$E_{\pi}/M_{\pi}$ and $S_{\pi}/M_{\pi}$ from an Analysis of $p(e, e'p)\pi^0$ in the region of the $\Lambda (1232)$ Resonance at $Q^2 = 3.2 \text{ (GeV/c)}^2$

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$E_{1+}/M_{1+}$ and $S_{1+}/M_{1+}$ from an Analysis of $p(e,e'p)\pi^0$ in the Region of the 
$\Delta(1232)$ Resonance at $Q^2 = 3.2$ (GeV/c)$^2$

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ABSTRACT

In this paper we present an analysis of exclusive $p(e,e'p)\pi^0$ data to determine the 
electromagnetic and scalar transition multipoles in the mass region of the $\Delta(1232)$ at the 
highest $Q^2$ value where data exist, $Q^2 = 3.2$ (GeV/c)$^2$.

1. Introduction

In recent years there have been numerous discussions about the applicability of perturbative QCD (pQCD) in the description of various exclusive reactions at moderately high energy and momentum transfers. Some authors$^1,2$ have argued that such a description may be applicable at momentum transfers as low as 3 to 5 (GeV/c)$^2$. Others$^3$ maintain that pQCD may only be applicable to exclusive reactions at much higher momentum transfers. We focus here on the $\gamma p\Delta(1232)$ transition, where dynamical quark models and pQCD make rather distinct predictions, and for which data at reasonably high $Q^2$ exist. A crucial test of our understanding of the $\Delta$ excitation, and the regions of validity of the different models, is to determine the electric and scalar quadrupole moments $E_{1+}$ and $S_{1+}$, and the magnetic dipole moment $M_{1+}$. The ratios of these multipoles are sensitive to the fundamental ingredients of the models. For example, in $SU(6)$ symmetric quark models, the $\gamma N\Delta$ transition is mediated by a single quark spin flip in the nucleon ground state leading to $M_{1+}$ dominance and $E_{1+} = S_{1+} \equiv 0$, while helicity conservation in pQCD requires $E_{1+} = M_{1+}$ as $Q^2 \to \infty$.

In this paper we present a brief description and discussion of the analysis, results for details and a list of references the reader should consult reference.$^4$

2. Analysis Method

We use a generalization of the model of Walker$^5$ which was used successfully for the analysis of photoproduction data.

The differential cross section for pion production can be written as:

$$\frac{d\sigma}{d\Omega_x} = \sigma_T + \epsilon \sigma_L + \epsilon \sigma_{TT} \cos 2\phi + \sqrt{\epsilon(1 + \epsilon)/2 \sigma_{TL} \cos \phi}$$

The cross section can be expressed in terms of 6 parity conserving helicity amplitudes $H_i$$^5$:

$$\sigma_T = 1/2 \cdot F \cdot (|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2)$$
$$\sigma_{TT} = 1/2 \cdot F \cdot Re(H_2 H_3^* - H_1 H_4^*)$$

1
\[
\sigma_L = F \cdot (|H_8|^2 + |H_6|^2)
\]
\[
\sigma_{TL} = \sqrt{2} \cdot F \cdot \text{Re}[H_8(H_4^* - H_1^*) + H_6(H_3^* + H_2^*)]
\]
with
\[
F = \frac{2MW|q^*_e|}{W^2 - M^2}
\]

where \(q^*_e\) is the pion 3-momentum in the hadronic cms frame. We define \(\bar{H}_i = H_i - H_i^{\text{Born}}\), where \(H_i^{\text{Born}}\) are the helicity amplitudes of the electric Born terms. At the tree level, only the s channel and u channel graphs contribute to \(\pi^0\) production. The Born terms are included as known contributions, to avoid unreasonably high partial waves in the partial wave expansion. The \(\bar{H}_i\) are expanded in terms of Legendre Polynomials for the pion orbital angular momentum \(l_\pi = l, l+1\), and the total resonance spin \(J = l_\pi \pm \frac{1}{2}\):

