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THE POTENTIAL FOR NEUTRINO PHYSICS
AT MUON COLLIDERS AND DEDICATED HIGH CURRENT
MUON STORAGE RINGS

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The Potential for Neutrino Physics at Muon Colliders and Dedicated High Current Muon Storage Rings

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Abstract:

Conceptual design studies are underway for both muon colliders and high-current non-colliding muon storage rings that have the potential to become the first true "neutrino factories". Muon decays in long straight sections of the storage rings would produce uniquely intense and precisely characterized two-component neutrino beams -- muon neutrinos plus electron antineutrinos from negative muon decays and electron neutrinos plus muon antineutrinos from positive muons. This article presents a long-term overview of the prospects for these facilities to greatly extend our capabilities for accelerator-based neutrino physics studies for both high rate and long baseline neutrino experiments. As the first major physics topic, recent experimental results involving neutrino oscillations have motivated a vigorous design effort towards dedicated neutrino factories that would store muon beams of energies 50 GeV or below. These facilities hold the promise of neutrino oscillation experiments with baselines up to inter-continental distances and utilizing well understood beams that contain, for the first time, a substantial component of multi-GeV electron-flavored neutrinos. In deference to the active and fast-moving nature of neutrino oscillation studies, the discussion of long baseline physics at neutrino factories has been limited to a concise general overview of the relevant theory, detector technologies, beam properties, experimental goals and potential physics capabilities. The remainder of the article is devoted to the complementary high rate neutrino experiments that would study neutrino-nucleon and neutrino-electron scattering and would be performed at high performance detectors placed as close as is practical to the neutrino production straight section of muon storage rings in order to exploit beams with transverse dimensions as small as a few tens of centimeters.

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We discuss the potential for three types of novel neutrino targets and detectors that, if realizable, could together usher in a new era of high precision neutrino physics studies: 1) fully-active vertexing and tracking targets surrounded by hermetic backing detectors that would potentially reconstruct at least of order 10^7 events per year per kilogram mass, 2) cryogenic polarized targets that should allow the first studies of spin physics with neutrino beams, and 3) fully active tracking liquid targets with masses of several tons for reconstructing the rarer neutrino-electron scattering events. The diverse areas of high rate neutrino physics considered in this article include: (i) the only known opportunity in any physics process to map out the detailed quark-by-quark 4-momentum and spin substructure of the nucleon, (ii) two precision tests of the weak mixing angle that characterizes the electroweak interaction, $\sin^2 \theta_W$, from both neutrino-electron scattering and deep inelastic neutrino-nucleon scattering, perhaps to an accuracy at the 10^{-3} to 10^{-4} level (iii) the most precise measurements of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$, (iv) additionally, beam energies well above the 25 GeV production threshold for b quark production might also yield uniquely precise measurements of the much more theoretically interesting elements $|V_{ub}|$ and $|V_{cb}|$, perhaps at the 1–3% level of precision, (v) several searches for rare and exotic processes, and finally, (vi) samples of hundreds of millions of precisely reconstructed charm decays would allow additional analyses outside neutrino physics. The expected experimental conditions at neutrino factories are so superior to today's neutrino experiments that the contents of this article must surely represent no more than a limited first attempt at understanding their full potential for neutrino physics.

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Chapter 1

Introduction and Physics Overview

1.1 Introduction

1.1.1 High Current Muon Storage Rings and their Use for Neutrino Physics

Muon colliders, storage rings in which muons are collided instead of electrons or protons, have been proposed as a means of circumventing the natural limitations on electron-positron storage rings due to synchrotron radiation. Synchrotron radiation by a muon beam is reduced by a factor of $(\frac{m_e}{m_\mu})^4 \simeq 5 \times 10^{-10}$ relative to a circulating electron beam of the same energy. Recent feasibility and design studies for future muon colliders [1, 2] have begun to focus attention on the exciting possibilities for physics experiments using the neutrino beams from high energy muon decays.

The proposals to use the neutrino beams emanating from straight sections in the magnet lattices of either muon colliders [3, 4] or muon storage rings designed specifically for neutrino physics [5] (dedicated “neutrino factories”) are reincarnations of suggestions for neutrino experiments at muon storage rings that date back at least to the 1960’s. The essential difference is the very large muon currents that might be produced and stored using muon collider technology [1, 2]. The resulting neutrino beams may be many orders of magnitude more intense than considered in the early references and, indeed, considerably more intense than today’s “conventional” neutrino beams from pion decays. Additional physics advantages will result from their unique and precisely understood flux composition: μ^- and μ^+ decays will produce pure 2-component neutrino beams containing $(\nu_\mu \bar{\nu}_e)$ or $(\bar{\nu}_\mu \nu_e)$, respectively.

This book explores the physics potential for these “neutrino experiments at

muon colliders”, or ν MCs for short, where this is implicitly includes the option of non-colliding muon storage rings dedicated to the production of neutrino beams.

The reader is referred to the specialist literature [1, 2] for the detailed technologies required for muon colliders and storage rings. These will be discussed only to the extent that they determine the properties of the neutrino beams. As a caveat on the physics potential of high intensity muon storage rings, it should always be borne in mind that the ambitious technologies of muon storage rings are still only at the level of either feasibility studies or early design studies and, at present, it is still not guaranteed that they will progress through to realization.

1.1.2 The Potential Role of ν MCs in High Energy Physics

Neutrinos are theoretically interesting because they are either massless or nearly so, with growing indirect evidence from neutrino oscillation experiments for masses of the order of eV or less, which is at least several orders of magnitude below the masses of their charged lepton partners.

The interactions of neutrinos are also worthy of study, both in their own right and as probes of the quark content and dynamics of nucleons. Several precise measurements involving the electroweak and strong interactions augment those from electron-positron colliders as a testing-ground for the consistency of the Standard Model of elementary particle physics and of Standard Model calculations using perturbative field theory. As well as looking for hints of new physics, the consistency of Standard Model calculations must be established in order to interpret the data from high energy colliders and to distinguish background processes from possible signatures for new physics.

In combination therefore, muon storage rings have wide-ranging potential to make advances in many areas of elementary particle physics, both in precision tests of the Standard Model and in searches for evidence of new physics beyond the Standard Model.

1.1.3 Some Preliminaries on Layout and Notation

The currently volatile status of neutrino oscillation measurements makes it difficult to assess the potential for neutrino oscillation experiments at muon storage rings. Additionally, this topic has already been covered in some detail elsewhere [6]. Therefore, only one chapter, chapter 2, is devoted to long baseline oscillation experiments, despite the current interest in this topic and the impressive potential for studies at dedicated neutrino factories or muon colliders.

Instead, this book concentrates largely on the high-rate neutrino physics possibilities at detectors close to the muon straight section. Chapters 3 through 7 discuss each of the main classes of physics at ν MCs ; in order: deep inelastic scattering and QCD studies, the CKM mixing matrix, precision electroweak studies, rare and exotic processes and charm physics.

The concept of neutrino experiments at high current muon storage rings is novel enough that the notations to describe these facilities are not yet settled. As already mentioned, this book will refer to them as “ ν MCs”, short for “neutrino experiments at muon colliders”, with the implicit understanding that the term also covers experiments at “dedicated” neutrino factories. It is noted that the alternative name “MURINEs”, standing for MUon RIng Neutrino Experiments, is also in use. Another convention and abbreviation in use in this book will be to refer to neutrino oscillation experiments as “long baseline” or LB experiments and the complementary experiments that study neutrino interactions will be referred to as either high rate (HR) or short baseline (SB) experiments. As one final notational note, units are given in square brackets in equations throughout the book.

1.2 Experimental Overview

The experimental requirements will be rather different for the two broad classes of neutrino physics, i.e., studies of neutrino oscillations and of neutrino interactions. Neutrino oscillations will usually be studied at long baseline experiments while the other topics will be investigated through studies of neutrino-nucleon deep inelastic scattering (DIS) and neutrino-electron elastic scattering. These will be conducted at short baselines to maximize the event rates.

This section is split into three subsections. The first subsection gives a primer on the dominant interaction process of many-GeV neutrino, namely, neutrino-nucleon deep inelastic scattering (DIS). The middle subsection then discusses the characteristics of the neutrino beams for ν MCs and also takes advantage of the cross section information from the preceding subsection to define and quantify some parameters that will characterize the event rates at high rate and long baseline detectors. The detailed calculations for this subsection have been relegated to an appendix, appendix A. This subsection culminates with a table of predicted beam parameters and event rate parameters for some example muon storage ring parameters. Again, the justifications for the storage ring parameters have been relegated to appendix A. The section is rounded out with a subsection that discusses SB and LB detector design considerations and scenarios. Detector scenarios are combined with the tabulated information from the preceding subsection to predict order-of-magnitude event sample sizes for both short and long baseline experiments.

1.2.1 A Primer on the Deep Inelastic Scattering Interactions of Neutrinos

The dominant interaction of TeV-scale neutrinos is deep inelastic scattering (DIS) off nucleons (i.e. protons and neutrons), of which there are 2 types: neutral current (NC) and charged current (CC) scattering.

NC scattering comprises about 25 percent of the total interaction cross section. The neutrino is deflected by a nucleon (N) and loses energy with the

production of several hadrons (X):

$$\nu + N \rightarrow \nu + X. \quad (1.1)$$

For neutrino energies beyond a few GeV, DIS becomes rather well described by the quark-parton model. In this model, NC DIS is interpreted as elastic scattering off one of the many quarks inside the nucleon through the exchange of a virtual neutral Z boson:

$$\nu + q \rightarrow \nu + q. \quad (1.2)$$

CC scattering is similar to NC scattering except that the neutrino turns into its corresponding charged lepton:

$$\begin{aligned} \nu + N &\rightarrow l^- + X, \\ \bar{\nu} + N &\rightarrow l^+ + X, \end{aligned} \quad (1.3)$$

where l is an electron/muon for electron/muon neutrinos. In the quark-parton model this is interpreted as an exchange of a W boson between the neutrino and a quark (q), which is turned into another quark species (q') whose charge differs by one unit.

$$\begin{aligned} \nu + q &\rightarrow l^- + q', \\ \bar{\nu} + q' &\rightarrow l^+ + q. \end{aligned} \quad (1.4)$$

For NC and CC interactions alike, the final state quark always “hadronizes” at the nuclear distance scale, combining with quark-antiquark pairs to produce the several primary hadrons, X , that are seen in the detector.

The quark-parton model is a useful approximation to neutrino-nucleon DIS that will be used extensively throughout this book to describe the potential physics analyses at ν MCs. However, it should be noted that the actual analyses would be performed using more correct and detailed descriptions of the interactions for which the quark-parton model is only the leading term in perturbative QCD expansions and other sub-leading order processes are accounted for.

For TeV-scale neutrinos, the neutrino cross-section is approximately proportional to the neutrino energy, E_ν . The charged current (CC) and neutral current (NC) interaction cross sections for neutrinos and anti-neutrinos have numerical values of [7]:

$$\sigma_{\nu N}[\text{cm}^2] \text{ for } \begin{pmatrix} \nu - CC \\ \nu - NC \\ \bar{\nu} - CC \\ \bar{\nu} - NC \end{pmatrix} \simeq \begin{pmatrix} 0.72 \\ 0.23 \\ 0.38 \\ 0.13 \end{pmatrix} \times E_\nu[\text{TeV}] \times 10^{-35}. \quad (1.5)$$

Much of the interesting short baseline physics involves heavy flavor production and the fractional production of heavy flavors as functions of neutrino and anti-neutrino energy are shown in figure 1.2.

The cross section for neutrino-electron scattering is down by about three orders of magnitude from neutrino-nucleon scattering. It will be shown that this process can be studied at ν MCs in order to obtain a precise measurement of the weak mixing angle, $\sin^2 \theta_W$.

1.2.2 Neutrino Beam Characteristics and Event Rate Parameters at Long Baseline and High Rate Detectors

Neutrinos from muon decays

In current design scenarios for muon colliders [1, 2], large opposing bunches of a few times 10^{12} positive and negative muons will circulate in the collider ring until decay into neutrinos and electrons:

$$\begin{aligned}\mu^- &\rightarrow \nu_\mu + \bar{\nu}_e + e^-, \\ \mu^+ &\rightarrow \bar{\nu}_\mu + \nu_e + e^+.\end{aligned}\tag{1.6}$$

Bunch sizes may be similar or smaller in the designs that are beginning to emerge for dedicated neutrino factories, and the bunches may well be partitioned into bunch trains.

The neutrinos from muon decays in straight sections of the muon storage ring will line up as uniquely intense and pure 2-component beams: $(\nu_\mu \bar{\nu}_e)$ beams from negative muon decays and $(\bar{\nu}_\mu \nu_e)$ beams from positive muons. Depending on the production option, the “production straight section” (PSS) used for neutrino experiments can be either the longest straight section in a muon collider, the linac section in a recirculating-linac accelerating structure or the dedicated PSS in a dedicated muon storage ring. These possibilities are discussed in detail in appendix A, along with a detailed characterization of the neutrino spectra produced and simple approximate formulae for the neutrino fluxes and event rates expected at short and long baseline neutrino detectors.

The decay of muons is a purely electroweak processes that is accurately predictable and is also very well-tested through the experimentally observed energy spectrum of the daughter electrons. Further, the characteristics of the muon beam in the straight section can be well understood and modeled through a knowledge of the focusing magnet lattice and through beam monitoring. To illustrate, beam current measurements using current loops should allow prediction of the neutrino flux normalization to at least the 1 part per mil level [8].

Neutrino spectra and fluxes at long baseline and high rate detectors

The spectra of the pure 2 component beams can be relatively accurately predicted using simple analytic formulae for the decays of non-polarized muons in perfect monochromatic, divergenceless pencil beams. An overview of the results follows; the details are given in appendix A. As a caveat, it should be noted that the idealizations used in the calculations of appendix A are not always good approximations, particularly for SB detectors. In particular, the spectra

will be distorted whenever the length of the production straight section becomes comparable to the distance to the detector.

The neutrino energy spectrum at an angle θ to an idealized muon pencil beam with energy E_μ is given by:

$$E_\nu(x, E_\mu, \theta) \equiv xE_\mu(1 + \beta \cos \theta')/2, \quad (1.7)$$

where the angle, θ' , in the muon rest frame is related to the laboratory angle by

$$\sin \theta = \frac{\sin \theta'}{\gamma(1 + \beta \cos \theta')}, \quad (1.8)$$

x is a scaling variable ranging from 0 to 1 and the scaled muon velocity, β , is essentially unity and will be neglected in some future formulae. Explicit formulae for the distributions in x of muon- and electron-neutrinos (or anti-neutrinos) are given in appendix A.

LB detectors will sample the forward-going neutrinos, at $\theta' = \theta = 0$ while SB detectors will sample a range in θ . For simplicity, we will normally assume that the neutrino targets of SB detectors intercept the boosted forward hemisphere of the decaying neutrinos, i.e., out to an angle:

$$\theta' = \frac{\pi}{2} \Leftrightarrow \theta_\nu \simeq \sin \theta_\nu = 1/\gamma = \frac{m_\mu c^2}{E_\mu} \simeq \frac{10^{-4}}{E_\mu[\text{TeV}]}. \quad (1.9)$$

Figure 1.1 shows the neutrino event sample energies for a long baseline detector which subtends only the very forward region in the center of mass and of a short baseline detector which subtends the forward 90 degrees in the center of mass. The short baseline detector spectrum is a convolution of the scaled spectra at each angle while the long baseline spectrum reflects the spectrum expected at maximal energy.

The values in table 1.1 demonstrate that both the mean beam and event energies are appreciable fractions of their maximum value, and that the beam and event energy spread are comparatively small. Both of these characteristics are favorable for neutrino physics.

Table 1.1: Average neutrino energies and rms energy spreads as fractions of the maximum neutrino energy at any chosen polar angle relative to the neutrino beam axis. The “beam” row gives the fraction for the neutrino beam itself while the “events” column is weighted by the energy-dependent neutrino cross-section. (See the text for details.)

	ν_e or $\bar{\nu}_e$	ν_μ or $\bar{\nu}_\mu$
scaled average energy of beam	0.60	0.70
scaled average energy of events	0.67	0.76
scaled rms energy spread of beam	0.20	0.21
scaled rms energy spread of events	0.18	0.18

A short-baseline neutrino target centered on the beam axis and subtending a fixed solid angle will produce neutrino events in proportion to the beam strength

Neutrino Fluxes from a 50 GeV Muon Beam

Figure 1.1: Neutrino event spectra (weighted by the interaction cross section) for muon neutrinos and electron anti-neutrinos from a negative muon beam. The solid curves indicate the spectra for decays at zero degrees in the center of mass system. This is the spectrum expected for a detector located very far from the muon decay region. The dashed curves indicate the spectra for decays within the forward hemisphere in the center of mass frame. This is what would be expected for a detector close enough to the muon decay region to subtend an angle of $\frac{1}{\gamma}$.

and to the depth of the target, l , given in units of g.cm^{-2} . Thus, a convenient benchmark for the size of neutrino event samples the beam can produce is N^{sb} , defined as

$$\text{no. of } \nu \text{ events} = N^{\text{sb}}[\text{events/yr}/(\text{g.cm}^{-2})] \times l[\text{gm.cm}^{-2}] \times Y, \quad (1.10)$$

with Y the number of years of data-taking.

Appendix A.2 derives the following numerical expression for N^{sb} in the case where the target is rotationally symmetric around the beam axis and extends out to the polar angle $\theta^{\text{max}} = 1/\gamma$:

$$N^{\text{sb}}[\text{events/yr}/(\text{gm.cm}^{-2})] = 2.13 \times 10^{-15} \times E_\mu[\text{GeV}] \times N_\mu^{\text{ss}}[\text{yr}^{-1}]. \quad (1.11)$$

The parameter N_μ^{ss} is the number of forward-going muons (as opposed to muons circulating in the opposite direction in, e.g., a collider ring) decaying in the production straight section per year. It is related to the total number of muons in the collider by equation A.12 of appendix A.2.

In contrast to short baseline detectors, a long baseline detector will subtend a small solid angle at the center of the neutrino beam so the number of interactions in the detector will vary in proportion to the target mass, $M[\text{kg}]$, and inversely as the square of the baseline length, $L[\text{km}]$. This leads to a definition, analogous to equation 1.10, for the event rate benchmark $N^{\text{lb}}[\text{events/yr}/(\text{kg.km}^{-2})]$:

$$\text{no. of } \nu \text{ events} = N^{\text{lb}}[\text{events/yr}/(\text{kg.km}^{-2})] \times \frac{M[\text{kg}]}{(L[\text{km}])^2} \times Y. \quad (1.12)$$

Appendix A.2 predicts a numerical value at the beam center of

$$N^{\text{lb}}[\text{events/yr}/(\text{kg.km}^{-2})] = \frac{1.62 \times 10^{-20} \times N_\mu^{\text{ss}}[\text{yr}^{-1}] \times (E_\mu[\text{GeV}])^3}{(\gamma \cdot \delta\theta)^2}. \quad (1.13)$$

(Note the appearance of an angular divergence scaling factor, $\gamma \cdot \delta\theta$; this is discussed in appendix A.2.) As an aside, it is a useful conversion for long baseline values at the higher energy parameters that the numerical value of N^{lb} is the same in units of $\text{events/yr}/(\text{kg.km}^{-2})$ or of $\text{events/yr}/(\text{kT} \cdot (1000\text{km})^{-2})$.

Event rates for example scenarios

Table 1.2.2 gives a summary of the beam parameters and some benchmark rate parameters, such as N^{sb} and N^{lb} , for short and long baseline detectors and with specific neutrino factory and muon collider scenarios spanning a muon energy range from 20 GeV to 5 TeV. The motivation for the parameters in each of the scenarios is discussed in section A.5 of appendix A and the implications for event rates in detectors will be followed up on in the following subsection. The integrated luminosity corresponding to 10^{10} events in a SB detector is also given in table 1.2.2. From the discussion in appendix A, its numerical value is easily seen to be given by

$$\int L dt [\text{cm}^{-2}] = 6.022 \times 10^{23} \times N_\mu^{\text{ss}}[\text{yr}^{-1}] \times (l \times Y)[\text{g.cm}^{-2} \cdot \text{yr}]. \quad (1.14)$$

Table 1.2: Neutrino fluxes and event rates for example parameter sets for muon storage rings (muSR's) or muon colliders.

description muon energy, E_μ	muSR 20 GeV	Higgs 50 GeV	top 175 GeV	frontier 500 GeV	2nd gen. 5 TeV
storage ring specs.:					
$(\mu^- \text{ or } \mu^+)/\text{year}, N_\mu [10^{20}]$	3.0	6.0	6.0	3.2	3.6
time to beam dump, $t_D [\gamma\tau_\mu]$	no dump	no dump	no dump	0.5	no dump
ring circumference [m]	300	345	900	2000	15 000
str. section length [m]	90	40	110	150	450
fract. str. sect. length, f_{ss}	0.30	0.12	0.12	0.12	0.03
ν beam specs.:					
ν from str. sect./year $[10^{20}]$	1.8	1.4	1.4	0.30	0.22
ν ang. diverg. $[\gamma \cdot \delta\theta_\nu]$	1	1	1	10	1
ν ang. diverg. [mrad]	5.3	2.1	0.60	2.1	0.021
SB exptal. specs.:					
$N^{sb} [\text{evt/yr}/(\text{g} \cdot \text{cm}^{-2})]$	3.8×10^6	6.5×10^6	2.7×10^7	2.3×10^7	1.0×10^8
$(l \times Y) [\text{g}/\text{cm}^2]$ for 10^{10} evt	2600	1500	370	430	100
$\int \mathcal{L} dt [10^{46} \text{cm}^{-2}]$ for 10^{10} evt	14	6.5	1.6	0.65	0.065
LB exptal. specs.:					
$N^{lb} [\text{evt/yr}/(\text{kg} \cdot \text{km}^{-2})]$	1.2×10^4	1.4×10^5	6.2×10^6	5.0×10^5	2.2×10^{10}

1.2.3 Neutrino Detectors for ν MCs

The SB and LB detectors needed for ν MCs would be expected to differ considerably from each other in both their physics motivation and their design. SB detectors will study the physics of neutrino interactions and they will be placed as close as backgrounds allow in order to maximize the event rate and to subtend the neutrino beam with a target of small cross-section. In contrast, the LB experiments will study the flavor oscillations and other potential baseline-dependent properties of the neutrinos themselves.

The event rate parameters for these detectors, given in table 1.2.2, are very impressive. They correspond to neutrino beams that are orders of magnitude stronger than today's beams, with the gains biggest at the higher energies. The values of $l \times Y$ needed for 10^{10} events in the high rate detectors seem to be feasible, as will be discussed further in appendix A. Many of the physics discussions in chapters 3 through 7 assume an event sample size of 10^{10} events, while samples of 10^{11} events and above can be seriously contemplated for higher mass targets at the highest energy neutrino beams that can be contemplated for ν MCs [9]. The steeply rising value of the long baseline rate parameter, N^{lb} , with increasing energy reflects the cubic energy dependence in equation 1.13. As discussed in chapter 2, it is natural to increase the detector baseline with increasing energy so the event rate will rise less steeply with energy than N^{lb} does.

Long baseline detectors for studies of neutrino oscillations

Long baseline experiments will be able to operate with both much higher event rates and baselines than today's accelerator-based experiments. They will also benefit from extremely well understood and pure two-component beams with accurately predictable energy spectra, angular divergences and intensities. The electron-neutrino and electron-antineutrino components of these beams have not previously been available at high energies and high fluxes. As one consequence that is discussed in chapter 2, this tends to emphasize the importance of ν_μ appearance experiments that are sensitive to the oscillation process $\nu_e \leftrightarrow \nu_\mu$.

As is further outlined in chapter 2, the design parameters for the detectors will depend on the specific oscillation process and range of oscillation parameters under study. The scenarios that are currently attracting the most intense interest revolve around the evidence for neutrino oscillations at the Super-Kamiokande experiment. The LB experiments relevant to these studies can probably be best performed using dedicated muon storage rings with long, steeply-inclined straight sections pointed at kiloton-scale detectors that may be up to several thousand kilometers away. Event rates at ν MC oscillation experiments should not be too dissimilar to those in today's experiments – perhaps 10^3 or 10^4 events per year – because any increase in the total neutrino beam flux will likely be absorbed by increasing the experimental baseline. Chapter 2 concludes from this that the LB detector technologies and experimental techniques at ν MCs might also be similar to those employed today, with the sole significant exception that the 2-component beams provide strong motivation for a magnetic spectrometer to distinguish muon charge signs.

High rate detectors for studying neutrino interactions

Unlike LB detectors, SB detectors will continue to be placed as close as is practical to the production straight section, so the effects at the detector of the increasing beam intensity will not be diluted by an increasing base-line. The unprecedented collimation of the neutrino beams for ν MCs will further amplify the beam intensity seen by SB neutrino targets.

The intense pencil beams for short baseline experiments hold the promise of event samples of order 10^{10} events in high-performance tracking targets that might perhaps be only tens of centimeters across and of order 100 g.cm^{-2} deep. Therefore, novel high-performance SB detectors will be highly desirable to fully exploit the new physics potential gained from the huge event statistics, and will anyway be required to cope with the unprecedented data sample sizes.

As another difference from LB experiments, the physics motivation for SB experiments will unambiguously improve with increases in the neutrino beam energies. Muon colliders with beam energies of order 100 GeV or above look to provide the best beam-lines for HR neutrino experiments due to the linear increase with energy of neutrino interaction cross sections and to the turn-on of beauty production (see figures 1.2).

Three types of HR neutrino targets/detectors will be discussed in this book:

Figure 1.2: Plots for the fractional production of charm and beauty for neutrinos and anti-neutrinos scattering off nucleons. Fractional production is plotted versus neutrino energy and the calculations were performed at leading order.

