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AN INVESTIGATION OF TIME EFFICIENCY IN WAVELET-BASED MARKOV PARAMETER EXTRACTION METHODS

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Abstract

This paper investigates the time efficiency of using a wavelet transform-based method to extract the impulse response characteristics of a structural dynamic system. Traditional time domain procedures utilize the measured disturbances and response histories of a system to develop a set of auto and cross correlation functions. Through deconvolution of these functions, or matrix inversion, the Markov parameters of the system may be found. By transforming these functions into a wavelet basis, the size of the problem to be solved can be reduced as well as the computation time decreased. Fourier transforms are also used in this capacity as they may increase the time efficiency even more, but at the cost of accuracy. This paper will therefore compare the time requirements associated with a time, wavelet, and Fourier-based method of Markov parameter extraction, as well as their relative accuracy in modeling the system.

1. Introduction

The basis for extracting the Markov parameters of a system, $\mathbf{h}$, from vibration data involves the deconvolution of the input excitation, $\mathbf{u}$, from the output response, $\mathbf{y}$:

$$\mathbf{y}(t) = \int_0^{T} \mathbf{h}(\tau)\mathbf{u}(t-\tau)d\tau \tag{1}$$

Classically, the fast Fourier transform (FFT) has been used extensively in solving the deconvolution problem due to the time efficiency obtained by its ability to transform the time-domain convolution to a simple frequency multiplication. However, FFT procedures are prone to a variety of problems, including leakage and aliasing, that limit their ability to extract Markov parameters effectively.

Time procedures, on the other hand, do not suffer from the limitations associated with FFTs since the deconvolution problem is solved entirely in the time domain. These procedures essentially reduce to solving a set of linear equations given by:

$$\mathbf{Y} = \mathbf{h}\mathbf{U} \tag{2}$$

The output matrix $\mathbf{Y}$, the impulse response matrix $\mathbf{h}$ and the input matrix $\mathbf{U}$ are given, respectively, by:

$$\begin{aligned}
\mathbf{Y} &= \langle \mathbf{y}(0) \quad \mathbf{y}(1) \quad \dots \quad \mathbf{y}(s-1) \rangle \quad [m \times s] \\
\mathbf{h} &= \langle \mathbf{h}(0) \quad \mathbf{h}(1) \quad \dots \quad \mathbf{h}(rp) \rangle \quad [m \times r(p+1)] \\
\mathbf{U} &= \\
&\begin{bmatrix}
\mathbf{u}(0) & \mathbf{u}(1) & \dots & \mathbf{u}(p) & \dots & \mathbf{u}(s-1) \\
0 & \mathbf{u}(0) & \dots & \mathbf{u}(p-1) & \dots & \mathbf{u}(s-2) \\
0 & 0 & \dots & \mathbf{u}(p-2) & \dots & \mathbf{u}(s-3) \\
0 & 0 & \dots & \dots & \dots & \dots \\
0 & 0 & 0 & \mathbf{u}(0) & \dots & \mathbf{u}(s-p-1)
\end{bmatrix} \\
&\qquad [r(p+1) \times s]
\end{aligned} \tag{3}$$

in which m, s, r and p are the number of measurement vectors, the number of measurement samples, the number of input signals, and the length of the impulse response, respectively. The direct method of solving (2) is to multiply both sides of the equation by $\mathbf{U}^{-1}$. Unfortunately, depending on the kind of input used, the matrix $\mathbf{U}$ may be strongly ill-conditioned, leading to an exploding impulse response. Also, for multiple input systems, the additional DOFs form a matrix $\mathbf{U}$ that has

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more rows than columns, meaning the matrix will be underdetermined. Most cases therefore require the use of a pseudo inverse rather than a direct one to overcome the underdetermined state, performed through the creation of auto and cross correlation functions:

$$\begin{aligned} Cross-Correlation: \quad & \mathbf{R}_{YU} = \mathbf{Y}\mathbf{U}^T \\ Auto-Correlation: \quad & \mathbf{R}_{UU} = \mathbf{U}\mathbf{U}^T \end{aligned} \tag{4}$$

The Markov parameters, $\mathbf{h}$, are then found by:

$$\mathbf{h} = \mathbf{R}_{YU}\mathbf{R}_{UU}^{-1} \tag{5}$$

which is also the definition of the pseudo inverse.

