DOE/ER/13916--T3

FINAL REPORT on DOE GRANT No. DE-F-G02-88ER13916A000:

Turbulence and Spatio-Temporal Chaos.

by C. Jayaprakash and F. Hayot

Physics Department, Ohio State University

174 W. 18th. Avenue, Columbus Ohio 43210

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

OVERVIEW.

The main object of our studies since 1991 has been the study of extended dynamical systems that exhibit both chaotic behavior and scale invariance. One aspect of our study involved the equivalence at large scales of two very different partial differential equations: the Kuramoto- Sivashinsky (KS) and the Kardar- Parisi- Zhang (KPZ) equations. The KS equation applies to flame front propagation and chemical reactions, and also appears in the Benjamin- Feir instability of a complex Ginzburg- Landau equation. The KPZ equation provides a generic description of interfacial growth on large scales.

Turbulence phenomena encompass many behaviors at many scales, from chaotic behavior at small scales to coherent structures at large scales. The crucial question is how different scales interact. We have sought clues to a general answer by raising the following questions: can a deterministic chaotic system described by a deterministic PDE (the KS equation) be characterized at large scales by the same static and dynamic scaling exponents which determine a stochastic equation (the KPZ equation) ? Through an extensive numerical study in both one and two dimensions we have concluded that truly, on large scales, the KS and KPZ equations belong to the same universality class. The procedure (pioneered by Zaleski) involves coarse graining in Fourier space. Through a detailed analysis of its features, and in particular

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

the effective noise- noise correlation, we are able to comprehend the magnitudes of the physical parameters, and in particular estimate the cross- over times from free to interacting behavior.

Within the whole complex of questions which relate to scaling behavior and pattern formation we have also investigated the following problems (with results verified through studies of coupled map lattices):

- non gaussian probability distributions for scalars being advected turbulently.

- generic scale invariance and self-organized criticality in extended dynamical systems with chaotic behavior

- domain motion for a complex, non phase invariant, Ginzburg- Landau equation with a complex external magnetic field.

- stabilization of turbulent domains in annular flows.

and previously from 1988 to 1991 the related set of problems:

- Saffman-Taylor type instability of the interface between two immiscible fluids in a microscopic model.

development of an algorithm implementing Lévy walks in lattice gas hydrodynamics, and application to turbulent flows.

- spiral turbulence in Taylor-Couette flow.

- study of the effect of conservation laws on the collective behavior of coupled map systems, and investigation of transitions from long ranged spatio-temporal order to chaos.

- general arguments for the stability of temporally periodic states in discrete time, synchronously updated, stochastic systems with short range interactions based on anisotropic domain interface motion and a study of a new class of models illustrating these ideas.

- generalization to continuous time, lattice models and time dependent Ginzburg- Landau equations of Toom's probabilistic cellular automata: these models are argued analytically and numerically to exhibit phase coexistence generically by exploiting anisotropic domain wall dynamics.

- new and efficient method for the characterization of unstable periodic orbits in low dimensional chaotic attractors and repellers including the Hénon map.

I. SUMMARY OF WORK DONE FROM 1988 to 1991.

Fernand Hayot

Lattice gas hydrodynamics adopts a microscopic approach to fluid flow. What makes it attractive is its computational simplicity, which includes treatment of complicated boundaries, numerical stability and an obviously parallelizable algorithm. The approach has its problems though, the main one being the difficulty of reaching high Reynolds numbers. Another one concerns the occurrence of noise, which is intrinsic to the algorithm.

The aim of my work over the last few years (1988-1991) has been to push the microscopic viewpoint into challenging new directions in order to test its ingredients, and better circumscribe its domain of applicability. The problem dealt with mainly is that of the Saffman - Taylor instability.

Saffman - Taylor Instability

The Saffman - Taylor instability is the fingering instability which occurs at the interface between two immiscible fluids, when the less viscous one, such as water, pushes the more viscous one, such as oil. It happens in Hele-Shaw cell flow and in porous media. The basic length scale is the width of the cell or a scale much larger than any pore diameter.