\[
\bar{H}_1 = \frac{1}{2} \sqrt{2} \sin \theta \cos \frac{\theta}{2} \sum_{l=0}^{\infty} (B_{l+} - B_{l+1-}) \cdot (P''_l - P''_{l+1})
\]
\[
\bar{H}_2 = \sqrt{2} \cos \frac{\theta}{2} \sum_{l=0}^{\infty} (A_{l+} - A_{l+1-}) \cdot (P'_l - P'_{l+1})
\]
\[
\bar{H}_3 = \frac{1}{2} \sqrt{2} \sin \theta \sin \frac{\theta}{2} \sum_{l=0}^{\infty} (B_{l+} + B_{l+1-}) \cdot (P''_l + P''_{l+1})
\]
\[
\bar{H}_4 = \sqrt{2} \sin \frac{\theta}{2} \sum_{l=0}^{\infty} (A_{l+} + A_{l+1-}) \cdot (P'_l + P'_{l+1})
\]
\[
\bar{H}_5 = \sqrt{2} \cos \frac{\theta}{2} \sum_{l=0}^{\infty} (C_{l+} - C_{l+1-}) \cdot (P'_l - P'_{l+1})
\]
\[
\bar{H}_6 = 2 \sin \frac{\theta}{2} \sum_{l=0}^{\infty} (C_{l+} + C_{l+1-}) \cdot (P'_l + P'_{l+1})
\]

where \(\theta\) is the pion cms production angle. For the \(\Delta(1232)\), the multipoles \(E_{l+}, M_{l+}, S_{l+}\) are related to the partial wave helicity amplitudes like:

\[
2A_{l+} = M_{l+} + 3E_{l+}
\]
\[
B_{l+} = E_{l+} - M_{l+}
\]
\[
C_{l+} = 2\frac{\sqrt{Q^*}}{|Q^*_e|} S_{l+}
\]

where \(Q^*\) is the photon 3-momentum in the cms frame. The resonant helicity elements are described by sums of relativistic Breit-Wigner amplitudes with energy-dependent widths, and a phenomenological background which has been assumed to be real has the correct threshold behavior and small energy dependence.
Fig. 1. (a): Examples of our fit to the pion center-of-mass angular distribution at fixed values of $W$ and $\phi$. (b): Examples of our fit to the $\pi^0$ electroproduction data in the $W$ plane ($\mu$barn/sterad) for different $\theta$ and $\phi$ bins.

3. Results and Summary

The data used in the analysis are differential cross sections from DESY$^6$ on $\pi^0$ production off protons, at fixed $Q^2 = 3.2$ (GeV/$c$)$^2$. They extend over an invariant mass range of 1.145 GeV/$c^2$ to 1.715 GeV/$c^2$. Figure 1 shows examples of the $\pi^0$ differential cross section data together with our best fit.

The ratio of the electric quadrupole and the magnetic dipole transition moments is determined to be:

$$E_{1+}/M_{1+} = 0.06 \pm 0.02(\text{fit}) \pm 0.03(\text{syst.}),$$

and for the scalar quadrupole transition we find:

$$S_{1+}/M_{1+} = 0.07 \pm 0.02(\text{fit}) \pm 0.03(\text{syst.}).$$
Fig. 2. Examples of the fit using the pQCD constraint $E_{1+} = M_{1+}$ in the $W$ plane (μbarn/sterad) for different $\theta$ and $\phi$ bins.

The systematic uncertainties were estimated using various parametrizations of the non-resonant amplitudes and by including or omitting higher mass resonant states, and shifting the mass of the $\Delta$ resonance. The results for $R_{EM}$ and $R_{SM}$ are stable within the given systematic uncertainties.

Fig. 2 shows the fit using the pQCD constraint $E_{1+} = M_{1+}$ indicating a clear failure to reproduce the data. The results rule out that perturbative QCD governs the dynamics of the $\gamma p\Delta$ transition at such $Q^2$. They are more consistent with predictions of relativistic extensions of the non-relativistic constituent quark model. For the $M_{1+}$, which describes the dominant magnetic dipole transition to the $\Delta(1232)$ we find the value $|M_{1+}| = 0.681 \pm 0.017 \pm 0.025$ $\sqrt{\mu b}$ at $Q^2 = 3.2 (GeV/c)^2$.

References