the entire plasma is rapidly shifted ($\Delta x/\Delta t \geq 30$ cm/60 ms) horizontally. With a high sampling rate (500kHz), the ECE channels can measure the electron temperature and its fluctuations at many plasma positions, much more than what would be allowed with a stationary plasma. In addition, this provides the possibility for $\tilde{T}_e(x)$ of the whole island region around the rational surface to be measured by a single ECE channel. Thus, the systematic between-channel calibration error can be avoided.

Using a code developed by Mike Zarnstorff of PPPL, the ECE data can be mapped back to the position before the "jog". If a tearing mode is present, it can be seen clearly

from the $\tilde{T}_e(x)$ profile (Fig. 6.1). To apply Eqs (5.14) and (5.15) to this data, we need to know the minor radius of the rational surface r_* . The location of the rational surface is at the minimum of the fluctuation level shown in Fig. (6.1). Using the major radius of this location and the magnetic axis, r_* can be determined. In some cases, both inboard and outboard islands are recorded by different channels. The average of the minor radii of the two minima in the fluctuations in the two islands is r_* . The final determined value of Δ' is not sensitive to r_* .

Figure 6.1: The electron temperature fluctuation profile for a 3/2 mode from a "jog" experiment (Shot 82443). The rational surface is at the minimum in the fluctuation amplitude. The noise level is around 10 eV.

6.2 Fitting the experimental data

A nonlinear fitting code [38] is used to fit Eqs. (5.14) and (5.15) to the ECE data to find Δ' for the tearing mode observed in TFTR. A typical fit is shown in Fig. 6.2. The Δ' found from the fitting for 7 cases is shown in Table 6.1. Zuoyang Chang helped find the modes and identify the mode number from the external magnetic coils. It can be seen that the Δ' values for the 3/2 and 4/3 modes are negative. Only the 2/1 mode can have a positive Δ' . This shows that the 3/2 and 4/3 are not Δ' -driven. Instead they are neoclassical tearing modes.

Table 6.1: Values of $r_*\Delta'$ obtained from two different methods. The "fitting" values are obtained using the formulas developed in this thesis. The last column lists the values of $r_*\Delta'$ calculated from the q -profile. Except for shot 86138, the data are from a single ECE channel. The data for shot 86138 are from two adjacent channels. "SN" means using the q -profile from the SNAP code, while "TR" means from TRANSP. Usually the ones using the TRANSP q -profiles are more reliable.

Shot	Mode	$r_*\Delta'$ (fitting)	$r_*\Delta'$ (q -profile)
73413	2/1	-1.3	-2.3 (SN)
82443	3/2	-3.7	-2.7 (TR)
82444	4/3	-5.9	-5.8 (TR)
82445	3/2	-4.3	-4.0 (TR)
84665	2/1	0.52	1.4 (TR)
86138	2/1	4.2	10.2 (SN)
91621	3/2	-7.0	-5.2 (SN)

Chang has also calculated the Δ' for these cases by numerically integrating the mode equation with spline-interpolated q -profile from TRANSP or SNAP, which is a code similar to TRANSP but only models one time slice. Generally, the q -profile from TRANSP is more accurate than that from SNAP. The results are compared with the

Figure 6.2: Comparison of experimental data and fitted data (using Eqs. (5.14) and (5.15)) for the electron temperature fluctuation profile for a 3/2 mode (shot 82443, channel 12, 240-point duration for fluctuation calculation). The fitting parameters are: $A = 57.5$, $w = 1.66$, $1/\alpha_+ = -0.0763$, $1/\alpha_- = 0.107$, $\epsilon_1 = 0.235$. Therefore, $r_s \Delta' = -3.7$.

experimental results from ECE measurement in Table 6.1. The general agreement validates the use of the TRANSP q -profile in determining Δ' in Ref [19].