It is a macroscopic problem par excellence. The basic ingredients are two typical macroscopic laws, Darcy's and Laplace's. Darcy's law relates fluid velocity to pressure gradient, and Laplace's law introduces surface tension

into the expression for interfacial pressure difference. Deriving these laws from the microscopic dynamics of a discrete binary gas is undoubtedly a significant step.

In a first paper ¹ we showed how Darcy's law emerges in a two dimensional flow, when the restrictions to the flow caused by pores are mimicked by a low density of scatterers. In two further papers ^{2,3} we constructed algorithms for an interface between two immiscible fluids, and studied its properties. The approach taken is semi-macroscopic. The interface is a continuous chain of links, lying along a hexagonal lattice, which deforms under the impact of fluid particles. The two algorithms we developed essentially describe how deformations take place. The crucial point is to ascertain that the interface has the right physical properties. We investigated both static properties establishing Laplace's law, and the fluctuations of the interface in the heat bath provided by the two fluids. In particular we showed that the interface behavior follows that described by a noisy Langevin equation. This point is worth emphasizing, namely we were able to corroborate our numerical results by a series of calculations on interface properties, such as surface tension.

Having established Darcy's law and introduced a successful interface algorithm, we are then able ⁴ to attack the problem of the Saffman-Taylor instability of fluid flow. In a series of simulations we show how the instability occurs at long wavelength, following the pattern derived from linear stability analysis, and takes the form of a finger penetrating from one fluid

into the other. Because of the presence of noise, which is intrinsic to the method, and results from the spatial averaging, the finger itself, however, is unstable and exhibits tip splitting. This occurs at large capillary number, where the phenomenon is also observed in actual experiments. This is a case where a mechanism intrinsic to the algorithmic procedure manifests itself as a physical phenomenon.⁵ The lattice gas thus duplicates the real world, with the disadvantage that the presence of noise prevents an in-depth quantitative study.

Cylinder Wake

We did an analysis ⁶ of flow dynamics behind a cylinder, studying first the time development of wake length at low Reynolds numbers, then lift and drag on the cylinder once the von Karman street exists. Dependence of the Strouhal number on the ratio of cylinder diameter to channel width was considered.

Two points emerge: one is the crucial presence - in order to obtain agreement with data- of the special, density dependent, modification of the convective Navier - Stokes term, which arises in lattice gas hydrodynamics. The other is the obvious stability of the lattice gas algorithm, when going from low (several tenths) to high (up to 200) Reynolds numbers, which is not the case of the usual algorithms , which cannot deal with the wake and the von Karman street.

Lévy Walk and Turbulent Channel Flow⁷

A statistical mechanics approach to enhanced diffusion, as it occurs in fully developed turbulence, was recently proposed. The approach is based on so-called Lévy walks. Imagine a fluid particle jumping from site to site in a flow. If the mean square displacement is finite, the probability density will ultimately be a gaussian. If it is not, it will tend towards a Lévy distribution, A Lévy walk is a generalization , where to a Lévy distribution for distances is added a completion time for each jump. For a sufficiently slowly falling distribution for distances, one recovers Richardson' law, namely that the mean square displacement increases cubically in time.

My work consists in the implementation of a Lévy walk at the microscopic level, namely in lattice gas hydrodynamics. It provides a way to go to effectively much higher Reynolds numbers than the size of the system itself would allow. Lattice gas hydrodynamics lends itself naturally to such an implementation, since particles can be exchanged among lattice sites according to some space and time dependent distribution. The algorithm contains one parameter only, once the relation connecting the number of momentum exchanges to distances is chosen. This parameter is the exponent in the probability distribution for distances. The case considered is that of pressure driven channel flow. The approach provides the equivalent of a closure approximation to the Navier - Stokes equation. The flattened velocity profile, flattened relative to a parabolic one, looked at in scaled variables, corresponds

to Reynolds numbers of order several ten thousands, although the one derived from channel width and maximum velocity is much smaller. The shape and magnitude of the turbulent logarithmic velocity profile is also recovered from this microscopic implementation of a Levy walk.

