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BOUND ON $\Delta B = 1$ COUPLINGS IN THE SUPERSYMMETRIC STANDARD
MODEL

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Abstract

The most general supersymmetric model contains baryon number violating terms of the form $\lambda_{ijk} \bar{D}_i \bar{D}_j \bar{U}_k$ in the superpotential. We reconsider the bounds on these couplings, assuming that lepton number conservation ensures proton stability. These operators can mediate $n \rightarrow \bar{\mu}$ oscillations and double nucleon decay. We show that neutron oscillations do not, as previously claimed, constrain the $\lambda_{abc}$ coupling; they do provide a bound on the $\lambda_{abc}$ coupling, which we calculate. We find that the best bound on $\lambda_{abc}$ arises from double nucleon decay into two kaons; the calculation is discussed in detail. There are no published limits on this process; experimenters are urged to examine this nuclear decay mode. Finally, the other couplings can be bounded by the requirement of perturbative unification.

In the standard electroweak model, conservation of baryon number and lepton number arises automatically from gauge invariance. This is not the case in supersymmetric models, however. In the most general low-energy supersymmetric model, one has terms which violate lepton number and terms which violate baryon number [1]. Since the presence of both of these may lead to unacceptably rapid proton decay (unless the couplings are extraordinarily small), one or both must generally be suppressed by a discrete symmetry [2]. In the most popular model, $R$-parity, given by $(-1)^{N_b + L}$, where $N_b$, $L$, and $F$ are the baryon number, lepton number and fermion number, is imposed, leading to baryon and lepton number conservation. However, there is no a priori reason that $R$-parity must be imposed (other than a desire to conserve baryon number, lepton number and to simplify phenomenology); it is quite possible that only one of the quantum numbers is conserved. There has been extensive discussion of the possibility that lepton number is violated [3], but relatively little investigation of the possibility that lepton number is conserved and baryon number is violated.

In this case, baryon number conservation will be violated in the low energy theory. A term will appear in the superpotential given by

$$\lambda_{ijk} \bar{D}_i \bar{D}_j \bar{U}_k$$

where the indices give the generation number and the chiral superfields are all right-handed isosinglet antiquarks. Since the term is symmetric under exchange of the first two indices, it must be antisymmetric in color, and it is antisymmetric in the first two flavor indices, leaving nine couplings, which will be designated (in an obvious notation) as $\lambda_{12u}, \lambda_{13u}, \lambda_{12d}, \lambda_{13d}, \lambda_{23u}, \lambda_{23d}, \lambda_{12e}, \lambda_{13e}$, and $\lambda_{23e}$.

There are many models in which low energy baryon number is violated and yet lepton number is conserved; the most familiar are some left-right symmetric models (see Ref. [4] for a comprehensive discussion). It is widely believed that the strongest bound on $B$-violating operators comes from neutron oscillations, which violate $B$ by two units. The first discussion [5] on the effects of some of these operators in supersymmetric models used neutron...
oscillations to bound the $\lambda_{\text{dm}}$ and $\lambda_{\text{hn}}$ couplings; this result remains widely cited today [6,7].

The purpose of this Letter is to determine the most stringent bounds that can be placed on these nine couplings. First, we will point out that neutron oscillations do not provide any significant bound at all on the $\lambda_{\text{dm}}$ coupling, due to a suppression factor which was neglected in the original calculation. This suppression is less severe for the $\lambda_{\text{hn}}$ coupling, and we will obtain a bound in that case. We will then note that the strongest bound on the $\lambda_{\text{dm}}$ coupling will come from limits on double nucleon decay (in a nucleus) into two kaons of identical strangeness, and will estimate the bound. A recent work of Brahmacari and Roy [7] noted that the $\lambda_{\text{dm}}$ and $\lambda_{\text{hn}}$ couplings can be bounded by requiring perturbative unification; we will extend their work to cover all of the additional couplings. Finally, we will comment on various bounds which can be obtained by considering products of some of the couplings.\(^1\)

\(^1\)Severe bounds on some of the couplings have been suggested from cosmological arguments, however. Dreiner and Ross [8] have shown that these bounds can be evaded.