6.3 Error estimation

An estimate of the appropriate error bars for these results is difficult. However, to illustrate the degree of reliability of the fitting results in Table 6.1, an example is given in Table 6.2 in which three ECE channels (Ch.12-14) have scanned one entire

island. The fluctuations at a certain radial position are calculated from the raw data for two different durations—every 240 points (0.48 ms) or every 360 points (0.72 ms). (The results in Table 6.1 were obtained using the 240 point duration). With the data collection rate of 500 kHz and the mode rotation frequency on the order of 10 kHz, the 360 point duration corresponds to surveying the island about 7 times during the calculation of the fluctuations. The plasma moves radially in this duration only about 0.3 cm due to the “jog”; thus the radial resolution is not compromised. Six possible values of $r_s \Delta'$ for the same mode are listed in Table 6.2. The systematic uncertainty of the fitting results is thus seen to be about 25%. The sign of Δ' for this 3/2 mode is shown unequivocally to be negative.

Table 6.2: Values of $r_s \Delta'$ for one island from 3 different ECE channels (Ch.12-14). Here, “240” and “360” denote the number of points used in the local fluctuation calculation from the raw data. The average $r_s \Delta'$ is -3.9.

	Ch.12			Ch.13			Ch.14		
No. of points	240	360	240	360	240	360	240	360	360
$r_s \Delta'$	-3.7	-4.2	-4.1	-4.9	-3.5	-3.1			

6.4 Discussion

The underlying assumptions used in the derivation of the analytical expression Eqs. (5.14) and (5.15) are: 1) the electron temperature is constant on a magnetic flux surface; and 2) the heat flux from diffusion is constant across the flux surface in the narrow region near the rational surface. Thus, this method can be applied to any quantity that is

a surface function and has a continuous diffusive flux, such as the density or pressure fluctuations. As a verification of these assumptions, we apply Eqs. (5.14) and (5.15) to the pressure fluctuation data generated by a full three-dimensional initial value resistive/neoclassical MHD code [39]. The initial equilibrium is set up to be classically stable (with $r_s\Delta' = -4.4$ at the $q = 2/1$ surface) but neoclassically unstable for the $2/1$ mode. Then, the pressure is allowed to equilibrate on the perturbed magnetic flux surfaces. The final pressure fluctuations at a certain minor radius are analyzed in a similar way to the analysis of ECE electron temperature fluctuations in the previous chapter. Shown in Table 6.3 are the values of $r_s\Delta'$ obtained by applying Eqs. (5.14) and (5.15) to the simulated pressure fluctuation data for different ratios of parallel to perpendicular heat conductivity. As the parallel heat conductivity becomes higher and the pressure becomes more of a surface function, the method developed here can obtain ever more accurate measurements of Δ' .

Table 6.3: Values of $r_s\Delta'$ obtained by applying Eqs. (5.14) and (5.15) to the computer-simulation-generated pressure fluctuation data for different $\chi_{||}/\chi_{\perp}$, the ratio of parallel to perpendicular heat conductivity. On TFTR, $\chi_{||}/\chi_{\perp} \approx 10^9$. The theoretical input value of $r_s\Delta'$ is -4.4 .

$\chi_{ }/\chi_{\perp}$	10^5	10^6	10^7	10^8
$r_s\Delta'$	-1.87	-2.37	-2.79	-3.04

Eqs. (5.14) and (5.15) are derived under the assumption of a flat temperature profile inside the island (Fig. 5.1b). This assumption is based on theoretical considerations [40] and has experimental support [18]. However, a similar fit function for a non-flat profile

can be obtained easily using the method developed here, requiring only a modification of the $|x| \leq |x_{\pm}|$ part in Eq.(5.5) (and Eq. (5.15) correspondingly).

The intrinsic band-width of a single ECE channel limits its radial resolution to about 2 cm (full width). However, the "jog" technique and the assumed linear radial temperature profile near the rational surface (Fig. 5.1b) allow the fluctuation profile to be measured with a radial resolution less than 2 cm (see the data in Fig. 6.2). A simulation shows that the finite radial resolution of the ECE channels affects little of the measurement of the data outside the two peaks. It is the data points outside the two peaks that mainly determine the value of Δ' .