Bubble Motion And Bubble Streams

This is work done by my student Don Burgess, who is now at the Center for Nonlinear Studies in Los Alamos. In a first paper (with M. Foster)⁸ Burgess investigated three dimensional effects as they manifest themselves in effective boundary conditions for two dimensional Hele - Shaw bubbles. The method used is singular perturbation theory. He computed in particular the pressure drop from the front to the back of the bubble, due to a thin wetting film, and showed the existence of a new scaling region for film thickness along the bubble sides. In a second paper (with S. Tanveer)⁹ he presented exact solutions for a steady stream of bubbles in a Hele - Shaw cell, when surface tension is neglected. When surface tension is included, numerical and analytical evidence shows that there is a dramatic increase in bubble speeds when the distance between bubbles becomes of the order of a bubble diameter.

Spiral Turbulence And Wetting

These works deal with two important nonlinear problems. One¹⁰ concerns spiral turbulence in Taylor - Couette flow. We proposed for the first time

a phase equation for the turbulent spiral which coexists with laminar flow. This is a breakthrough for this subcritical bifurcation, which shows strong hysteretic effects. A second one¹¹ deals with wetting of a defect plane in the mean field approximation to a lattice model for two fluids. It shows the existence of a new multicritical point in the relevant phase space.

Ciriyam Jayaprakash

The program of elucidating general features of extended, non-equilibrium systems by studying models using a statistical mechanical approach has been continued fruitfully.

Collective Behavior of a Coupled Map System

With a Conserved Quantity

The main motivation for this study¹² arises from the fact that both past studies and recent investigations in the context of self-organized criticality have emphasized the ramifications of conservation laws for spatial and temporal correlations in nonequilibrium systems. We have carried out an extensive numerical investigation of the behavior of coupled sine maps with a locally conserved variable on a square lattice. We have provided supporting analytic arguments based on the renormalization group for the results given below:

1) By varying the parameters of the model parameters we found two distinct “conservation-induced” routes to chaos: One is a first-order transition from a periodic 2-cycle phase with broken time-translation invariance and long-range, striped, spatial order, to a chaotic, spatially-disordered phase in which time-translation invariance is restored; this transition is accompanied by a discontinuous jump in the maximum Lyapunov exponent, λ_{max} . The other is a continuous onset of temporal chaos without any change in the spatial symmetry of the phase, which has striped long-range order on both sides of the transition. In this latter case, λ_{max} increases continuously from zero as the control parameter, R , is increased. In the presence of noise, the numerics show λ_{max} growing *linearly*, in agreement with simple arguments. We believe that these routes to chaos can occur in a broad class of systems. 2) We have argued analytically that, in the presence of noise, spatial and temporal correlations decay algebraically, both in the chaotic and nonchaotic phases, though the arguments for the chaotic phases are heuristic. Numerical evidence (strong for the nonchaotic and weaker for the chaotic phases) has been accumulated to demonstrate the algebraic decay (t^{-1} or t^{-2} depending on the model) in time of autocorrelations.

Stability of Temporally Periodic States

Physical systems that are not driven by time-dependent external forces and described by time-translationally invariant equations of motion can ex-

hibit macroscopic properties varying periodically in time. We have studied¹³ discrete-time, synchronously updated, stochastic systems with short-ranged interactions. We have shown that periodic k -cycle states with period $k > 2$ are typically metastable losing spatial coherence through the nucleation and growth of droplets. We have constructed rules that exploit lattice anisotropy to suppress droplet fluctuations and so stabilize higher period cycles. We have also argued for unusually rapidly decaying temporal autocorrelations in such systems.

Generic Nonergodic Behavior in Continuous Systems:

Probabilistic cellular automata that exhibit non-ergodicity in finite regions of parameter space have been constructed by the Russian mathematician Toom. We have constructed¹⁴ (partial) differential equations that are nonequilibrium variations of time-dependent Ginzburg-Landau models; these are argued to exhibit analogous behavior. The models incorporate asymmetric terms that have been identified as the crucial ingredient leading to anisotropic domain wall dynamics that in turn causes nonergodicity. We have studied these models in continuous time both on a discrete lattice and in a spatial continuum (i.e. in a stochastic PDE). By analyzing the motion of domain walls we have analytically demonstrated their anisotropic velocity. The effect of curvature was studied by solving the PDE numerically and verifying that domains of opposite sign are eroded linearly in time.