Due to both the ill-conditioning problems and the large time requirements for inversion of the input matrix, we have proposed the use of wavelets for solving the set of linear equations (Robertson, 1997). Wavelets are used to transform the time domain deconvolution problem into a set of wavelet bases where their special features can be used to solve the equations more effectively. The transformation of the input or input auto-correlation matrix (depending on the method used) is the most important, since this matrix must be inverted to find a solution. The change of bases can improve various aspects of the problem including the conditioning of the matrices and the ability to reduce the size of the matrices. This paper explores these various attributes when using wavelet transforms and how they can be used to decrease the overall computation time required for finding the Markov parameters of a system.

2. Reduction in Matrix Sizes via Wavelet Multiresolution

The size of the input and output matrices are dictated by the size of the ensemble used in the analysis, and by the number of Markov parameters needed. Noisy or lightly damped systems necessitate the use of a rather large ensemble in order to obtain a converged solution. One method that has been used to help decrease the size of the matrix needed for a lightly damped system is the Observer Kalman Identification method(Juang, 1993). This method actually induces damping into the system so that the Markov parameters will decay more quickly, allowing for a smaller ensemble size to be used.

Wavelets can be used to decrease not the size of the ensemble but the input or input-correlation matrix that is formed. There are a variety of ways to apply wavelets to the deconvolution problem, of which three approaches will be examined in this paper. A more detailed explanation of these procedures may be found in (Robertson, 1997). The first method is the transformation of the rows of the $\mathbf{U}$ and $\mathbf{Y}$ matrices, which when substituted back into the convolution equation (2), appear as:

$$\mathbf{Y}^{\mathrm{DWT}} = \mathbf{h} \cdot \mathbf{U}^{\mathrm{DWT}} \tag{6}$$

Second is the transformation of the rows of the $\mathbf{UU}'$ and $\mathbf{YU}'$ matrices:

$$[\mathbf{Y} \cdot \mathbf{U}^T]^{\mathrm{DWT}} = \mathbf{h} \cdot [\mathbf{U} \cdot \mathbf{U}^T]^{\mathrm{DWT}} \tag{7}$$

And last is the 2D transformation of the $\mathbf{UU}'$ matrix and 1D transformation of $\mathbf{YU}'$:

$$[\mathbf{Y} \cdot \mathbf{U}^T]^{\mathrm{DWT}} = \mathbf{h} \cdot [\mathbf{U} \cdot \mathbf{U}^T]^{\mathrm{DWT2}} \tag{8}$$

where $DWT2$ represents the 2D wavelet transformation. With all these methods, it is possible to reduce the size of the matrix, $\mathbf{U}$ or $\mathbf{UU}'$, to be inverted by truncating the wavelet coefficients. Truncation of the input matrix will help reduce the condition number, the amount of time needed for inversion, and in some cases allow for the inversion of a matrix which was previously not possible due to ill-conditioning

The construction of the wavelet transform of a signal can be expressed in the following form:

$$[a_0][a_1][a_2a_3][a_4 \ldots a_7][a_8 \ldots a_{15}] \ldots [a_{n/2+1} \ldots a_{n-1}] \tag{9}$$

where n is the length of the signal and each bracket represents a frequency band. The frequency bands increase from left to right as they also double in width. Therefore, the last frequency band represents frequencies from one-half the Nyquist frequency to the Nyquist frequency. Any of these frequency bands (or levels) may be truncated, provided there is no significant information contained in that band. For instance, if the highest frequency band were truncated, the result would be a vector that is half the original size. For the input matrix, this will result in a matrix that is now half the width of the original time domain matrix. With the truncation of the wavelet-transformed input matrices, the resulting size of the Markov parameters are the same, since only the inner dimensions are being altered. Take for instance a system with 256 time points, the change in the size of the matrices due to truncation of the second half of the wavelet transform is shown in equation 10.