We should point out that the ECE data used here are usually taken when the magnetic island width is greater than the resistive layer width—the so-called Rutherford regime [6]. Although we fit the data in the whole island region, we find that the determined value of Δ' depends most sensitively on the data points outside the two maximum fluctuation points [41]. This indicates that the Δ' measured here is really the nonlinear Δ' in the presence of an island of finite width. This result [13] represents the first direct measurement of such a Δ' within a plasma.

Chapter 7

"Almost ideal MHD" constraint, and stability and saturation of tearing mode

7.1 Introduction

In Chapter 2 the central role of Δ' in the study of the tearing mode was explained. In this chapter, we present a new approach to the stability and saturation of the tearing mode.

In the study of the tearing mode [5, 6], usually the initial equilibria are one dimensional with two ignorable coordinates and the perturbed equilibria are two dimensional with one ignorable coordinate. Therefore, magnetic flux surfaces exist for both the initial and the perturbed equilibria (Fig. 7.1). As an intrinsically nonlinear approach, the use of the "almost ideal MHD" constraint [42] is well suited to study the nonlinear properties of the tearing mode. In this chapter, as a validation of the method, we study two characteristics of the tearing mode using the "almost ideal MHD" constraint: 1) the linear stability condition for the initial one dimensional equilibrium; and 2) the final saturation level for the unstable case. Here, we only consider the simplest case where no gradient of pressure or current density exists at the mode resonant surface.

The tearing mode cannot exist under the ideal MHD constraint, which does not

Figure 7.1: Magnetic flux surfaces for two equilibria: a) initial one-dimensional equilibrium; b) final two-dimensional equilibrium with island.

permit changes in magnetic flux surface topology [1]. On the other hand, it is well known that the ideal MHD constraint is violated only in a narrow region near the mode resonant surface, where the magnetic field is perpendicular to the wave vector of the mode [5]. The "almost ideal MHD" constraint is a relaxation of the ideal MHD constraint to allow such local changes of the flux surface topology [42].

In principle one can find many equilibria with magnetic islands that are associated with the initial one dimensional equilibrium through the "almost ideal MHD" constraint. In general these island equilibria have a singular current at the X-point of the islands (Fig. 7.1b). Depending on the sign of this singular current, the equilibrium evolves, on the tearing mode time scale, in the direction of either increasing or decreasing the island width. We are interested in an equilibrium where the singular current

is zero and therefore is a final stationary state. If such a final equilibrium is found to include an island (it is believed that there is at most one such equilibrium), then the initial equilibrium is presumed to be tearing-mode unstable and the island width of the final state is the saturation width. If no possible final state with an island is found, then the initial equilibrium is tearing-mode stable. Because we do not use a small amplitude expansion, this approach is intrinsically nonlinear.

In the rest of the chapter, we use an example to illustrate the application of the "almost ideal MHD" constraint to the problem of tearing mode stability and saturation. We will show that the stability criterion is the same as the linear " Δ' theory" [5] and give the saturation width for an unstable initial equilibrium. As mentioned earlier, this is only done for simple initial equilibria.

7.2 An example

We consider a simple case where there is no pressure gradient, $\nabla p = 0$. Then, the force-balance equation for the equilibrium is just $\mathbf{J} \times \mathbf{B} = 0$. We use Cartesian coordinates where $\hat{\mathbf{z}}$ is an ignorable coordinate ($\partial/\partial z = 0$) for both the initial and final equilibrium. The initial equilibrium is also independent of y . The magnetic field $\mathbf{B}$ is expressed through functions $B_z(x, y)$ and $\psi(x, y)$ as

$$\mathbf{B} = B_z \hat{\mathbf{z}} + \nabla \psi \times \hat{\mathbf{z}}.$$

Then, the force-balance equation becomes

$$B_z \nabla B_z + (\nabla^2 \psi) \nabla \psi + \hat{\mathbf{z}} (\nabla \psi \times \nabla B_z \cdot \hat{\mathbf{z}}) = 0.$$

The $\hat{\mathbf{z}}$ -component of the above equation gives $B_z = B_z(\psi)$. The rest of the equation is

$$\nabla^2 \psi + B_z \frac{dB_z}{d\psi} = 0. \quad (7.1)$$

Equation (7.1) determines both the initial and final equilibrium, once $B_z(\psi)$ and the boundary conditions are given. For the boundary conditions, we assume that there are two perfectly-conducting walls at $x = \pm 1$ where $\psi(\pm 1, y) = 0$. The boundary condition in the y -direction is set to be periodic with a period of L , $\psi(x, y) = \psi(x, y + L)$.

