Instantaneous Reactive Power and Power Factor of Instantaneous Phasors

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Abstract

The unique property of instantaneous phasors is that at any instant the instantaneous three-phase currents and voltages can be represented by a set of balanced phasors. The instantaneous reactive power and the concept of instantaneous power factor can be clearly understood from the instantaneous phasors. This provides a theoretical foundation for power quality monitoring, diagnostics, and compensation methods.

I. INTRODUCTION

The instantaneous phasor method originated by the author has a unique symmetrical property. Regardless of how unbalanced the three-phase situation is, the instantaneous phasors of one phase can be used to represent three phases[1-4]. Three-phase currents and voltages can be represented by a set of balanced instantaneous phasors, respectively.

Traditionally, the concept of power factor is the ratio of active power and the product of the root-mean-square (rms) values of current and voltage over a period of time. Recent interesting developments on the instantaneous reactive power and instantaneous power concept [5-7] have been proven to be useful for power quality and utilization improvements.

The unique instantaneous phasors discussed in this paper not only provide a clear picture of the instantaneous reactive power and the instantaneous power factor, they also give a clear overall power quality picture that is not limited to the instantaneous instant. The roundness of the trajectory of instantaneous phasors of a fundamental cycle indicates the quality of currents and voltages. The instantaneous phasors provide theoretical foundation for power quality monitoring, diagnostics, and improvements.

The instantaneous phasors of voltages and currents derived in [1] can either be presented in a vector format or in a complex number format. The arbitrarily chosen complex number format of the instantaneous phasors, such as the voltages, \( V_a, V_b, \) and \( V_c \), are given in (1). They have the same magnitude but are 120-degrees apart.

\[
V_a = (v_a - v_0) + jv_{aq}, \\
V_b = (v_b - v_0) + jv_{bq}, \text{ and} \\
V_c = (v_c - v_0) + jv_{cq},
\]

where the zero-sequence component for the three-phase voltages is

\[
v_0 = \frac{1}{3}(v_a + v_b + v_c).\]

Alternatively, a more general expression of (1) to include the zero-sequence components in the equations by shifting the origins of phasors can be adapted.

The real values of the instantaneous phasors are simply the instantaneous phase values without the zero-sequence component as given in (3).

\[
(v_a - v_0), \\
(v_b - v_0), \text{ and} \\
(v_c - v_0).
\]

The instantaneous phasors' imaginary values denoted as \( v_{aq}, v_{bq}, \) and \( v_{cq} \) can be obtained from (4).
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The numerators of equation (4) are actually the instantaneous line to line voltages that are not affected by the zero-sequence component.

The instantaneous phasor magnitude, \( V \), of \( V_a, V_b, \) and \( V_c \) can be derived from (2), (3), and (4). The result is that

\[
\|V_a\| = \|V_b\| = \|V_c\| = V,
\]

where

\[
V = \sqrt{\left(\frac{v_a}{\sqrt{3}} - \frac{v_b}{\sqrt{3}}\right)^2 + \left(\frac{v_b}{\sqrt{3}} - \frac{v_c}{\sqrt{3}}\right)^2 + \left(\frac{v_c}{\sqrt{3}} - \frac{v_a}{\sqrt{3}}\right)^2}.
\]

Alternatively, the instantaneous phasor magnitude \( V \), of \( V_a, V_b, \) and \( V_c \) can be derived from a phase, for instance, from phase-a

\[
V = \|V_a\| = \sqrt{(v_a - v_0)^2 + v_{aq}^2},
\]

from phase-b

\[
V = \|V_b\| = \sqrt{(v_b - v_0)^2 + v_{bq}^2},
\]

and from phase-c

\[
V = \|V_c\| = \sqrt{(v_c - v_0)^2 + v_{cq}^2}.
\]

II. ROUNDNESS AND COMPONENTS OF TRAJECTORIES OF INSTANTANEOUS PHASORS

A. Roundness Of Trajectories Of Instantaneous Phasors

The voltages and currents obtained from a field test of a 7.5-hp, 4-pole induction motor are shown in Fig. 1. The voltages are slightly unbalanced, and the currents are significantly more unbalanced. The roundness of a trajectory indicates how balanced the three phases are.