Characterization of Unstable Periodic Orbits

In Low-Dimensional Chaotic Attractors and Repellers

A postdoctoral fellow Dr. Ofer Biham working with me and a student of mine, Wolfgang Wenzel have devised¹⁵ a numerical technique to determine unstable periodic orbits in the chaotic regime of low- dimensional maps. The clever algorithm consists of deriving the trajectories from a suitably defined Hamiltonian and an associated artificial dynamics whose evolution leads to an extremum of the Hamiltonian that corresponds to an exact periodic orbit of the map. This technique has been applied successfully to the complex Henon map¹⁶; this has also enabled the definition of a new symbolic dynamics which holds for real and complex periodic orbits. Recently, we have applied¹⁷ the technique to the standard dissipative map and deduced scaling relations. Another postdoctoral fellow associated with me Mark Kvale and O. Biham have applied¹⁸ this technique to the stadium problem and studied the efficacy of the Gutzwiller trace formula for the quantum behavior.

Ginzburg-Landau Theory

For Oil-Water-Surfactant Mixtures

Oil-water-surfactant mixtures exhibit a variety of phases and phase equilibria. Surfactant molecules reduce the bare oil-water interfacial tension leading to the formation of bulk phases with a large density of microscopic oil-water interfaces. We have constructed¹⁹ a Landau-Ginzburg theory with

a scalar order parameter for the oil-water subsystem and a vector for the the surfactant part. A detailed density-functional calculation yields three-phase coexistence between oil-rich, water-rich, and microemulsion phases and modulated phases including lamellar, columnar, and cubic phases. The computed structure-factor for water in the microemulsion phase shows a variety of trends in qualitative accord with experiments.

II.SUMMARY OF WORK FROM 1991 TO 1994.

C. Jayaprakash

The objective of understanding various aspects of extended, non-equilibrium dynamical systems that typically exhibit spatio-temporal chaos has been pursued with a variety of projects that focus on the asymptotic (long-distance, long-time) properties of correlation functions including the occurrence of scale-invariant behavior (algebraic decays or algebraic growth in interface models), the nature of short distance properties such as long tails in local probability distribution functions, and the extent to which deterministic chaos can play the role of external noise. I have studied discrete interface models, continuum time-dependent Ginzburg -Landau models, coupled map lattices, Josephson junction arrays and partial differential equations such as the Kuramoto-Sivashinsky equation.

Scale-invariant behavior in dynamical systems

The conditions for the occurrence of generic scale invariance — correlations that decay algebraically in space and time without tuning external parameters — in non-equilibrium systems with external noise include local conservation and lattice anisotropies. This analysis tacitly assumes modest nonlinearities and hence, non-chaotic fluctuations. In systems driven strongly enough to produce chaos both in the presence and absence of noise the occurrence of scale invariance is an interesting issue. We have proposed²¹ heuristic

arguments to suggest that in both these cases conserving chaotic systems do exhibit generic scale invariance in $d > 1$. We have devised and simulated coupled map lattices in $d = 2$ that are chaotic and shown power law decays with exponents identical with those predicted analytically for noisy non-chaotic models, in agreement with our heuristic arguments. If this holds rather generally, as we suspect, then it constitutes a powerful tool for analyzing the macroscopic properties of conserving chaotic systems.

Scale invariance in multicomponent systems

In collaboration with G. Grinstein and J. E. S. Socolar (IBM)

An interesting question concerns the issue of generic scale invariance in multi-component systems where some of the variables are not conserved. We have studied²² coupled two-mode systems in which one of the modes is conserving and the other is not, e.g. temperature and density in fluids. We have analyzed a simple Langevin model with two coupled variables, one conserved and the other not and shown that the conserved mode induces power law spatial correlations in the non-conserved one. We first obtained the result at the linear level and then verified that it persists in the presence of nonlinearities by a renormalization group analysis. We have found numerical confirmation of this result in coupled lattice models.