The one-dimensional initial equilibrium can be described by $\psi = \psi_i(x)$. For simplicity, we further assume $\psi_i(x) = \psi_i(-x)$ and that ψ_i decreases monotonically from the center ($x = 0$) toward the two conducting walls (at $x = \pm 1$). The resonant surface is at $x = 0$ where ψ reaches its maximum value. By assuming left-right symmetry, we only have to study the $x \geq 0$ region. The approach can be generalized to the asymmetrical case [42].

One family of such initial equilibria can be described by $B_z B_z' = a^2 \psi + b$, which results in $\psi = -(b/a^2)(1 - \cos ax / \cos a)$, with $0 < a < \pi$ and $b / \cos a > 0$. The value of Δ' , which is the key linear stability parameter [5], for these initial equilibria can readily be found analytically, yielding $\Delta' = -2\kappa \cot \kappa$ with $\kappa^2 = a^2 - k^2$ and $k = 2\pi/L$. A positive Δ' means instability.

The ideal MHD constraint for this $\nabla p = 0$ case can be represented by requiring that the B_z flux enclosed by a flux surface $\psi = \psi_1$,

$$\phi(\psi_1) = \int B_z S[\psi(x, y) - \psi_1] dx dy,$$

$$S[u] = \begin{cases} 1 & u > 0 \\ 0 & u < 0, \end{cases}$$

be a conserved function. In a tokamak, this is equivalent to the conservation of the safety factor. For simplicity, we consider the limit where $B_z \rightarrow \infty$, $B_z' \rightarrow 0$ and $B_z B_z'$ remains finite. Then, the conservation of $\phi(\psi)$ is equivalent to the incompressibility of the plasma. That is, the area enclosed by a flux surface remains constant; i.e.,

$$A(\psi_1) = \int S[\psi(x, y) - \psi_1] dx dy$$

is a conserved function.

To relax this constraint to the "almost ideal MHD" constraint, we only require a finite number of "moments" of $A(\psi)$ to be conserved. That is, the set of integrals,

$$g_n = \int G_n(\psi) dA, \quad n = 1, 2, \dots, N,$$

where G_n is a set of base functions, are conserved. The base functions G_n should be sufficiently diverse in ψ -space. The number of base functions is finite to allow magnetic field line reconnection. When the number of base functions approaches infinity, the "almost ideal MHD" constraint approaches the ideal MHD constraint. We choose the following set of the "tent" base functions,

$$\frac{dG_n}{d\psi} = \begin{cases} (N-1)(\psi/\psi_{max} - \frac{n-2}{N-1}) & \text{for } \frac{n-2}{N-1} \leq \psi/\psi_{max} \leq \frac{n-1}{N-1} \\ 1 - (N-1)(\psi/\psi_{max} - \frac{n-1}{N-1}) & \text{for } \frac{n-1}{N-1} \leq \psi/\psi_{max} \leq \frac{n}{N-1} \\ 0 & \text{otherwise,} \end{cases}$$

where ψ_{max} is the maximum value of ψ . So formally, the "almost ideal MHD" constraint can be written as

$$g_{nf} = g_{ni}, \quad n = 1, 2, \dots, N \quad (7.2)$$

where g_{nf} and g_{ni} are the integrals of the n th base function over the final and initial state, respectively. The set g_{ni} can be calculated from the given initial equilibrium and is the only information needed from it. Our goal is to find a final state $\psi(x, y)$ that satisfies both (7.1) and (7.2), by choosing the right form of $B_z B_z'$.