Higgs and charginos) in the diagram\(^2\). This flavor change is produced by the box diagram of Figure 1. One can also replace the $W$ with a charged Higgs boson, and the $\tilde W$ should be the lightest chargino, but we will consider the case in which the charged Higgs boson is much heavier than the $W$, and there is little mixing in the chargino sector; for an order of magnitude estimate, this will be sufficient. Since only isodoublets participate in the weak interactions, and the baryon number violating term only has isosinglets, there must be mass insertions (at least two, in the simplest case) which give a transition from $\tilde d_R$ to $\tilde d_L$. This mass insertion will be proportional to the mass of the associated quark. Thus, there will be electroweak interactions in the diagram, and an additional suppression factor of the order of $m^2_s/m^2_W$, where $m_s$ is the strange quark mass. This makes the contribution of the $\lambda_{\text{dm}}$ highly suppressed.

The contribution involving the term $\lambda_{\text{dm}} D B U$ will be suppressed by a factor of $m^2_W/m^2_s$, which is not negligible, and this could give a significant contribution to neutron oscillations.

The relevant diagram is given in Figure 2, where the flavor changing box subdiagram is represented by a blob; the two possible contractions of the legs of the box give an equal contribution. The resulting dimension nine effective operator can be written as (greek indices correspond to color)

$$T_{\alpha\beta\gamma} \epsilon_{\alpha'} \epsilon_{\beta'} \epsilon_{\gamma'} W_{\nu R} d_R \overline{\tilde d}_L A_{\nu'} W_{\nu R} d_{R'}$$

The diagram is calculated at zero external momenta and yields

$$T = \frac{3g^4}{8\pi^2} m^2_W m_{\tilde d_R} M^2_{\nu R} m_{\tilde d_L} A \delta_{ij} J(M^2_{\mu 1}, M^2_{\mu 2}, M^2_{\nu R}, M^2_{\nu L})$$

where the mass term $M_{\mu R}$ which mixes $\tilde b_L$ and $\tilde b_R$ is given by $M_{\mu R} = A m_{\nu R}$, $A$ is the soft supersymmetry breaking parameter (there is also an F-term contribution which we absorb

\(^2\)Should additional sources of strangeness violation exist, such as tree-level flavor changing neutral currents due to an extended Higgs sector, then electroweak interactions would be unnecessary.
into A); j and j' are generation indices, ξ_{jj'} is a combination of KM angles (we assume the left-handed squark KM matrix is the same as that of the quarks):

$$\xi_{jj'} = V_{ij} V_{ij'}^\dagger V_{ki} V_{kj'}^\dagger$$

(4)

and

$$J(m_1^2, m_2^2, m_3^2, m_4^2) = \sum_{m_1} \frac{m_1^4 \ln(m_1^2)}{\prod_{m_2}(m_2^2 - m_1^2)}$$

(5)

The neutron oscillation time is then given by \(\tau = 1/\Gamma\), where \(\Gamma = T\psi(0)^2\). \(\psi(0)^2\) gives the matrix element of the operator in (2); we use the estimate given by P. Pasupathy [8]. \(\psi(0)^2 = 3 \times 10^{-4}\) GeV, but it should be noted that other evaluations [9] differ by more than an order of magnitude (the bound on \(\lambda_{ts}\) will vary as the square root of \(\psi(0)^2\)). From the experimental limit on the neutron oscillation time [10], \(\tau > 1.2 \times 10^6\) sec., we can obtain the bound on \(\lambda_{ts}\). The results depend on the Kobayashi-Maskawa angles, which are taken to be the central values of the allowed ranges, and the squark masses. It is assumed, as is the case in most models, that the charm and up squark masses are degenerate. The bound is plotted in Figure 3 as a function of the top squark mass for various charm squark masses. We keep \(A = m_t = 200\) GeV throughout. Note the peaks which correspond to GIM cancellations in the box diagram. Unless the parameters are tuned to this cancellation, the upper bound on \(\lambda_{ts}\) is between 0.002 and 0.1 if the squark masses are between 200 and 600 GeV, with more stringent limits resulting for lighter masses. As stated above, the bound on \(\lambda_{ts}\) will be weaker by roughly a factor of \(m_t/m_s\), and will not be competitive with the bound in the following paragraph. The bound on the square root of the product \(|\lambda_{ts}\lambda_{td}|\) is suppressed instead by a factor \(\sqrt{m_t/m_s}\). As Figure 3 suggests, unless the supersymmetric particles are lighter than 1 TeV, no useful bounds result from \(a \rightarrow \pi^0\) oscillations. In fact, better bounds can then be derived from perturbative unification as we discuss later.