1. figure 1.3 illustrates a general purpose HR neutrino detector with a CCD tracking target that would be suitable for wide-ranging studies concentrating mainly on DIS interactions. Such a detector will be assumed for most of the physics analysis discussions in chapters 3 through 7. It is discussed in more detail in appendix B.
2. polarized solid protium-deuterium targets would allow studies of spin physics. Such a target could well use the same detector as the preceding general purpose detector if it was placed immediately upstream of the CCD target. It is discussed further in the section on spin physics, section 3.5.
3. a more massive liquid target with electronic tracking may well be the technology of choice for precision studies of neutrino-electron scattering, as is discussed in section 5.2 of chapter 5.

Table 1.3 gives a summary of some of the characteristics of each of these three targets and also gives plausible but very approximate integrated luminosities and event sample sizes for 2 of the illustrative ν MC energies in table 1.2.2: 50 GeV and 500 GeV.

The event samples in table 1.3 are truly impressive. It is seen, for example, that high performance detectors with fully-active tracking neutrino targets might collect and precisely reconstruct data samples with many billions of neutrino-nucleon DIS interactions

1.3 Physics Analysis Topics

1.3.1 Neutrino Oscillations

The potential for LB oscillation experiment is currently the topic involving ν MCs that is of most interest to the high energy physics community. Despite this, Chapter 2 documented that both the experimental and theoretical status of neutrino oscillations are in such a state of flux that long-term predictions for ν MCs could be stated only in general terms.

Long baseline experiments at ν MCs might well provide a definitive follow-up to the recent intriguing evidence for neutrino oscillations. There will presumably already have been some progress in the verification or refutation of today's oscillation signals by the time long baseline ν MCs come on-line. Even so, ν MCs will still clearly be important for more probative follow-up studies to characterize the form and phenomenology of any observed oscillation signals. For example, ν MCs might help to determine whether an observed oscillation signal is consistent with mixing between 3 neutrino families or whether a fourth, sterile neutrino is required. In the former case, long baseline ν MCs can nail down the values of the mass-squared differences, search for CP violation and look for matter effects in oscillations in the Earth's interior. Clearly, the spectrum of possible studies would be richer still in the case of 4-neutrino mixing.

Figure 1.3: Example of a general purpose neutrino detector, reproduced from reference [4]. Its scale is illustrated by a human figure in the lower left corner. The neutrino target is the small horizontal cylinder at mid-height on the right hand side of the detector. Its radial extent corresponds roughly to the radial spread of the neutrino pencil beam, which is incident from the right hand side. The illustration is partially schematic in that the geometries of the calorimeters and dipole magnet have been simplified for illustrative purposes, as discussed further in the text.

Table 1.3: Specifications, integrated luminosities and event rates for the HR targets discussed in this paper and for 50 GeV (500 GeV) muon storage rings. The approximation is made that the target is situated 100 m (1 km) downstream from a straight section that has $N_\mu^{ss} = 10^{20}$ decays of 50 GeV (500 GeV) muons and that its radial extent subtends an angle of $1/\gamma$ at the production straight section. The beam energy corresponds to average neutrino energies of 32.5 GeV (325 GeV) and to approximately 1 (2) years running for storage ring parameters given previously in the literature.

target purpose	general	polarized	$\nu - e$ scatt.
material	Si CCD's	solid HD	liquid CH ₄
ave. density	0.5 g.cm ⁻³	0.267 g.cm ⁻³	0.717 g.cm ⁻³
length	2 m	0.5 m	20 m
mass/area, l	100 g.cm ⁻²	13.4 g.cm ⁻²	1434 g.cm ⁻²
radius	0.2 m	0.2 m	0.2 m
mass	126 kg	16.8 kg	1800 kg
$\int L dt$	6.0×10^{45} cm ⁻²	8.1×10^{44} cm ⁻²	8.6×10^{46} cm ⁻²
no. DIS events:			
at 50 GeV	1.4×10^9	1.9×10^8	2.0×10^{10}
at 500 GeV	1.4×10^{10}	1.9×10^9	2.0×10^{11}
no. ν -e events:			
at 50 GeV	3.5×10^5	NA	7×10^6
at 500 GeV	3.5×10^6	NA	7×10^7

1.3.2 QCD and Deep Inelastic Scattering

This has traditionally been one of the major areas of study in neutrino physics, despite the experimental limitations imposed by the tiny neutrino cross-sections. The potential for improvements at high intensity muon storage rings is of order 10^4 in neutrino flux. Billions of well-reconstructed DIS events might well provide the best ever experimental laboratory for studying QCD through a scattering process.

Traditional and novel areas for potential study are discussed in chapter 3. They include:

1. one of the most precise and theoretically sound measurements of the strong coupling constant, α_s
2. stringent consistency checks for the predictions of perturbative QCD
3. nucleon structure functions – including spin-dependence, neutral current structure functions, flavor-specific structure functions and separate structure functions for protons and neutrons
4. verifications and refinements of various aspects of QCD calculations, such as the interplay between different mass scales in calculations involving heavy quarks and

5. first studies of nuclear QCD comparing neutrino interactions off of a variety of nuclear targets to high statistics H_2 and D_2 results.

1.3.3 The CKM Quark Mixing Matrix

Some of the most important high-rate measurements will involve the Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix that characterizes the charged current (CC) weak interactions of quarks. This topic is discussed in detail in chapter 4. Unique and theoretically clean CKM measurements with huge event statistics might well give muon storage ring neutrino sources a central role in characterizing and understanding this fascinating and experimentally challenging part of the SM, particularly for high energy muon colliders.

Neutrino-nucleon DIS offers unique and systematically independent measurements of CKM matrix elements to all other processes, since it is the only process using an external W probe rather than internal weak interaction inside a hadron. The higher momentum transfers from an external probe allow for a cleaner theoretical interpretation using only relatively small corrections from perturbative QCD. As a further theoretical advantage, the measurements are fully inclusive – i.e. summing over all final states with single charm or beauty production – which avoids uncertainties from hadronic branching ratios.

The relatively clean theoretical interpretations and huge samples of flavor tagged events, particularly for charm production, should allow impressive measurements of the absolute squares for several of the elements in the CKM quark mixing matrix. The estimated contributions to determining the CKM matrix elements are summarized in Table 1.4.

As perhaps the most impressive measurement, chapter 4 predicts a potential determination of $|V_{ub}|$ to better than 5%, conceivably even reaching the 1% level. This is an order of magnitude better than the current uncertainty and might well be better than can be achieved in any other single measurement.

B production should also allow for an extraction of $|V_{cb}|$ at the few percent level that is *systematically* different from studies of decay processes in both its experimental and theoretical aspects and is comparably accurate to anticipated future measurements from these processes.

The matrix element $|V_{cd}|$ is already best measured from CC charm production in today's neutrino experiments, based on event samples of several thousands of events. Using hundreds of millions of vertex-tagged charm events in a high-performance detector, $|V_{cs}|$ and $|V_{cd}|$ will surely be extracted with very much smaller uncertainties – *without* imposing three-family unitarity! Such measurements of $|V_{cd}|$, in particular, would be uniquely precise.

1.3.4 Precision Electroweak Physics

Chapter 5 demonstrates that high intensity neutrino sources should be able to provide two types of precision measurements of the weak mixing angle, $\sin^2 \theta_W$, from the ratio of neutral current (NC) to charged current (CC) DIS events and from neutrino-electron scattering.

Table 1.4: Absolute squares of the elements in the Cabbibo-Kobayashi-Maskawa (CKM) quark mixing matrix. The second row for each quark gives current [10] percentage 1-sigma uncertainties in the absolute squares and speculative projections of the uncertainties after analyses from a ν MC operating with neutrino energies well above the B production threshold. The measurements of $|V_{cd}|^2$ and $|V_{cs}|^2$ might be comparably good for a 50 GeV ν MC but $|V_{ub}|^2$ and $|V_{cb}|^2$ would not be measured. The uncertainties assume that no unitarity constraints have been used.

	d	s	b
u	0.95 $\pm 0.1\%$	0.05 $\pm 1.6\%$	0.00001 $\pm 50\% \rightarrow \mathcal{O}(1\%)$
c	0.05 $\pm 15\% \rightarrow \mathcal{O}(1\%)$	0.95 $\pm 35\% \rightarrow \mathcal{O}(\text{few}\%)$	0.002 $\pm 15\% \rightarrow \mathcal{O}(\text{few}\%)$
t	0.0001 $\pm 25\%$	0.001 $\pm 40\%$	1.0 $\pm 30\%$

Both types of determinations will be a far extrapolation in event statistics and experimental technique from today's best neutrino results. They will allow vigorous consistency checks of the SM and provide sensitivity to several potential possibilities for new physics.

With its huge statistics, the DIS measurement will presumably eventually be systematically limited by theoretical hadronic uncertainties. It might anyway become several times more precise than today's best neutrino measurements, which are already equivalent to about a 100 MeV uncertainty on the W mass.

For neutrino-electron scattering, by contrast, no theoretical uncertainties enter into this simple scattering process between two elementary point particles and so the measurements will be limited only by statistics (3 orders of magnitude down from DIS) and experimental ingenuity. The measurements using neutrinos from positive and negative muon beams are theoretically distinct because of the additional scattering diagrams from annihilation of the $\bar{\nu}_e$'s in the $(\nu_\mu \bar{\nu}_e)$ beam and from W exchange of the ν_e 's in the $(\bar{\nu}_\mu \nu_e)$ beam.

One can contemplate neutrino-electron scattering event samples of order 10^8 events using novel detectors that have been optimized for the reconstruction of low-multiplicity events. This corresponds to impressive statistical uncertainties in $\sin^2 \theta_W$ of order 10^{-4} and sensitivity to new contact interactions to energy scales up to approximately 25 TeV. The biggest experimental challenges may come from normalizing the neutrino beam flux. If the experimental uncertainties could be reduced to the extremely challenging level of the statistical uncertainties then this process holds the potential for measurements of $\sin^2 \theta_W$ that might potentially be as good as or better than the best current measurements from collider experiments.

1.3.5 Rare and Exotic Processes

The potential for studying rare and exotic processes at ν MCs is limited relative to collider experiments due to the more modest center-of-mass energies. Nevertheless chapter 6 shows that there will still be some opportunities to both further our knowledge of the Standard Model by studying rare processes and to search for processes not predicted by the Standard Model.

1.3.6 Charm Decays

As an interesting bonus not directly involving neutrino interactions, ν MCs should be rather impressive factories for the study of charm decays – with a clean, well reconstructed sample of several times 10^8 charmed hadrons produced in 10^{10} neutrino interactions. Chapter 7 points out several interesting physics motivations for charm studies at a ν MC. Measurement of charm decay branching ratios and lifetimes are useful for both QCD studies and for the theoretical calibration of the physics analyses on B hadrons. Charm decays also provide a “clean laboratory” to search for exotic physics contributions since the SM predicts 1) tiny branching ratios for rare decays, 2) small CP asymmetries and 3) slow $D^0 \rightarrow \bar{D}^0$ oscillations, with only of order 1 in 10^4 oscillating before decay.

It is a unique advantage of CC-induced charm production in neutrinos that the production sign of the charm quark is tagged with very high efficiency and purity by the charge of the final state lepton:

$$\begin{aligned}\nu q &\rightarrow l^- c \\ \bar{\nu} \bar{q} &\rightarrow l^+ \bar{c},\end{aligned}\tag{1.15}$$

where $q = d$ or s and $\bar{q} = \bar{d}$ or $\bar{s}$. This is of particular benefit to oscillation and CP studies, as is the expected precise vertexing reconstruction of the proper lifetime of decays.

As an example of the advances in charm studies that might result at ν MCs, both particle-antiparticle mixing and CP violation have yet to be observed in the charm sector and chapter 7 shows it to be quite plausible that a ν MC could provide the first observations of these phenomena and additionally provide some context for their proper interpretation.

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Chapter 2

Neutrino Oscillation Experiments with a Muon Storage Ring/Neutrino Factory

In this chapter we briefly describe the current experimental status of neutrino oscillation measurements, and review the 3-generation mixing formalism. After describing what the next generation of neutrino experiments hopes to achieve we discuss what a muon storage ring could add to this field of study, and what experimental issues would arise in such unique neutrino beams.

2.1 Evidence for Neutrino Oscillations

In a modern theoretical context, one generally expects nonzero neutrino masses and associated lepton mixing. Experimentally, there has been accumulating evidence for such masses and mixing. All solar neutrino experiments (Homesake, Kamiokande, SuperKamiokande, SAGE, and GALLEX) show a significant deficit in the neutrino fluxes coming from the Sun [11]. This deficit can be explained by oscillations of the ν_e 's into other weak eigenstate(s), with Δm_{sol}^2 of the order 10^{-5} eV² for solutions involving the Mikheev-Smirnov-Wolfenstein (MSW) resonant matter oscillations [12, 13] or of the order of 10^{-10} eV² for vacuum oscillations. Accounting for the data with vacuum oscillations (VO) requires almost maximal mixing. The MSW solutions include one for small mixing angle (SMA) and one with essentially maximal mixing (LMA).

Another piece of evidence for neutrino oscillations is the atmospheric neutrino anomaly, observed by Kamiokande [14], IMB [15], SuperKamiokande [16] with the highest statistics, and by Soudan [17] and MACRO [18]. This data can be fit by the inference of $\nu_\mu \rightarrow \nu_\tau$ oscillations with $\Delta m_{atm}^2 \sim 3.5 \times 10^{-3}$ eV²

[16] and maximal mixing $\sin^2 2\theta_{atm} = 1$. The identification $\nu_x = \nu_\tau$ is preferred over $\nu_x = \nu_{sterile}$ at about the 2.5σ level [19], and the identification $\nu_x = \nu_e$ is excluded by both the Superkamiokande data and the Chooz experiment [20, 21].

In addition, the LSND experiment [22] has reported observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ oscillations with $\Delta m_{LSND}^2 \sim 0.1 - 1 \text{ eV}^2$ and a range of possible mixing angles, depending on Δm_{LSND}^2 . This result is not confirmed, but also not completely ruled out, by a similar experiment, KARMEN [23].

Although these three signatures imply three different mass differences and hence four neutrinos, we consider for the moment the three-generation mixing scenario. Some proposals for the form of the mixing matrix invoke only 3 generations of neutrinos to account for all signatures, while others invoke a fourth sterile neutrino.

2.2 Neutrino Oscillation Formalism

In the simplest theoretical context, there are three light electroweak doublet neutrinos. Although electroweak singlet neutrinos may be present in the theory, we will generally assume that they are much heavier than the mass scales of interest here, in accordance with the seesaw mechanism, as discussed further below. If this is the case, then the neutrino mixing can be described by the matrix

$$U = R_{23} K R_{13} K^* R_{12} K' \quad (2.1)$$

where R_{ij} is a rotation matrix in the ij subsector. Writing this out gives

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - s_{13}c_{12}c_{23}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} K' \quad (2.2)$$

where R_{ij} is the rotation matrix in the ij subspace, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, $K = \text{diag}(e^{-i\delta}, 1, 1)$ and K' involves further possible phases due to Majorana mass terms that will contribute here. Thus, in this framework, the neutrino mixing depends on the four angles θ_{12} , θ_{13} , θ_{23} , and δ , and on two independent differences of squared masses, $\Delta m_{atm.}^2$, which is $\Delta m_{32}^2 = m(\nu_3)^2 - m(\nu_2)^2$ in the favored fit, and $\Delta m_{sol.}^2$, which may be taken to be $\Delta m_{21}^2 = m(\nu_2)^2 - m(\nu_1)^2$. Note that these quantities involve both magnitude and sign; although in a two-species neutrino oscillation in vacuum the sign does not enter, in the three species oscillations relevant here, and including both matter effects and CP violation, the signs of the Δm^2 quantities do enter and can, in principle, be measured.

For our later discussion it will be useful to record the formulas for the various relevant neutrino oscillation transitions. In the absence of any matter effect, the probability that a (relativistic) weak neutrino eigenstate ν_a becomes ν_b after

propagating a distance L is

$$P(\nu_a \rightarrow \nu_b) = \delta_{ab} - 4 \sum_{i>j=1}^3 \text{Re}(K_{ab,ij}) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) + 4 \sum_{i>j=1}^3 \text{Im}(K_{ab,ij}) \sin\left(\frac{\Delta m_{ij}^2 L}{4E}\right) \cos\left(\frac{\Delta m_{ij}^2 L}{4E}\right) \quad (2.3)$$

where

$$K_{ab,ij} = U_{ai} U_{bi}^* U_{aj}^* U_{bj} \quad (2.4)$$

and

$$\Delta m_{ij}^2 = m(\nu_i)^2 - m(\nu_j)^2 \quad (2.5)$$

Recall that in vacuum, CPT invariance implies $P(\bar{\nu}_b \rightarrow \bar{\nu}_a) = P(\nu_a \rightarrow \nu_b)$ and hence, for $b = a$, $P(\bar{\nu}_a \rightarrow \bar{\nu}_a) = P(\nu_a \rightarrow \nu_a)$. For the CP-transformed reaction $\bar{\nu}_a \rightarrow \bar{\nu}_b$ and the T-reversed reaction $\nu_b \rightarrow \nu_a$, the transition probabilities are given by the right-hand side of (2.3) with the sign of the imaginary term reversed. (Below we shall assume CPT invariance, so that CP violation is equivalent to T violation.)

We shall concentrate on the case with the usual three flavors of active neutrinos, with no light sterile (=electroweak-singlet) neutrinos. This is sufficient to describe the solar and atmospheric neutrino deficit. If one were also to include the LSND experiment, then, to obtain a reasonable fit, one would be led to include light electroweak-singlet neutrinos. Since the LSND experiment has not so far been confirmed, we shall, while not pre-judging the outcome of the BOONE experiment, not include this in our fit. We calculate oscillation probabilities in the full 3×3 mixing case and we study when Δm_{sol}^2 can be relevant. In most cases there is only one mass scale relevant for long baseline neutrino oscillations, $\Delta m_{atm}^2 \sim \text{few} \times 10^{-3} \text{ eV}^2$ and we work with the hierarchy

$$\Delta m_{21}^2 = \Delta m_{sol}^2 \ll \Delta m_{31}^2 \approx \Delta m_{32}^2 = \Delta m_{atm}^2 \quad (2.6)$$

In this case, CP (T) violation effects are negligibly small, so that in vacuum

$$P(\bar{\nu}_a \rightarrow \bar{\nu}_b) = P(\nu_a \rightarrow \nu_b) \quad (2.7)$$

$$P(\nu_b \rightarrow \nu_a) = P(\nu_a \rightarrow \nu_b) \quad (2.8)$$

In the absence of T violation, the second equality (2.8) would still hold in matter, but even in the absence of CP violation, the first equality (2.7) would not hold. With the hierarchy (2.6), the expressions for the specific oscillation transitions are

$$P(\nu_\mu \rightarrow \nu_\tau) = 4|U_{33}|^2|U_{23}|^2 \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right)$$

$$= \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \quad (2.9)$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= 4|U_{13}|^2|U_{23}|^2 \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \\ &= \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \end{aligned} \quad (2.10)$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\tau) &= 4|U_{33}|^2|U_{13}|^2 \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \\ &= \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \end{aligned} \quad (2.11)$$

In neutrino oscillation searches using reactor antineutrinos, i.e. tests of $\bar{\nu}_e \rightarrow \bar{\nu}_e$, the two-species mixing hypothesis used to fit the data is

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2(2\theta_{reactor}) \sin^2\left(\frac{\Delta m_{reactor}^2 L}{4E}\right) \quad (2.12)$$

where $\Delta m_{reactor}^2$ is the squared mass difference relevant for $\bar{\nu}_e \rightarrow \bar{\nu}_x$. In particular, in the upper range of values of Δm_{atm}^2 , since the transitions $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ contribute to $\bar{\nu}_e$ disappearance, one has

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2(2\theta_{13}) \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \quad (2.13)$$

i.e., $\theta_{reactor} = \theta_{13}$, and the Chooz reactor experiment yields the bound [20]

$$\sin^2(2\theta_{13}) < 0.1 \quad (2.14)$$

which is also consistent with conclusions from the SuperK data analysis [16].

Further, the quantity “ $\sin^2(2\theta_{atm})$ ” often used to fit the data on atmospheric neutrinos with a simplified two-species mixing hypothesis, is, in the three-generation case,

$$\sin^2(2\theta_{atm}) \equiv \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \quad (2.15)$$

Hence for small θ_{13} , as implied by (2.14), it follows that, to good accuracy, $\theta_{atm} = \theta_{23}$.

2.3 Types of Neutrino Masses, Seesaw Mechanism

We review here the theoretical background concerning neutrino masses and mixing. In the standard $SU(3) \times SU(2)_L \times U(1)_Y$ model (SM) neutrinos occur

in $SU(2)_L$ doublets with $Y = -1$:

$$\mathcal{L}_{L\ell} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}, \quad \ell = e, \mu, \tau \quad (2.16)$$

There are no electroweak-singlet neutrinos (often called right-handed neutrinos) $\chi_{R,j}$, $j = 1, \dots, n_s$. Equivalently, these could be written as $\overline{\chi}_{L,j}^c$. There are three types of possible Lorentz-invariant bilinear operator products that can be formed from two two Weyl fermions ψ_L and χ_R :

- Dirac: $m_D \bar{\psi}_L \chi_R + h.c.$ This connects opposite-chirality fields and conserves fermion number.
- Left-handed Majorana: $m_L \psi_L^T C \psi_L + h.c.$ where $C = i\gamma_2\gamma_0$ is the charge conjugation matrix.
- Right-handed Majorana: $M_R \chi_R^T C \chi_R + h.c.$

The Majorana mass terms connect fermion fields of the same chirality and violate fermion number (by two units). Using the anticommutativity of fermion fields and the property $C^T = -C$, it follows that a Majorana mass matrix appearing as

$$\psi_i^T C (M_{maj})_{ij} \psi_j \quad (2.17)$$

is symmetric in flavor indices:

$$M_{maj}^T = M_{maj} \quad (2.18)$$

Thus, in the SM, there is no Dirac neutrino mass term because (i) it is forbidden as a bare mass term by the gauge invariance, (ii) it cannot occur, as do the quark and charged lepton mass terms, via spontaneous symmetry breaking (SSB) of the electroweak (EW) symmetry starting from a Yukawa term because there are no EW-singlet neutrinos $\chi_{R,j}$. There is also no left-handed Majorana mass term because (i) it is forbidden as a bare mass term and (ii) it would require a $I = 1$, $Y = 2$ Higgs field, but the SM has no such Higgs field. Finally, there is no right-handed Majorana mass term because there is no $\chi_{R,j}$. The same holds the minimal supersymmetric standard model (MSSM) and the minimal $SU(5)$ grand unified theory (GUT), both for the original and supersymmetric versions.

However, it is easy to add electroweak-singlet neutrinos χ_R to the SM, MSSM, or $SU(5)$ GUT; these are gauge-singlets under the SM gauge group and $SU(5)$, respectively. Denote these theories as the extended SM, etc. This gives rise to both Dirac and Majorana mass terms, the former via Yukawa terms and the latter as bare mass terms. In the extended SM:

$$-\mathcal{L}_{Yuk} = \sum_{i=1}^3 \sum_{j=1}^{n_s} h_{ij}^{(D)} \bar{\ell}_{L,i} \chi_{R,j} \phi + h.c. \quad (2.19)$$

The electroweak symmetry breaking (EWSB), with

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad (2.20)$$

where $v = 2^{-1/4} G_F^{-1/2} \simeq 250$ GeV, yields the Dirac mass term

$$\sum_{i=1}^3 \sum_{j=1}^{n_s} \bar{\nu}_{L,i} (M_D)_{ij} \chi_{Rj} + h.c. \quad (2.21)$$

with

$$(M_D)_{ij} = h_{ij}^{(D)} \frac{v}{\sqrt{2}} \quad (2.22)$$

The Majorana bare mass terms are

$$\sum_{i,j=1}^{n_s} \chi_{Ri}^T C (M_R)_{ij} \chi_{Rj} + h.c. \quad (2.23)$$

For compact notation, define the flavor vectors $\nu = (\nu_e, \nu_\mu, \nu_\tau)$ and $\chi = (\chi_1, \dots, \chi_{n_s})$ and observe that one can equivalently write ν_L or ν_R^c and χ_R or χ_L^c , where $\psi^c = C \bar{\psi}^T$, $\bar{\psi} = \psi^\dagger \gamma^0$. One can write the full set of Dirac and Majorana mass terms in the compact matrix form

$$-\mathcal{L}_m = \frac{1}{2} (\bar{\nu}_L \quad \bar{\chi}_L^c) \begin{pmatrix} M_L & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \chi_R \end{pmatrix} + h.c. \quad (2.24)$$

where M_L is the 3×3 left-handed Majorana mass matrix, M_R is a $n_s \times n_s$ right-handed Majorana mass matrix, and M_D is the 3-row by n_s -column Dirac mass matrix. In general, all of these are complex, and $(M_L)^T = M_L$, $(M_R)^T = M_R$. Because the extension of the SM to include χ_R does not include a $I = 1$, $Y = 2$ Higgs field allowing a renormalizable, dimension-4 Yukawa term that would yield a left-handed Majorana mass, one may take $M_L = 0$ at this level (but see below for dimension-5 contributions). The diagonalization of this mass matrix yields the neutrino masses and corresponding transformation relating the neutrino weak eigenstates to the mass eigenstates.

The same comments apply to the extended MSSM and SU(5) GUT. In the extended SU(5) GUT, the Dirac neutrino mass term arises most simply from the Yukawa couplings of the 5_R with a 5-dimensional Higgs representation H^α (in terms of component fields):

$$\bar{\psi}_{R\alpha} M_D \chi_L^c H^\alpha + h.c. \quad (2.25)$$

and the bare Majorana mass term $\chi_R^T M_R \chi_R + h.c.$.