$$\begin{aligned} Original: \quad & \underset{(1\times 256)}{\mathbf{Y}} = \underset{(1\times 256)}{\mathbf{h}}\ \underset{(256\times 256)}{\mathbf{U}} \\ Truncated: \quad & \underset{(1\times 128)}{\mathbf{Y}} = \underset{(1\times 256)}{\mathbf{h}}\ \underset{(256\times 128)}{\mathbf{U}} \end{aligned} \tag{10}$$

For the 2D transform, on the other hand, the dimensions of the Markov parameters (MPs) are being altered by half. Therefore, when the MPs are being constructed from their wavelet transform (WT), the signal should be padded with zeros so that the resulting MPs are of the correct size.

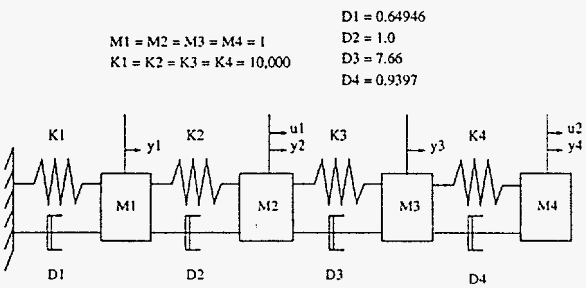

FIGURE 1: FOUR DOF SPRING-MASS SYSTEM

The conditioning and computation time improvements to be made by a decrease in the size of the input matrices will now be examined. A four DOF spring-mass system with damping as shown in figure 1 will be used for the analysis. The system is excited by four sinusoidal inputs of 256 time points at each of the DOFs. Therefore, the size of the input matrix is 1024×256 ($rn \times n$) and the input auto-correlation matrix 1024×1024.

Table 1 shows both the conditioning of either the input or input auto-correlation matrix in the wavelet domain and the time required to invert the matrix. The times were determined using MATLAB (Mathworks, 1989), a mathematical software package. The first row, $\mathbf{U}w$, refers to the wavelet transformation of the rows of the $\mathbf{U}$ matrix, and $\mathbf{U}wt$ the truncation of the second half of the matrix. The third row contains $\mathbf{UU}w$, referring to the row transformation of the $\mathbf{UU}'$ matrix, while $\mathbf{UU}wt$ is the truncation of the second half of the transform. Lastly, the fifth and sixth rows, $\mathbf{UU}w2$ and $\mathbf{UU}w2t$, involve the 2D transformation of the $\mathbf{UU}'$ matrix and its truncation of half of the rows and columns.

TABLE 1: CONDITION VALUES AND INVERSION TIME FOR INPUT AND AUTOCORRELATION MATRICES

Method	Cond.	Time (sec)
$\mathbf{U}w$ (1024×256)	3.2082e3	78.83
$\mathbf{U}wt$ (1024×128)	1.6015e3	18.61
$\mathbf{UU}w$ (1024×1024)	2.1312e22	1048
$\mathbf{UU}wt$ (1024×512)	1.4676e18	294.8
$\mathbf{UU}w2$ (1024×1024)	7.2083e21	1048
$\mathbf{UU}w2t$ (512×512)	2.0734e20	103.7

In table 1, one can see the drastic reduction in the amount of time required for inversion when the size of the input matrix is reduced. For the $\mathbf{U}w$ and $\mathbf{UU}w$ inversions, the time needed is reduced to less that 1/3 of the original, while for the 2-D transform, time is dropped to 1/10 the original. Though the conditioning of the matrix also drops with the reduction in inversion time, the most important feature for the time reduction seems to be the size of the matrix.

With these drops in time, however, there is also a concern about the accuracy of the Markov parameters determined from the truncated matrices. Therefore, results from the truncation of the $\mathbf{UU}w$ matrix will be displayed. Figure 2 shows a comparison between using the full $\mathbf{UU}w$ matrix versus the truncation of the second half of the wavelet transform as compared the the exact, analytical FRFs. Very little error is induced, though the size of the matrix for inversion is reduced by half.