Local Probability Distribution Functions in extended dynamical systems

In collaboration with R. Bhagavatula (OSU)

Recently local probability distribution functions (PDF) of velocity gradients and passive scalars (temperature) have been observed to display exponential tails in various turbulence experiments. A question of interest is the reason for this ubiquitous presence of exponential tails in PDFs. We have argued²³ non-Gaussian behavior of local fluctuations can be caused by having intermittent amplitude fluctuations of the terms (external or deterministic) on a time scale larger than or comparable to the intrinsic diffusive scale. We have demonstrated this mechanism using discrete systems where the passive scalar is advected by chaotic velocity field generated from stream functions that are (a) Random noise variables with amplitude fluctuations that are arranged to be intermittent and (b) deterministic coupled map models with different time scales. We have also studied continuum models with Gaussian random velocity fields and shown that both modified Lorentzian and Gaussian PDFs can occur. We have also studied analytically the Fokker-Planck equation for systems with one or two degrees of freedom that yield different forms of steady-state PDFs that occur in extended systems.

Chaotic Phenomena in Josephson Junction Arrays

In collaboration with R. Bhagavatula and C. A. Ebner (OSU) We have investigated the dynamical behavior of 2-d capacitive Josephson Junction Arrays driven by external ac and dc currents from the point of view of spatially extended dynamical systems.

(i) We have identified²⁴ chain states that are stable for a broad range of junction parameters in which there are zero currents in the transverse junctions for zero noise and the superconducting phase difference across a longitudinal junction, along the line of the input current, is quasiperiodic. We have computed the $I - V$ characteristics of such states in the presence of thermal fluctuations.

(ii) In the chaotic regimes we found²⁵ that while local variables exhibit chaotic behavior, macroscopically-averaged quantities, e.g., voltage difference between two junctions far apart and the spatial average of the Josephson current, are periodic with the period of the external ac currents. This is in accord with expectations based on our previous work.

(iii) Motivated by experiments in several fluid systems, we have studied probability distributions of “local” junction voltages.²⁵ We find that the voltages across transverse junctions have exponential tails in the chaotic states. This is the first non-fluid system where such behavior has been seen. We have shown that the mechanism suggested by us²³ applies here: the amplitude of the effective noise that couples to the equations of motion at a coarse grained level show slow variations. This in turn is caused by the the random generation and motion of vortices. Good agreement is found between parameters that characterize our mechanism and the slopes of the exponential distributions.

Self-Organized Criticality in non-conserving models

Work done with G. Grinstein and J. Socolar (IBM).

It has recently been argued that certain nonequilibrium model systems evolve generically to a scale-invariant steady state characterized by algebraic decays of correlations and a power law distribution of the sizes of relaxation events. This phenomenon, termed self-organized criticality, has been conjectured to occur commonly in spatially-extended, dissipative systems that are driven slowly relative to the system's relaxation rate. We have studied²⁶ two models with nonconserving dynamics and continuous, deterministic driving that show evidence of self-organized criticality. When periodic boundary conditions are applied, the models correspond to special points in parameter space that divide regions of qualitatively different noncritical behaviors. The results elucidate the role of boundary conditions in self-organizing nonconserving systems. We have also identified interesting new regimes where spatially averaged variables exhibit periodic or chaotic temporal oscillations. We conjecture that in such states scale invariant correlations arise due to breaking of continuous time-translation symmetry; this is a new mechanism for the generation of scale invariance in nonconserving systems.

Interface Models of Turbulence

Work done with T. Bohr (Niels Bohr Institute), G. Grinstein (IBM), and M. Jensen (NORDITA) and J. Krug (Jülich).

