ESTIMATING THE UNCERTAINTY IN REACTIVITY ACCIDENT NEUTRONIC CALCULATIONS

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ABSTRACT

A study of the uncertainty in calculations of the rod ejection accident in a pressurized water reactor is being carried out for the U.S. Nuclear Regulatory Commission. This paper is a progress report on that study. Results are presented for the sensitivity of core energy deposition to the key parameters: ejected rod worth, delayed neutron fraction, Doppler reactivity coefficient, and fuel specific heat. These results can be used in the future to estimate the uncertainty in local fuel enthalpy given some assumptions about the uncertainty in the key parameters. This study is also concerned with the effect of the intra-assembly representation in calculations. The issue is the error that might be present if assembly-average power is calculated, and pin peaking factors from a static calculation are then used to determine local fuel enthalpy. This is being studied with the help of a collaborative effort with Russian and French analysts who are using codes with different intra-assembly representations. The U.S. code being used is PARCS which calculates power on an assembly-average basis. The Russian code being used is BARS which calculates power for individual fuel pins using a heterogeneous representation based on a Green's Function method.

INTRODUCTION

Background

This study (and others) have been carried out for the U.S. Nuclear Regulatory Commission (NRC) to understand fuel behavior at burnups beyond the current licensing limits. When behavior is sufficiently understood, new acceptance criteria may be proposed for design-basis reactivity initiated accidents (RIAs) in high burnup fuel (and perhaps for fuel at burnups already experienced in operating plants). Acceptance criteria have traditionally been expressed in terms of maximum fuel pellet enthalpy, and hence it is of interest to know what is the fuel

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1This work was performed under the auspices of the U.S. Nuclear Regulatory Commission.
enthalpy during an RIA. "Best-estimate" methods are available to answer this question, but it is necessary to also understand the uncertainty in the calculated fuel enthalpy.

The above explains the principal motivation for this study which focuses on the design-basis reactivity accident for a pressurized water reactor (PWR). However, it should also be noted that NRC licensees may be moving away from the traditional, conservative methods for calculating fuel enthalpy toward best-estimate methods, and these methods will require an uncertainty analysis. Hence, the study reported upon herein may have a role in regulatory decisions approving new methods for, and the results of, analyzing RIAs.

In a previous study at Brookhaven National Laboratory, the uncertainty in calculating fuel enthalpy for the rod drop accident in a boiling water reactor (BWR) was addressed [1]. That study indicated that the random error in the calculated fuel enthalpy could be approximately ±75% at the 2σ level. It also showed that there could be an additional systematic error of 25°A due to the way the intra-assembly power peaking was calculated. This latter error in combination with the random error meant that the calculated fuel enthalpy has to be increased by approximately 100% to obtain results at the 95-95 confidence level.

The current study looks at the RIA for a PWR—namely, the rod ejection accident (REA). Because the intra-assembly calculation of power was important for the BWR, it was felt that this should be studied for the PWR and with more rigorous methods than were used for the BWR study. Calculations done as part of a recent Russian study have shown that the peak fuel pin enthalpy during an REA may not be found in the assembly with the peak average fuel enthalpy [2]. In the West, it is typical to use methods that homogenize the assembly and then calculate the assembly response to the REA. The state-of-the-art is changing as flux reconstruction methods are being introduced to improve the intra-assembly representation. Nevertheless, the Russian results are another reason to study the effect of the intra-assembly representation.

Objective

The objective of this study is to improve our understanding of the uncertainty in fuel enthalpy calculated for the REA. The approach is twofold. Sensitivity studies are to be carried out to determine the effect on calculated fuel enthalpy of uncertainties in the important parameters which determine the outcome of the REA. The ultimate objective is to use the sensitivity to estimate the random error in the fuel enthalpy due to random errors in these key parameters.

The second approach in this study is to compare the results for the REA using a code that treats the assembly as an homogenized region and a code that represents each pin in the assembly explicitly. This would give an estimate of the uncertainty in results due to the intra-assembly representation. The PARCS code (Purdue Advanced Reactor Core Simulator) [3], like several other nodal codes in use in the West, treats each assembly as an homogenized region. A newer version of PARCS, with a flux reconstruction method, will soon be available and can be used to obtain additional information about the effect of the intra-assembly representation. The BARS code [2], developed at the Russian Research Centre - Kurchatov Institute (RRC-KI), uses a heterogeneous method wherein each fuel pin is represented explicitly. Hence, it is an
objective of this study to compare results for the REA from PARCS and BARS in order to understand the effect of the intra-assembly representation. It should also be noted that since PARCS is a relatively new addition to the computer tools used by the NRC, the comparison of PARCS and BARS is likely to contribute to the code assessment carried out for PARCS.