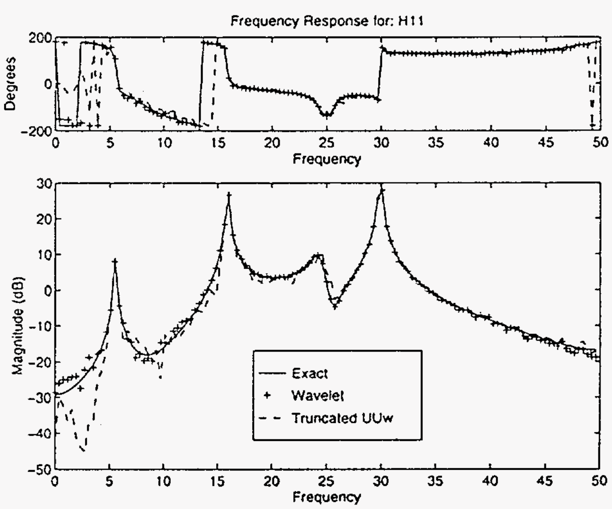

FIGURE 2: MARKOV PARAMETERS FOR 4 DOF SPRING-MASS SYSTEM

3. FFT Approximation for Matrix Inversion

Another method of reducing the computation time for the deconvolution procedure will be examined. FFT methods are known for their time efficiency due to their unique ability of transforming convolution to multiplication. This ability can also be used to form an approximate solution for the wavelet method of Markov parameter extraction. This is achieved by first determining the auto and cross-correlation functions in the wavelet domain:

$$[\mathbf{Y} \cdot \mathbf{U}^T]^{\mathrm{DWT}} = \mathbf{h} \cdot [\mathbf{U} \cdot \mathbf{U}^T]^{\mathrm{DWT}} \qquad (11)$$

and then applying the Fourier transform to only the top rows of these matrices:

$$\left\{[\mathrm{Y}\cdot\mathrm{U}^T]^{\mathrm{DWT}}(1,1:n)\right\}^{\mathrm{FFT}} = \mathrm{h}^{\mathrm{FFT}}\cdot\left\{[\mathrm{U}\cdot\mathrm{U}^T]^{\mathrm{DWT}}(1,1:n)\right\}^{\mathrm{FFT}} \tag{12}$$

where the index $(1, 1 : n)$ represents the top row of the matrix. Division of the cross correlation by the auto correlation function may then be carried out just as it is done in spectral methods:

$$\mathrm{h}^{\mathrm{FFT}}(i) = \left\{[\mathrm{Y}\cdot\mathrm{U}^T]^{\mathrm{DWT}}(1,1:n)\right\}^{\mathrm{FFT}}(i) \cdot\left[\left\{[\mathrm{U}\cdot\mathrm{U}^T]^{\mathrm{DWT}}(1,1:n)\right\}^{\mathrm{FFT}}(i)\right]^{-1} \tag{13}$$

where i denotes a matrix of size $s \times r$ for both h and $\mathrm{Y}\cdot\mathrm{U}^T$ and of size $r \times r$ for $\mathrm{U}\cdot\mathrm{U}^T$. The formula is stepped through n times, the total number of time points. The Markov parameters are then found through the inverse Fourier transform of the FRFs. This is similar to a correlated FFT procedure described in (Juang, 1994), except that we are doing ensemble-averaging in the wavelet domain rather than the frequency.

The approximate solution is very similar to doing the true inversion of the auto-correlation matrix, with only a small amount of error. Figure 3 shows a comparison between the standard wavelet method of Markov parameter extraction and the approximate. To see how these results compare to just the standard Fourier transform method, figure 4 compares the wavelet and FFT procedures for the same example problem found in the last section. The approximate solution is not as accurate as the standard wavelet method, but still performs much better than the standard Fourier method. The next section will discuss the improvements in computation time that this procedure creates.