The fact that the best bound on \(\lambda_{ts}\) comes from double nucleon decay into two kaons of identical strangeness was noted some time ago [11,12]. It is easy to see that the mass insertions and electroweak interactions are unnecessary, since the process does violate both \(B\) and \(S\) by two units. In Ref. 12, a rough order of magnitude estimate was given for the bound (with no details of the estimate). In Ref. 11, it was assumed that, in a nucleus, a neutron could "oscillate" into a \(\Xi\) through two applications of the operator, and that the \(\Xi\) annihilates with another neutron in the nucleus to produce two kaons. Here, we will consider the process \(NN \rightarrow KK\) directly. The relevant diagram is shown in Figure 4, which reduces to dimension nine operators of the form:

$$\frac{16}{3} g_2^2 \frac{\lambda_{ts}^2}{M_H M_T} \int d^6k_1 d^6k_2 \rho(k_1) \rho(k_2) \nu_{\text{rel}}(1 - \overrightarrow{v_1} \cdot \overrightarrow{v_2}) \sigma_{\text{int}}(NN \rightarrow X)$$

(6)

where the ellipsis indicate all possible permutations between the symbols \((u, d, s)\) and the symbols \((u', d', s')\).

The final state must contain minus two units of strangeness, plus some pions. The strange component of the final state can be any of the following: \(K^{*0}\Lambda K^{*0}, K^{*0}\Xi\), \(K^{*0}\Lambda\). Since for each possible final state the corresponding amplitude contains a large number of possible arrangements of the quark legs of the effective operators, a good estimate of the individual rates is very difficult. For the purpose of giving a bound on the order of magnitude of \(\lambda_{ts}\), a rough estimate of these rates should suffice, as they are proportional to the fourth power of \(\lambda_{ts}\). We, therefore, make the following simplifying assumptions: a) The nine terms in (6) add up roughly incoherently, and give similar contributions to the total rate; b) The amplitudes for individual rates are estimated using dimensional arguments, where the relevant scale is a hadronic scale \(\Lambda\), which we will let vary within a generous range; c) It is sufficient for our purpose to consider only a few final states, in particular, the final state \(KK\). We have checked that the addition of other final states like \(KK\pi\) and \(KK\pi\pi\) hardly affect our results.

The total rate is given by:

$$\Gamma = \frac{1}{(2\pi)^3 p_N} \int d^6k_1 d^6k_2 \rho(k_1) \rho(k_2) \sigma_{\text{int}}(NN \rightarrow X)$$

(7)

where \(\rho_N\) is the average nucleon density, \(\rho(k)\) is the nucleon density in momentum space, and the nucleon velocities are taken in the following as small. Using our assumptions, the cross section is approximately given by:
Here the final state phase space was taken to be that for two massless particles. $C_{KK}$ has dimension ten and is approximated by $\Lambda^{10}$. This scale is hard to estimate; direct annihilation of the two nucleons by the dimension nine operator is suppressed due to hard-core repulsion, while the contribution due to $t$-channel $\Xi$ exchange may be the dominant piece. With this we finally obtain the following bound:

$$ \Gamma \sim \rho \pi \sigma^2 \frac{\lambda_{\text{dim}}^4}{M_N^2} \frac{\Lambda^{10}}{M_N^3 M_N^3}. $$

(9)