Further information on the effect of the intra-assembly representation will come from the same analysis being carried out by the French Institute for Nuclear Safety and Protection (IPSN) and from calculations expected to be carried out using the version of PARCS with the flux reconstruction model.

Scope of this Paper

The results of the sensitivity analysis for the REA are presented in the following section. In the future, these results will be applied to give an estimate of the uncertainty in the calculated fuel enthalpy based on the random errors expected in the key parameters which enter into the calculation. The comparison being carried out between PARCS and BARS is discussed in the next section. This includes some of the limitations of that comparison.

SENSITIVITY STUDIES

Introduction

Although fuel enthalpy is the parameter of interest, the calculations shown in this paper were of the sensitivity of energy deposition. If the event is adiabatic, then energy deposition and fuel enthalpy are essentially identical. The calculations were for total energy deposition rather than for energy deposition at the position of peak power. At a later time, when the edits become available in PARCS, local fuel enthalpy will be considered.

The sensitivity is the relative change in energy deposition \( Q \) per relative change in key reactor parameter \( x \). Hence, the sensitivity to \( x \) is

\[
S_x = \frac{\delta Q/Q}{\delta x/x}
\]

The parameters of interest are well known from previous studies (e.g., [1]) and are related to the reactivity insertion above the prompt critical condition and the negative reactivity feedback from the energy deposition. There are four key parameters which control these phenomena. The first is the reactivity worth of the ejected control rod, \( \rho_p \). This parameter is determined by the core design and by the operating procedures which determine the extent of insertion of the rod and the placement of other control rods at any operating condition. The second parameter is the delayed neutron fraction, \( \beta \). For a given core design, this parameter changes significantly as the fuel burnup changes. The fuel feedback can be expressed in terms of the fuel temperature (or Doppler) reactivity coefficient, \( \alpha \), and the specific heat, \( C_p \), of the pellet which translates energy into temperature. The Doppler coefficient is determined by core
design and the time during the fuel cycle whereas the specific heat changes relatively little with burnup.

**Sensitivity to Key Parameters**

The sensitivity to control rod worth is given in Figure 1 where it is plotted versus control rod worth in units of $, i.e., \( R = \rho_0/\beta \). The data points on the graph come from PARCS calculations of different reactivity insertion events. Only events with rod worths greater than $1 are of interest. The energy deposition is that deposited during the initial power pulse.

**Figure 1 Sensitivity of Energy Deposition to Reactivity Insertion**

It is well-known that the power excursion resulting from a reactivity insertion above prompt critical consists of an initial power pulse turned around by feedback followed by a slow decrease in power, due to delayed neutrons, at a level that is still significant as far as energy deposition is concerned. This relatively simple behavior is amenable to simple models.

Using the point kinetics model and the Nordheim-Fuchs approximation an expression for the sensitivity to rod worth can be derived for the time up to the end of the initial power pulse. That expression is \( S_p = R/(R-1) \) which is plotted on Figure 1. It can be seen that the data points are in good agreement with the theory for the range shown. As the rod worth approaches $1 (from above), corresponding to prompt critical, the simplified expression has
a singularity and is no longer valid. Nevertheless, the sensitivity does increase as rod worth approaches $1$. Although this is true, it is also true that the energy deposition becomes smaller and this sensitivity may become less important. The fact that the energy deposition gets smaller has been shown by many analysts. The corresponding result for the energy deposition from the simplified model introduced above also shows this trend:

\[ Q = 2(p_0 - \beta)/(\alpha C_p) \]

The sensitivity of energy deposition to delayed neutron fraction is shown in Figure 2 as a function of reactivity insertion. The data points from PARCS are plotted for both the initial power pulse and by accounting for energy deposition out to three seconds. In addition, the graph shows the curve obtained from the simplified model which predicts a sensitivity of \( S_\beta = -1/(R-1) \). Again it is seen that the results for the initial power pulse are in agreement with the theoretical results for the range shown except that as rod worth approaches $1$ from above the agreement begins to fail. The sensitivity to 3 s is less than for the initial power pulse as the energy deposited after the initial power pulse is not dependent on rod worth but only on the delayed neutron decay.