In the extended SM, MSSM, or SU(5) GUT, one could consider the addition of the χ_R fields as *ad hoc*. However, a more complete grand unification is

achieved with the (SUSY) SO(10) GUT, since all of the fermions of a given generation fit into a single representation of SO(10), namely, the 16-dimensional spinor representation ψ_L . In this theory the states χ_R are not *ad hoc* additions, but are guaranteed to exist. In terms of SU(5) representations (recall, $\text{SO}(10) \supset \text{SU}(5) \times \text{U}(1)$)

$$16_L = 10_L + \bar{5}_L + 1_L \quad (2.26)$$

so for each generation, in addition to the usual 15 Weyl fermions comprising the 10_L and 5_R , (equivalently $\bar{5}_L$) of SU(5), there is also an SU(5)-singlet, χ_L^c (equivalently, χ_R). So in SO(10) GUT, electroweak-singlet neutrinos are guaranteed to occur, with number equal to the number of SM generations, inferred to be $n_s = 3$. Furthermore, the generic scale for the coefficients in M_R is expected to be the GUT scale, $M_{GUT} \sim 10^{16}$ GeV.

There is an important mechanism, which originally arose in the context of GUT's, but is more general, that naturally predicts light neutrinos. This is the seesaw mechanism [24]. The basic point is that because the Majorana mass term $\chi_R^T C M_R \chi_R$ is an electroweak singlet, the associated Majorana mass matrix M_R should not be related to the electroweak mass scale v , and from a top-down point of view, it should be much larger than this scale. Denote this generically as m_R . This has the very important consequence that when we diagonalize the joint Dirac-Majorana mass matrix above, the eigenvalues (masses) will be comprised of two different sets: n_s heavy masses, of order m_R , and 3 light masses. We illustrate this in the simplest case of a single generation and $n_s = 1$. Then the mass matrix is simply

$$-\mathcal{L}_m = \frac{1}{2}(\bar{\nu}_L \ \bar{\chi}_L^c) \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \chi_R \end{pmatrix} + h.c. \quad (2.27)$$

The diagonalization yields the eigenvalues

$$\lambda = \frac{1}{2} \left[m_R \pm \sqrt{m_R^2 + 4m_D^2} \right] \quad (2.28)$$

Since $m_D \sim h^{(D)} v$ while m_R is naturally $\gg v$ and hence $m_R \gg m_D$, we can expand to get

$$\lambda_{>} \simeq m_R \quad (2.29)$$

and

$$\lambda_{<} \simeq -\frac{m_D^2}{m_R} \left[1 + O\left(\frac{m_D^2}{m_R^2}\right) \right] \quad (2.30)$$

(The $-$ sign is not physically important.) The largeness of m_R then naturally explains the smallness of the masses (or, most conservatively, upper bounds on masses) of the known neutrinos. This appealing mechanism also applies in the physical case of three generations and for $n_s \geq 2$.

However, at a phenomenological level, without further theoretical assumptions, there is a large range of values for the light m_ν , since (1) the actual scale of m_R is theory-dependent, and (2) it is, *a priori*, not clear what to take for m_D since the known (Dirac) masses range over 5 orders of magnitude, from $m_e, m_u \sim \text{MeV}$ to $m_t = 174 \text{ GeV}$, and this uncertainty gets squared.

For the full case with 3 generations and $n_s > 1$, and assuming, as is generic, that $\det(M_R) \neq 0$ so that M_R^{-1} exists, the set of 3 light neutrino mass eigenstates is determined by the matrix analogue of eq. (2.30):

$$M_\nu = -M_D M_R^{-1} M_D^T \quad (2.31)$$

The seesaw mechanism is appealing, but detailed predictions are rather model-dependent.

A different way to get neutrino masses is to interpret the SM as a low-energy effective field theory, as is common in modern quantum field theory. One thus allows (nonrenormalizable) operators $\mathcal{O}$ in the Lagrangian of mass dimension $d_{\mathcal{O}} > 4$, provided that their coefficients, of dimension $4 - d_{\mathcal{O}}$ in mass units, are sufficiently small. Then one observes that the dimension-5 operator [25]

$$\mathcal{O} = \frac{1}{M_X} \sum_{a,b} h_{L,ab} (\epsilon_{ik} \epsilon_{jm} + \epsilon_{im} \epsilon_{jk}) [\mathcal{L}_{aL}^{Ti} C \mathcal{L}_{bL}^j] \phi^k \phi^m + h.c. \quad (2.32)$$

(where a, b are flavor indices, i, j, k, m are $\text{SU}(2)$ indices) is an electroweak singlet. Upon electroweak symmetry breaking (EWSB), this operator yields a left-handed Majorana mass term

$$\sum_{a,b=1}^3 \nu_{L,a}^T C (M_L)_{ab} \nu_{L,b} + h.c. \quad (2.33)$$

with

$$(M_L)_{ab} = \frac{(h_L)_{ab} (v/\sqrt{2})^2}{M_X} \quad (2.34)$$

Since the SM is phenomenologically very successful, one should have $M_X \gg v$, so again these dimension-5 operators lead naturally to light neutrinos. The diagonalization of the above operator determines the unitary transformation relating the mass eigenstates to the weak eigenstates,

$$\nu_{\ell_a} = \sum_{i=1}^3 U_{ai} \nu_i, \quad \ell_1 = e, \ell_2 = \mu, \ell_3 = \tau \quad (2.35)$$

i.e.,

$$\begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (2.36)$$

One often denotes $U_{1i} = U_{ei}$, etc.

For the case of electroweak-singlet neutrinos and resultant seesaw, because of the splitting of the masses into the light set and the heavy set, the observed weak eigenstates of neutrinos are again, to a very good approximation, linear combinations of the three light mass eigenstates, so that the full $(3+n_s) \times (3+n_s)$ mixing matrix breaks into block diagonal form involving the 3×3 U matrix and an analogous $n_s \times n_s$ matrix for the heavy sector. In terms of the flavor vectors, this is

$$\begin{pmatrix} \nu_\ell \\ \chi^c \end{pmatrix} = \begin{pmatrix} U & 0 \\ 0 & U_{heavy} \end{pmatrix} \begin{pmatrix} \nu_i \\ \chi_{i,m}^c \end{pmatrix} \quad (2.37)$$

If one accepts all of the data indicating neutrino masses, including the solar neutrino deficiency, atmospheric neutrinos, and LSND experiments, then one needs light sterile (electroweak-singlet) neutrinos with masses of $\sim \text{eV}$ or smaller. These are usually considered unnatural, because electroweak-singlet neutrinos naturally have masses $\sim m_R \gg M_{ew} = v$. If such light χ_R 's exist, this tends to undermine the reasoning on which the seesaw mechanism is based.

2.4 Tests for Neutrino Masses in Decays

Given the focus of this booklet, we shall not review the well-known kinematic tests for neutrino masses except to mention that these are of three main types. There are direct tests, which search for the masses of the dominantly coupled neutrino mass eigenstates emitted in particle and nuclear decays; these yield the current upper bounds on these eigenstates for the three dominantly coupled mass components in ν_e , ν_μ , and ν_τ . Second, there are tests for rather massive neutrinos emitted, via lepton mixing, in particle and nuclear decays. Third, there are searches for neutrinoless double beta decay, which would occur if there are massive Majorana neutrinos.

2.5 Models for Neutrino Masses and Mixing

We discuss the seesaw mechanism in further detail here. In the SM, a single Higgs field ϕ breaks the gauge symmetry and gives masses to the fermions. In the MSSM, one needs two $T = 1/2$ Higgs fields H_1 and H_2 with opposite hypercharges $Y = 1$ and $Y = -1$ to do this. GUT theories may have more complicated Higgs sectors; one typically uses different Higgs to break the gauge symmetry and give masses to fermions. In the context of the weakly coupled heterotic string, there is a connection between the Kac-Moody levels for the factor groups $SU(3)$, $SU(2)$, $U(1)$'s, and the corresponding representations of the chiral matter superfields. Indeed, for GUT's in a string context, this connection has yielded a tight constraint on the Higgs chiral superfields. Starting first by allowing the full range of Higgs fields in $SO(10)$; then for the Clebsch-Gordan

decomposition of the representations in the fermion mass term

$$16 \times 16 = 10_s + 120_a + 126_s \quad (2.38)$$

Hence, *a priori*, one considers using Higgs of dimension 10, 120, and 126. The coupling to the 10-dimensional Higgs fields yields Yukawa terms of the following form (suppressing generation indices).

$$\psi_L^T C \psi_L \bar{\phi}_{10} = (\bar{d}_R d_L + \bar{e}_R e_L) \phi_{10}(\bar{5}) + (\bar{u}_R u_L + \bar{\nu}_R \nu_L) \phi_{10}(5) \quad (2.39)$$

The coupling to the 126-dimensional Higgs yields a term

$$\chi_R^T C \chi_R \phi_{126}(1) \quad (2.40)$$

together with other linear combinations of $\bar{u}_R u_L$, $\bar{\nu}_R \nu_L$, $\bar{d}_R d_L$, and $\bar{e}_R e_L$ times appropriate SU(5)-Higgs; these four types of terms are also produced by the coupling to a 120-dimensional Higgs. Hence, in this approach, one expects some similarity in Yukawa matrices, and thus Dirac mass matrices, for $T_3 = +1/2$ fermions, i.e. the up-type quarks u, c, t and the neutrinos:

$$M^{(u)} \sim M_D^{(\nu)}, \quad M^{(d)} \sim M_D^{(l)} \quad (2.41)$$

However, in many string-inspired models, one avoids high-dimension Higgs representations such as the 120- and 126-dimensional representations in SO(10). Instead, one constructs the neutrino mass terms from nonrenormalizable higher-dimension operators. A review with references to a large number of papers with theoretical models is [26].

To get a rough idea of the predictions, one may pretend that M_D and M_R are diagonal and let m_R denote a typical entry in M_R . Denote $m_{u,1} = m_u$, $m_{u,2} = m_c$, $m_{u,3} = m_t$. Then, (neglecting physically irrelevant minus signs)

$$m(\nu_i) \simeq \frac{m_{u,i}^2}{m_R} \quad (2.42)$$

This is the quadratic seesaw. For $m(\nu_3)$, one gets

$$m(\nu_3) \sim \frac{m_t^2}{m_R} \simeq \left(\frac{175 \text{ GeV}}{10^{16} \text{ GeV}} \right) (1.75 \times 10^{11} \text{ eV}) \sim 10^{-3} \text{ eV} \quad (2.43)$$

which, given the uncertainties in the inputs, is comparable to the value

$$m(\nu_3) \simeq \sqrt{\Delta m_{32}^2} \simeq 0.05 \text{ eV} \quad (2.44)$$

inferred from the SuperK data with the assumption $\nu_\mu \rightarrow \nu_\tau$ and $m(\nu_2) \ll m(\nu_3)$. This gives an idea of how the seesaw mechanism could provide a neutrino mass in a region relevant to the SuperKamiokande data.

In passing, we note that string theories allow a low string scale, perhaps as low as 100 TeV. These models have somewhat different phenomenological implications for neutrinos than conventional models with a string scale comparable to the Planck mass.

2.6 Lepton Mixing

We proceed to consider off-diagonal structure in M_R , as part of the more general topic of lepton mixing. Neutrino mass terms naturally couple different generations and hence violate lepton family number; the Majorana mass terms also violate total lepton number. Lepton mixing angles are determined by diagonalizing the charged lepton and neutrino mass matrices, just as the quark mixing angles in the CKM (Cabibbo-Kobayashi-Maskawa) matrix are determined by diagonalizing the up-type and down-type quark mass matrices. Before the atmospheric neutrino anomaly was reported, a common expectation was that lepton mixing angles would be small, like the known quark mixing angles. This was one reason why theorists favored the MSW mechanism over vacuum oscillations as an explanation of the solar neutrino deficiency – MSW could produce the deficiency with small lepton mixing angles, whereas vacuum oscillations needed nearly maximal mixing. It was long recognized that an explanation of the atmospheric neutrino anomaly requires maximal mixing, and while neutrino masses are not surprising or unnatural to most theorists, the maximal mixing has been something of a challenge for theoretical models to explain.

Denoting the lepton flavor vectors as $\ell = (e, \mu, \tau)$ and $\nu = (\nu_e, \nu_\mu, \nu_\tau)$, we have, for the leptonic weak charged current,

$$J^\lambda = \bar{\ell}_L \gamma^\lambda \nu_L \quad (2.45)$$

The mass terms are

$$\bar{\ell}_L M_\ell \ell_R + \bar{\nu}_L M_\nu \nu_R^c + h.c. \quad (2.46)$$

where, as above, $M_\nu = -M_D M_R^{-1} M_D^T$ and we have used the splitting of the neutrino eigenvalues into a light sector and a very heavy sector. We diagonalize these so that, in terms of the associated unitary transformations, with the notation $\ell_m = (e_m, \mu_m, \tau_m)$ and $\nu_m = (\nu_1, \nu_2, \nu_3)$, for charged lepton and neutrino mass eigenstates, the charged current is

$$J^\lambda = \bar{\nu}_{mL} U_L^{(\nu)} \gamma^\lambda U_L^{(\ell)\dagger} \ell_{mL} = \bar{\nu}_{mL} U \gamma^\lambda \ell_{mL} \quad (2.47)$$

where the lepton mixing matrix is

$$U = U_L^{(\nu)} U_L^{(\ell)\dagger} \quad (2.48)$$

For some background, we recall the types of predictions that theorists obtained from plausible models before the advent of the atmospheric neutrino anomaly. For this purpose we consider a simple two-generation Fritzsch-type model for leptons. The diagonalization of the charged lepton mass matrix yields

$$\theta_\ell = \arctan\left(\sqrt{\frac{m_e}{m_\mu}}\right) \simeq \sqrt{\frac{m_e}{m_\mu}} \quad (2.49)$$

For the neutrino sector, consider first the ν_e, ν_μ subsector, with

$$\begin{pmatrix} \nu_e & \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} \\ -\sin \theta_{12} & \cos \theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad (2.50)$$

We have

$$M_\nu = -M_D M_R^{-1} M_D^T = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} M_R^{-1} \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} \quad (2.51)$$

where in an SO(10) GUT, one expects $M_D \sim M^{(up)}$, so $A \sim \sqrt{m_u m_c}$, $B \sim m_c$ and hence $A \ll B$. If M_R is roughly diagonal, $M_R = m_R \text{diag}(1, 1)$, then

$$M_\nu = -\frac{1}{m_R} \begin{pmatrix} A^2 & AB \\ AB & A^2 + B^2 \end{pmatrix} \quad (2.52)$$

where Diagonalizing yields

$$m(\nu_1) \simeq \frac{m_u^2}{m_R}, \quad m(\nu_2) \simeq \frac{m_c^2}{m_R} \quad (2.53)$$

$$\theta_\nu \simeq \sqrt{\frac{m_{\nu_1}}{m_{\nu_2}}} \simeq \frac{m_u}{m_c} \quad (2.54)$$

and hence for the overall lepton mixing angle

$$\theta_{12} \simeq \left| -\theta_\ell + \theta_\nu e^{i\phi} \right| = \left| -\sqrt{\frac{m_e}{m_\mu}} + \sqrt{\frac{m(\nu_1)}{m(\nu_2)}} e^{i\phi} \right| = \left| -\sqrt{\frac{m_e}{m_\mu}} + \frac{m_u}{m_c} e^{i\phi} \right| \quad (2.55)$$

Since $\sqrt{m_e/m_\mu} = 0.07$ and $m_u/m_c \simeq 0.004$, this gives $\theta_{12} \simeq 0.07$, close to the value obtained from the preferred MSW small-angle fit to the solar neutrino data, $\sin^2 2\theta \simeq 0.006$, which yields $\theta_{12} \simeq 0.04$. This may be considered a success for such a simple model. Taking M_R to have a more general form (with $\det(M_R) \neq 0$) would yield similar results. We can also apply this sort of simple model for $\nu_\mu - \nu_\tau$ mixing relevant to the atmospheric neutrino anomaly, and we get, in the same way,

$$\theta_{23} \simeq \left| -\sqrt{\frac{m_\mu}{m_\tau}} + \sqrt{\frac{m(\nu_2)}{m(\nu_3)}} e^{i\phi} \right| = \left| -\sqrt{\frac{m_\mu}{m_\tau}} + \frac{m_c}{m_t} e^{i\phi} \right| \quad (2.56)$$

i.e., $\theta_{23} \simeq 0.2 \sim \theta_C$. Thus, this class of models tends to give small mixing in the $\nu_\mu - \nu_\tau$ sector, though not as small as in the $\nu_e - \nu_\mu$ sector.

In order to get large or maximal mixing, as is necessary to account for the atmospheric neutrino anomaly, one must hypothesize a special form for either the charged lepton mass matrix M_ℓ or neutrino mass matrix M_ν . As a two-species example, considering the $\nu_\mu - \nu_\tau$ sector to fit the SuperK data, this form would be

$$M_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.57)$$

with $a = c$, so that the rotation angle that diagonalizes this matrix,

$$\tan 2\theta = \frac{2b}{a - c} \quad (2.58)$$

will be $\theta = \pi/4$. As a simple example, take the charged lepton mass matrix to be diagonal and let M_ν have the special form. Working backwards from the diagonal matrix $M_{\nu,diag.} = \text{diag}(\lambda_2, \lambda_3)$, one finds, with $\theta_{23} = \pi/4$

$$M_\nu = O(\theta_{\nu,23})^T M_{\nu,diag.} O(\theta_{\nu,23}) = \frac{1}{2} \begin{pmatrix} \lambda_2 + \lambda_3 & \lambda_2 - \lambda_3 \\ \lambda_2 - \lambda_3 & \lambda_2 + \lambda_3 \end{pmatrix} \quad (2.59)$$

where

$$O(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2.60)$$

Assuming that $M_R = m_R \text{diag}(1, 1)$ and solving for M_D , one finds

$$M_D = \frac{\sqrt{m_R}}{2} \begin{pmatrix} \sqrt{\lambda_2} + \sqrt{\lambda_3} & \sqrt{\lambda_2} - \sqrt{\lambda_3} \\ \sqrt{\lambda_2} - \sqrt{\lambda_3} & \sqrt{\lambda_2} + \sqrt{\lambda_3} \end{pmatrix} \quad (2.61)$$

Thus, with this special assumption, one gets maximal mixing in the $\nu_\mu - \nu_\tau$ sector with arbitrary values of the mass eigenvalues λ_2 and λ_3 . One can change the form of M_D and M_R provided that one keeps the product $M_D M_R^{-1} M_D^T$ fixed to the desired form.

A number of fits have been made to the existing neutrino data. The fit by the SuperKamiokande collaboration to its data yields a minimum in the χ^2 at $\sin^2(2\theta_{atm}) = 1$, with an allowed region of $0.8 \lesssim \sin^2(2\theta_{atm}) \lesssim 1$. In terms of the basic angles in the lepton mixing matrix, this implies that θ_{23} is close to $\pi/4$ and allows a small, nonzero θ_{13} , consistent with the bound from CHOOZ. As will be discussed below, a major physics capability of the muon storage ring/neutrino factory is the ability to measure θ_{13} . Note that since one infers from the data only the total lepton mixing matrix U and not $U^{(l)}$ or $U^{(\nu)}$ individually, it is not possible to deduce the forms of M_l or M_ν separately. Rather, the theoretical model building consists of examining various assumed structures for these mass matrices.

2.7 Near Term Future Experiments

There are currently intense efforts to confirm and extend the evidence for neutrino oscillations in all of the various sectors – solar, atmospheric, and accelerator. Some of these experiments are running; in addition to SuperKamiokande and Soudan-2, these include the Sudbury Neutrino Observatory, SNO, and the K2K long baseline experiment between KEK and Kamioka. Others are in development and testing phases, such as BOONE, MINOS, the CERN - Gran Sasso

program, KAMLAND, and Borexino [27]. Among the long baseline neutrino oscillation experiments, the approximate distances are $L \simeq 250$ km for K2K, 730 km for both MINOS, from Fermilab to Soudan and the proposed CERN-Gran Sasso experiments. The sensitivity of these experiments is projected to reach down roughly to the level $\Delta m^2 \sim 10^{-3} \text{eV}^2$. Experiments that are planned as part of this program include [28] ICANOE and OPERA [29]. Although they are expected to begin operation after MINOS, they will involve somewhat different detector designs and plan to focus on establishing τ appearance. This, then, is the program of research for the next several years.

2.8 Oscillation Experiments at a Neutrino Factory

Although a neutrino factory based on a muon storage ring will turn on several years after this near-term period in which K2K, MINOS, and the CERN-Gran Sasso experiments will run, we believe that it has a valuable role to play, given the very high-intensity neutrino beams of fixed flavor-pure content, including, in particular, ν_e and $\bar{\nu}_e$ beams as well as the conventional ν_μ and $\bar{\nu}_\mu$ beams. The potential of the neutrino beams from a muon storage ring is that, in contrast to a conventional neutrino beam, which, say, from π^+ decay, is primarily ν_μ with some admixture of ν_e 's and other flavors from K decays, the neutrino beams from the muon storage ring would have extremely high purity: μ^- beams would yield 50 % ν_μ and 50 % $\bar{\nu}_e$, and so forth for the charge conjugate case of μ^+ beams. Furthermore, these could be produced with extremely high intensities, of order 10^{20} to 10^{21} neutrinos per year.

Given the form of the oscillation probabilities, a neutrino beam for an oscillation experiment would optimally be made from the lower end of the spectrum of energies considered in this book. Because of the lower requirements on beam focusing and acceleration, making a muon storage ring for the purposes of an oscillation experiment has been proposed as a first step towards developing a muon collider. Energies being considered range from 20 GeV to 50 GeV, and the geometry of the final muon ring is very different from a traditional collider ring, in that the straight section that points to a neutrino experiment comprises between 25 and 40% of the "circumference" of the ring.

The types of neutrino oscillations that can be searched for with the neutrino factory based on a muon storage ring are listed below for the case of μ^- , decaying as $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$:

1. $\nu_\mu \rightarrow \nu_\mu, \nu_\mu \rightarrow \mu^-$ (survival)
2. $\nu_\mu \rightarrow \nu_e, \nu_e \rightarrow e^-$ (appearance)
3. $\nu_\mu \rightarrow \nu_\tau, \nu_\tau \rightarrow \tau^-; \tau^- \rightarrow (e^-, \mu^-) \dots$ (appearance*)
4. $\bar{\nu}_e \rightarrow \bar{\nu}_e, \bar{\nu}_e \rightarrow e$ (survival)
5. $\bar{\nu}_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\mu \rightarrow \mu^+$ (appearance)

6. $\bar{\nu}_e \rightarrow \bar{\nu}_\tau, \bar{\nu}_\tau \rightarrow \tau^+; \tau^+ \rightarrow (e^+, \mu^+) \dots$ (appearance*)

It is clear from the list of processes above that since the beam contains both neutrinos and antineutrinos, the only way to determine what the parent neutrino was is to measure the charge of the final state lepton. The $\nu_\mu \rightarrow \nu_e$ oscillation will produce a wrong-sign e^- as will the $\nu_\mu \rightarrow \nu_\tau$ oscillation followed by τ decay to e^- . The easiest wrong-sign lepton signatures to detect arise from the oscillations $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$, giving a μ^+ and from $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$, giving a τ^+ which will decay part of the time to μ^+ . If one is searching for τ final states, muon storage ring energies above 30 GeV should be used to avoid threshold kinematic suppression.

To get a rough idea of how the sensitivity of an oscillation experiment would scale with energy and baseline length, recall that the event rate in the absence of oscillations is simply the neutrino flux times the cross section. First of all, neutrino cross sections in the region above about 10 GeV (and slightly higher, for τ production) grow linearly with the neutrino energy. Secondly, the beam divergence is a function of the initial muon storage ring energy; this divergence yields a flux, as a function of θ_d , the angle of deviation from the forward direction, that goes like $1/\theta_d^2 \sim E^2$. Combining this with the linear E dependence of the neutrino cross section and the overall $1/L^2$ dependence of the flux far from the production region, one finds that the event rate goes like

$$\frac{dN}{dt} \sim \frac{E^3}{L^2} \quad (2.62)$$

To set the scale, consider an experiment that sees 2×10^{20} 30 GeV muon decays in a straight section pointed at a detector 2800km away. In the absence of oscillations, the $\nu_\mu(\bar{\nu}_e)$ charged current rate would be 52,500 (22,600) events per 10 kton [5]. Figure 2.1 shows the relative ν_e and $\bar{\nu}_\mu$ statistics for two configurations: one is a 20 GeV μ^+ ring with a detector at 2800 km, the other is a 50 GeV μ^+ ring with a detector at 9100 km, which is close to the distance from either Fermilab or CERN to Kamiokande.

Now recall the general formula for the probability of a two-species neutrino oscillation in vacuum, say of $\nu_e \rightarrow \nu_\mu$. Putting in units, it is, from (2.10),

$$P(\nu_e \rightarrow \nu_\mu) = 4|U_{13}|^2|U_{23}|^2 \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right) \quad (2.63)$$

Where the $\sin^2$ term is small, if one expands it, there is a factor of $(L/E)^2$, which cancels the L dependence in eq. (2.62) and reduces the E dependence to a linear factor. Underlying these considerations of optimal energy is the fact that even if one designs an experiment for τ appearance, the overall event rate of detected τ 's may be rather small.

It is safe to assume that when a neutrino factory turns on, Δm_{atm}^2 and $\sin^2(2\theta_{23})$ will be known at the 10-30% level. Although a neutrino factory could undoubtedly improve the precision on those two parameters, the novel physics

that can be addressed is a determination of θ_{13} , and the sign of Δm_{atm}^2 . By using matter effects, and a comparison of ν_μ versus $\bar{\nu}_\mu$ oscillations (by switching the muon storage ring from μ^- to μ^+), the sign of Δm_{atm}^2 can be determined.