Using nuclear matter density $\rho = 0.25 \text{fm}^{-3}$, $\alpha_s \sim 0.12$ and a lower bound for nuclear matter lifetime of $\tau_N \sim 10^{40}$ years, we obtain the following bound on $|\lambda_{\text{dim}}|$ in terms of the ratio between the hadronic and the supersymmetric scales $R = \frac{\Lambda}{M_N^3}$:

$$ |\lambda_{\text{dim}}| < 10^{-15} R^{-1/2} $$

(10)

This ranges from as low as $10^{-7}$ for $R \sim 10^{-3}$, to 1 for $R \sim 10^{-4}$). Our bound is comparable to the bound obtained by Barbieri and Masiero [11] by considering the transition $N \rightarrow \Xi$ in nuclei: they obtained a bound approximately given by $5 \times 10^{-18} R^{-1/2}$. Although their hadronic scale is not necessarily identical to ours, we expect them to be similar in value.

What can be said about the other seven couplings? It was recently noted by Brahmacari and Roy [7] that in a unified theory precise bounds can be obtained by requiring that the couplings be perturbative up to the unification scale. They looked at only the $\lambda_{ab}^{ab}$ and $\lambda_{ab}^{\lambda} a$ couplings, although their results can easily be generalized to all of the others. Specializing to real couplings, the renormalization group equations for the $\lambda_{ab}$ are given by

$$ \frac{1}{2\pi} \frac{\partial}{\partial \ln \lambda_{ab}} \lambda_{ab} = \gamma_a' \lambda_{ab} + \gamma_b' \lambda_{ab} + \gamma_{ab}' \lambda_{ab}, $$

(11)

where $\gamma_a' = (2\pi)^{-1/2} \partial \beta_0 (2\pi)^{1/2}$ and $Z_1$ relates the renormalized superfield $\Phi^1$ to the unrenormalized $\Phi_0$. For example, the renormalization group equation for $\lambda_{ab}$ is

$$ \frac{1}{2\pi} \frac{\partial}{\partial \ln \lambda_{ab}} \lambda_{ab} = \lambda_{ab} (\gamma_a' + \gamma_b' + \gamma_{ab}' + \lambda_{ab} \gamma_a' + \lambda_{ab} \gamma_b' + \lambda_{ab} \gamma_{ab}'). $$

(12)

The anomalous dimensions can be obtained from the formulae listed by Martin and Vaughn [13]. The diagonal $\gamma$'s are given by

$$ 16\pi^2 \gamma_a^1 = 2(\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda}) - \frac{8}{3} g_s^2 \frac{2}{15} \delta_{a}^2 \delta_{a}^2 $$

(13)

$$ 16\pi^2 \gamma_* = 2(\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda}) - \frac{8}{3} g_s^2 \frac{8}{15} \delta_{a}^2 \delta_{a}^2 $$

(14)

for the $u$ and $d$ superfields; the generalization to the other generations is obvious by permutation. There is an additional term $2\delta_{a}^2$ on the right hand side of the expression for $16\pi^2 \gamma_*^1$ due to the top quark Yukawa coupling. The off-diagonal $\gamma$'s are given, for example, by

$$ 16\pi^2 \gamma_{ab}^1 = 2(\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda}) $$

(15)

$$ 16\pi^2 \gamma_* = 2(\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda}) $$

(16)

If we define $Y = (\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda})/4\pi$, $Y_3 = (\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda})/4\pi$ and $Y_1 = (\lambda_{ab}^{ab} + \lambda_{ab}^{\lambda} + \lambda_{ab}^{\lambda})/4\pi$, then the renormalization group equations can be combined to yield expressions for $Y_1$, $Y_2$ and $Y_3$. These equations, which in general cannot be written in terms of the $Y_i$'s alone, take a particularly simple form when one of the $Y_i$'s dominates (e.g., when the couplings in $Y_1$ and $Y_2$ do not significantly contribute to the beta-function for $Y_3$), we have

$$ \frac{\partial Y_i}{\partial \ln \Lambda} = 6 Y_i^2 - 8 \alpha_s Y_i + 2 \delta_{a}^2 \delta_{a}^2 Y_i. $$

(17)

We now simply require that the $Y_i$ not become nonperturbative (i.e. $Y_i < 1$) by the unification scale; this leads to a bound at low energies on the $Y_i$. The renormalization group equations for $Y_1$, $Y_2$ and (if the top quark Yukawa coupling is small) $Y_3$, using $\alpha_s$ as it results from the one loop beta function, can be solved exactly yielding

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3The factors of 2 in front of the $\lambda$ terms were given as 6 in Ref. 7. We thank Herbi Dreiner for pointing out the correct expressions.