7.3 Numerical algorithm

Finding the exact solution of Eqs. (7.1) and (7.2) is difficult. Therefore, a numerical algorithm is used to find an approximate solution. The function $B_z B_z'$ is parameterized by a set of parameters p_m , $m = 1, 2, \dots, M$. The aim is to adjust the p_m 's to minimize the quantity $Q = \sum_{n=1}^N (g_{nf} - g_{ni})^2$. The quantity Q is a function of the p_m 's and $dQ \approx \sum_{m=1}^M (\partial Q / \partial p_m) dp_m$. If the p_m 's change an amount $dp_m = -s(\partial Q / \partial p_m)$ with the step s being a sufficiently small positive number, then Q decreases an amount of $dQ \approx -s \sum_{m=1}^M (\partial Q / \partial p_m)^2$. Given a set of p_m 's, one can calculate the Q and $\partial Q / \partial p_m$'s by first solving Eq. (7.1) and then finding the g_{nf} 's from the solution of Eq. (7.1). This procedure of decreasing Q can be repeated until Q is minimized. The final ψ thus obtained is the final equilibrium of the initial state and it can then be examined to see whether it has an island structure.

Obviously, the initial island-free equilibrium itself is always a possible final state. A successful algorithm must find a final state other than the initial one if the initial equilibrium is unstable to the tearing mode. It can be shown that the Jacobi algorithm [43] (or its variants like the Gauss-Seidel and Successive Over-Relaxation algorithms)

used to solve Eq. (7.1) converges to the initial state only if the Δ' of the initial state is negative [44]. So if the initial equilibrium is tearing-mode unstable ($\Delta' > 0$), the algorithm moves the solution away from it towards a different equilibrium.

Numerically, Eq. (7.1) is solved on a 25 by 49 grid with a grid size of $\Delta x = \Delta y = 1/24$. The length of the grid in the y -direction is half of the period L plus $2\Delta y$ (so $L=23/6$) and the boundary conditions in the y -direction are $\partial\psi(x, 0)/\partial y = \partial\psi(x, L/2)/\partial y = 0$. The number of base functions N is taken to be 14. The form of $B_z B'_z$ is taken to be

$$B_z B'_z = p_1 + p_2 \left(1 - \frac{\psi}{\psi_{max}}\right)^{1/2} + \sum_{i=3}^M p_i \left(1 - \frac{\psi}{\psi_{max}}\right)^{i-2},$$

with $M = 7$. The final result is found not to depend sensitively on the grid size, the number of base functions (N), or the number of the parameters (M) in the expression of $B_z B'_z$.

The relative amount of reconnected flux, $\delta\psi \equiv 1 - \psi_z/\psi_{max}$, is a measure of the size of the island. Here, ψ_z is the ψ value at the X-point. The following relation may be used to relate $\delta\psi$ and the full island width W_{sat} when the island width is small,

$$\delta\psi = \frac{1}{2} \left(\frac{W_{sat}}{2} \right)^2 |\psi''_i/\psi'_i|_{s=0}, \quad (7.3)$$

where ψ_i is the ψ of the initial equilibrium. Plotted in Fig. 7.2 is $\delta\psi$ calculated using the algorithm described here for the family of initial equilibria (different Δ') described in Sec. 7.2. We can see that the island only exists ($\delta\psi > 0$) when $\Delta' > 0$. Thus, the "almost ideal MHD" approach gives the same linear stability criterion as conventional tearing

mode theory [5]. As a comparison, the saturated island width can also be estimated using the assumption that the mode saturates when the value of the nonlinear version of the stability parameter Δ' vanishes [17]. For the family of the initial equilibria used in our example, the saturation width is found analytically to be

$$W_{sat} = -\frac{2}{\kappa} \arctan(\cot \kappa). \quad (7.4)$$

The $\delta\psi$ estimated from Eqs (7.4) and (7.3) is also plotted in Fig. (7.2). The saturation level from the "almost ideal MHD" approach is found to be somewhat smaller than that estimated from the combination of Eqs. (7.4) and (7.3).

Figure 7.2: The reconnected flux $\delta\psi$ for different initial equilibria (different Δ') calculated by using the "almost ideal MHD" constraint and by using the requirement $\Delta'(W_{sat}) = 0$. In both cases, an island exists ($\delta\psi > 0$) only when $\Delta' > 0$.