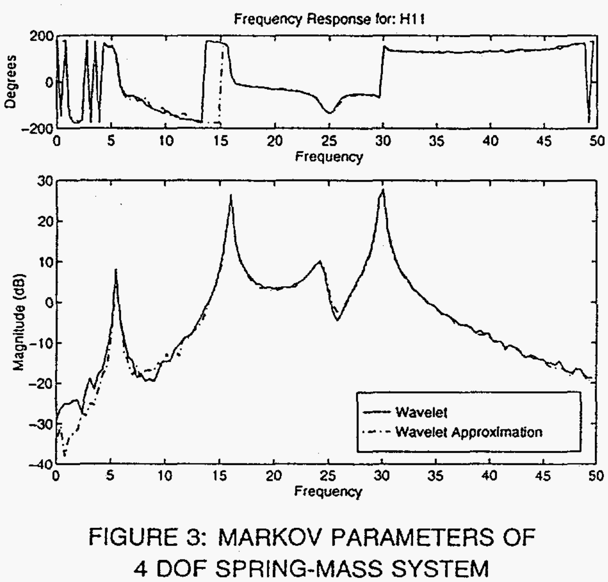

FIGURE 3: MARKOV PARAMETERS OF 4 DOF SPRING-MASS SYSTEM

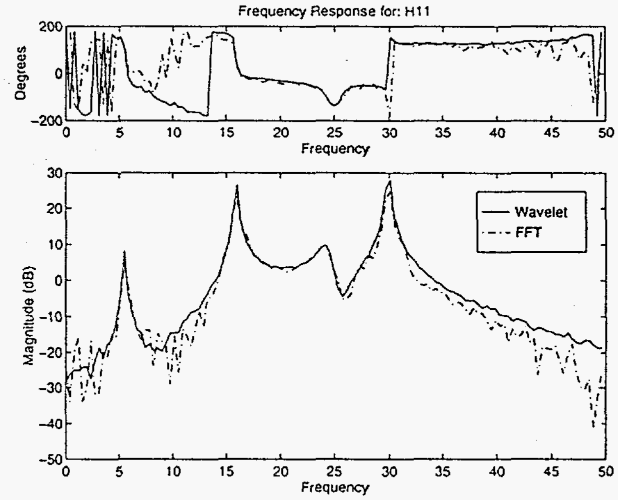

FIGURE 4: MARKOV PARAMETERS OF 4 DOF SPRING-MASS SYSTEM

4. Comparison of Computation Time for Time, Wavelet, and FFT Methods

A thorough comparison will now made between the computation time required for the time, wavelet, and FFT methods. The 4 DOF spring-mass problem is once again used for this comparison, but with only one input and one output for a total of 4096 time points. Ensemble lengths of 256, 512, and 1024 points are used. Two different wavelet methods, the general time domain method, and the FFT method, are used to find the Markov parameters. The first wavelet method, referred to as "Wavelet 1" in figures, involves the wavelet transform of the auto and cross-correlation matrices. A full input matrix is used for this procedure. The second method, referred to as "Wavelet 2" in the figures, involves the wavelet transform of the rows of the Y and U matrices, with the U matrix being upper triangular. Obviously, the Fourier method is much quicker than the time and wavelet methods, but the concern more is whether the time involved is unreasonable.

Since the computation time for the FFT methods is much lower, the times for the wavelet and time methods are normalized by the FFT computation time. Figure 5 shows a comparison of the normalized time for the three methods mentioned above as a function of ensemble length: 256, 512, and 1024 points. The plot shows the time required per ensemble as being anywhere from 10 to 250 times greater than the FFT method, with the second wavelet method requiring the least amount of time. This plot only considers the time to create the auto and cross correlation matrices, it does not include the time to invert the auto-correlation matrix and then find the Markov parameters. This will be shown in the next plot. The steps in the three procedures that contribute most to the computation time are: the creation of the input matrix, which for a 256 point ensemble is a 256x256 matrix; the multiplication of this matrix by its transpose to form the auto-correlation matrix; and then the wavelet transform of each row or column of the matrices. It is important to note that these programs have not been optimized very well for time efficiency. There is much room for improvement in the time, especially by using MEX files in MATLAB, files that call C functions to be more time efficient.