4We ignore the hypercharge term, since it is clearly smaller than the uncalculated two-loop strong interaction corrections.
\[ Y(\mu) = \frac{\bar{\gamma}(\mu)}{5 \alpha_s(\mu) + C_Y} \]  

(18)

where we replaced \( t = \frac{1}{v} \log(\frac{1}{x}) \), and \( C_Y \) is given in terms of \( Y(\mu = M_W) \)

\[ C_Y = \alpha_s^2(M_W) \left( \frac{1}{Y(M_W)} - \frac{6}{5} \alpha_s^2(M_W) \right). \]  

(19)

For \( \alpha_s(M_W) = 0.125 \), the requirement of perturbative unification yields a bound of \( Y_t(M_W) < 0.124 \) giving an upper bound of 1.25 on \( \lambda_{ae}, \lambda_{ab}, \lambda_{au}, \lambda_{db}, \lambda_{db} \) and \( \lambda_{de} \). The bound on \( \lambda_{ae}, \lambda_{ab} \) and \( \lambda_{de} \) does depend on the top quark Yukawa coupling and must be integrated numerically. As shown by Brahmchari and Roy, however, the resulting bound is very insensitive to \( \tan \beta \) and is also insensitive to the top quark mass—using a top quark mass below about 180 GeV changes the upper bound by less than ten percent, and using a heavier top quark causes the top quark Yukawa coupling to become nonperturbative by the unification scale. It is worthwhile to mention that \( Y(\mu) \) decreases up to values of \( \mu \) around \( 10^7 \) GeV, an effect due to the running of \( \alpha_s \). Note that the first two couplings are bounded already from neutron oscillations or nuclear decay, but the bound of approximately 1.25 is the strongest bound on the remaining seven. If one relaxes the assumption that only one \( Y_t \) dominates, it is easy to show that the bounds, in all cases, become strengthened. This is because all the new contributions to the RGE of equation (17) are positive.

The bounds we have obtained are on the individual couplings. On the other hand, bounds can be obtained on the products of various couplings. These bounds are discussed in detail by Barbieri and Maierhofer [11], who considered the effects of these interactions in \( K - \bar{K} \) mixing, on \( c'/c \) and on the neutron electric dipole moment. The first of these gives bounds on \( \lambda_{ae} \lambda_{ab} \) and on \( \lambda_{ab} \lambda_{ae} \); the latter two give bounds on the imaginary part of the product of two couplings. The reader is cautioned that these bounds were obtained for a top quark mass of 45 GeV, although changing them to incorporate a heavier top quark is simple.

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Figure 1: Box diagram giving the necessary neutral flavor mixing necessary in $\nu-\bar{\nu}$ oscillations. Similar diagrams appear with $\tilde{b}$ substituted by $\tilde{s}$.

Figure 2: Diagram mediating $\nu-\bar{\nu}$ oscillations. All particles with no index are right handed. The big blob represents the insertion of the box in Figure 1, and the small blobs in the $\tilde{b}$ propagators are insertions of $M_{\tilde{b}}$. 
Figure 3: Bound on $|\lambda_{\Delta n}|$ from $n-\bar{n}$ oscillations as a function of $M_{\tilde{f}}$. The solid line corresponds to $M_\ell = M_\ell' = 200 \text{ GeV}$, the dashed line to $400 \text{ GeV}$ and the dotted line to $600 \text{ GeV}$.

Figure 4: $\Delta B = -2$, $\Delta S = -2$ diagrams contributing to double nucleon decay. $(q_i, q_j, q_k)$ corresponds to permutations of $(u, d, s)$, and similarly for the primed quarks.