The trajectories of instantaneous voltage phasors, \( V_a, V_b, \) and \( V_c \), of (1) are shown in Fig. 2, where the three phasors are always identical in magnitude but are 120-degrees apart. The same observations can be drawn from the trajectories of instantaneous current phasors, \( i_a, i_b, \) and \( i_c \), shown in Fig. 3.
Figs. 2 and 3 also show that for the polluted voltages and currents, the instantaneous phasor trajectories are not circles because of harmonics and negative-sequence content. The instantaneous phasors are not rotating at a constant speed [1, 2]. The six peaks shown in the current trajectory suggest a strong fifth or seventh harmonic content.

The unique symmetrical property of the instantaneous phasors of three phases permits taking the voltage and current phasors of one phase to calculate the various power components of the entire motor.

**B. Components Of Trajectories Of Instantaneous Phasors**

If necessary, the trajectories of instantaneous phasors shown in Figs. 2 and 3 can be broken down into the phasors for various frequency components. Subsequently, the phasors of each frequency component can be further broken down into harmonics, positive and negative-sequence phasor components.

Fig. 4 shows the flow chart for obtaining the fundamental three-phase balanced components. The bottom left of the figure is the input example of three-phase voltages, $v_a$, $v_b$, and $v_c$. Accordingly, the phasor-$a$ imaginary portion, $v_{aq}$, and the real portion, $v_a - v_0$, can be calculated through (3) and (4). Consequently, the instantaneous phasor magnitude, $V$, is computed from (6), (7), (8), or (9).

The second harmonic of the instantaneous phasor can be obtained through a second-order-harmonic band pass filter. Since only zero crossing points of the second harmonic are of interest, a low-Q filter that normally has fast response is good enough. The positive and negative peaks of the second harmonic are 45-degrees apart from the zero crossing points and are used to find the initial and synchronization phasors' positions. Alternatively, a fast Fourier transform (FFT) may be used for the same purpose. The theory given in [2] is briefly described as follows. The maximum and minimum peaks of the second harmonic correspond to the in-phase positions of the fundamental-frequency positive and negative sequences [2] as shown in Fig. 5. When the positive-sequence phasor and the negative-sequence phasor coincide in the same direction, the instantaneous phasor has the maximum magnitude (points B and D) and is in phase with the positive-sequence component. When the positive-sequence phasor and the negative-sequence phasor coincide in opposite directions (points A and C), the instantaneous phasor has the minimum magnitude and is again in phase with the positive-sequence component.
The fundamental voltage and current phasors of phase $a$ are given in Figs. 6 and 7. They are not in perfect circles because of the unbalanced voltages and currents [1].

These instantaneous phasor components can be used for detailed investigations of the three-phase circuits.

III. INSTANTANEOUS ROOT-MEAN-SQUARE (rms) VALUES OF VOLTAGES OR CURRENTS

From Fig. 8 and (1) the instantaneous value of phase currents or voltages can be expressed by the projections of instantaneous phasors to the real axis.
The left-hand portion of the equal sign of the following equation, (10), is the instantaneous rms value of the three-phase voltages excluding the zero-sequence component. The rms value contains a "mean" process. For the instantaneous rms value, the "mean" refers to the averaging over three phases. This is different from the conventional rms that is averaging over a certain time period.

\[
\frac{(V \cos \phi)^2 + [V \cos(\theta + \frac{4\pi}{3})]^2 + [V \cos(\theta + \frac{2\pi}{3})]^2}{3} = \frac{V}{\sqrt{2}}.
\]

Detailed derivation proves that the instantaneous current or voltage phasor magnitude divided by \( \sqrt{2} \) equals the instantaneous rms value of three-phase currents or voltages excluding their zero-sequence component.