2.9 Matter Effects

With the advent of the muon storage ring, the distances one can place detectors are large enough so that for the first time matter effects can be exploited in accelerator-based oscillation experiments. Simply put, matter effects are the matter-induced oscillations which neutrinos undergo along their flight path through the Earth from the source to the detector. Given the typical density of the earth, matter effects are important for the neutrino energy range $E \sim O(10)$ GeV and $\Delta m_{atm}^2 \sim 10^{-3}$ eV² values relevant for the long baseline experiments. After the initial discussion of matter-induced resonant neutrino oscillations in [12], an early study of these effects including three generations was carried out in [30]. The sensitivity of an atmospheric neutrino experiment to small Δm^2 due to the long baselines and the necessity of taking into account matter effects was discussed, e.g., in [31]. After Ref. [13], many analyses were performed in the 1980's of the effects of resonant neutrino oscillations on the solar neutrino flux, and matter effects in the Earth were studied, e.g., [32] and [33], which also discussed the effect on atmospheric neutrinos (see also the review [34]). Recent papers on matter effects relevant to atmospheric neutrinos include [35, 36]. Early studies of matter effects on long baseline neutrino oscillation experiments were carried out in Early studies of matter effects on long baseline neutrino oscillation experiments were carried out in [37]. More recent analyses relevant to neutrino factories include [5, 38], [39]-[40]. In recent papers by one of the authors (RS) and I. Mocioiu, calculations were presented of the matter effect for parameters relevant to possible long baseline neutrino experiments envisioned for the muon storage ring/neutrino factory [41], [42]. In particular, these authors compared the results obtained with constant density along the neutrino path versus results obtained by incorporating the actual density profiles. They studied the dependence of the oscillation signal on both $E/\Delta m_{atm}^2$ and on the angles in the leptonic mixing matrix, and commented on the influence of Δm_{sol}^2 and CP violation on the oscillations. Additional recent studies include [43]-[44].

2.10 Detector Considerations

In order to measure oscillation parameters that describe the transitions above, ideally one would want a detector that could identify the existence and flavor of any outgoing lepton from the neutrino interaction, as well as the hadronic and leptonic energy in the event. When measuring very small oscillation probabilities, however, backgrounds must be taken into account. Naively one would think that simply detecting a muon of opposite charge of that in the storage ring is a signal for the electron neutrino oscillating. However, pions and kaons are

produced copiously in neutral and charged current neutrino interactions, and if one decays to a muon before it interacts in the detector this can constitute a significant background. At higher energies charmed mesons are also produced which decay immediately to muons 10% of the time. Ultimately detectors will need to be designed that have sufficient resolution on both the energies and the angles of the final state lepton and hadronic shower, in order to remove these backgrounds.

Although detectors exist which could identify all of the final state leptons and their charges, the challenge is to make them on the several kton scale. If the largest Δm^2 is in the LSND region, there will undoubtedly be more work done to optimize relatively low mass detectors that emphasize tau appearance. But the only detectors which have thus far been proposed on the 10-40 kton scale are for detecting wrong sign muon events. There are two detector technologies that have been considered in detail for oscillation experiments for a muon storage ring: one is a magnetized sampling calorimeter such as the one used by MINOS and one is a Liquid Argon TPC combined with a muon spectrometer such as the one proposed by the ICANOE collaboration for the CERN to Gran Sasso neutrino beam.

2.10.1 Magnetized Sampling Calorimeter

The magnetized sampling calorimeter consists simply of alternating layers of magnetized steel and readout, which traditionally consists of scintillator and/or drift chambers. The performance of the sampling calorimeter depends primarily on the sampling frequency of the detector, assuming the charged particle efficiency of the readout planes is close to 1.0. A steel/scintillator sandwich with sampling every 5 cm of steel would have a hadron energy resolution of approximately $.76/\sqrt{E_{had}(GeV)}$. With fine enough transverse segmentation, the hadron angular resolution is dominated by the hadron energy resolution, and can be made to be $17/\sqrt{E_{had}(GeV)} + 12/E_{had}(GeV)(rad)$. The muon energy and angular resolution is expected to be much better.

Although separating ν_e charged current events from neutral current events is difficult and determining the charge of the outgoing electron impossible in this detector, a muon in the final state can be easily and efficiently detected, and its charge, momentum, and initial outgoing angle can be determined once the muon traverses enough steel to be spatially separated from the hadronic shower. Kinematic cuts can be made on the muon momentum and its component transverse to the hadronic shower to reduce the background from charm production. With signal efficiencies from 25 to 30% the backgrounds can be reduced to a level of 10^{-5} to 10^{-6} , depending on the neutrino energy. At higher energy the backgrounds are larger, but since the rejection factor is also larger the overall background contribution is smaller [45].

2.10.2 Liquid Argon TPC

The ICANOE-type detector would consist of a large volume of Liquid Argon instrumented with time projection chambers (TPC's), followed by a much thinner volume of magnetized steel where a muon's charge and momentum can be determined. The TPC would have very small wire spacing (3mm) and would act much like an electronic bubble chamber. ν_e charged current interactions could be distinguished from neutral current interactions, although the electron charge could not be measured. By breaking up the event samples to four distinct classes: right sign muons, wrong sign muons, electron-like events, and neutral current events, one could fit all four distributions simultaneously to determine oscillation parameters. By looking at the acoplanarity of the event ν_τ 's might also be identified on a statistical basis.

The energy and angular resolutions of all the final state particles would be extremely good ($20\%/\sqrt{E}$ for hadrons, 150mrad for hadron shower angles). However, the ability of this detector to see wrong-sign muons would depend primarily on the segmentation between the liquid argon and the magnetized spectrometer and the thickness of the spectrometer itself. The thinner one makes the spectrometer, the more likely one is to have backgrounds from charge misidentification. The thicker the spectrometer, the less room there is for the liquid argon in a given volume. The thinner the liquid argon, the higher the acceptance for low energy muons (since muons lose approximately 0.211GeV/m in liquid Argon [46]), but the less target volume one has overall. Clearly optimization of this geometry is needed, and will depend somewhat on the energy of the muon storage ring.

2.10.3 Muon Detector Conclusions

Although the two types of detectors have different strengths, detailed studies have shown that both either would be adequate to make precise measurements of δm^2 and its sign, $\sin^2 \theta_{23}$, and extend the sensitivity of $\sin^2 \theta_{13}$ by 1-2 orders of magnitude (in the scenario where the largest Δm^2 is described by the atmospheric neutrino anomaly) [47, 38, 39, 42, 48].

2.10.4 Tau and Electron Detectors

Alternate technologies must be employed to achieve electron or tau identification event-by-event, or electron or tau charge measurements. If LSND is confirmed and the largest Δm^2 would suggest baselines on the order of 10's of km, then there is a much higher premium on designing detectors that can do tau and electron charge, and they will not have to be as massive. Detectors on the 1 kton scale could be quite adequate at these short baselines to make precision measurements on $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$. Even if LSND is not confirmed, efforts to make massive tau and electron charge identification detectors should not be abandoned—these two channels still comprise a large part of the mixing matrix and should be researched to confirm our understanding of neutrino mixing.

One category of new detectors uses thin ($\sim 100\mu m$) sheets of emulsion combined with low-density ($\sim 300\mu m$) spacers, and thin sheets of metal to give the detector mass. With emulsion one can measure the kink that occurs when a tau decays by comparing the slope of a track before and after the spacer. Such a geometry with lead as the mass is described in reference [49]. This would be very useful for identifying taus and electrons. However, for charge identification one needs to introduce a magnetic field. This could be done using an extremely large external magnet, such as the one used in ATLAS and thin steel plates, or using a coil and magnetized steel to make the mass [50]. Since the overhead for analysis of each event is high in this sort of detector, one would place it in a region where the tau appearance probability is maximized.

2.11 Conclusions

In conclusion, neutrino masses and mixing are generic theoretical expectations. The seesaw mechanism naturally yields light neutrinos, although its detailed predictions are model-dependent and may require a lower mass scale than the GUT mass scale. One of the most interesting findings from the atmospheric data has been the maximal mixing in the relevant channel, which at present is favored to be $\nu_\mu \rightarrow \nu_\tau$. Even after the near-term program of experiments by K2K, MINOS, the CERN-Gran Sasso experiments, and mini-BOONE, a high-intensity neutrino factor generating $10^{20} - 10^{21}$ neutrinos per year coupled with, ideally two long-baseline neutrino oscillation experiments that can take advantage of matter effects to amplify certain transitions and a massive detector that will identify μ 's and e 's with charge discrimination, will add greatly to our knowledge of the neutrino masses and lepton mixing matrix. In particular, it should be able to measure $\Delta m_{atm.}^2$ and $\sin^2(2\theta_{23})$ to the level of several per cent and also give important information about the sign of $\Delta m_{atm.}^2$ and about $\sin^2(2\theta_{13})$.

Figure 2.1: ν_e and $\bar{\nu}_\mu$ event rates per GeV as a function of E/L for two different experiments, and the L/E dependence of the oscillation probability, assuming the largest δm^2 is $2, 3.5, \text{ or } 5 \times 10^{-3} \text{GeV}^2$.

Chapter 3

Deep Inelastic Scattering and QCD Studies

3.1 Introduction

Neutrinos serve as an excellent probe of the strong interaction. Through their parity-violating charged current interactions, they access unique flavor combinations of partons not accessible to electroproduction experiments. Indeed, with sufficient statistics, a combination of $I = 0$ and $I = 1$ targets, and excellent final state lepton detection, a νMC deep inelastic scattering (DIS) experiment could *independently* measure with precision the left-handed, right-handed, and longitudinal components of all six light quark PDF over a Q^2 range set by the beam energy and final state lepton acceptance. In fact, one can seriously contemplate using neutrinos to extract completely new *polarized* neutrino structure functions. Neutrino beams permit detailed study of charm, and for a high energy νMC program, beauty quark production, a clean “two scale” test of QCD. Through heavy flavor production, one gains access to the nucleon’s gluon distribution. Finally, the beam intensities of a νMC offer novel probes of the strong interaction in the strong coupling regime of QCD. This section provides a brief introduction to the wealth of strong interaction physics accessible to a νMC program.

3.2 Deep Inelastic Scattering

Deep Inelastic Scattering (DIS) has long been a dominant process in understanding perturbative Quantum Chromodynamics (pQCD) through the study of the evolution of parton densities within the nucleon. Starting from the early SLAC eP experiments in the late 60’s, proceeding through the CERN and Fermilab Neutrino experiments of the 70’s through 90’s, the CERN and Fermilab $\mu - A$ experiments of the 80’s and 90’s, and the HERA experiments currently

underway, DIS has provided us with an increasingly accurate picture of the partonic structure of the nucleon. Although other pQCD - described processes such as Drell-Yan scattering, leptonic decays of the W and direct photon production contribute to this picture, global QCD analysis, which combine all such processes, still depend heavily on DIS results to anchor their conclusions. The reason neutrinos play such a dominant role in the studies of perturbative QCD is that they can effectively separate the valence and sea quark components in the nucleon as well as individual flavors of quarks when antineutrino scattering is included. Furthermore, the sea quarks themselves can be investigated and the strange sea distribution has been measured uniquely.

3.2.1 Kinematics of unpolarized DIS

To set up the notations, consider the total cross section for either charged or neutral current neutrino interactions with an unpolarized nucleon

$$\nu(p_l)N \rightarrow l^-[\nu](p_{l'})X, \quad (3.1)$$

with X being the inclusive final state. Its cross section is given by

$$d\sigma = \frac{4\alpha^2}{s} \frac{C_b}{(q^2 - M_b^2)^2} L^{\mu\nu} H_{\mu\nu}^{(X)} \frac{d^3 p_{l'}}{2|p_{l'}|}, \quad (3.2)$$

where $C_b = 1/(64 \sin^4 \theta_w)$, $M_b = M_W$ for the charged current interactions, $L^{\mu\nu} = 2(p_l^\mu p_{l'}^\nu + p_{l'}^\mu p_l^\nu - g^{\mu\nu}(p_l p_{l'})) + i\epsilon^{\mu\nu\alpha\beta} p_{l\alpha} q_\beta$ is a standard leptonic tensor, $q = p_{l'} - p_l$, m is a nucleon mass, and $H_{\mu\nu}^{(X)}$ is a hadronic tensor defined in terms of six invariant structure functions. Three of these vanish for the massless final state lepton upon contraction with the leptonic tensor $L^{\mu\nu}$ and will not be considered further on. The remaining ones can be defined as

$$\begin{aligned} \frac{1}{2} H_{\mu\nu}^{(X)} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) \\ &+ \left(p_\mu - pq \frac{q_\mu}{q^2} \right) \left(p_\nu - pq \frac{q_\nu}{q^2} \right) \frac{1}{pq} F_2(x, Q^2) \\ &- \frac{i}{pq} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F_3(x, Q^2). \end{aligned} \quad (3.3)$$

Here $Q^2 = -q^2$, $pq = \nu = m(E - E')$ in the laboratory rest frame, and $F_i(x, Q^2)$ are Lorentz scalars, the nucleon *structure functions*. Introducing the dimensionless variables

$$x = \frac{Q^2}{2m\nu}, \quad y = \frac{\nu}{E} \quad (3.4)$$

the differential cross section for neutrino nucleon interactions can be brought to

$$\frac{d^2 \sigma^{\nu(p)}}{dx dy} = \frac{\pi \alpha^2 Q^2}{4 \sin^4 \theta_w (Q^2 + M_W^2)} \left[y F_1^{\nu(p)} + \frac{(1-y)}{xy} F_2^{\nu(p)} + \delta_\nu \left(1 - \frac{y}{2} \right) F_3^{\nu(p)} \right]. \quad (3.5)$$

Here $\delta_\nu = +1(-1)$ for the incident neutrino (anti-neutrino). Please note that we have introduced superscripts to distinguish (in principle) different structure functions for ν and $\bar{\nu}$ scattering. Eq. (3.5) involves a set of nucleon's structure functions that exhibit scaling behavior for $Q^2 \rightarrow \infty$, $F_i(x, Q^2) \rightarrow F_i(x)$. The scaling behavior is broken by logarithmic evolution of the parton densities which can be calculated in perturbative QCD. The densities *themselves* cannot be calculated from the first principles at the moment and have to be extracted from the experimental data. We note that moments of structure functions of Eq. (3.3) can be rigorously expressed via the nucleon matrix elements of the set of operators arising from the operator product expansion of the product of currents building $H_{\mu\nu}^{(X)}$.

3.2.2 Kinematics of polarized DIS

High-intensity neutrino beams from the decaying muons would allow for truly novel measurements of parity-violating spin-dependent structure functions. The studies of these structure functions were not considered to be feasible at the conventional neutrino experiments due to the technical difficulties associated with fabrication of large polarized heavy targets required for sufficient statistics of these experiments. High-intensity neutrino beams from the muon storage ring should provide sufficient statistics even with the light targets, such as H_2 or D_2 . These will be considered later in this chapter.

In the case of polarized incident beam and polarized target, the hadronic and leptonic tensors defined in Eq. (3.3) are modified, as there is now another variable (target spin S^μ) that has to be taken into consideration. We define polarized structure functions in accord with [51]:

$$\begin{aligned} H_{\mu\nu} = H_{\mu\nu}(S=0) &+ \frac{m}{pq} \left\{ i\epsilon_{\mu\nu\rho\sigma} q^\rho S^\sigma g_1^i + i\epsilon_{\mu\nu\rho\sigma} q^\rho \left(S^\sigma - \frac{Sq}{pq} p^\sigma \right) g_2^i \right. \\ &+ qS \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) g_3^i + \frac{qS}{pq} \left(p_\mu - \frac{pq}{q^2} q_\mu \right) \left(p_\nu - \frac{pq}{q^2} q_\nu \right) g_4^i \\ &\left. + \frac{1}{2} \left[\left(p_\mu - \frac{pq}{q^2} q_\mu \right) \left(S_\nu - \frac{qS}{q^2} q_\nu \right) + \left(p_\nu - \frac{pq}{q^2} q_\nu \right) \left(S_\mu - \frac{qS}{q^2} q_\mu \right) \right] g_5^i \right\}. \end{aligned} \quad (3.6)$$

Terms proportional to q^μ or q^ν can be dropped in the definition of $H_{\mu\nu}$ because they give no contribution in the limit $\frac{m}{E} \rightarrow 0$ when contracted with the appropriate lepton tensors $L_{\mu\nu}$.

It is quite straightforward to calculate the differential cross-sections difference for the polarized case. Let us, as before, present them in the laboratory frame. Defining *longitudinal* target spin vector as $S_L^\mu = (0, 0, 0, 1)$ and *transverse* target spin vector as $S_T^\mu = (0, \cos \alpha, \sin \alpha, 0)$ (where α is an angle between the the direction of spin and the x -axis) we obtain the cross section difference

for $i = \nu$:

$$\frac{d^2(\sigma_{\rightarrow}^{\leftarrow} - \sigma_{\leftarrow}^{\leftarrow})^{\nu(\bar{\nu})}}{dx dy} = \frac{M_W^4 G_F^2 m E_\nu}{\pi(Q^2 + M_W^2)^2} \left\{ \pm y \left(1 - \frac{y}{2} - \frac{xy m}{2E}\right) x g_1 \mp \frac{x^2 y m}{E} g_2 \right. \\ \left. + y^2 x \left(1 + \frac{x m}{E}\right) g_3 + \left(1 - y - \frac{xy m}{2E}\right) \left[\left(1 + \frac{x m}{E}\right) g_4 + g_5\right] \right\}, \quad (3.7)$$

where $s = 2mE$ in the nucleon's rest frame. The transverse cross section ($p_\perp S = 0$) difference, integrated over α reduces to:

$$\frac{d^2(\sigma_{\uparrow}^{\leftarrow} - \sigma_{\downarrow}^{\leftarrow})^{\nu(\bar{\nu})}}{dx dy} = \frac{M_W^4 G_F^2 m}{16\pi^2(M_W^2 + Q^2)^2} \sqrt{xy m [2(1-y)E - xy m]} \\ \left\{ \mp 2xy \left(\frac{y}{2} g_1 + g_2\right) + xy^2 g_3 + \left(1 - y - \frac{xy m}{2E}\right) g_4 - \frac{y}{2} g_5 \right\}. \quad (3.8)$$

To obtain this result one should make use of the relations $pS = 0, p_\perp p = pq/y, 2p_\perp q = q^2, p^2 = p_\perp^2 = 0$ and $x = Q^2/2pq$. The transverse cross section is suppressed by m/Q with respect to the longitudinal cross section.

In the case of neutral current neutrino interactions with a polarized nucleon we have $(Q^2 + M_W^2)^{-2} \rightarrow (\sin(2\theta_w))^{-4} (Q^2 + M_Z^2)^{-2}$.

3.3 Measurements of α_s

While pQCD does not predict the parton distributions themselves, their evolution, as a function of Q^2 , is predicted. One of the most precise measurements of the strong coupling constant, α_s , comes from measuring this evolution. Similarly, the integral of the structure functions over all x can also give a measurement of α_s , with minimal QCD uncertainties.

Neutrino deep-inelastic scattering provides two clean measurements of the strong coupling constant α_s : one from the Q^2 dependence of the non-singlet structure function $x F_3$, and one from the Gross Llewellyn Smith (GLS) Sum Rule [52, 53],

$$\int_0^1 x F_3(x, Q^2) \frac{dx}{x} = 3 \left[1 - \frac{\alpha_s}{\pi} - a(N_f) \left(\frac{\alpha_s}{\pi}\right)^2 - b(N_f) \left(\frac{\alpha_s}{\pi}\right)^3 \right]. \quad (3.9)$$

Here $a(N_f)$ and $b(N_f)$ are known functions of x and number of flavors N_f . Both of these measurements are independent of the gluon distribution, and the latter has very small perturbative QCD uncertainties (possible higher twist corrections to Eq. (3.9) were calculated in [54, 55]). Currently, however, both measurements are limited by experimental systematic uncertainties: the $x F_3$ evolution is limited by the uncertainty in the energy calibration [56], and the GLS Sum Rule is limited by the uncertainty in both the overall neutrino cross section, and the ratio of neutrino to anti-neutrino cross sections [57]. Furthermore, both cross section measurements are themselves limited by systematic uncertainties [56]. A neutrino experiment at a muon collider could address all three of these issues in fundamentally different ways, because of the way the beam is formed.

Neutrinos from a monochromatic beam of muons have a very distinct spectrum peaked towards the high end of the available energy from the muon. Muon decay itself is very well understood, and the muon energies at the collider are predicted to be measurable to a few parts per million [58]. By calibrating the detector to the end point of the neutrino spectrum one should be able to achieve much better than the current energy scale uncertainty in neutrino experiments, which is about 1% [56].

Similarly, the muon currents in the beam-lines can be measured, and given the energy of the muons and the currents and the geometry of the beam-line, the neutrino flux at a given detector can be accurately predicted. The availability of an independent flux measurement for a neutrino experiment will be an important improvement and will dramatically change the nature of absolute neutrino cross section measurements, as well as the scale of the uncertainties in those measurements.

3.4 Measurement of Quark Densities

Information on the quark structure of the nucleon is extracted by measuring the differential cross section as a function of x and Q^2 . These differential cross sections are, in turn, related to the structure functions F_1 , F_2 and xF_3 of Eq. (3.5). We will concentrate on the F_2 and F_3 structure functions hereafter.

By plotting the cross section vs $(1-y)^2$, in each x and Q^2 interval, and measuring the slope and intercept of the resulting curve, one can measure the four structure functions. In the framework of the “naive” parton model these structure functions can be expressed in terms of quark densities (we assume an isoscalar target):

$$\begin{aligned} F_2^{\nu N} &= x [u(x) + d(x) + 2s(x) + \bar{u}(x) + \bar{d}(x) + 2\bar{c}(x)] , \\ F_2^{pN} &= x [u(x) + d(x) + 2c(x) + \bar{u}(x) + \bar{d}(x) + 2\bar{s}(x)] , \\ xF_3^{\nu N} &= x [u(x) + d(x) + 2s(x) - \bar{u}(x) - \bar{d}(x) - 2\bar{c}(x)] , \\ xF_3^{pN} &= x [u(x) + d(x) + 2c(x) - \bar{u}(x) - \bar{d}(x) - 2\bar{s}(x)] . \end{aligned} \quad (3.10)$$

Callan-Gross relation for F_j hold

$$F_2^i = 2xF_1^i. \quad (3.11)$$

Note that although there are four independent structure functions, no neutrino experiments to date has had sufficient statistics or sufficiently controlled systematics to measure all four uniquely and the assumption has been made that $F_2^{\nu} = F_2^p$ and $xF_3^{\nu} = xF_3^p$. In the case of F_2 , this implies that we assume $s(x) = \bar{s}(x)$ and $c(x) = \bar{c}(x)$, not an unreasonable assumption although, as evidence for non-asymmetry of the sea quarks mounts, an assumption that should be confirmed by measurement. For xF_3 the implications are somewhat more drastic since, even assuming $s = \bar{s}$ and $c = \bar{c}$ as above, $xF_3^{\nu} - xF_3^p = 4s(x) - 4c(x)$. Only at large x or considerably above charm

threshold is this a negligible quantity compared to the size of the valence quarks which $x F_3$ is presumed to measure.

Once we have the statistics and control of the systematics to measure all four structure functions we will have a new and powerful measure of both the valence quarks down to small x and the individual sea quarks. Note that the three equations

$$\begin{aligned} F_2^\nu - x F_3^\nu &= 2x \{ [\bar{u}(x) + \bar{d}(x)] + 2\bar{c}(x) \}, \\ F_2^p - x F_3^p &= 2x \{ [\bar{u}(x) + \bar{d}(x)] + 2\bar{s}(x) \}, \text{ and} \\ x F_3^\nu - x F_3^p &= 2x \{ s(x) + \bar{s}(x) - c(x) - \bar{c}(x) \} \end{aligned} \quad (3.12)$$

can be solved (with the assumption that $s(x) = \bar{s}(x)$ and $c(x) = \bar{c}(x)$) to yield the three quantities $[\bar{u}(x) + \bar{d}(x)]$, $s(x)$ and $c(x)$ at each x and Q^2 . Below charm threshold (assuming no inherent charm) the three equations yield a measure of $\bar{u}$, $\bar{d}$ and s .

Note also that the assumption that $s(x) = \bar{s}(x)$ can be tested explicitly by measuring the quantity $F_2^\nu - F_2^p$. This quantity may be dominated by systematic errors. Another method for measuring the difference between $s(x)$ and $\bar{s}(x)$ comparing F_2 measured by charged lepton scattering with F_2 measured with neutrinos as discussed in the low- x section below. Some recent result on the experimental extractions of these structure functions can be found in [59].

It is well known that the charged current neutrino nucleon interactions provide one of the best probes of intrinsic strangeness $[s(x)]$ and intrinsic charm $[c(x)]$ densities of the nucleon. Many of those measurements (as well as the determinations of V_{cs} and V_{cd} CKM matrix elements discussed further on) involve charm production. The simple language of the quark parton model has to be modified to take into account the non-zero value of the charmed quark mass in the final state. For instance, the ‘‘slow rescaling’’ model of Georgi and Politzer [60] amounts to redefinition of the scaling variable x by substituting it with

$$x \rightarrow \xi = x \left(1 + \frac{m_Q^2}{Q^2} \right), \quad (3.13)$$

where m_Q is a heavy quark mass. In QCD, the prescription that consistently takes into account final state quark mass was developed by M. Aivazis *et. al.* (ACOT) [61] and is based on the application of factorization theorems of perturbative QCD.

The essential problem of having a heavy quark in the final state is the fact that its mass brings about an additional scale to the problem at hand. In its simplest form, the factorization theorem of QCD separates the high scale set by the value of transferred momentum $\mu_h^2 \sim Q^2$ and low scale of hadronic physics $\mu_l^2 \sim \Lambda_{QCD}^2$. This formalism is based on the Operator Product Expansion (OPE) techniques and allows the summation of large logarithms of the form $\log(\mu_h^2/\mu_l^2)$ that multiply the expansion parameter (α_s) and might render perturbation theory expansion useless.