A rough interface exhibits a growing structure function $S(x - y) = \langle [h(x) - h(y)]^2 \rangle \sim |x - y|^{2\chi}$, where $h(x, t)$ is the position of the interface at x

and at time t . The coexistence of scale-invariant behavior of structure functions with chaotic behavior is characteristic of fully-developed turbulence. Motivated by this we have constructed²⁷ non-conserving one-dimensional models that are both rough and chaotic; we have computed the Liapunov spectrum and found that the attractor dimension grows linearly with the size of the system. The one-dimensional models show no non-trivial intermittency corrections. An important issue is the universality class of the long-wavelength behavior. Heuristic renormalization group suggest that the model should belong in the universality class of the Kardar-Parisi-Zhang (KPZ) model. However, numerically we find that the exponents are consistent with those of the free diffusive system; we argued that there is very slow crossover and this is due to the smallness of the nonlinear coupling constant.²⁸ We have also studied variants of the original model in which the crossover can be observed earlier. This work ties in nicely with that on the K-S equation in the effort to find a long-wavelength description of deterministic chaotic models in terms of stochastic equations with appropriate symmetries.

Dynamic Scaling in the Kuramoto-Sivashinsky Equation

Work done with K. Sneppen, M. Jensen, T. Bohr (Niels Bohr Institute), and J. Krug (IBM)

We have performed²⁹ extensive numerical simulations on the Cray-YMP and a connection machine (CM-2) of the one-dimensional Kuramoto-Sivashinsky equation. We have used the results to conduct a detailed crossover scaling

analysis which indicates that the large-scale fluctuations of the chaotic system are described by a noisy KPZ or Burgers equation depending on the representation used. This work complements the coarse-graining methods used by us elsewhere. A large effective interfacial tension causes the asymptotic behavior to be observed only after a long intermediate scaling regime. The skewness of the interfacial fluctuations is found to be a useful probe of the crossover phenomena.

Fluctuating hydrodynamics of microemulsions

Work done with S. Ramaswamy and R. Pandit (Bangalore)

Multi-component mixtures of oil, water, and surfactants have been studied extensively both experimentally and theoretically. We had proposed a continuum Landau-Ginzburg theory and explored static thermodynamic phenomena earlier. Recently, we have computed time-dependent correlation functions that are experimentally accessible. We have generalized our static model to a time-dependent Ginzburg-Landau equation incorporating conservation laws (of the various densities, momentum, etc.) and enforcing incompressibility. We have calculated the first perturbative correction to the density fluctuation spectrum of oil/water that is measurable by inelastic light scattering. We have also computed the corrections to the viscosity whose real and imaginary parts yield contributions to sound damping and the sound velocity.³⁰

F. Hayot

Non variational effects in a domain wall equation

Work done with L. Fourtune (Paris).

We have studied in two dimensions a Ginzburg- Landau equation for a complex amplitude, which represents magnetization, where non- variational effects are due to the presence of a weak complex external magnetic field³¹. These effects manifest themselves through the dynamics of domain wall motion. There is a rich phase space, with parameters related to the frequency and strength of the magnetic field, and the amount of anisotropy in the magnetic energy. Related experiments concern the technology of magnetic "bubble" materials.

We obtain striking results concerning the motion of domain boundaries.

These boundaries have the shape of spirals or connected spirals, which are turning around Bloch points, which are zeroes of the amplitude, and act as point sources of the spirals. The velocity of propagation of the front between two opposite magnetization increases as one moves away from the Bloch point along the spiral, with the asymptotic velocity growing approximately as the square of the amplitude of the magnetic field. Moreover for circular domains of one magnetization embedded in a region of opposite magnetization , we show that there exists a critical radius above which these domains expand in the presence of the complex rotating magnetic field. There thus exist the

conditions for the occurrence of nucleation phenomena.

Turbulent domain stabilization in annular flows.

Work done with Y. Pomeau (Paris).

We point³² out a new mechanism for stabilizing expanding turbulent domains in annular flows. This nonlocal mechanism is explained within the context of a Ginzburg-Landau equation for a real amplitude which is related to the amplitude of turbulent fluctuations. The expression for the nonlocal term can be derived by analogy with existing calculations in Taylor-Couette flow for Taylor vortices. We compare our numerical results with experiments on Couette flow.

F. Hayot and C. Jayaprakash

Effective stochastic behavior of deterministic systems at long wavelengths

Work done with C. Josserand (Paris) and R. Pandit(Bangalore).