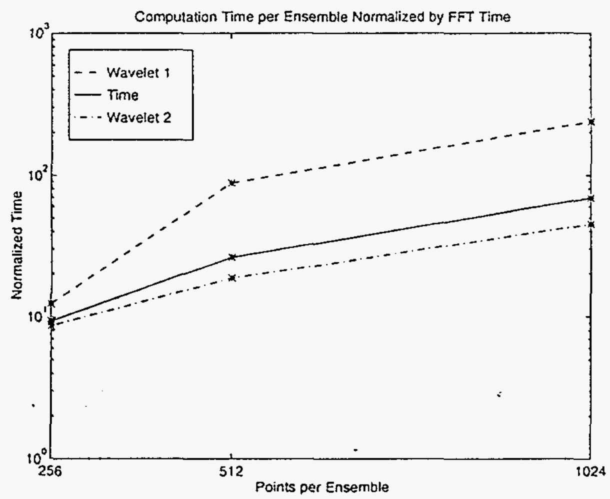

FIGURE 5: COMPUTATION TIME FOR DETERMINATION OF AUTO AND CROSS CORRELATION MATRICES

Figure 6 then shows how the computation time increases with the addition of the inversion step and the calculation of the Markov parameters. The total computation time required to determine the Markov parameters with 4096 time points is computed with 256, 512, and 1024 point ensembles. This means that 16, 8, and 4 ensembles were used respectively. For comparison we have used the fastest of the three methods examined above, the "Wavelet 2" method. Once again, the time is normalized by the time needed for the FFT method. The solid line shows the time needed without the inversion, while the dashed line includes it. Obviously, there is a great increase in time (more than 5 times for the 1024 point case) when the inversion is included.

The method of doing an approximate inversion via FFTs as discussed in the last section is therefore also analyzed. The time required for the approximate inversion is almost negligible as can be seen in figure 6. To get an idea of the actual times needed for these procedures, the FFT method requires 12.44 seconds to find the Markov parameters using a 512 point ensemble, while the "Wavelet 2" method requires 234 seconds (about 4 minutes) when the approximate inverse is used. For smaller sized ensembles, the time obviously is not of that great of an importance, however if larger ensembles are needed, then the computation time for wavelet and time procedures becomes a problem. This supports the belief that wavelet methods are better suited when only a small amount of vibration data is available.

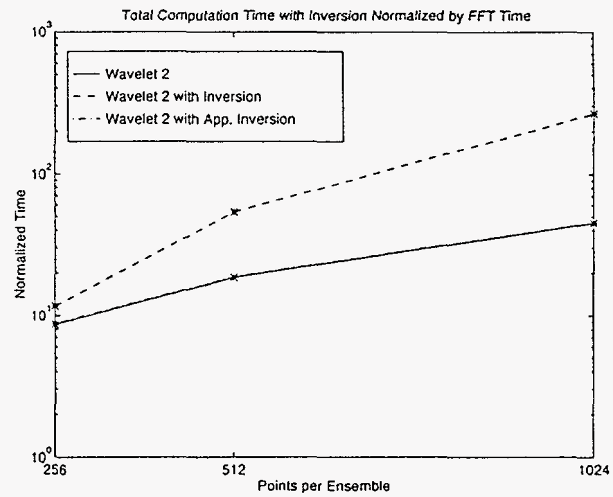

FIGURE 6: NORMALIZED COMPUTATION TIME FOR DETERMINATION OF MARKOV PARAMETERS

5. Comparison Between Error and Computation Time for Model Realization

It was shown in previous papers (Robertson, 1996) that with a given amount of simulated data, wavelets are able to determine a modal model of a system more accurately than FFTs. Comparisons were made between the error in the identified modes, mode shape, and damping of the system, for varying amounts of data. In this paper, however, it was shown that wavelet methods have a major drawback, a much larger computation time than FFTs. It is interesting therefore to examine how the improvement in accuracy weighs against the time needed to obtain it.