The presence of the additional scale introduced by the heavy quark mass brings back large logarithms of the form $\alpha_s \log(\mu_{ph}^2/m_Q^2)$ which have to be resummed without losing control over the region where $\mu_{ph}^2 \sim m_Q^2$. Here μ_{ph} is a “physical” scale of the process set by, for example, the value of transferred momentum Q^2 . A practical recipe for such re-summation is provided by the ACOT prescription.

One of the problems in describing heavy quark production is an ambiguity in treatment of heavy quarks at high energies. For example, in the neutral current charm production studies one can treat charm production as a “flavor-creation” process, $Z + g \rightarrow c\bar{c}$, which starts at the order α_s^1 in pQCD. In this case, the appropriate scheme for calculation of radiative corrections involve only three light flavors (the so-called three-flavor scheme) and all of the effects associated with the charm quark are isolated in the Wilson coefficients, i.e. taken into account perturbatively. On the other hand, at high energies when $Q^2 \gg m_c^2$, the mass of the charm quark is much smaller than the relevant physical scale, so the charm quark is effectively “light”. Here one can view the charm quark as a parton with the corresponding parton distribution function (so-called four-flavor scheme). In particular, the lowest order Wilson coefficient for charm production in this situation is a “flavor-excitation” process, $Z + c \rightarrow c$, which is an α_s^0 . One must make sure that the correct subtraction is performed to avoid double counting.

The ACOT prescription [61] introduces a *variable flavor number scheme* which treats the number of active flavors as a scale-dependent quantity. In particular, it suggests the use of a three-flavor evolution for $\mu < m_Q$ and a four-flavor evolution above m_Q with continuity at the break point. Thus, for a heavy quark Q , one should use a parton distribution function $f_{Q/N}(\xi, \mu)$ which vanishes for $\mu \leq m_Q$ and satisfies the usual $\overline{MS}$ QCD evolution equation (with massless kernel functions) for $\mu > m_Q$. Therefore, it is in principle important to additionally label the parton distribution functions by the number of active flavors.

In summary, the ability to extract all four weak structure functions is a powerful new tool for investigating the behavior of the both valence and sea quarks and provides a consistency test of the entire parton model.

Aside from the inherent value of understanding the partonic structure of the nucleon, another very compelling reason to improve DIS measurements with neutrinos involves future experiments searching for signals of new physics. Discovery channels for new physics are going to come from quark-anti-quark annihilation at high energy machines, and if one does not know the underlying quark/antiquark distributions then the ability to measure anything through this channel is severely compromised.

Since there is already a wealth of neutrino DIS data available, a fair question is how can neutrinos from a muon source improve the picture. The answer lies in the intensity and flavor of the neutrino beams. Up to now the intensity of neutrino beams has been rather low requiring massive heavy nuclear targets, such as Fe, to accumulate the high statistics needed for these studies. The

high statistics samples available today are for neutrino-iron scattering which differs from neutrino-nucleon scattering due to nuclear effects. With the intense neutrino beams available at a muon collider one can accumulate the necessary statistics using very light targets such as H_2 or D_2 .

Not only will a neutrino beam from a muon source yield reduced statistical errors, but such a facility would permit a significant reduction in one of the larger systematic errors plaguing contemporary neutrino experiments. Knowledge of the incoming neutrino spectrum in current experiments involves complicated Monte Carlo which follow the neutrino production mechanism from pA interactions in thick targets through poorly known π/K production spectra and modeling of a complicated target and focusing decay path surrounded by massive shielding. In the case of neutrino beams from a muon source, the prediction of the incoming spectra is reduced to an understanding of the muon phase space distribution along a straight section plus muon decay kinematics, a much simpler process.

Another improvement will come from the necessity of accumulating high statistics of anti-neutrinos as well as neutrinos for precise measurements of all four structure functions. Traditionally accumulating antineutrino events has required separate experimental runs which have been considerably less efficient (by a factor of, roughly, 5) in yielding events than neutrino runs. This is due to the reduced admixture of negative π 's and K 's coming from pA interactions coupled with the smaller antineutrino cross section ($\sigma_{\bar{\nu}} \approx 2\sigma_{\nu}$). This has resulted in the statistical precision of structure function measurements being limited by the antineutrino statistics. If we can utilize both the muon neutrinos and the electron antineutrinos in a beam made from parent muons, and get comparable efficiency and resolution, then the antineutrino event statistics will already be half of the neutrino statistics without the necessity of running opposite charged muons.

3.5 Polarized Parton Densities and Spin Structure of Nucleon

A unique new feature of a high intensity neutrino factory would be the availability of sufficiently intense beams to allow scattering on a polarized target for the first time with neutrino beams. This in turn allows access to four polarized nucleon structure functions, and should be able to answer many unresolved questions about the spin structure of the nucleon. The nucleon spin ($\frac{1}{2}$) can be decomposed in terms of quarks and gluon contributions:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g, \quad (3.14)$$

where $\Delta\Sigma = \Delta u + \Delta d + \Delta s + \dots$ and Δg are the net quark and gluon helicity along the nucleon spin direction, while L_i are their relative orbital angular momentum.

To date, the only experiments which address this question are charged lepton polarized deep-inelastic scattering experiments (PDIS) running at energies where only photon exchange is relevant. Thus, only the parity conserving polarized structure functions g_1^f and g_2^f have been studied so far.

The results are surprising: from the first moment of g_1^f , referred to as Γ_1 , it is concluded that only 30% of the nucleon spin is carried by quarks, assuming that $\Delta g = 0$, i.e., $\Delta \Sigma = 0.3$. In addition, the “strange sea” must be somehow anti-aligned with the nucleon spin $\Delta s = -0.1$. Alternatively, one could conclude that the gluons themselves are polarized, i.e., $\Delta g \neq 0$. This latter conclusion is supported by a perturbative QCD analysis of the Q^2 evolution of the available data on g_1^f .

In the QCD improved parton model the polarized structure functions are represented in terms of “polarized” parton densities, $\Delta q = \delta q + \delta \bar{q}$:

$$\delta q(x, Q^2) = q^{\uparrow\uparrow}(x, Q^2) - q^{\uparrow\downarrow}(x, Q^2), \quad (3.15)$$

$$\delta \bar{q}(x, Q^2) = \bar{q}^{\uparrow\uparrow}(x, Q^2) - \bar{q}^{\uparrow\downarrow}(x, Q^2), \quad (3.16)$$

where δq is the difference between the parton densities of quarks polarized parallel or anti-parallel to the nucleon spin, and which measures how much the parton of flavor q “remembers” from its parent proton polarization. To leading order (LO) in QCD, g_1^f and Γ_1 can be decomposed into a flavor non-singlet (NS) and singlet (S) component:

$$\begin{aligned} g_1^f(x, Q^2) &= g_{1,NS}(x, Q^2) + g_{1,S}(x, Q^2), \\ &= \sum_q \frac{(e_q^2 - \langle e^2 \rangle)}{2} (\delta q + \delta \bar{q}) + \frac{\langle e^2 \rangle}{2} \sum_q (\delta q + \delta \bar{q}) \end{aligned} \quad (3.17)$$

$$s_\mu \Gamma_1^{p(n)}(Q^2) = s_\mu \frac{C_1^{NS}(Q^2)}{6} \left[+(-)\frac{1}{2}a_3 + \frac{1}{6}a_8 \right] + s_\mu \frac{C_1^S(Q^2)}{9} a_0, \quad (3.18)$$

where C_1^{NS} and C_1^S are the non-singlet and singlet coefficient functions, respectively. The axial charge matrix elements, $s_\mu a_i(Q^2) = \langle ps | \bar{q}_i \gamma_5 \gamma_\mu q_i | ps \rangle$, are:

$$\begin{aligned} a_3 &= \Delta u - \Delta d = F + D, & a_8 &= \Delta u + \Delta d - 2\Delta s = 3F - D, \\ a_0 &= \Delta u + \Delta d + \Delta s, \end{aligned} \quad (3.19)$$

where a_3 and a_8 can be expressed in terms of the $SU(3)$ coupling constants F and D , obtained from neutron and hyperon β -decays. The singlet combination a_0 has already naively been identify with the spin content of the nucleon carried by quarks, but this will be modified in the QCD improved QPM where the parton distributions will have contributions coming from gluon radiation ($q \rightarrow qg$) and gluon-initiated ($g \rightarrow q\bar{q}$) subprocesses. This causes the appearance of gluon distributions in the nucleon: $\Delta g(x, Q^2)$. Since massless quark flavor u, d, s are produced by gluons at the same rate, there is no gluon contribution to the non-singlet state. Therefore, even though the gluon distribution does not contribute directly to $g_1^f(x, Q^2)$ at LO, the flavor-singlet part will be affected indirectly by gluons via the Q^2 -evolution equations and has to be taken into

account if we want to conclude which fraction of the nucleon spin is carried by quarks. The result of the most commonly used decomposition to do this is shown in Fig. 3.1. This last step is highly model dependent, but due to the ability of neutrinos to select specific quark flavors, it should be possible to constrain the models with PDIS neutrino scattering.

As described in 3.2.2, the cross section for neutrinos scattering off polarized nucleons, $\nu N \rightarrow l^- X$, can be decomposed into $\sigma = \bar{\sigma} - \frac{1}{2}h_N\{\Delta\sigma_{PC} + \Delta\sigma_{PV}\}$, where h_N is the helicity of the nucleon and the ν is assumed to be left handed. $\bar{\sigma}$ is described by the unpolarized structure functions F_1, F_2 and F_3 , $\Delta\sigma_{PC}$ is described by two parity conserving polarized structure functions g_1 and g_2 , and $\Delta\sigma_{PV}$ by three parity violating polarized structure functions g_3, g_4 and g_5 ¹.

These polarized structure functions in the naive parton model can be described by the individual polarized parton distributions,

$$g_1^{\nu p}(x, Q^2) = \delta d(x, Q^2) + \delta s(x, Q^2) + \delta \bar{u}(x, Q^2) + \delta \bar{c}(x, Q^2), \quad (3.20)$$

$$g_1^{\bar{\nu} p}(x, Q^2) = \delta u(x, Q^2) + \delta c(x, Q^2) + \delta \bar{d}(x, Q^2) + \delta \bar{s}(x, Q^2). \quad (3.21)$$

If one assumes that isospin predicts that $u_n = d_p$, then the cross sections for scattering from neutrons can be rewritten in terms of the proton parton distributions as:

$$g_1^{\nu n}(x, Q^2) = \delta u(x, Q^2) + \delta s(x, Q^2) + \delta \bar{d}(x, Q^2) + \delta \bar{c}(x, Q^2), \quad (3.22)$$

$$g_1^{\bar{\nu} n}(x, Q^2) = \delta d(x, Q^2) + \delta c(x, Q^2) + \delta \bar{u}(x, Q^2) + \delta \bar{s}(x, Q^2). \quad (3.23)$$

The g_2 structure function has a twist-2 (g_2^{WW}) and a twist-3 ($\bar{g}_2$) contribution and has no simple parton model interpretation,

$$g_2 = g_2^{WW} + \bar{g}_2; \quad g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \quad (3.24)$$

For g_3 and g_{4+5} one can get the parton model expressions by an explicit calculation of the lowest order processes and by comparing it to the general form of the hadron tensor (3.7):

$$g_{4+5}^{\nu p}(x, Q^2) = 2xg_3^{\nu p}(x, Q^2) \quad (3.25)$$

$$= -[\delta d(x, Q^2) + \delta s(x, Q^2) - \delta \bar{u}(x, Q^2) - \delta \bar{c}(x, Q^2)],$$

$$g_{4+5}^{\bar{\nu} p}(x, Q^2) = 2xg_3^{\bar{\nu} p}(x, Q^2) \quad (3.26)$$

$$= -[\delta u(x, Q^2) + \delta c(x, Q^2) - \delta \bar{d}(x, Q^2) - \delta \bar{s}(x, Q^2)]$$

$$g_{4+5}^{\nu n}(x, Q^2) = 2xg_3^{\nu n}(x, Q^2) \quad (3.27)$$

$$= -[\delta u(x, Q^2) + \delta s(x, Q^2) - \delta \bar{d}(x, Q^2) - \delta \bar{c}(x, Q^2)],$$

$$g_{4+5}^{\bar{\nu} n}(x, Q^2) = 2xg_3^{\bar{\nu} n}(x, Q^2) \quad (3.28)$$

$$= -[\delta d(x, Q^2) + \delta c(x, Q^2) - \delta \bar{u}(x, Q^2) - \delta \bar{s}(x, Q^2)],$$

¹These parity violating polarized structure function are also important in polarized $lN \rightarrow \ell' X$ at HERA energies where $W^\pm$ and Z exchange has to be taken into account.

Figure 3.1: Model dependent decomposition of singlet term into quarks and gluon based on PDIS data, $a_0 \rightarrow \Delta q - 3\frac{\alpha_s}{2\pi}\Delta g$. Also shown are the QPM expectation and the results from a NLO fit to most of the available data on g_1^f

As shown, like in the unpolarized case parton model description, g_1 , g_3 and g_{4+5} is found from the flavor interchanges $d \leftrightarrow u$ and $s \leftrightarrow c$.

Test of $\Delta\Sigma$ and Δg based on PSF:

The sum

$$\int_0^1 dx (g_1^{\nu N} + g_1^{\bar{\nu} N}) = \delta u + \delta \bar{u} + \delta d + \delta \bar{d} + \delta s + \delta \bar{s} + \delta c + \delta \bar{c}, \quad (3.29)$$

$$\Rightarrow \Delta\Sigma - C\Delta g \quad (3.30)$$

is a measure of the axial vector singlet current matrix element, a_0 . The factor C is model dependent, one possibility would be $C = N_f \frac{a_0}{2\pi}$. Since we have knowledge of $\Delta\Sigma$ from other data, this measurement would be a 'clean' way to verify this decomposition. Note this can be done without relying on the low energy determination of the matrix elements a_3 and a_8 , in contrast to polarized charged lepton scattering. The values of these matrix elements come from beta decay and rely on isospin and SU(3) symmetry, respectively.

Test of Δs and Δc based on PSF:

The structure functions g_3 and g_{4+5} probe only non-singlet combinations of parton densities, so that they do not get a contribution from the gluon density! Consider scattering on an isoscalar target. Then

$$(g_3^{\nu N} - g_3^{\bar{\nu} N})_p + (g_3^{\nu N} - g_3^{\bar{\nu} N})_n = 2(\delta c + \delta \bar{c} - \delta s - \delta \bar{s}). \quad (3.31)$$

Assuming that the charm contribution is much smaller than the strange, this allows to us see how large the polarized strange quark sea is.

Test of Δs and Δc independent of PSF:

The strange spin contribution can be studied via charm production. At a 50 GeV muon storage ring, 5% of events have charm in the final state and 1% have a detectable semi-leptonic decay. An experiment with 20 M neutrino interactions thus will have 200K semi-leptonic charm events before kinematic cuts. These should be sufficient for a precise measurement of δs and $\delta \bar{s}$.

The structure functions $g_3 - -g_5$, like F_3 are non-singlet functions in which contribution from gluons cancel. Comparison of the non-singlet functions with the single functions g_1 and F_2 is an indirect way of measuring the contribution of gluons Δg .

An ideal polarized target will be one that has a mass of at least 10 kg, high polarization, large fraction of polarizable material, f and has the capability of having both protons and neutron polarized which is required to study the 'sea quark' contribution to the nucleon spin. 'ICE' targets are a new technology which looks very promising. 'ICE' is a HD solid polarizable compound material.

Both the H and the D can be polarized, separately or simultaneously. The expected polarization and dilution are $P_H=80\%$, and $f_H = 1/3$ for H, and $P_D=50\%$ and $f_D = 2/3$ for deuteron. A 7 kg ($\rho_t=1.1\text{gr/cm}^2$) polarized target with the qualities mentioned above can be built out of an ICE-target (20 cm in radius, 50 cm thick) mounted in the detector proposed for structure functions studies. Raw event rates in such a detector would be around 20M per 10^{20} muon decays. Another study for ν MCs [62] showed that a 200 kg target of polarized butanol with 10% polarization could measure the strange sea polarization to about 3% in a one year run in a beam downstream of a 250 GeV muon collider with a 10 m straight section.

In order to measure g_1 both ν and $\bar{\nu}$ beams are needed. A double spin difference with longitudinally polarized nucleon and evaluated eq. is measured:

$$A_{g_1} = \{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}\}_\nu - \{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}\}_{\bar{\nu}}. \quad (3.32)$$

This difference should be measured not only as a function of x but also as a function of y in order to see the effect of different suppression factors of unwanted structure functions like g_2 . Similarly, for g_3 we evaluate $A_{g_3} = \{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}\}_\nu + \{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}\}_{\bar{\nu}}$. Cross checks can be made with the target transversely polarized where the suppression factors for the different structure functions are different.

Analysis of the double difference can be done in 10-bins in x , if the acceptance for events is $> 40\%$ then the error on the asymmetry in each x bin would be: $\delta A_{g_1} \simeq (f P_T \sqrt{N})^{-1} \sim 1\%$.

If the neutrino beam intensities and polarized target described above are feasible, the physics motivations would be very strong. We will be able to do measurements where we can cleanly separate singlet (gluon-dependent) from non-singlet (gluon-free) terms. Furthermore, due to the nature of the neutrino charged interactions it will be possibility to perform a measurement of the polarization of the proton's quarks by flavor, with sea and valence contributions separated. That is, it will be possible to : (1) separate quark and anti-quark targets by using the y distributions of the events; (2) pick one flavor of a quark doublet by taking data with both neutrinos and anti-neutrinos; and (3) tag strange quarks by identifying charm state.

This is clearly an area where much further study could be performed if the polarized target program for neutrinos becomes a reality.

3.6 Studying Nuclear Effects with Neutrinos

In the past, with high statistics neutrino experiments only possible off heavy nuclear targets (Fe), it has been necessary to view nuclear effects as problems to be overcome in the study of neutrino-nucleon physics. In this section we wish to review the physics of nuclear-QCD and show that, on the contrary, nuclear effects are in themselves an interesting field of study and we could add a great deal of knowledge to this topic from a high-statistics experiment on a variety of heavy nuclear targets as well as H_2 and D_2 .

As we discuss possible physics topics we will refer to an experiment which assumes a 50 GeV storage ring with a detector 50 m away from an 800 m straight section. With 10^{20} mu decays per year, we would expect $4 \times 10^6 \nu_{\mu}$ and $1.7 \times 10^6 \bar{\nu}_e$ per kg-year in a fiducial volume of radius 10 cm. The detector would consist of a liquid D_2 target followed by a rotating support of different A targets - similar to the geometry of the Fermilab E-665 Tevatron muon experiment. To acquire 10^7 events per year in all targets we would need targets with fiducial volume $r = 10$ cm and length of 50 cm for D_2 3.7 cm for C and 0.45 cm for W. The liquid H_2 or D_2 targets could be CCD coupled active liquid targets (ie Bubble Chambers!) which could provide tracking within the target, followed by the higher A targets interspersed with tracking chambers, followed by appropriate calorimetry and muon identification/measurement techniques.

If we consider the behavior of the ratio R_A of $F_2(x, Q^2)$ measured on a nucleus (A) to $F_2(x, Q^2)$ measured on a nucleon as a function of x we pass through four distinct regions in going from $x = 0$ to $x = 1.0$:

1. "shadowing" up to $x = 0.1$,
2. "anti-shadowing" for $0.1 < x < 0.2$,
3. the "EMC Effect" for $0.2 < x < 0.7$ and
4. "Fermi motion effects" for $x > 0.7$.

These nuclear effects have been studied extensively using muon and electron beams but have only been glanced at in low-statistics bubble chamber experiments using neutrinos.

3.6.1 Low x : PCAC and Nuclear Shadowing

A pertinent question is to ask what additional things can we learn when using a weak probe. In the shadowing region there are several effects that should yield a different ratio R_A when using neutrinos as the probe. In the limit $Q^2 \rightarrow 0$, the vector current is conserved and goes to 0, however the axial-vector part of the weak current is only partially conserved (PCAC) and $F_2(x, Q^2) \rightarrow$ a non-zero constant as $Q^2 \rightarrow 0$. According to the Adler theorem [63] the cross section of $\nu_{\mu} - N$ can be related to the cross section for $\pi i - N$ at $Q^2 = 0$. What effect does a nuclear environment have on the Adler theorem?

As we increase Q^2 from 0 but keep it under 10 GeV^2 in the shadowing regime we enter the region of vector meson dominance (VMD) in $\mu/e - A$ scattering. The physics concept of VMD is the dissociation of the virtual photon into a $q - \bar{q}$ pair one of which interacts strongly with the "surface" nucleons of the target nucleus (thus the "interior" nucleons lie in the "shadow" of the "surface" nucleons). With neutrino scattering we not only have a VMD effect but an additional contribution from axial-vector mesons such as the A_1 . There are additional non-perturbative effects that appear as nuclear shadowing (mainly in large nuclei) which involve gluon recombination from the neighboring nucleons

shifting the parton distributions towards higher values of x . Boros et al [64] predict that the resulting shadowing effects in ν - A scattering will be roughly 1/2 that measured in μ/e - A scattering.

In a more quantitative analysis, Kulagin [65] used a non-perturbative parton model to predict shadowing effects in ν - A scattering. At 5 GeV^2 he predicts equal or slightly more shadowing in ν - A scattering than in μ/e - A scattering. He also attempts to determine quark flavor dependence of shadowing effects by separately predicting the shadowing observed in $F_2(x, Q^2)$ (sum of all quarks) and $xF_3(x, Q^2)$ (valance quarks only). Fig. 3.2 shows the results of a 2 year run using predictions of Kulagin's model for F_2 and xF_3 . The proposed comparison of $W:D_2$ would exhibit a much sharper decrease in the ratio as x decreases. As can be seen, the predicted difference between the shadowing on sea and valence quarks is clearly visible down to $x = 0.02 - 0.03$.

Figure 3.2: $R_{Ca:D_2}$ for both F_2 and xF_3 as measured in a 2 year exposure at the facility described in the text.

3.6.2 Mid x : Anti-shadowing and the EMC Effect

Drell-Yan experiments have also measured nuclear effects. Their results are quite similar to DIS experiments in the shadowing region. However, in the anti-shadowing region where R_A makes a brief but statistically significant excursion above 1.0 in DIS, Drell-Yan experiments see no effect. This could be an indication of difference in nuclear effects between valence and sea quarks. Eskola et al [66] quantified this difference by using a leading order/leading twist DGLAP model which used initial nuclear parton distributions from CTEQ4L and GRV-LO and assumed scale evolution of nuclear parton distribution is perturbative. The model predicts that the difference between nuclear effects in $xF_3(x, Q^2)$ and $F_2(x, Q^2)$ persist through the anti-shadowing region as well. Taking the work of Kulagin and Eskola together implies that nuclear effects in $xF_3(x, Q^2)$ should be quite dramatic with more shadowing than $F_2(x, Q^2)$ at lower x and then R_A rising fairly rapidly to yield significant anti-shadowing around $x = 0.1$. In our standard 2 year experiment we should be able to measure anti-shadowing effects and the difference between shadowing effects in $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ to the 6 s.d. statistical level

To determine individual quark contributions to the EMC-effect will be challenging since the participation of sea quarks, and thus the difference between $F_2(x, Q^2)$ and $xF_3(x, Q^2)$, shrinks rapidly with increasing x . However, Eskola's predictions for this region indicate that the contribution of $\bar{q}$ and $\bar{d}$ to R_A in the Q^2 range of this experiment remains well below 1 so that the quantity $R(2)_A - R(3)_A$ should remain negative well into the EMC-effect region.

3.6.3 High x : Multi-quark Cluster Effects

When working in the "fermi-motion" region it has been shown that we need to add more than the Fermi gas model to a simple nucleon to reproduce the behavior of $F_2(x, Q^2)$ at high x_{Bj} . Few-nucleon-correlation models and multi-quark cluster models allow quarks to have a higher momentum which translates into a high- x_{Bj} tail. In this region $F_2(x, Q^2)$ should behave as $e^{-ax_{Bj}}$. There have been analysis of this behavior in similar kinematic domains using $\mu + C$ and $\nu + Fe$. The muon experiment finds $a = 16.5 \pm 0.5$ while the neutrino experiment finds $a = 8.3 \pm 0.7 \pm 0.7$ (systematic). Is the value of a dependent on the nucleus? One would expect any few nucleon correlation or multi-quark effects to have already saturated by Carbon. Is a dependent on the probe?

3.7 Higher Twist

There are also very important open questions at the high end of the x scale, having nothing to do with nuclear effects, that can have serious effects on the backgrounds at high energy colliders. This comes about since uncertainties at high x and low Q^2 evolve to uncertainties at much lower x as Q^2 increases to collider values.

The high x uncertainties are of two types; the value of $d(x)/u(x)$ as $x \rightarrow 1$, and the role of leading power corrections (higher twist) in the extraction of the high x behavior of the quarks.

There are several recent analyses of electroproduction off hydrogen and deuterium targets which use nuclear models to correct for the nuclear (binding) effects of deuterium. Using such a model, Yang and Bodek's analysis demonstrates how a comparatively simple x dependent correction which has the $d(x)/u(x)$ ratio asymptotically approaching 0.2 (instead of $d(x)/u(x) \rightarrow 0$) as $x \rightarrow 1$ improves the QCD fits to the high- x NMC data and the CDF W -decay asymmetry. On the other hand the recent CTEQ global QCD analysis which resulted in the CTEQ5 parton distribution functions finds that the quality of fit does not change significantly by including a Yang-Bodek type correction. There is something happening at high- x which is not clear from presently available data sets. A direct way of measuring the $d(x)/u(x)$ ratio at high x , which is entirely free of any nuclear corrections, is with the ratio of neutrino-proton to antineutrino-proton cross sections which, aside from sea quark contributions which are negligible at high x , measures the $d(x)/u(x)$ valence quark ratio directly. A high statistics exposure to a H_2 target is all that is required to answer this question.

