FORM FACTORS OF KAON SEMILEPTONIC DECAYS

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A calculation of the semi–leptonic decays of the kaon ($K_{l3}$) is presented. The results are direct predictions of a covariant model of the pion and kaon introduced earlier by Ito, Buck, Gross. The weak form factors for $K_{l3}$ are predicted with absolutely no parameter adjustments of the model. We obtained for the form factor parameters: $f_-(q^2 = m_l^2)/f_+(q^2 = m_l^2) = -0.28$ and $\lambda_+ = 0.028$, both within experimental error bars. Connections of this approach to heavy quark symmetry will also be discussed.

A covariant model based on coupled Bethe–Salpeter and Schwinger–Dyson equations with an NJL–like interaction was successfully used earlier to describe electromagnetic properties of pions and kaons.

Using this model, we analysed form factors of $K_{l3}$ decays (describing the processes $K \rightarrow \pi l\nu$). Assuming that only the vector weak current contributes to the decays, the matrix element may be written as

\[ J_\mu = \frac{G_F \sin \theta_C}{\sqrt{2}} [f_+(t)(P_K + P_\pi)_\mu + f_-(t)(P_K - P_\pi)_\mu], \]

where $P_K$ and $P_\pi$ are the four–momenta of the $K$ and $\pi$ mesons, $G_F$ is a Fermi coupling constant, $\theta_C$ is a Cabibbo angle, and $f_+$ and $f_-$ are dimensionless form factors which can depend only on $t = (P_K - P_\pi)^2$. For the decay channel, the physical region is limited by the masses of the lepton, pion and kaon: $m_l^2 \leq t \leq (m_K - m_\pi)^2$.

In our model, the pion and the kaon both share the same light quark mass of 250 MeV. It is this mass, together with the corresponding mesonic vertex functions (wave functions) employed in the calculation of the elastic and transition form factors of the pion and kaon that is now employed in the

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$K_{l3}$ decays addressed here. The pion calculation provides a non-perturbative solution to the axial anomaly for space-like $Q^2$ and the charged pion form factor, while the kaon wave function provides a good description of the charged and neutral kaon form factors.

The main results of the predictive calculation of the $K_{l3}$ decays are illustrated in the Table 1 and compared to selected theoretical models and experiment (see also Ref. 3). Results are in quite good agreement with available empirical data for all observables studied and, when one quark is infinitely heavy, we reproduce the heavy quark symmetry limit for the ratio $f_{-}/f_{+}$.

Table 1: Model predictions for the parameters of $K_{l3}$ decay form factors.

<table>
<thead>
<tr>
<th></th>
<th>CPT</th>
<th>VMD</th>
<th>ISGW2</th>
<th>This work</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0.0312</td>
<td>0.0245</td>
<td>0.019</td>
<td>0.028</td>
<td>0.0286±0.0022 $(K_{e3})$</td>
</tr>
<tr>
<td>$\xi_A$</td>
<td>$-0.164±0.047$</td>
<td>$-0.28$</td>
<td>$-0.28$</td>
<td>$-0.28$</td>
<td>$-0.35±0.15 (K_{\mu3})$</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>$0.017±0.004$</td>
<td>0.0</td>
<td>$-0.005^{**}$</td>
<td>0.0026</td>
<td>$0.004±0.007 (K_{\mu3})$</td>
</tr>
</tbody>
</table>

It should be noted that Jefferson Lab measurements of $\pi$ charge form factor $(E=93-021)$, kaon charge form factor $(E=93-018)$, a $\gamma p \rightarrow \gamma p$ measurement at the $\pi^0$ pole, and $K_{l3}$ transitions will further constrain theoretical models addressing all electroweak pion–kaon reactions. More to the point, all of these experiments are connected; in fact, we contend that they must be connected without new parameter fits or adjustments.

It is speculated that to employ these theoretical techniques in the study of the nucleon, our quark masses must be accompanied by a rather large interquark kinetic energy in order to bind the nucleon and extract form factors.

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References