![Figure 2](image.png)

**Figure 2**  Sensitivity of Energy Deposition to Delayed Neutron Fraction
The sensitivity of energy deposition to fuel heat capacity is shown in Figure 3 as a function of reactivity insertion. Energy deposition is assumed to be to either the end of the initial power pulse or to 3 s to obtain the points plotted on the graph. The corresponding sensitivity from the simplified model is $S_C = 1.0$ which is also drawn on the graph.

![Figure 3 Sensitivity of Energy Deposition to Heat Capacity](image)

It is not possible to plot the sensitivity to the Doppler coefficient as the Doppler coefficient is not a parameter that can easily be extracted from a space-dependent kinetics calculation.

**Uncertainty Analysis**

The above results are the first step in obtaining an estimate of the local fuel enthalpy during an REA. The sensitivity will be different when it is the local fuel enthalpy rather than the core energy deposition (as above) that is being assessed. This difference will be quantified in the future after the appropriate edits are available in PARCS. An estimate of the uncertainty in fuel enthalpy can be made from the sensitivity if the uncertainty in the fundamental parameters is known. For example, from Figure 2, the sensitivity of energy deposition to delayed neutron fraction is as high as -6 at the end of the initial power pulse and declines to -3 at 3 s. This is the maximum assuming a rod worth just above prompt critical. If we use the -3 value and assume that the uncertainty at the 1σ level for the delayed neutron fraction is approximately 10% then the uncertainty in energy deposition is ±30% due to this key parameter. This type of analysis would have to be done for all parameters in order to complete the analysis.
EFFECT OF DIFFERENT INTRA-ASSEMBLY REPRESENTATIONS

The effect of the intra-assembly representation on the uncertainty in fuel enthalpy is being assessed by comparing codes with different models. The principal codes involved are PARCS the code being used by NRC and BARS, a code developed by the RRC-KI. However, in addition, CRONOS, a code being used by the French Nuclear Protection and Safety Institute (IPSN) and a new version of PARCS with flux reconstruction, will be used to make comparisons.

The PARCS code uses a nodal approximation wherein each assembly is homogenized and the power (flux) is calculated for each axial region in the assembly (or perhaps in a quadrant of the assembly if the mesh is smaller than an assembly). The assembly is homogenized so that neutron cross sections are uniform across the assembly and the thermal-hydraulic parameters are calculated for an average channel representing the assembly. The power in individual fuel rods is obtained by overlaying power peaking factors obtained from an auxiliary calculation. Traditionally, this auxiliary calculation is from a static assembly calculation with reflective boundary conditions, i.e., without consideration of what is happening in adjacent assemblies.

The BARS code uses a Green’s Function approach wherein each fuel pin is represented explicitly in the time-dependent calculation. The Green’s Functions are based on diffusion theory. Although each pin is represented explicitly in the neutronics calculation, the fuel temperature for each pin is based on an assembly-average calculation. This model will be based on a pin-by-pin calculation in the future.

Although this comparison will provide insight into the effect of the intra-assembly representation, there are other differences between PARCS and BARS which will have a bearing on the calculation of fuel enthalpy. Although the same ENDF/B nuclear data files are used, the data processing codes are different and the nuclear data that enters into PARCS and BARS has a different theoretical basis. For PARCS the CASMO-3 code was used to generate two-group data whereas for BARS, the TRIFON code is used to produce the necessary lambda matrices. The data for BARS is for five energy groups rather than the two that are used in PARCS because the heterogeneous representation puts greater demands on representing changes in spectrum between different regions.

Other differences between the two codes include the axial representation, the time integration method, and the thermal-hydraulics model. Each of these are not expected to have a large effect but the cumulative effect could be significant.

At the time of this paper the specifications of the problem have been completed but no comparisons have yet been done. The specifications refer to the reactor composition and the list of calculations that will be compared. Since the codes do not use the same modeling it is not possible to start from a given cross section data set as has been done in several benchmark exercises in the past. Rather one must specify the composition and geometry of every region within the core: pellet, clad, guide tube, burnable poison rod, control rod, etc. The reactor model is for a core with exposures into the 50 GWd/t range and hence each pellet has a different composition due to burnup.
Prior to comparing results for an REA, results will be compared for steady state calculations of reactivity coefficients, control rod worths, and power distributions. It is only when the differences in results for these parameters are assessed that it makes sense to progress to comparing transient results.

REFERENCES