The question of leading power corrections is closely related to the measurement of quark densities at high x . Leading power corrections are proportional to $1/Q^{2n}$ and reflect the fact that quarks have transverse momentum within the nucleon and that the probe becomes larger as Q^2 decreases thus increasing the probability of multi-quark participation in an interaction. An attempt to measure these effects in electroproduction was made by Milzstein by combining SLAC data with BCDMS muo-production data. They found that the size of the twist-4 contribution rapidly increased above $x = 0.4$ and was equal in magnitude to the leading $1/\log Q^2$ term for $x > 0.75$. The only measurements of this higher-twist term in neutrino experiments have been two low-statistics bubble chamber experiments in Gargamelle with freon and BEBC with NeH_2 . Both the analyses are complicated with nuclear corrections at high- x . However, both analyses found a twist-4 contribution that is smaller in magnitude than the muo-production analysis and, most significantly, preferentially negative. The CTEQ global QCD analysis which combines neutrino and muo production up to $x = 0.75$ finds that NO higher-twist term is required for values of Q^2 down to 0.7 GeV^2 but the analysis uses a $W > 4 \text{ GeV}$ cut (mass of the hadronic system) which could exclude any twist-4 contribution. Most recently the Yang-Bodek analysis mentioned above reanalyzed electroproduction data looking for a higher twist contribution. They find that by incorporating a NLO QCD analysis, as opposed to the LO analysis used by Milzstein, the higher twist contribution becomes much smaller. However, they have also included the $d/u \rightarrow 0.2$ model in their extraction of higher twist which will reduce the size of any extracted higher twist term. Furthermore, in a recent CTEQ paper Guo and Qiu [67] have predicted the leading (due to the factor $1/(1-x)$) x dependence to the higher-twist term in $F_2(x, Q^2)$ in the region of large x and have found that the higher-twist contribution is *different* for u and d quarks. Were this to be

true, then the $d(x)/u(x)$ analysis and the leading power correction analysis are directly intertwined. Interestingly enough, this prediction depends on two non-perturbative parameters only and therefore is highly constrained. It can be tested at CEBAF and/or the ν MC facility.

As one can see, there is interesting physics in the region of high x which can be characterized as the interface between perturbative and non-perturbative QCD. A region that requires much additional study with both electroproduction and the weak current of neutrino nucleon interactions. A high statistics neutrino/antineutrino exposure in H_2 and D_2 is the most direct way to study this rich region of phase space.

3.8 J/ψ Production and Gluon Distribution Functions

Due to its clean experimental signature, one of the most important charmonium production processes is J/ψ production. A neutrino facility at a Muon Collider would allow studies of the total cross section and the differential Q^2 distribution of J/ψ 's. These studies can help to determine several important non-perturbative parameters (matrix elements of non-relativistic QCD (NRQCD) operators) that describe hadronization of $c\bar{c}$ quark pair into charmonium state. These matrix elements are usually determined from hadron collider experiments, but the processes involved are sensitive only to various linear combinations of these matrix elements. The problem is aggravated by the usually very large theoretical uncertainties involved in these calculations (on the order of 50 – 100%), due to higher twist effects, uncalculated and/or incalculable higher order perturbative and nonperturbative contributions. In this situation any independent determination of these quantities should be welcome. On the other hand, J/ψ produced in sufficiently high numbers can be used to study gluon distribution function in a wide range of x [68] if the NRQCD parameters governing $c\bar{c}$ fragmentation are measured at the future Tevatron experiments.

A major advantage of using a neutrino beam is that, at leading order in α_s , the spin structure of the νZ coupling selects a certain combination of NRQCD octet operators. The largest contribution is from the one with the quantum numbers $^3S_1^{(8)}$. Of course, order of magnitude measurements of the size of the matrix elements of this operator have already been performed for the J/ψ and ψ' , as well as for the χ_{cJ} states. The estimates of these matrix elements mostly come from Tevatron fits to hadroproduction cross sections for the J/ψ and χ_{cJ} and yield, with large theoretical errors [69],

$$\begin{aligned} \langle 0 | \mathcal{O}_8^{J/\psi} (^3S_1) | 0 \rangle &\sim 0.01 \text{ GeV}^3, \quad \text{and} \\ \langle 0 | \mathcal{O}_8^{\chi_{c0}} (^3S_1) | 0 \rangle &\sim 0.01 \text{ GeV}^3. \end{aligned} \quad (3.33)$$

These values are consistent, within a $\sim 50\%$ accuracy level, with the value found from Z decay at LEP [70] (the latter does not separate cascade and

direct production, so the value of 0.019 GeV^3 is understandably larger than the one in Eq. (3.33)).

The differential cross section was calculated in [68], and we quote the result in a form involving the structure functions $h_n(Q^2)$:

$$\begin{aligned} \frac{d\sigma(s, Q^2)}{dQ^2} &= \frac{\pi^2 \alpha^2 \alpha_s}{3 \sin^4 2\theta_W} \frac{1}{(Q^2 + m_Z^2)^2} \\ &\times \sum_n \frac{\langle 0 | \mathcal{O}_n | 0 \rangle}{m_c^3} \int_{\frac{Q^2 + 4m_c^2}{s}}^1 dx f_{g/N}(x, Q^2) h_n(y, Q^2), \end{aligned} \quad (3.34)$$

where s is the total invariant mass of the νN system, x is the momentum fraction of the incoming gluon, $-Q^2$ is the momentum-squared transferred from the leptonic system, $y = \frac{Q^2 + 4m_c^2}{sx}$, and $f_{g/N}(x, Q^2)$ is a gluon distribution function in the nucleon. The charmonium structure functions are given by

$$\begin{aligned} h_{1S_0^{(8)}}(y, Q^2) &= (g_V^c)^2 \times 6 \frac{Q^2 m_c^2}{(Q^2 + 4m_c^2)^2} (y^2 - 2y + 2) \\ h_{3S_1^{(8)}}(y, Q^2) &= (g_A^c)^2 \times 2m_c^2 \frac{Q^2(y^2 - 2y + 2) + 16(1 - y)m_c^2}{(Q^2 + 4m_c^2)^2} \\ h_{3P_0^{(8)}}(y, Q^2) &= (g_V^c)^2 \times 2Q^2 \frac{(Q^2 + 12m_c^2)^2}{(Q^2 + 4m_c^2)^4} (y^2 - 2y + 2) \\ h_{3P_1^{(8)}}(y, Q^2) &= (g_V^c)^2 \times 4Q^4 \frac{Q^2(y^2 - 2y + 2) + 16(1 - y)m_c^2}{(Q^2 + 4m_c^2)^4} \\ h_{3P_2^{(8)}}(y, Q^2) &= (g_V^c)^2 \times \frac{4}{5} Q^2 \left[\frac{(y^2 - 2y + 2)Q^4}{(Q^2 + 4m_c^2)^2} \right. \\ &\quad \left. + \frac{48(1 - y)Q^2 m_c^2 + 96(y^2 - 2y + 2)m_c^4}{(Q^2 + 4m_c^2)^2} \right], \end{aligned} \quad (3.35)$$

where $g_A^c = \frac{1}{2}$ and $g_V^c = \frac{1}{2} (1 - \frac{8}{3} \sin^2 \Theta_W)$ are the vector and axial couplings of the c -quark. Clearly, the coupling constants favor the $3S_1^{(8)}$ contribution, which is due to the large axial coupling (a similar contribution is, of course, absent in the case of J/ψ lepto and photoproduction). Indeed a numerical estimate [68] shows that this matrix element dominates the total cross section, and also the differential cross section unless $Q^2 \gg m_c^2$. At large Q^2 , the relative Q^4 enhancement of the P -wave structure functions makes them dominant. These structure functions should be incorporated in the specific Monte Carlo generators built for the particular detector design.

Using approximate values for the NRQCD matrix elements satisfying the restrictions imposed by heavy quark symmetry and compatible with the existing extractions [69, 71, 72] the behavior of differential cross section can be studied. The energy dependence of the total cross section, taken at various representative

incident neutrino energies, is shown in Table 3.1. The total cross section is clearly very sensitive to the neutrino beam energy E_ν . The origin of this strong dependence can be traced to the fact that the main contribution to the integral in Eq. (3.34) comes from the smallest available $x \approx \frac{Q^2 + 4m_c^2}{s}$, so that larger s probes a smaller region of x where the gluon PDF's are enhanced. It becomes obvious that in order to have a reasonably large event number, the neutrino beams with the highest possible energy should be employed. As seen from the

$E_\nu[\text{GeV}]$	7.5	25	120	450
$\sigma[\text{nb}]$	7.8×10^{-13}	6.9×10^{-10}	1.3×10^{-8}	5.5×10^{-8}

Table 3.1: Total cross sections for the J/ψ production in $\nu N \rightarrow J/\psi X$ for various representative incident neutrino energies.

Eqs. (3.34), the differential cross section quickly decreases with Q^2 . However, the decrease is not as fast as in the leptonproduction case. Empirically,

$$\left(\frac{d\sigma}{dQ^2}\right)_{\nu N} \propto \left(\frac{d\sigma}{dQ^2}\right)_{eN} \times \frac{Q^2(Q^2 + \beta m_c^2)}{m_Z^4} \quad (3.36)$$

where $\beta \approx 6$. The upshot is that the absence of a photon propagator results in a much wider tail of the Q^2 distribution than in leptonproduction, allowing for better discrimination between S - and P -wave contributions.

Fig. 3.3 shows the ratio of the contributions from these matrix elements as a function of Q^2 . At low Q^2 , where the contribution into the cross section is large, the $^3S_1^{(8)}$ dominates but a determination of the differential cross section in the range $\sim 2 \text{ GeV} < Q < 4 \text{ GeV}$, where the event number is still sizable, would allow a reliable extraction of both $^3S_1^{(8)}$ and $^3P_J^{(8)}$ matrix elements. Higher Q^2 's will only measure a linear combination of these, as the ratio in Fig. 3.3 becomes Q^2 -independent.

3.9 Λ_C^+ Form Factors

In addition to the studies of nucleon via charmed meson production, the νMC facility could provide a good opportunity to study charmed baryon production rates and form factors. For instance, the reactions

$$\nu_\mu n \rightarrow \mu^- \Lambda_C^+, \quad (3.37)$$

$$\nu_e n \rightarrow e^- \Lambda_C^+, \quad (3.38)$$

have cross sections that reach an energy independent $\sigma \simeq 0.4 \text{ fb}$ at high energies [73]. This represents approximately 0.06 (100 GeV/ E)% of all CC events, and about 6×10^6 of these events should be produced for 10^{10} CC interactions.

The Λ_C^+ differential cross section $d\sigma/dQ^2$ can be described by three form factors [74], two vector and one axial, in analogy to quasielastic scattering. The

form factors can be measured by the joint Q^2, E distributions of Λ_C^+ production with an accessible Q^2 range $0 \leq Q^2 \leq 2ME$ bounded from above by the beam energy. This is in sharp contrast to form factor measurements of charmed mesons from semileptonic decay in which the upper limit on momentum transfer is determined by the parent hadron mass. This lack of Q^2 in semileptonic decay physics has in turn resulted in little discrimination between models [75].

In principle this reaction can be identified with a good vertex detector supplemented by enough particle ID to identify protons. One searches for a two-prong primary vertex followed by a decay vertex which emits either a proton or Λ baryon. The challenge will be to eliminate contributions from continuum Λ_C^+ production and production from excited Λ_C^+ baryons.

Figure 3.3: The contributions of the various matrix elements as a function of Q^2 for FMC-Ring, with baseline-sized matrix elements. Observe that the contributing linear combination of matrix elements drastically changes with Q^2 within a region where the rate is still sizable, so one can realistically extract both the $^3S_1^{(8)}$ and the $^3P_1^{(8)}$ matrix elements even without requiring exceedingly high statistics.

Chapter 4

Studies of the CKM Quark Mixing Matrix

4.1 Introduction

The CKM quark mixing matrix represents a set of fundamental parameters of the SM that reflect the dynamics shaping the generation of particle masses. Furthermore, they provide essential input for predictions on various CP asymmetries in B decays and these predictions should be sharpened as much as possible to increase the sensitivity for the presence of new physics.

It is likely that medium energy ν MCs could make a very substantial contribution to measurements of quark mixing, with systematically unique measurements of 4 of the 9 moduli in the CKM matrix that should be comparable to or better than the best future measurements at other experiments. The relevant moduli are $|V_{cd}|$, $|V_{cs}|$, $|V_{ub}|$ and $|V_{cb}|$, to be extracted from charm production off d and s quarks, and from beauty production off u and c quarks, respectively. If muon colliders ever reach the 100 TeV energy range then they could further provide a unique opportunity for precise direct measurements of the CKM elements involving the top quark, as is discussed in reference [9].

This chapter is organized as follows. Subsection 4.1.1 gives a summary of the current experimental status and future expectations before the advent of ν MCs. Subsection 4.1.2 then gives a general overview of the method for extracting CKM information at ν MCs. Sections 4.2 and 4.3 then give more specific details on ν MC experimental analyses involving the production of charm and bottom quarks, respectively. Section 4.5 summarizes the expected improvements from ν MCs to our knowledge of the CKM matrix.

4.1.1 Current experimental knowledge of the CKM matrix and future expectations before ν MCs

The allowed ranges at 90 % C.L. on the moduli of the elements of the CKM matrix read as follows when one allows for more than three families [76]

$$|\mathbf{V}_{CKM}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| & \dots \\ |V_{cd}| & |V_{cs}| & |V_{cb}| & \dots \\ |V_{td}| & |V_{ts}| & |V_{tb}| & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

$$= \begin{pmatrix} 0.9722 - 0.9748 & 0.216 - 0.223 & 0.002 - 0.005 & \dots \\ 0.199 - 0.233 & 0.784 - 0.976 & 0.037 - 0.043 & \dots \\ 0.0 - 0.09 & 0 - 0.55 & 0 - 0.9993 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \quad (4.1)$$

The values for $|V_{cd}|$ and $|V_{cs}|$ have been extracted from charm production in deep inelastic scattering supplemented by information gathered in semileptonic D decays; $|V_{cb}|$ and $|V_{ub}|$ have been obtained from semileptonic B decays.

Imposing three-family-unitarity constraints gives much tighter bounds on the moduli of the CKM elements [76]:

$$|\mathbf{V}_{CKM}| = \begin{pmatrix} 0.9742 - 0.9757 & 0.219 - 0.226 & 0.002 - 0.005 \\ 0.219 - 0.225 & 0.9734 - 0.9749 & 0.037 - 0.043 \\ 0.004 - 0.014 & 0.035 - 0.043 & 0.9990 - 0.9993 \end{pmatrix}. \quad (4.2)$$

Equations 4.1 and 4.2 show that $|V_{cs}|$ and $|V_{cd}|$ are already well known numerically *if* one imposes 3-family unitarity; yet, otherwise, the constraints are much looser.

In the next few years *exclusive* semileptonic charm decays like $D \rightarrow \ell \nu K / K^* / \pi / \rho$ will be measured more precisely. However it is not clear at present, whether theoretical technologies like QCD simulations on the lattice will improve sufficiently to fully utilize such data for extracting more accurate values of $|V_{cs}|$ and $|V_{cd}|$.

At present $|V_{cb}|$ is known with about a 5% *theoretical* and a 5% or better *experimental* uncertainty as shown above. The two most reliable determinations are from the semileptonic B width and the form factor for $B \rightarrow \ell \nu D^*$ at zero recoil. One can reasonably expect the theoretical uncertainty to go down to about 2% [77].

The situation is considerably less satisfactory for $|V_{ub}|$. Most of the analyses employed so far contain a large dependence on theoretical models the accuracy of which is hard to evaluate. One can hope that studies of both exclusive and inclusive semileptonic B decays done at the B factories will yield values for $|V_{ub}|$ with a *theoretical* accuracy of no more than 10% by 2005; however this is not guaranteed.

It is highly desirable to improve on this situation by obtaining more precise and reliable values for these CKM parameters. For they are a consequence of the generation of quark masses which in turn reflects dynamics operating at presumably ultra-high energy scales. It turns out that qualitatively quite distinct so-called ‘textures’ for the Yukawa couplings assumed to apply at GUT scales lead to CKM parameters that are numerically relatively similar at electroweak scales. Secondly, various CP asymmetries in B decays will be measured presumably to better than 5% over the next ten years; their predicted values depend crucially on the size of V_{ub} , V_{cb} (and V_{td}). To exploit the discovery potential for new physics to the fullest one wants to match up experimental and theoretical accuracy.

To be more specific:

- One wants at least to confirm the values for $|V_{cs}|$ and $|V_{cb}|$ by a *systematically* different method.
- One would like to determine a precise value for $|V_{cd}|$ *without* imposing three-family-unitarity.
- It is an important goal to extract the value of $|V_{ub}|$ with considerably less than 10% uncertainty.

It appears that ν MCs are up to these tasks and possibly more:

- If in the future a ν MC in the 100 TeV energy range were to be realized, then even top quark production could be studied, which would allow the extraction of V_{td} and V_{ts} in a relatively straightforward way!

4.1.2 Extracting CKM matrix elements in ν MCs : an overview

The measurements of the moduli of the CKM matrix elements at ν MCs will be analogous to, but vastly superior to, current neutrino measurements of $|V_{cd}|^2$ that use dimuon events for final state tagging of charm quarks. The experimental quantities used for all the quark mixing measurements in ν MCs are the production cross sections and kinematic distributions of heavy flavor final states from CC interactions. These are related back to the quark couplings through a parameterization of the initial state quark distributions and a model of the production processes that includes threshold suppressions due to the quark masses.

In principle, the struck quark can be converted into any of the three final state quarks that differ by one unit of charge: a down (d), strange (s), or bottom(b) quark can be converted into an up (u), charmed (c), or top (t) quark and vice versa. In practice, production of the heavy top quark is kinematically forbidden except from neutrinos at energies above 16 TeV and the production of other quark flavors is influenced by their mass. After correcting for this kinematic suppression, the Standard Model (SM) predicts the probability for the interaction to be proportional to the absolute square of the appropriate element in the CKM matrix. The most relevant theoretical uncertainty lies in the treatment of the production thresholds. Measuring these cross sections at

different energies and separately for CC and NC reactions should enable us to understand the threshold behavior.

4.2 Analyses involving charm production: extraction of V_{cd} and V_{cs}

The extraction of $|V_{cd}|$ will be the cleanest of the four CKM measurements at ν MCs discussed here and is the one CKM element whose modulus is already best determined in neutrino-nucleon scattering. It is measured from charm production off valence d quarks in isoscalar targets, whose distribution can accurately be determined from the nucleon SF measurements – see chapter 3.

Figure 1.2 of chapter 1 shows that charm production occurs in of order 10% of CC interactions from 100 GeV neutrinos, and will occur at slightly lower/higher rates for lower/higher energies according to the changing level of mass threshold suppression. This shows that realistic event sample sizes will result in of order 10^8 statistics for the production of charm from valence d quarks.

The feature of this analysis that makes it systematically rather clean is that charm production from valence quarks occurs for neutrinos but not for antineutrinos. The valence quarks have a much harder x distribution than the sea contributions from both s and (at a lower level) d quarks. It is also helpful that the s and d seas seen by neutrinos are also closely equal to their antiquark counterparts that are probed by neutrinos, as can be verified in SF analyses of the ν MC data set – see chapter 3. This allows use of the antineutrino sample as an effective background subtraction of the sea contributions to the charm production by neutrinos.

As in analyses of today’s neutrino experiments, the d and s contributions to the charm event sample will in practice be separated from one another in a fit involving the measured x distributions of the events etc. and with the charm quark mass as a fitted parameter.

With such huge statistics, the statistical uncertainty in the measurement will be only at the 10^{-4} level so the measurement will be dominated by *systematic* uncertainties. Major contributions to the uncertainty are likely to be:

- estimation of the charm tagging efficiency. This error is minimized by the clean, high efficiency charm tagging that is possible with high performance vertexing and only a tiny B hadron background – see appendix B.
- the charm production model. Since one is actually measuring charm production relative to any production in that kinematical regime, one is sensitive to the kinematical suppression of charm states, but not to the initial parton distribution functions. Mass threshold corrections are applied according to the “slow-rescaling” model and additional smaller “fast-rescaling” corrections account for the final state hadron mass suppressions. Since these corrections can be largely measured and checked using the data itself, this uncertainty should initially decrease with in-

creasing sample size. It should be noted that the mass parameter contained in the threshold corrections cannot be identified with the charm mass extracted from charmonium spectroscopy and charm decays.

- estimation of the extent to which the sea is symmetric in quarks versus antiquarks. This uncertainty is lessened because the signal is largely at higher x than the background. If helpful, a cut in x value could be applied to the fit to further reduce this uncertainty.

With experimental handles on the major systematic uncertainties, it may be guessed that the irreducible *theoretical* uncertainty on $|V_{cd}|^2$ due to violations of quark-hadron duality might be at the percent level.

The value of $|V_{cs}|^2$ will come out of the same fit, influenced mainly by the data at low x . This measurement will also benefit from of order 10^8 statistics and clean charm tagging. The major difficulty here will be an incomplete knowledge of the initial strange sea distribution. This can presumably be estimated by using a “leading particle tagging” analysis of neutral current interactions with a resonant $\phi(1020)$ final state. A rough guess is that the resulting measurement accuracy on $|V_{cs}|$ might be at the few percent level.

4.3 Analyses involving bottom production: extraction of V_{ub} and V_{cb}

A similar analysis to the previous section will be applied for the two transitions with a b quark in the final state, $u \rightarrow b$ and $c \rightarrow b$, that give measurements of V_{ub} and V_{cb} , respectively.

Figure 1.2 shows that statistics for CC b quark production might be only perhaps of order 10^4 events or above, depending on the energy of the muon storage ring and the consequent threshold suppression due to the b quark mass. Besides the threshold suppression, other reasons for the low B production seen in figure 1.2 are that production from u quarks is suppressed because the value of $|V_{ub}|^2$ is only about 10^{-5} while production from charm quarks is suppressed by the low charm content of the nucleon (either an intrinsic charm sea or charm production through higher order Feynman diagrams).

Given the nearly optimal vertexing geometry possible at a ν MC, the vertexing discrimination between b and c hadrons should be able to separate out much of the fractionally small b hadron event sample, as discussed in appendix B.

Assuming that the B tagging efficiency and purity are well known, then the B production analysis should be a relatively straightforward copy of the charm production analysis, involving a simultaneous fit to the u and c contributions to B production from neutrino and antineutrino scattering.

The precision of the $|V_{ub}|^2$ measurement may approach the statistical limit of around 1 percent. In contrast, the c -to- b transition has the additional challenge of estimating the initial charm quark spectrum using NC charm events and/or CC scattering involving the c -to- s transition, and this may well limit the $|V_{cb}|^2$ measurement accuracy to the few percent level.

An alternative and maybe more useful approach will be to use the value of V_{cb} as extracted from semileptonic B decays to analyze the mass suppression in beauty production close to threshold and use this information as input to the extraction of V_{ub} as sketched above. It is widely expected that $|V_{cb}|$ will anyway be known from B decays to a few percent or better by 2005 [77].

4.4 V_{cd}/V_{cs} via Diffractive Charmed Vector Meson Production

It is possible to diffractively produce charmed vector mesons via W -boson-pomeron scattering

$$\nu_\mu A \rightarrow \mu^- (W^+ \mathcal{P}) A \rightarrow \mu^- D_S^{*+} A, \quad (4.3)$$

$$\nu_\mu A \rightarrow \mu^- (W^+ \mathcal{P}) A \rightarrow \mu^- D^{*+} A. \quad (4.4)$$

The D_S^{*+} cross section is of order 0.01 fb/nucleus, while that for D^{*+} should be $|V_{cd}/V_{cs}|^2$ smaller. If the $W - D_S^*$ and $W - D^*$ dynamical couplings are identical, the relative rate can measure the ratio of CKM matrix elements. For 10^{10} CC events, one expects of order 1.5×10^6 D_S^* events and 7500 D^* events.

The experimental signature is fairly unique: a two- or three-prong muon-meson vertex with a D^+/D^0 or D_S^+ secondary. The cross section will peak at low momentum transfer, and there should be no evidence for nuclear breakup.

The chief theoretical uncertainty likely has to do with evaluating SU(3)-breaking effects in the W -vector meson couplings. Again an alternative perspective can be taken: use the information available on $|V_{cd}/V_{cs}|^2$ to analyze the SU(3) pattern of diffractive D^* vs. D_S^* production.

4.5 Improvement of CKM knowledge from ν MCs

PDG2000 [76] assigns the following uncertainties to the four CKM parameters that were the main focus of our discussion:

$$|\Delta V_{cd}|_{now} = 17\% \quad [3\%] \quad (4.5)$$

$$|\Delta V_{cs}|_{now} = 20\% \quad [2\%] \quad (4.6)$$

$$|\Delta V_{ub}|_{now} = 40\% \quad (4.7)$$

$$|\Delta V_{cb}|_{now} = 8\% \quad (4.8)$$

where the numbers for V_{cd} and V_{cs} quoted in square brackets hold after 3-family unitarity has been imposed.

The future before a ν MC could contribute can be sketched as follows:

- Some improvements can be expected over the next several years in *direct* extractions of V_{cd} and V_{cs} (i.e. those that do not impose 3-family unitarity constraints). However it is very unlikely that they could come close to the 2 - 3 % level.