7.4 Discussion

We have shown in this chapter that the "almost ideal MHD" constraint can be applied to the tearing mode problem and gives the correct linear stability criterion. For the unstable case, the final nonlinear saturation island width can also be obtained; it is found to be smaller than that estimated from $\Delta'(W_{sat}) = 0$.

We cannot explain this difference in the island saturation width at this time. Including higher order terms, the theory of White *et al.* predicts that (Eq. (28) in Ref. [16]):

$$\frac{dW}{dt} = 1.66\eta(r_s)[\Delta'(W) - \alpha W].$$

The parameter α is a function of the initial current profile and other quantities. However, α is less than zero for the initial equilibria used here. Thus, the theory of White *et al.* with this α term would predict an even bigger island saturation width and make the discrepancy between the two approaches bigger.

The initial equilibria studied here are symmetric and have no pressure gradient. The "almost ideal MHD" constraint approach can be extended to include a pressure or a current density gradient at the resonant surface of the initial equilibrium [42]. It is possible that under these more complicated situations linearly stable but nonlinearly unstable cases, similar to the neoclassical tearing mode, can be found [45].

Chapter 8 Summary

The tearing mode is an important resistive MHD mode whose magnetic island structure can degrade the plasma confinement and reduce fusion reaction rate. This thesis studies several tearing mode problems—concerning the stability, saturation, and mode structure—from both theoretical and experimental points of view. It can be divided roughly into two parts: a study combining theoretical modeling and experimental data analysis of tearing mode structure using ECE data from tokamaks (Chapter 3-6); and a theoretical study of a new way to obtain the stability boundary and nonlinear saturation level of the tearing mode (Chapter 7).

The new approach to obtaining the tearing mode stability boundary and saturation level uses the "almost ideal MHD" constraint [42]. The key point is to relax the ideal MHD frozen-flux constraint to allow a change of the magnetic field topology. Nonetheless, the "almost ideal MHD" constraint still recognizes the fact that except in a narrow region near the rational surface the mode structure can be described by ideal MHD. The approach is nonlinear and is able to give both the stability boundary and the saturation level. The numerical results for a sheared slab model and simple equilibrium show that the obtained linear stability boundary is the same as that from Δ' -theory [5] and the saturation level is somewhat smaller than that of Ref. [16]. This method is

potentially suitable for studying the nonlinear stability properties of the tearing mode in the presence of pressure gradient.

The major part of this thesis is to demonstrate that ECE is an excellent diagnostic for studying an MHD mode structure and its properties in tokamak plasmas. It has been shown that the MHD mode structure can be detected from electron temperature fluctuations measured by ECE (Chapter 3). The profiles of the amplitude and phase of the fluctuations contain detailed information about the mode structure. Specifically, two examples were presented to show what can be learned about the tearing mode in the tokamak.

The first example is the identification of the magnetic island deformation due to the combination of sheared flow and viscosity from the ECE fluctuation phase profile (Chapter 4). A model has been presented to relate qualitatively the observed phase gradient to the local magnetic field, flow velocity and viscosity in a 2D slab geometry, using an ideal Ohm's law and the plasma momentum equation with flow and viscosity. Numerical solution of the resultant Grad-Shafranov-like equation describing the deformed island shows that the experimentally observed value of the phase gradient can be obtained under realistic parameters for the flow velocity and viscosity.

The second example is about measuring Δ' , the stability parameter for the tearing mode, from the ECE fluctuation amplitude profile (Chapters 5-6). A fit function with Δ' as one of the fitting parameters is derived for the ECE fluctuation amplitude profile. The theoretical function is then fitted to the experimental data from TFTR. The obtained

values of Δ' for 3/2 and 4/3 modes on TFTR are negative, supporting the previous findings that these modes are neoclassical tearing modes [19]. The method developed in this thesis is more direct and reliable than previous methods. It also represents the first direct determination of Δ' from internal plasma diagnostics.

These two examples demonstrate the advantage of combining theoretical modeling and experimental data analysis. Theoretical models can relate important but hard to measure plasma parameters to experimentally measurable quantities. Therefore, the limited experimental data can be utilized to reveal more secrets of the complicated plasma behavior.

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