The three parameters: modes, mode shapes, and damping were found using the Eigensystem Realization Algorithm (Juang, 1985) and are used to compare the amount of error in the determined model of the system. The 4 DOF spring-mass system with damping used throughout this paper is the basis of comparison, but with random excitation. Further analysis was done on other examples to support the results shown here, but only the random input results will be presented. For this example, there are 2 inputs and 4 outputs to the system. Based on the fastest wavelet method, it is found that each ensemble of the wavelet procedure takes approximately 20 times as long as the FFT for 256 point ensembles. This is higher than the results shown previously in this paper since two inputs are used in this case instead of one. The time axes in these plots are scaled based on this ratio of 20:1, each ensemble for FFTs was given one time unit, while the wavelet was given 20.

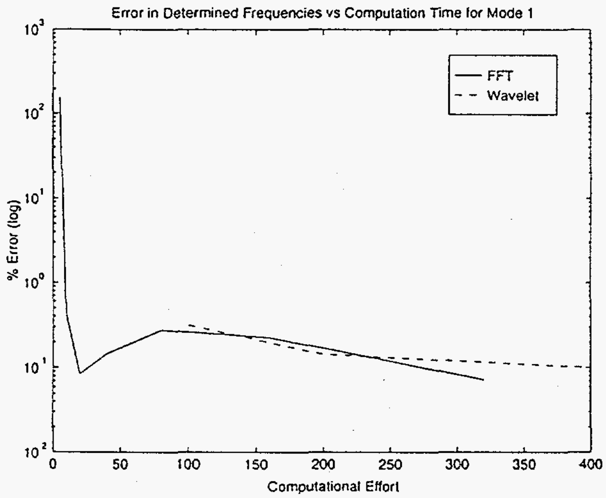

FIGURE 7: ERROR IN DETERMINED MODES VERSUS COMPUTATION TIME

The first plot, 7, shows the percent error found in the determined frequencies or modes of the system, based on the amount of time taken for both wavelet and FFT procedures. Due to the larger amounts of time needed in the wavelet methods, the FFT and wavelet methods barely intersect for a reasonable amount of data, 40960 points total. The FFT curves with more data do not change dramatically, while the wavelet curves do continue to decrease for times greater than shown in these figures. For 40 ensembles both methods have basically converged to their presumed solution, though 160 is the last point for FFT methods shown in this graph, while only 20 ensembles are shown for the wavelet method. In this first plot, there is not a clear distinction as to whether using wavelets is beneficial. The wavelet error does seem to decrease slightly, but the FFT error does also and at a quicker rate. It is known that the wavelet error continues to decrease at larger amounts of time, but this data is not available for the FFT method. Based on the intersections of these lines, it appears that wavelets may be able to achieve smaller amounts of error than FFT methods ever will, but it will take more time to get this improvement. It should be noted that the % error on this graph is of log scale, so differences are larger than they appear.

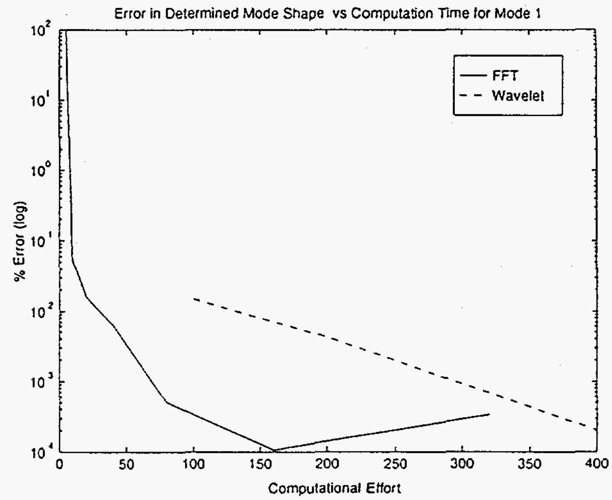

FIGURE 8: ERROR IN DETERMINED MODE SHAPES VERSUS COMPUTATION TIME

The % error found in the determined mode shapes of the system can then be found in figure 8, also on a log scale. In this case, there seems to be a steady decrease in the error with the more time taken for both wavelet and FFT methods, except when the FFT method reaches about 160 on the time scale, after which the error increases. It may be difficult to tell at which point the error might start to increase, which can cause problems in the realization of a system, especially if the error starts rising significantly with the more data used. The amount of error for the wavelet method does decrease further for times greater than shown here, but it is evident that FFTs can obtain good estimates of the modes shapes with far less computation time. The error is so low for this graph, 1x10-3% after only a few ensembles, that extra time need not be taken.