- A combination of more detailed data on semileptonic B decays and further refinement of heavy quark expansions will yield very significant improvements in V_{cb} and V_{ub} ; not unreasonable expectations are

$$|\Delta V_{cb}|_{pre-nuMC} \simeq 4\% , \quad |\Delta V_{ub}|_{pre-nuMC} \simeq 10 - 15\%. \quad (4.9)$$

Our discussion suggests that potential ν MC analyses could have an essential impact on central aspects of the SM by meeting the goals stated in the beginning of this chapter:

- provide a *systematically* different determination of $|V_{cs}|$ and $|V_{cb}|$ that is as good as can be achieved in charm and beauty decay studies:

$$|\Delta V_{cs}|_{nuMC} \sim \mathcal{O}(\text{few}\%) , \quad |\Delta V_{cb}|_{nuMC} \sim \mathcal{O}(\text{few}\%) ; \quad (4.10)$$

- yield a value for $|V_{cd}|$ through *direct* observation that is about as good as otherwise achieved *only* through imposing 3-family-unitarity

$$|\Delta V_{cd}|_{nuMC} \sim \mathcal{O}(1\%) ; \quad (4.11)$$

- lower the theoretical uncertainty in $|V_{ub}|$ by about an order of magnitude:

$$|\Delta V_{ub}|_{nuMC} \sim \mathcal{O}(1\%) ; \quad (4.12)$$

this would enable us to predict various CP asymmetries in B decays with order 1% accuracy; next generation experiments like LHC-B and BTeV are expected to measure them with such an experimental accuracy which would allow us to exhaust the discovery potential for New Physics in B decays;

- improve dramatically our numerical information on the CKM parameters involving top quarks if a very high energy ν MC could be built.

Chapter 5

Precision Electroweak Studies

5.1 Introduction

Neutrino scattering is a natural place to study the structure of the weak interaction. Historically, neutrino scattering has played an important role in establishing both the basic structure of the weak interaction[78] and in providing the first precision tests of electroweak unification[79]. It is natural, as planned high-intensity muon beams make available a new level of statistics in high energy neutrino interactions, that one considers a new generation of experiments to probe the weak interaction.

At the same time, given the ambitious physics programs of LEP I, SLD, LEP II and the Tevatron which will be complete at the time of such an experiment and given the physics program of the LHC that will be ongoing, the goals of such experiments must be correctly focused.

5.1.1 Knowledge at the time of ν MC

Between today and the advent of a ν MC, it is safe to assume that little improvement in amazingly precise measurement of Z^0 decay and production parameters [80] will have been achieved. However, it is extremely likely that LEP II and the Tevatron Run II will have produced a measurement of M_W to a 20 MeV precision[82, 83] and that the mass of the top quark will be known with a precision of 1 GeV[83]. Furthermore, although it is no sure bet, the Higgs boson may be discovered at the Tevatron or at the LHC. Of course, the odds of such a discovery are much higher if the Higgs is very light, as present data seems to suggest [80]. An observation of a Standard Model Higgs, along with a precise prediction of its mass driven by the W and top quark mass measurements, will

make an elegant *pièce de resistance* of the electroweak physics program at the energy frontier for the period between 1990 and 2005.

During this time, there may also be improvements in low-energy tests of the electroweak Standard Model. Atomic Parity Violation experiments may be able to yet again improve significantly in their ability to provide precision tests of weak interactions if experiments utilizing trapped unstable Francium become possible[84]. Also, SLAC E-158[85], a proposed polarized Moller scattering experiment, may be able to probe $\sin^2 \theta_W$ at a precision of 0.0008 at a $Q^2 \sim 10^{-2} \text{ GeV}^2$.

5.1.2 Goals of a fixed-target electroweak physics programs

The primary goals of a low-energy test of a high-energy theory remain largely the same as they have been in the past. Unification of the interactions of the on-shell weak bosons with their low-energy manifestations in weak interactions at high precision remains an appealing and elegant test of the model.

The low-energy experiments also allow access to some aspects of the theory that cannot be readily observed at high energies. For example, a demonstration of the running of the weak coupling now only awaits precise low-energy data[86], given the high precision at the scale of the weak boson masses. Another powerful use for low-energy data is the sensitivity to interference between new physics and tree-level processes. For example, if Z' bosons are discovered at the energy frontier, observation of the interference between the standard model Z and the Z' at low-energy may be one of the most powerful tests constraining models that relate the two interactions.

5.1.3 Electroweak processes with neutrinos

To provide a precision test of the Standard Model, a process must be reasonably common and precisely calculable. The two useful processes for these sorts of studies are neutrino deep inelastic scattering (neutrino-quark scattering), and neutrino-electron elastic and quasi-elastic scattering. The former, of course, wins on large cross-section, but the latter wins on simplicity of target and therefore minimal theoretical uncertainties. The possibilities using these processes are described below.

Processes not considered include neutrino tridents (three lepton final states resulting from internal conversion of a virtual photon), which are effectively a test of boson-boson scattering. While these processes are interesting, they do not provide a stable basis on which to form a precision probe of the model because of large theoretical uncertainties in the cross-section. See Chapter 6 for further discussion of this and other processes.

5.2 Elastic and Quasi-Elastic Neutrino-Electron scattering

Perhaps the most promising reaction for studying electroweak interactions of neutrinos is $\nu - e$ scattering. Neutrino-electron scattering possesses one significant advantage over deep inelastic scattering for precision electroweak studies, namely that the target is point-like and its structure does not introduce uncertainties in extracting parameters of the fundamental interaction from observed cross-sections.

In the muon and electron neutrino and anti-neutrino beams at a muon collider, the following $\nu - e$ scattering reactions occur:

$$\nu_\mu e^- \rightarrow \nu_\mu e^-, \quad (5.1)$$

$$\nu_\mu e^- \rightarrow \nu_e \mu^-, \quad (5.2)$$

$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-, \quad (5.3)$$

$$\nu_e e^- \rightarrow \nu_e e^-, \quad (5.4)$$

$$\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-, \quad (5.5)$$

$$\bar{\nu}_e e^- \rightarrow \bar{\nu}_\mu \mu^-, \bar{\nu}_\tau \tau^-, \bar{u}d \dots \quad (5.6)$$

Reactions 5.1 and 5.3 are purely neutral-current processes mediated by the exchange of a Z^0 . Reaction 5.4 has both charged current ($W^\pm$ exchange) and neutral current components and reaction 5.5 has neutral current t -channel and charged-current s -channel components. Reactions 5.2 and 5.6 can result in the production of a single muon and therefore have a significant muon mass threshold in the cross-section. Figure 5.1 shows the Feynman diagrams for these processes.

Because of the small ratio of the electron to proton mass, the cross-section for neutrino-electron scattering is much smaller than that for neutrino deep inelastic scattering. The total cross-section is of order $10^{-41} \text{ cm}^2 \text{ GeV}^{-1} \times E_\nu$, and the leading order differential cross-section with respect to $y = E_e/E_\nu$ is given by

$$\frac{d\sigma(\nu e^- \rightarrow \nu e^-)}{dy} = \frac{2G_F^2 m_e E_\nu}{\pi} [g_L^2 + g_R^2(1-y)^2], \quad (5.7)$$

where terms of $\mathcal{O}(m_e/E_\nu)$ are neglected. g_L and g_R are process dependent because of their exchange in the neutral current process under $\nu \leftrightarrow \bar{\nu}$, and because of the addition of the charged current process for electron neutrino induced reactions. The values of g_L and g_R are given in table 5.1. The numerical values for the cross-sections after integrating over y are:

$$\sigma(\nu e^- \rightarrow \nu e^-) = 1.6 \times 10^{-41} \times E_\nu [\text{GeV}] \times \left[g_L^2 + \frac{1}{3} g_R^2 \right], \quad (5.8)$$

where the values for the final term are given in the final column of table 5.1. Radiative corrections have been calculated to 1-loop for this process [87], and theoretical techniques exist to extend this calculation to higher orders.

Figure 5.1: Feynman diagrams for the $\nu - e$ scattering processes expected in neutrino beams from a muon collider: (a) NC ν - e scattering, (b) CC ν_e - e scattering, (c) CC $\bar{\nu}_e$ - e annihilation, (d) inverse muon decay.

Reaction	g_L	g_R	$g_L^2 + \frac{1}{3}g_R^2$
$\nu_\mu e^- \rightarrow \nu_\mu e^-$	$-\frac{1}{2} + \sin^2 \theta_W$	$\sin^2 \theta_W$	0.091
$\nu_e e^- \rightarrow \nu_e e^-$	$\frac{1}{2} + \sin^2 \theta_W$	$\sin^2 \theta_W$	0.551
$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$	$\sin^2 \theta_W$	$-\frac{1}{2} + \sin^2 \theta_W$	0.077
$\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$	$\sin^2 \theta_W$	$\frac{1}{2} + \sin^2 \theta_W$	0.231

Table 5.1: g_L and g_R by $\nu - e$ scattering process listed for the cross-section formula in Eqn. 5.7. The numerical values in the final column correspond to the combination of couplings that appears in the tree-level total cross-section for $\sin^2 \theta_W = 0.23$.

Figure 5.2: Integral cross-section for neutrino-electron scattering processes above $y > y_{\text{cut}}$, and assuming $E_\nu = 30$ GeV.

The differential cross-section for inverse muon decay, reaction 5.2, is

$$\frac{d\sigma}{dy} = \frac{G_F^2(s - m_\mu^2)}{4\pi}, \quad (5.9)$$

and the differential cross-section for $\bar{\nu}_e e^- \rightarrow \bar{\nu}_\mu \mu^-$ is

$$\frac{d\sigma}{dy} = \frac{G_F^2(s - m_\mu^2)}{4\pi} \left[\frac{s}{s - m_\mu^2} y(1 - y) - \frac{m_\mu^2}{s} \right]. \quad (5.10)$$

Experimentally, the measured quantity is the number of observed events consisting of a forward-going electron track with no hadronic activity and with an energy above the threshold, E_{cut} ,

$$N_{\nu-e} = \int \theta(yE_\nu - E_{\text{cut}}) \sigma \Phi(E_\nu) dE_\nu. \quad (5.11)$$

Figure 5.3: Change in the integral cross-section with respect to $\sin^2 \theta_W$ for neutrino-electron scattering processes above $y > y_{\text{cut}}$, assuming $E_\nu = 30$ GeV.

Figure 5.2 shows the integral cross-section for the reactions above $y > y_{\text{cut}}$, and figure 5.3 shows the change in the cross-section above $y > y_{\text{cut}}$ as a function of $\sin^2 \theta_W$. Statistical sensitivity to $\sin^2 \theta_W$ in any given channel is proportional to $\sqrt{\sigma}/(d\sigma/d\sin^2 \theta_W)$, which is shown in figure 5.4. In a neutrino beam produced by muon decays, the observed rate of visible electrons will include scattering of both neutrinos and anti-neutrinos. Note that an undesirable feature of the μ^- beam for measuring $\sin^2 \theta_W$ is that the dependences of the integral cross-sections on $\sin^2 \theta_W$ for ν_μ and $\bar{\nu}_e$ have opposite signs. The resulting sensitivity in integral cross-sections for beams from $\mu^\pm$ decay, in the adequate approximation that the muon and electron type neutrino fluxes are assumed equal, is shown in figure 5.5. The exact sensitivity for this sort of summed measurement would depend on beam polarization, particularly for the μ^- beam.

5.2.1 Current Measurements

The best measurement of neutrino-electron scattering to date was performed in the CHARM II experiment in the CERN Sp $\bar{p}$ S neutrino beam. The beam was predominantly ν_μ and $\bar{\nu}_\mu$ with an event sample of 2000 events in both the ν_μ and $\bar{\nu}_\mu$ beams. This led to a measurement of the weak mixing angle of [88]

$$\sin^2 \theta_W = 0.2324 \pm 0.0058(\text{stat}) \pm 0.0059(\text{syst}). \quad (5.12)$$

Not surprisingly, the systematic errors primarily result from normalization and background uncertainties.

Figure 5.4: Statistical uncertainty in $\sin^2 \theta_W$ for an integrated luminosity of 10^{46}cm^{-2} by process above $y > y_{\text{cut}}$, assuming $E_\nu = 30 \text{ GeV}$. The statistical uncertainty in a given experiment is proportional to the inverse square root of the average energy.

Figure 5.5: Statistical uncertainty in $\sin^2 \theta_W$ for an integrated luminosity of 10^{46}cm^{-2} by process above $y > y_{\text{cut}}$ assuming beams with equal neutrino and anti-neutrino fluxes and $E_\nu = 30 \text{ GeV}$.

5.2.2 Experimental Concerns

The signature for $\nu - e$ scattering is a single negatively charged lepton with very low p_t , $p_t \lesssim \sqrt{m_e E_\nu}$. In the case of a beam generated by μ^- decay, muons as well as electrons will be produced by inverse muon decay and $\bar{\nu}_e - e^-$ annihilation processes. This beam may also be preferred because the background from quasi-elastic electron neutrino scattering produces positrons rather than electrons.

The best types of detectors for such a measurement are likely to be target/detectors with inherent tracking capabilities and very little dead material. Dead material will not only worsen resolution due to multiple scattering, but will enhance the possibility of confusion of $\gamma \rightarrow e^- e^+$ with single electron production. With this type of detector, rare low p_t backgrounds, such as coherent single π^0 production which were a significant problem in the high-mass CHARM II neutrino detector[88], should not be difficult to identify. To minimize possible e/γ confusion, it is *crucial* to establish the presence of an electron track before the electron can begin to shower in the detector. This requirement may be at odds with the desire to identify the charge of the lepton in order to reduce backgrounds from $\bar{\nu}_e$ charged current interactions; however, the charge measurement also provides a cross-check of sign-symmetric detector backgrounds, such as $\gamma \rightarrow e^+ e^-$.

To remove quasi-elastic $\nu_e N$ scattering backgrounds, which cannot be separated by electron charge if the beam is produced by μ^+ decay, it will be important to resolve the different p_t distributions. ν_e quasi-elastic scattering has a characteristic p_t scale of $\sqrt{m_N E_\nu}$. In this case, signal and background suppression can be achieved by fitting the observed single electron p_t distribution, which therefore must be measured with resolution $\ll \sqrt{m_N E_\nu}$.

Normalization of the cross-section is also a significant issue since this probe of weak couplings is only as good as the normalization of the beam flux. The best candidate process for normalization is single muon production in neutrino-electron scattering. In the μ^- produced beam where these processes occur, there is a significant background from quasi-elastic ν_μ interactions, which can only be separated by a precise measurement of the outgoing lepton p_t . At E_ν of 30 GeV, the quasi-elastic cross-section is approximately 5 times greater (on an isoscalar target) than the inverse muon decay cross-section; therefore at this energy, a p_t cut at 100 MeV would leave a well-measured background of about 10% under the inverse muon decay peak. Detectors discussed in the following subsection make this goal seem achievable in a detector of modest mass. It should be noted that uncertainty in beam polarization could make this measure of ν_μ flux difficult to relate to $\bar{\nu}_e$ flux.

Normalization to the muon beam flux itself is another possibility. Theoretical predictions of the decay process would likely not limit this normalization technique, and it is likely that a precise measurement of the number of muons in the ring could be made. However, uncertainties in beam dynamics, such as spot sizes, orbits, divergences and polarization if not averaged to zero will likely limit the precision of a beam-based normalization.

Figure 5.6: Monte-carlo generated picture of a high energy neutrino-electron scattering interaction in liquid helium. (Note the distorted scale that is indicated by the relative lengths of the x and z axes.) The view is perpendicular to both the beam direction and a 0.1 Tesla magnetic field.

5.2.3 Detector considerations for neutrino-electron elastic scattering

The signal of low p_t single electrons strongly suggests the use of a fully active tracking detector in a very low Z material to maximize the radiation length. A natural choice is a time projection chamber (TPC) filled with a low-Z liquid. Unfortunately, liquid hydrogen is ruled out because of insufficient electron mobility.

Liquid helium also suffers from poor mobility and potential difficult operation because it lacks the ability to self-quench; however, it deserves further consideration because it has a radiation length of 7.55 m, allowing very well resolved events (Figure 5.6). Liquid methane appears to be a good candidate for the TPC medium. Its favorable electron transport properties have led to it being suggested for TPC detectors of up to several kilotons [89]. It is liquid at atmospheric pressure between -182.5 and -161.5 degrees centigrade and has a density of 0.717 g/cm^3 and a radiation length of 65 cm. Heavier alkanes that are liquid at room temperature, such as octane, would be superior for safety and convenience if their electron transport properties are good.

Even if most DIS events are trivial to distinguish from the signal events, the detector must still cope with the high interaction rate from the dominant background of DIS neutrino-nucleon interactions, which has a cross-section three orders of magnitude larger than the signal processes. This problem is made even worse because the interactions from up to hundreds of turns (depending on experimental details) may pile up in the TPC read-out. This suggests the need for additional fast read-out of the events, which could come from, e.g., planes of scintillating fibers within the TPC volume. (The feasibility of including the fibers within the electric field must be established.)

5.2.4 Estimates of Sensitivity

For a μ^- beam, the cross-section calculations above show estimated sensitivities of approximately $\delta \sin^2 \theta_W \sim 0.0007$ for an integrated luminosity of 10^{46} cm^{-2} and $\langle E_\nu \rangle \equiv 30 \text{ GeV}$, which corresponds to approximately 1.5×10^9 DIS charged current events (c.f., Table 1.3). The μ^+ beam statistical sensitivity is about a factor of three better, with the caveat that beam flux monitoring is required for normalization. With this luminosity, expected event samples would be approximately 1.5×10^6 for a μ^- beam and 3×10^6 for a μ^+ beam. Normalization and background uncertainties must therefore be kept at the $\text{few} \times 10^{-4}$ level in order to achieve this precision.

This result can also be interpreted in terms of a sensitivity to a high mass contact interaction of the form

$$\mathcal{L} = \sum_{H \in \{L, R\}} \frac{\pm 4\pi}{\Lambda_{H_e H_\nu}^\pm} (\Lambda_{H_e H_\nu}^\pm) \bar{e}_{H_e} \gamma^\mu e_{H_e} \bar{\nu}_{H_\nu} \gamma_\mu \nu_{H_\nu},$$

where the H indices represent helicity states of the electron and neutrino. The statistics given above would probe contact interactions at mass scales $\Lambda \sim 10 \text{ TeV}$, again assuming statistical uncertainties dominate.

5.3 $\sin^2 \theta_W$ from DIS

Neutrino-nucleon deep inelastic scattering experiments provide an excellent testing ground for the Standard Model because of the high statistics possible even with contemporary neutrino beams. Nucleons, however, make for a most unappealing and difficult target, and it is necessary to consider ratios of observable processes in order to make sense of the results.

5.3.1 Previous Measurements

The CCFR ν experiment extracted $\sin^2 \theta_W$ [90, 91] through a measurement of the ratio of the cross sections for NC and CC interactions, as expressed in the following Llewellyn-Smith formula [92] :

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \rho^2 \left(\frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W \left(1 + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\bar{\nu})}} \right) \right), \quad (5.13)$$

where the value of ρ depends on the nature of the Higgs sector and has the value $\rho = 1$ in the SM. This method, although it removed much of the uncertainty due to QCD effects in the target, does leave some rather large uncertainties associated with heavy quark production from the quark sea of the nucleon target.

The successor experiment, NuTeV (FNAL-E815) experiment, has improved upon this measurement by using separate neutrino and anti-neutrino beams.

Separation of neutrino and anti-neutrino neutral current events allows the utilization of the Paschos-Wolfenstein relationship [93]:

$$R^- = \frac{\sigma_{NC}^\nu - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^\nu - \sigma_{CC}^{\bar{\nu}}} = \frac{R^\nu - \tau R^{\bar{\nu}}}{1 - \tau} = \rho^2 \left(\frac{1}{2} - \sin^2 \theta_W \right), \quad (5.14)$$

where

$$\tau = \frac{\sigma(\bar{\nu}, CC)}{\sigma(\nu, CC)} \simeq 0.5.$$

R^- is, by construction, sensitive only to scattering from valence quarks in the proton, and reduces the theoretical uncertainties associated with the target considerably. NuTeV has presented a preliminary result[94] of

$$\sin^2 \theta_W = 0.2253 \pm 0.0019(\text{stat}) \pm 0.0010(\text{syst}). \quad (5.15)$$

Table 5.2 compares various uncertainties on the $\sin^2 \theta_W$ measurements from NuTeV and the expected ν MC uncertainties. (The detailed discussion of these ν MC uncertainties will be discussed in the following few sections.) There were two dominant systematic uncertainties in the CCFR experiment: 1) ν_e flux and 2) CC charm production. These two major systematic uncertainties in CCFR have been reduced in the NuTeV experiment, by utilizing sign selected neutrino beam, so that the remaining dominant uncertainty is the statistical uncertainty. Note that these past neutrino fixed target experiments used dense material as neutrino targets in order to increase the interaction rate. The calorimetric nature of the targets did not allow one to distinguish electron neutrino induced charged current interactions (CC) from neutral current (NC) interactions. Thus, these old-style heavy target experimental set-ups will be fatal for ν MC analyses because there always will be both ν_μ ($\bar{\nu}_\mu$) and $\bar{\nu}_e$ (ν_e) in the beam. Further discussion will assume the parameters of the high performance short baseline (SB) detector of appendix B.

5.3.2 Experimental Technique

In this section, we discuss the measured quantity used to reconstruct $\sin^2 \theta_W$, which is defined to be the ratio of the number of NC to CC events, the detector and beam requirements, and other experimental systematic and background issues that have to be overcome for precision measurements of $\sin^2 \theta_W$. The discussion assumes the conceptual detector discussed in appendix B and the ν MC beam configuration discussed in section 1.2.

The Ratio of NC to CC Events Used to Extract $\sin^2 \theta_W$

As in previous neutrino experiments, the measured quantity used to determine $\sin^2 \theta_W$ at ν MCs will be the ratio of NC to CC DIS events. However, the NuTeV-style ratio, R^- , is not accessible at ν MCs because $\nu_e(\bar{\nu}_e)$ and $\bar{\nu}_\mu(\nu_\mu)$

Table 5.2: Expected $\sin^2\theta_W$ uncertainties in ν MC beam, compared to NuTeV preliminary result.

SOURCE OF UNCERTAINTY	NuTeV	ν MC
Data Statistics	0.0019	$\sim 1.9 \times 10^{-5}$
ν_e flux	0.00045	Irrelevant
Transverse Vertex	0.0004	negl.
Energy Measurement		
Energy Scale	0.00049	negl.
Muon Energy Loss in Shower	0.00012	negl.
Energy Resolution	0.00006	negl.
Primary lepton ID	N.A.	negl.?
Event Length		Irrelevant
Hadron Shower Length	0.00022	
Counter Fiducial Size	0.00024	
Counter Efficiency & Noise	0.00011	
Vertex Determination	0.00012	
TOTAL EXP. SYST.	0.00078	negl.?
Charm Production, $\bar{s}$ ($m_c = 1.31 \pm 0.24$ GeV)	negl.	< 0.0003
Higher Twist	0.00011	< 0.00011
Longitudinal Cross-Section	0.00004	negl.
Charm Sea, ($\pm 100\%$)	0.00002	negl.
Non-Isoscalar Target	0.00017	0 (D_2 Target) 9×10^{-5} (CCD + Al Circuit)
Structure Functions	0.0001	negl.
Rad. Corrections	0.00051	< 0.0005
$\sigma^{\bar{\nu}}/\sigma^{\nu}$	0.00021	negl.
TOTAL PHYSICS MODEL	~ 0.00069	< 0.0005
TOTAL UNCERTAINTY	0.0022	< 0.0005
ΔM_W	$0.11 \text{ GeV}/c^2$	$< 0.025 \text{ GeV}/c^2$

neutrinos come in pairs and so NC events from neutrinos and those from anti-neutrinos are not distinguishable on an event-by-event basis. Instead, the two accessible ratios are essentially linear combinations of the Llewellyn-Smith ratios for neutrinos and antineutrinos given in equation 5.13:

$$R_{\nu}^{\mu-} = \frac{\sigma(\nu_{\mu}, NC) + \sigma(\bar{\nu}_e, NC)}{\sigma(\nu_{\mu}, CC) + \sigma(\bar{\nu}_e, CC)} = \frac{R^{\nu} + grR^{\bar{\nu}}}{1 + gr} \quad (5.16)$$

for the $(\nu_{\mu}\bar{\nu}_e)$ beam, and

$$R_{\bar{\nu}}^{\mu+} = \frac{\sigma(\bar{\nu}_{\mu}, NC) + \sigma(\nu_e, NC)}{\sigma(\bar{\nu}_{\mu}, CC) + \sigma(\nu_e, CC)} = \frac{R_{\nu_{\mu}} + g^{-1}rR_{\bar{\nu}_{\mu}}}{1 + g^{-1}r} \quad (5.17)$$

for the $(\bar{\nu}_{\mu}\nu_e)$ beam. The expression of $R_{\nu}^{\mu-}$ and $R_{\bar{\nu}}^{\mu+}$ in terms of r , R^{ν} and $R^{\bar{\nu}}$ assumes lepton universality, and the second of the two equations differs from the first only in the replacement of g by g^{-1} , where g is the energy-weighted flux ratio between ν_{μ} and $\bar{\nu}_e$ or, equivalently, between $\bar{\nu}_{\mu}$ and ν_e :

$$g \equiv \frac{\int \Phi(E_{\bar{\nu}_e})E_{\bar{\nu}_e}dE_{\bar{\nu}_e}}{\int \Phi(E_{\nu_{\mu}})E_{\nu_{\mu}}dE_{\nu_{\mu}}} = \frac{\int \Phi(E_{\nu_e})E_{\nu_e}dE_{\nu_e}}{\int \Phi(E_{\bar{\nu}_{\mu}})E_{\bar{\nu}_{\mu}}dE_{\bar{\nu}_{\mu}}}. \quad (5.18)$$

The equivalence between the expressions for the $(\nu_{\mu}\bar{\nu}_e)$ and $(\bar{\nu}_{\mu}\nu_e)$ beams implicitly assumes no polarization. (More generally, the equivalence is still valid for equal polarizations of the parent μ^{-} and μ^{+} beams.) Appendix A predicts the numerical value of g for an unpolarized beam to be close to unity:

$$g = \frac{\langle x \rangle^e}{\langle x \rangle^{\mu}} = \frac{6}{7}.$$

Therefore, $R_{\nu}^{\mu-}$ and $R_{\bar{\nu}}^{\mu+}$ are nearly identical linear combinations of R^{ν} and $R^{\bar{\nu}}$:

$$R_{\nu}^{\mu-} \simeq 0.70R^{\nu} + 0.30R^{\bar{\nu}}$$

and

$$R_{\bar{\nu}}^{\mu+} \simeq 0.63R^{\nu} + 0.37R^{\bar{\nu}}.$$

The numerical values of R^{ν} and $R^{\bar{\nu}}$ are anyway rather similar. Equation 5.13 predicts that $R^{\nu} = 0.317$ and $R^{\bar{\nu}} = 0.359$ for $\sin^2 \theta_W = 0.225$ so the numerical values of the two measurements will be nearly identical:

$$R_{\nu}^{\mu-} \simeq 0.330$$

and

$$R_{\bar{\nu}}^{\mu+} \simeq 0.332.$$

Similarly, the dependences of R^{ν} and $R^{\bar{\nu}}$ on $\sin^2 \theta_W$ are prescribed by equation 5.13 as:

$$\frac{dR_{\nu}}{d\sin^2 \theta_W} = -1 + \frac{10}{9}\sin^2 \theta_W(1+r) \simeq -0.625 \quad (5.19)$$

and

$$\frac{dR_{\bar{\nu}}}{d\sin^2\theta_W} = -1 + \frac{10}{9}\sin^2\theta_W \frac{(1+r)}{r} \simeq -0.250, \quad (5.20)$$

giving the corresponding sensitivities:

$$\frac{dR_{\nu}^{\mu-}}{d\sin^2\theta_W} = -0.513$$

and

$$\frac{dR_{\bar{\nu}}^{\mu+}}{d\sin^2\theta_W} = -0.487.$$

(Obviously, the sensitivities of both $R_{\nu}^{\mu-}$ and $R_{\bar{\nu}}^{\mu+}$ to $\sin^2\theta_W$ are intermediate between the sensitivities of R^{ν} and $R^{\bar{\nu}}$.)