An important issue is whether a system described by a deterministic, chaotic equation can, on large scales, be represented by a stochastic equation where the noise arises from the small-scale chaotic fluctuations. We have answered this question for the Kuramoto-Sivashinsky (K-S) equation. This equation has both chaotic behavior (finite density of Liapunov exponents)

and structure functions growing with distance, a situation reminiscent of fully developed turbulence in the inertial range. Our work deals with this equation in one³³ and two³⁴ dimensions. The effective stochastic equation with the right symmetries is the Kardar-Parisi-Zhang(KPZ) equation, a generic model for interfaces. The question then is: do the two systems display the same static and dynamic scaling behavior at large scales ? In $d = 1$, after our work and that of others, it is now believed that the answer is yes; in 2 dimensions the answer is still controversial: we have argued that the behaviors are the same but Procaccia and co-workers have argued that they are not.

We have used a coarse-graining procedure pioneered by Zaleski, which removes short-wavelength degrees of freedom of the K-S equation, and generates an effective equation of the KPZ form. The procedure renormalizes the “viscosity”, multiplying the diffusive term from a destabilizing negative value to a stable positive value ν_{eff} and introduces an effective noise. First we examined various aspects of this procedure in $d = 1$. We computed the noise-noise correlation function and demonstrated that at large scales it is additive and has a white noise structure. We determined the largest Liapunov exponent λ_{max} and found that the noise-noise correlation time is proportional to $1/\lambda_{max}$. From the strength of the noise-noise correlation, the renormalized viscosity and the value of the nonlinear term, we estimated the time and length scales at which one crosses over from free field to interacting behavior. These are large in $d = 1$ (in agreement with 1.2.7) but exponentially large

in $d = 2$. The large values of the crossover are partly due to the large value of ν_{eff} . We found that ν_{eff} is determined by the time scale $1/\lambda_{max}$ and the length scale of the cell structure.

Hydrodynamic instability in the Kuramoto- Sivashinsky equation

We have done a large amount of preliminary work on a nonlinear integro-differential equation that is a generalization of the Kuramoto- Sivashinsky equation. This equation adds to the K-S equation a nonlocal term (in Fourier space a $\text{mod}(k)$ term) which takes into account the Darrieus-Landau hydrodynamic instability of flame fronts. One can thus consider a continuum of models which interpolate between the Kuramoto- Sivashinsky equation proper, which deals with the purely diffusive instability of front flames controlled by the Lewis number, and a model with only the hydrodynamic instability. If ϵ denotes the interpolation parameter, let $\epsilon = 0$ correspond to the K-S equation, and $\epsilon = 1$ to the purely hydrodynamic case. It is known that for the whole range of ϵ up to values very close to 1, the largest Lyapunov exponent remains positive, and that a large scale structure with cusps develops as ϵ approaches 1, the nature of which has been studied analytically at exactly $\epsilon = 1$, and described through numerical simulations.

In the spirit of our approach to the K-S equation, we have investigated the scaling properties of this class of new models with the aim of checking whether here also an effective stochastic equation describes the large scale

properties or if the nonlocal terms prevent such a description.

Despite the large amount of preliminary work no clear comprehensive picture has yet emerged. Resolving computationally the cusp structure is computationally expensive. Isolated cusps emerge, move and disappear, and on top of the large scale structure, of which they are part, there occur the chaotic fluctuations of the K-S equation. We have found that for $\epsilon \rightarrow 1$, the scaling behavior is anomalous and the putative exponents different from those which characterize the purely diffusive instability. Since this is uncharted territory the accumulation of high statistics data is required to pin down the scaling behavior. We have yet been unable to accomplish the equivalent of the coarse-graining approach so successful in the pure Kuramoto-Sivashinsky approach. Nevertheless we have written down large scale effective equations which to some extent reproduce the observed scaling behavior. It turns out, interestingly, that the noise term in these equations has peculiar features of the type which are required in the renormalization group approach to turbulence initiated by Yakhot and Orszag.