Combining (6), (7), (8), (9), and (10) the following relationship given in (11) holds true.

\[
\text{Magnitude of instantaneous phasor of voltage}
\]

\[
= \sqrt{\frac{2}{3} [(v_a - v_b)^2 + (v_b - v_c)^2 + (v_c - v_a)^2]}
\]

\[
= \sqrt{\frac{2}{3} [(v_a - v_b)^2 + \frac{(v_b - v_c)^2}{\sqrt{3}}]}
\]

\[
= \sqrt{\frac{2}{3} [(v_b - v_c)^2 + \frac{(v_c - v_a)^2}{\sqrt{3}}]}
\]

\[
= \sqrt{\frac{2}{3} [(v_c - v_a)^2 + \frac{(v_a - v_b)^2}{\sqrt{3}}]}
\]

\[
= \frac{\sqrt{2}}{3} [(v_a - v_b)^2 + (v_b - v_c)^2 + (v_c - v_a)^2]
\]

\[
= \sqrt{2} \text{ (Instantaneous rms value of three-phase voltages).}
\]

Similar expression can be derived for the instantaneous current magnitude, I.

\[
P_{\text{phasor}} = (v_a - v_0) \cdot (i_a - i_0) + (v_b - v_0) \cdot (i_b - i_0) + (v_c - v_0) \cdot (i_c - i_0).
\]

IV. INSTANTANEOUS ACTIVE AND REACTIVE POWERS

A. Instantaneous Active Power Excluding Zero-Sequence Components

The instantaneous active power excluding zero-sequence components of three phases is the summation of products of phase voltages and currents without zero-sequence components.

\[
\left\| V_a \right\| \cos(\theta + \phi) \left\| I_a \right\| \cos \theta
\]

\[
+ \left\| V_b \right\| \cos(\theta + \frac{4\pi}{3}) \left\| I_b \right\| \cos(\theta + \frac{4\pi}{3})
\]

\[
+ \left\| V_c \right\| \cos(\theta + \frac{2\pi}{3}) \left\| I_c \right\| \cos(\theta + \frac{2\pi}{3})
\]

\[
= \frac{3}{2} VI \cos \phi.
\]

Simplifying the left-hand portion of the equal sign of (13) gives the right-hand term of (13).

From (12) and (13) we have

\[
P_{\text{phasor}} = \frac{3}{2} VI \cos \phi.
\]

This equation, (14), says that the instantaneous active power of three-phase phasors equals 3 times \( \frac{V}{\sqrt{2}} \) times \( \frac{I}{\sqrt{2}} \) times the cosine of the angle between the voltage and current phasors. The V and I are the magnitudes of voltage and current phasors excluding the zero-sequence components. This expression of instantaneous phasor power for either balanced or unbalanced situations is similar to the format of conventional average power of balanced three phases.
B. Three-Phase Instantaneous Active Power Including Zero-Sequence Components

The three-phase instantaneous active power, \( p \), calculated from the real instantaneous voltages and currents including zero-sequence components is

\[
p = v_a i_a + v_b i_b + v_c i_c. \tag{15}
\]

We have the three-phase instantaneous power

\[
p = p_{\text{phasor}} + 3v_0i_0. \tag{16}
\]

C. Three-Phase Instantaneous Apparent Power of Instantaneous Phasors

The three-phase instantaneous apparent power, \( s_{\text{phasor}} \), is the product of 3 times the rms voltage and current.

\[
s_{\text{phasor}} = \frac{3}{2} V I \tag{17}
\]

D. Three-Phase Instantaneous Reactive Power of Instantaneous Phasors

From Fig. 8 the instantaneous reactive power, \( q_{\text{phasor}} \), of the phasors is given by.

\[
q_{\text{phasor}} = \frac{3}{2} V I \sin \phi \tag{18}
\]

E. Instantaneous Power Factor

The instantaneous power factor, \( \cos \phi \), is defined by the ratio of the active and the apparent instantaneous powers.

\[
\cos \phi = \frac{p_{\text{phasor}}}{s_{\text{phasor}}} \tag{19}
\]

VI. CONCLUSIONS

The unique property of instantaneous phasors is that at any instant the instantaneous three-phase currents and voltages can be represented by a set of balanced phasors. The instantaneous reactive power and the concept of instantaneous power factor can be clearly understood from the instantaneous phasors. This provides a theoretical foundation for power quality monitoring, diagnostics, and power compensation methods. More general expressions to include the zero-sequence components in the equations by shifting the origins of phasors can be adapted.

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VIII. REFERENCES


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