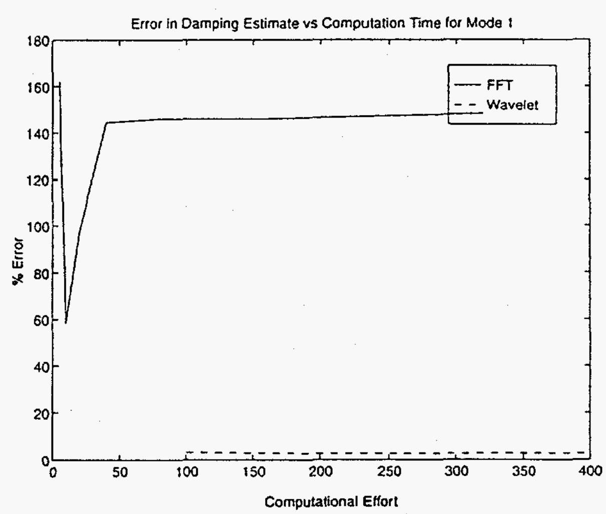

FIGURE 9: ERROR IN DETERMINED DAMPING VERSUS COMPUTATION TIME

This nice convergence is contrary to the next figure, 9, where the error in the damping estimates is given. FFT methods, as in all cases examined in this thesis, never seem to be able to capture the correct damping. Increases in time or amounts of data only add to the randomness of the determined damping ratio. Wavelets, on the other hand, do consistently get lower and lower damping error as more time is used. The damping error in this figure is consistent, and is shown to give much more accurate estimates with less amount of time taken.

There seems to be contradiction between the different parameters as to whether FFTs or wavelet methods are more efficient at identifying structural models. The clear convergence or reduction of error in the wavelet models indicates that one can know that with an increase in the amount of data used, or time taken, that results will improve. This is not always true in the case of FFT methods. In fact, there is no clear improvement at all for damping estimates with an increase in data. So, if damping estimates are of importance, wavelet methods are definitely the superior choice, otherwise, time efficiency versus accuracy importance must be taken into consideration in the choice of methods.

Conclusions

In this paper it was shown that wavelets can be used to decrease the size of the input matrix to be inverted by truncating the wavelet transform by half. This essentially results in the truncation of the upper half of the frequencies of the signal, which can be detrimental if important information is contained in the higher frequencies. This procedure should only be used for matrices with frequencies in the lower half of the frequency band, such as correlation matrices. Both the decrease in conditioning and size of the input matrices allows for a quicker solution of the deconvolution problem using wavelets. To further decrease the time needed for wavelet methods, an approximation method for the inversion of the input matrices was introduced. This method was able to effectively invert the input matrix in a fraction of the original time with only a small change in the determined Markov parameters.

A thorough comparison of the time required for the time, wavelet, and Fourier methods was then made. It was found that certain wavelet methods without any truncation or approximation techniques require less time than the general time domain method. Inversion of the input matrix was found to add considerable time to the overall process, so if possible, the approximated inverse method should be used. The increase in time for wavelet methods as compared to the FFT was anywhere from 10-250 times greater, depending largely on the size of ensemble and the number of inputs used.

A comparison was then made between the amount of time required and the resultant error in the determined modal model. There was no clear conclusion as to whether wavelets were more effective than FFT methods. For some parameters, the FFT method was able to get lower amounts of error in less time, but in other cases the WT method could get lower error than ever obtained by FFT methods. Damping was shown conclusively to belong to WT methods, with the increase in time or amount of data not improving the FFT results at all. This paper also demonstrated the steady convergence of error by wavelet methods with an increase in amount of data or time used. In general, the trade-offs between the accuracy obtained versus the computation time required are up to the user to decide.

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