References

- [1] K. Balasubramanian, F. Hayot and W. F. Saam, Phys. Rev. A 36 (1988) 2248.
- [2] D. Burgess, F. Hayot and W. F. Saam, Phys. Rev. A 38 (1988) 3589.
- [3] D. Burgess, F. Hayot and W. F. Saam, Phys. Rev. A 39 (1989) 4695.
- [4] D. Burgess and F. Hayot, Phys. Rev. A 40 (1989) 5187.
- [5] F. Hayot, Physica D 47, (1991) 64
- [6] F. Hayot and M. Raj Lakshmi, Physica D 40 (1989) 415.
- [7] F. Hayot, Phys. Rev. A 43 (1991) 806
- [8] D. Burgess and M. R. Foster, Phys. Fluids.
- [9] D. Burgess and S. Tanveer, Phys. Fluids, A 7 (1990) 1105.
- [10] J. Hegseth, C. D. Andereck, F. Hayot and Y. Pomeau, Phys. Rev. Lett. 62 (1989) 257.
- [11] C. Ebner, F. Hayot and J. Cai, Phys. Rev. B 42 (1990) 8187
- [12] G. Grinstein, Y. He, C. Jayaprakash, and B. Bolker, Phys. Rev. A 44, 4923 (1991).

- [13] C.H. Bennett, G. Grinstein, Y. He, C. Jayaprakash and D. Mukamel,
Phys. Rev. A 41, 1932 (1990).
- [14] Y. He, C. Jayaprakash, and G. Grinstein, Phys. Rev. A 42, 3348, (1990).
- [15] O. Biham and W. Wenzel, Phys. Rev. Lett. 63, 819, (1990).
- [16] O. Biham and W. Wenzel, Phys. Rev. A 42 , 4639, (1990).
- [17] W. Wenzel, O. Biham, and C. Jayaprakash, Phys. Rev. B **43**, 6550,
(1990).
- [18] O. Biham and M. Kvale, , Phys. Rev. A **46**, 6334 (1992).
- [19] K. Chen, C. Jayaprakash, R. Pandit, and W. Wenzel, Phys. Rev. Lett.
65, 2736, (1990).
- [20] A. Sinha and C. Jayaprakash, Physica D 39, 352 (1989).
- [21] R. Bhagavatula, Y. He, C. Jayaprakash and G. Grinstein, Phys. Rev.
Lett. **69**, 3483 (1992).
- [22] J. E. S. Socolar, G. Grinstein, and C. Jayaprakash, Phys. Rev. E **47**,
2366 (1993).
- [23] R. Bhagavatula and C. Jayaprakash, Phys. Rev. Lett. **71**, 3657 (1993).
- [24] R. Bhagavatula, C. Ebner, and C. Jayaprakash, Phys. Rev. B **45**, 4774
(1992).

- [25] R. Bhagavatula, C. Ebner, and C. Jayaprakash, Phys. Rev. B **50** 9376 (1994).
- [26] G. Grinstein, C. Jayaprakash, and J. E. S. Socolar, Phys. Rev. E **48**, RC 643 (1993).
- [27] T. Bohr, G. Grinstein, C. Jayaprakash, M. H. Jensen and D. Mukamel, Phys. Rev. A **46**, 4791 (1992).
- [28] K. Sneppen, J. Krug, M. H. Jensen, C. Jayaprakash and T. Bohr, Phys. Rev. A **46**, R7351 (1992).
- [29] T. Bohr, G. Grinstein, C. Jayaprakash and D. Mukamel, Physica D **59**, 177 (1992).
- [30] C. Jayaprakash, R. Pandit, and S. Ramaswamy, "Fluctuating Hydrodynamics of Microemulsions", *in preparation*.
- [31] F. Hayot and L. Fourtune, Physica A **199**, 75 (1993).
- [32] F. Hayot and Y. Pomeau, Phys. Rev. E **50**, 2019 (1994)
- [33] F. Hayot, C. Jayaprakash and Ch. Josserand, Phys. Rev. E **47**, 911 (1993).
- [34] C. Jayaprakash, F. Hayot and R. Pandit, Phys. Rev. Lett. **71**, 12 (1993).

- [35] R. Bhagavatula, K. Chen, and C. Jayaprakash, Phys. Rev. B **50** 9376 (1994).