The numerical similarities between the complementary variables $R_{\nu}^{\mu-}$ and $R_{\bar{\nu}}^{\mu+}$, from $(\nu_{\mu}\bar{\nu}_e)$ and $(\bar{\nu}_{\mu}\nu_e)$ beams respectively, mean that the two measurements can be regarded as physically nearly identical but with slightly different experimental systematics due to the approximate interchange of electron and muon energy spectra in the CC final states. (Because of the different kinematics for neutrino vs. antineutrino interactions, the $(\nu_{\mu}\bar{\nu}_e)$ beam will have more soft muons than electrons and vice versa for the $(\bar{\nu}_{\mu}\nu_e)$ beam.) The comparison of two physically similar measurements with different experimental challenges will be a valuable cross-check on the analyses.

Detector and Beam Requirements

Probably the most demanding requirement on the detector for this analysis is the ability to distinguish events with primary electrons or muons from neutrino CC interactions from the purely hadronic showers produced in NC interactions. (Traditional heavy target neutrino detectors could not distinguish ν_e induced CC interactions from NC interactions.)

Other particularly relevant capabilities are the use of a light isoscalar target that can provide the charm and strange sea measurements from the same experiment, as this will be very useful in reducing the remaining systematic uncertainties.

In order to satisfy the above requirements, the detector needs to have:

- Good electromagnetic and hadronic shower identification.
- High electron detection efficiency along with high efficiency particle identification.
- Good charged particle momentum measurement.
- Good electromagnetic and hadronic shower energy containment and measurement.

- Muon identification and momentum measurement.

More detailed discussion of the detector and target options can be found in appendix B.

Regarding the beam, a crucial requirement is the ability to reverse the polarity of the ring so that one can choose between $(\nu_\mu \bar{\nu}_e)$ or $(\bar{\nu}_\mu \nu_e)$ beams at any given time. This capability provides the opportunity for studying possible systematics due to the beam of a given flavor. That this should be achievable is explained in A.1.1 of appendix A, which also discusses other beam-related issues such as background events from the beam.

Experimental Uncertainties

The high-performance detectors for ν MC experiments will greatly suppress the experimental uncertainties in the $\sin^2 \theta_W$ measurement relative to today's analyses conducted using coarse-granularity calorimetric neutrino target.

Probably the most important remaining experimental systematic in this measurement will be the e/μ identification efficiency. Since the expected and meaningful total error in this measurement is on the order of 10^{-3} , it is extremely vital to control the overall efficiency of the particle identification system to much better than 10^{-3} . Otherwise every ν_e CC event where the electron escapes identification would be a direct background to NC interactions for both ν_μ and ν_e . Thus the unambiguous identification of the primary leptons for neutrino interactions is of the utmost importance in measurements of $\sin^2 \theta_W$. The positive identification of the primary lepton and backgrounds are generally both more difficult for low energy leptons, so $\sin^2 \theta_W$ measurement would benefit by using cuts on the minimum lepton energy. However, it is desirable for theoretical reasons to reduce the cut value to as low an energy as possible. Section B.3 shows that the particle identification for both electrons and muons will be outstanding and further demonstrates that one should also be able to control backgrounds from secondary electrons or background muons from pion decay that fake primary leptons. Despite this, identification of the primary lepton may well be the largest experimental uncertainty in a measurement dominated by theoretical uncertainties.

5.3.3 Theoretical Issues

Theoretical uncertainties will be expected to dominate the $\sin^2 \theta_W$ measurement at ν MCs, and an improved understanding of several potential theoretical uncertainties will be required for a meaningful interpretation of a sub-25 MeV equivalent W mass error.

Charm production uncertainties

The CC-charm cross section error needs to be minimized. In principle, this error can be reduced by measuring the CC production cross section of charm directly from the same experiment using vertex-tagged charm events.

To set the scale for the systematic uncertainty coming from the charm mass affecting the CC cross section, one can assume linear scaling of the uncertainties from those in the CCFR experiment. The Next-to-Leading-Order (NLO) measurement of the charm mass from CCFR [95] obtained:

$$m_c = 1.71 \pm 0.19 \pm 0.02 \text{ GeV}/c^2.$$

This resulted in a contributing uncertainty for that experiment of

$$\delta \sin^2 \theta_W = 0.003.$$

Since the statistics at a ν MC might be 3 or more orders of magnitude larger than in the CCFR experiment, a statistical improvement in the uncertainty by a factor of 10 could be considered very conservative. Therefore, we might expect $\delta \sin^2 \theta_W$ to be significantly less than 0.0003, at least to the extent that it is fair to scale the $\sin^2 \theta_W$ error linearly with the charm event statistics.

Electroweak Radiative Corrections

At ± 25 MeV, this is the largest current theoretical error in the NuTeV measurement. The current state of corrections suffers from the fact that only one attempt at a complete calculation exists, by Bardin and Dokuchaeva[96] (B-D), which, curiously, is unpublished. This calculation includes electroweak effects to one loop, is lengthy and complicated, and it would benefit from independent confirmation. Further, it has several theoretical shortcomings, as follows.

The B-D result depends explicitly on quark masses, which introduces a spurious model dependence. Similar calculations of EW radiative corrections in W mass production in $\bar{p}p$ annihilation show that quark mass effects can be absorbed into parton distribution and fragmentation functions[97]. The calculation is only approximately valid to leading log in QCD. In particular, scaling violation effects in important diagrams involving muon bremsstrahlung are neglected. It also neglects contributions from longitudinal partons and effects from the target mass. Further, it does not incorporate heavy quark effects in CC and NC charm production and, finally, is not differential in the final state radiated photon momentum vector.

A new EW radiative correction program should be pursued with the aim of ± 1 MeV residual error on the effective W mass. It may be necessary to re-sum large lepton logs, to include order $\alpha\alpha_S$ contributions, and to apply EM radiative correction to input pdf sets to realize this goal. The new program should provide capability for explicit generation of $\gamma\mu q$ final states.

Other Corrections

Further effects which require investigation include:

- Nucleon isospin-violating effects[98]. These arise from electromagnetic and quark-mass contributions which break the generally assumed relations $u_p(x, Q^2) = d_n(x, Q^2)$ and $u_n(x, Q^2) = d_p(x, Q^2)$. They will be present even in a deuterium or other pure isoscalar targets.

- Strange quark mass effects in $\nu_\ell \bar{u} \rightarrow \ell^- \bar{s}$ and $\nu_\ell s \rightarrow \nu_\ell s$. Because lattice gauge calculations predict that m_s has the awkward value $m_s \simeq 300 \text{ MeV} \simeq \Lambda_{QCD}$, its effects are difficult to reliably establish.
- Effects from “radiative higher twist processes” such as $\nu_\ell n \rightarrow \gamma \ell^- p$. The radiative photon present in CC events generates a CC/NC asymmetry and can boost the apparent Q^2 of events to high enough values to evade cuts designed to suppress higher twist.
- other higher twist effects, which are assessed as one of the dominant errors in today’s measurements. However, a recent study [99] indicates that most the higher twist effects might be higher order QCD corrections in GLS measurements.

5.3.4 Improvement in $\sin^2 \theta_W$ Measurement

Using the short baseline ν MC beam parameters discussed in subsection 1.2.2 for a mean muon energy of 175 GeV, one expects approximately $4.5 \times 10^8 \nu_\mu$ events per year per 1 m long D_2 target with a radius of 1 m. Since 90% of the beam is contained within 60 cm radius for this energy [100] ~ 500 m away from the end of the straight section of ν MC, the 1 m radius D_2 target is sufficient to contain the entire beam.

With a detector that can distinguish electrons and muons resulting from CC interactions, the effective neutrino interaction statistics doubles, resulting in a total of 9×10^8 events per year. Since the number of neutrino events from the NuTeV experiment (with 3×10^{18} protons-on-target) was on the order of 1 million events for ν_μ , the expected 9×10^8 events per year from the ν MC would cause a reduction in the statistical uncertainty by approximately a factor of thirty. This enormous increase in statistical power also helps to minimize many of the systematic uncertainties in Table 5.2 dramatically.

The remaining error of 0.0004 due to ν_e flux no longer exists for a detector that is capable of distinguishing CC interactions of ν_e from hadronic showers resulting from NC interactions by identifying the outgoing electrons. The uncertainties resulting from energy scale are also irrelevant because they are associated with the muon range through the muon length cut, which will not be relevant for the ν MC analysis. The errors in event length, in principle, also no longer contribute, because with the detector described in the following section, one can distinguish CC from NC interactions on an event-by-event basis. The above expectations will enable the experiment to reduce the statistical and experimental systematic uncertainties on $\sin^2 \theta_W$ to the equivalent M_W uncertainty of ~ 20 MeV.

5.3.5 Outlook and Comparisons of $\sin^2 \theta_W$ Measurements

Despite the issues pointed out in section 5.3.2 and 5.3.3, some of which are very serious, with sufficient advancement in techniques, one can conceive of bright

prospects for the measurement. We have investigated the possibility of measuring $\sin^2 \theta_W$ in an ν MC experiment. With a suitable detector that uses a light isoscalar target along with excellent electron and muon identification, one can expect to collect $\sim 10^9$ events per year, due to the utilization of both ν_μ and ν_e induced events resulting from the ν MC beam. Using this high intensity neutrino beam, along with theoretical help in reducing higher twist effects, we expect the precision of $\sin^2 \theta_W$ to be less than about 30 MeV on the W mass in the on-shell scheme, and perhaps much less than this if the theoretical uncertainties due to radiative corrections can be controlled, as discussed above.

Since the statistical and experimental systematic uncertainties will reduce dramatically using an adequate detector, it becomes crucial to reduce the remaining theoretical uncertainties. Calculations to minimize the higher twist effects, radiative correction, and the uncertainty from the longitudinal structure function R_L need to be dealt with.

However, assuming that one controls these issues, Figure 5.7 shows the expected status of various W and top quark mass measurements in the year 2010. Although the level of the bands and the actual slope of the curves might change depending on other parameters in the measurements, this figure gives a good idea as to what the actual level of accuracy one can expect.

The most precise measurement of M_W in the year 2010 is expected from direct measurements at TeV33. The contour represents the 68% confidence level from the TeV33 expectations of $\delta M_W = 20 \sim 30$ MeV and $\delta M_t = 1 \sim 2$ GeV, with $\int \mathcal{L} dt = 10 fb^{-1}$ using the traditional M_T method [101]. The ν MC measurements of the W mass would be of similar precision after one year of running with the beam parameters provided to us. As can be seen in figure 5.7, since the errors from both the direct measurements and the ν MC are going to be extremely small, and the ν MC measurement provides the SM-based band in M_W - M_T plane, the measurements would be complementary to each other in testing the SM and pinning down the SM Higgs mass to better than $\sim 20\%$.

5.4 Summary and Conclusions

This work has demonstrated that precision test of the electroweak Standard Model can be expected to play an important part in the physics program of a neutrino experiment at a muon collider. Two possible test were discussed, neutrino-nucleon and neutrino-electron scattering. The former process will allow tests of unparalleled statistical accuracy, but will likely suffer from substantial QCD and experimental systematics. By contrast, neutrino-electron scattering may be a statistical challenge even at a muon collider, but offers the possibility of a very clean target if experimental systematics associated with normalization and backgrounds can be controlled by proper selection of a detector.

Figure 5.7: Expected uncertainties of TeV 33 and νMC M_W vs M_T in the year 2010.

Chapter 6

Rare and Exotic Processes

6.1 Introduction

Despite the impressive direct and indirect searches for new physics available at the higher center of mass energies at the Tevatron, LEP, HERA and LHC colliders, some searches are quite unique to neutrino experiments and could be improved quite significantly with the much higher event statistics and improved experimental conditions promised at ν MCs. This chapter presents several examples of such processes, emphasizing the complementarity of these studies to already existing programs.

6.2 Studies of Low Energy QCD

6.2.1 $\bar{\nu}_e e^-$ Annihilation

As was pointed out in the previous chapter, neutrino-electron interactions can be used to extract electroweak parameters such as $\sin^2 \theta_w$ from purely leptonic interactions

$$\begin{aligned}\frac{d\sigma(\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-)}{dy} &= \frac{G_F^2 s}{4\pi} \left[4\sin^2 \theta_W + (1 + 2\sin^2 \theta_W)^2 (1-y)^2 \right], \\ \frac{d\sigma(\bar{\nu}_e e^- \rightarrow \bar{\nu}_\mu \mu^-)}{dy} &= \frac{G_F^2 s}{\pi} (1-y)^2 \left(1 - \frac{m_\mu^2}{s} \right)^2 \theta(s - m_\mu^2), \\ \frac{d\sigma(\bar{\nu}_e e^- \rightarrow \bar{\nu}_\tau \tau^-)}{dy} &= \frac{G_F^2 s}{\pi} (1-y)^2 \left(1 - \frac{m_\tau^2}{s} \right)^2 \theta(s - m_\tau^2),\end{aligned}\tag{6.1}$$

final state	E_ν (GeV)
$\bar{\nu}_\mu \mu^-$	11
$\gamma \pi^-$	19
$\pi^0 \pi^-$	76
$(\pi \pi \pi)^-$	172
γK^-	240
$(K \pi)^-$	358
$(K \pi \pi)^-$	446

Table 6.1: Threshold neutrino energy for various hadronic final states.

where $s = 2m_e E_\nu$. The second reaction occurs in combination with the inverse muon decay process with cross section

$$\frac{d\sigma(\nu_\mu e^- \rightarrow \nu_e \mu^-)}{dy} = \frac{G_F^2 s}{\pi} \left(1 - \frac{m_\mu^2}{s}\right)^2.$$

Electron antineutrino-electron annihilation is expected to show considerable complexity once the center of mass energy exceeds the threshold for annihilation into hadronic final states. The situation is much like $e^+e^- \rightarrow \text{hadrons}$ with the ρ, ω, ϕ and other low-lying vector resonances replaced by the π^-, ρ^- , and a_1^- . The axial component of the weak current produces coupling to axial vector resonances and, at low energy, to the pion. These latter couplings are absent in electron-positron annihilation, and thus the weak annihilation of the electron offers a novel complement to electron-positron physics. Direct measurements of $\bar{\nu}_e e^-$ annihilation would complement detailed studies of hadronic tau decay which currently provide some of the most powerful tests of QCD at low momentum transfer.

These measurements bridge two asymptotic limits of QCD: the perturbative regime operative at high Q^2 and the chiral limit ($m_u = m_d = m_s = 0$) that is approached at low $\sqrt{s}$, and provide some important information about the QCD spectral functions used in reducing theoretical hadronic uncertainties in the studies of CP-violation in kaon decays, in particular in the interpretation of ϵ'/ϵ measurements [102] (for a recent analysis see, for example, [103]). Yet, since the $\tau^- \nu_\tau$ threshold occurs at $E_\nu = 3.1$ TeV, there is a long way to go in neutrino energy to completely overlap the $\sqrt{s}$ region probed by tau decay.

Table 6.2.1 summarizes neutrino energy thresholds for s -channel final states. These thresholds for all channels are quite high. Nevertheless, one can see that a 50 GeV muon storage ring could provide access to the lowest lying final state and explore the interesting region near $\sqrt{s} = m_\pi$ to provide a clean test of PCAC, and a 250 GeV muon beam would extend the reach up to the threshold for kaon production.

6.2.2 W^*/Z^* -photon Scattering

Just as intense neutrino beams open up the possibility of directly exploring weak annihilation of leptons in analogy to e^+e^- annihilation, so too does one acquire access to the analog of two-photon physics: the scattering of virtual W or Z beams from quasi-real photons in the Coulomb field of the nucleus. The general reaction is of the form

$$\nu_\ell A \rightarrow f_\ell F \bar{f} A,$$

where $F, \bar{f}$ are fermions, and $f_\ell = \nu_\ell$ or ℓ^- . For $F\bar{f} = \nu_\ell \ell^+$ both CC and NC diagrams contribute. Otherwise $f_\ell = \ell^-$ proceeds through $W\gamma$ scattering and $f_\ell = \nu_\ell$ through $Z\gamma$ scattering.

The creation of lepton pairs in the Coulomb field of the nucleus have already been explored experimentally, in particular, by CCFR collaboration [104]. These measurements are sensitive to $W - Z$ interference and provide a direct test of the standard model. The purely coherent part of the cross section for $\nu_{\ell_1} A \rightarrow \ell_1^- \nu_{\ell_2} \ell_2^+$ at asymptotically large energies is [105, 106]

$$\begin{aligned} \sigma_{\ell_1 \ell_2}(E \rightarrow \infty) &= \frac{4Z^2 \alpha^2 G_F^2}{9\pi^3} E K_{\ell_1 \ell_2}(E, A) (1 - \chi), \\ K_{\ell_1 \ell_2}(E, A) &= \frac{5\pi\beta(A)}{32} \left(\log \frac{2E\beta(A)}{\rho_{\ell_1 \ell_2}} + \frac{1}{3} \log \frac{2E\beta(A)}{\rho_{\ell_1 \ell_2}} - R_{\ell_1 \ell_2} \right) \end{aligned} \quad (6.2)$$

$K_{\ell_1 \ell_2}(E, A)$ is a reaction and nucleus-dependent form factor with $\beta(A) \simeq A^{-1/3} \text{ fm}^{-1} = 6A^{-1/3} \text{ GeV}$ set by the nuclear size, and $\rho_{\ell_1 \ell_2}, R_{\ell_1 \ell_2}$ simple functions of either $\beta(A)$ or final state fermion masses. The factor $Z^2 \alpha^2$ reflects the coherent electromagnetic nature of the process; and the scale of the cross section is set by $G_F^2 \beta_I E$. Since the effective center-of-mass energy is $\sqrt{s} \simeq \sqrt{2\beta_I E}$ and $\beta_I \gg m_e$, all possible leptonic final states are accessible (including those with $\tau^\pm$) to a 250 GeV muon beam derived neutrino beam.

The factor χ incorporates effects of neutral currents, including interference in the $\ell^+ \ell^-$ final states. A nice electroweak test is to measure χ through the ratios

$$\frac{\sigma_{\mu\mu}(\nu_\mu A \rightarrow \mu^- \mu^+ \nu_\mu A)}{\sigma_{\mu e}(\nu_\mu A \rightarrow \mu^- e^+ \nu_e A)} = (1 - \chi) \frac{K_{\mu\mu}(E, A)}{K_{\mu e}(E, A)};$$

where $K_{\mu\mu}(E, A)/K_{\mu e}(E, A)$ will depend only weakly on nuclear form factor and energy.

Hadronic resonances are also possible. We expect similar states as are produced in $\gamma\gamma$ collisions, namely $0^{-+}, 0^{++}, 2^{++}, \dots$ with $I = 0$ and $I = 1$. Some Cabibbo-allowed examples include

$$\begin{aligned} \nu_e A &\rightarrow e^- \pi^+ A \\ \nu_e A &\rightarrow \nu_e \pi^0 A, \nu_e \eta A, \nu_e \eta' A \\ \nu_e A &\rightarrow e^- a_1^+ A \\ \nu_e A &\rightarrow \nu_e f_0 A, \nu_e a_1^0 A \\ \nu_e A &\rightarrow e^- D_S^+ A. \end{aligned} \quad (6.3)$$

Unfortunately there is no unique signature for some of these hadronic resonances. Single pion production, for example, can occur through the diffractive process $\nu_e A \rightarrow e^- \pi^+ A$ mediated by a collision of the pion component of the virtual W boson with a pomeron from the nucleus or nucleon.

Some experimental issues include:

- Backgrounds from charged current charm production

$$\nu_\mu N \rightarrow \mu^- c X,$$

where the charm quark fragments into a charmed hadron which takes all of the events hadronic energy and then decays semi-leptonically into a final state where leptons carry all the energy.

- Backgrounds from coherent meson production mediated by W/Z -pomeron scattering.

6.3 New Physics Sensitivity

6.3.1 Flavor Changing Neutral Currents

Neutrino physics at the muon collider offers many opportunities, not only to continue testing current predictions of the standard model, but also to search for new phenomena in yet unexplored physical regions. Neutral current production of a *single* heavy quark

$$\begin{aligned} \nu_\mu N &\rightarrow \nu_\mu c X_{C=0}, & \nu_\mu N &\rightarrow \nu_\mu \bar{c} X_{C=0}, \\ \nu_\mu N &\rightarrow \nu_\mu b X_{B=0}, & \nu_\mu N &\rightarrow \nu_\mu \bar{b} X_{B=0} \end{aligned} \quad (6.4)$$

would signal the presence of flavor changing neutral current (FCNC) processes. These processes provide an important constraint on the New Physics processes as they occur in the Standard Model only at the one loop level. The FCNC vertices $\nu\nu sb$ of Eq. (6.4) will be extensively studied in exclusive and inclusive B -decays $B \rightarrow X_s \nu \bar{\nu}$ at the B-factories as well as at CERN [107]. The inclusive processes such as $B \rightarrow X_s \nu \bar{\nu}$ and $\nu_\mu N \rightarrow \nu_\mu b X_{B=0}$ have considerable advantage over the corresponding FCNC processes involving charged leptons simply because these processes are mostly dominated by the short-distance physics. This ensures robustness of the theoretical predictions. In the framework of the standard model of particle interactions single b quark production is enhanced by the high mass of intermediate top quark states by virtue of the GIM mechanism

$$\sigma(\nu N \rightarrow \nu b X_{B=0}) \propto G_F^4 X_0^2 \left(\frac{m_t^2}{M_W^2} \right) \left[|V_{td}|^2 D + |V_{ts}|^2 S \right] \eta(m_b^2/2ME),$$

where D and S represent the down and strange quark contributions from the nucleon, respectively, $X_0^2(x_t) \sim x_t \log x_t$ (with next-to-leading order QCD corrections available [108]), and $\eta(m_b^2/2ME)$ represents a kinematic threshold

suppression from the heavy b quark mass. D and S would likely make similar contributions. Single charm quark production amplitude is suppressed, apart from the CKM factors, by m_b^4/m_t^4 relative to single b production and is more sensitive to the long-distance QCD effects that are not calculable at the moment from first principles.

It is the fact that FCNC occur in the standard model at one loop level only that makes their studies quite a sensitive test of New Physics. These new physics effects can occur both at tree and one loop level. The former include new intermediate bosons with FCNC quark couplings, while later include a wide class of new physics models such as supersymmetry and technicolor. Some of these models are already constrained from other measurements (for a recent review see, for example, [108]). Clearly, ν MC would have tough time competing with high statistics B and K experiments studying these FCNC interactions.

There is probably only one clear advantage of ν MC over the B-factory experiments. The point is that B-factories really study the processes like $B \rightarrow X_s E$, where E is some missing energy. It means that any new physics process like $B \rightarrow X_s P_i$ where P_i is(are) some weakly-interacting particle(s) which escape(s) the detector and therefore, automatically identified with a pair of neutrinos unless complicated angular analysis is performed. Combining the B-factory measurements with the ν MC results should identify $\nu\nu sb$ -vertex contributions to $B \rightarrow X_s E$ decay and therefore tighten constraints on the potential new physics contributions.

Unfortunately, gains in sensitivity to this type of new physics increase slowly. For instance, in the models with new tree-level FCNC interactions, such as string-inspired models with neutral Z' bosons, the FCNC rate due to the Z' exchange is $\sim 1/M_{Z'}^4$, and so sensitivity to $M_{Z'}$ only improves at best as the fourth root of rate increases.

Some potential experimental issues include:

- Backgrounds from $y \rightarrow 1$ allowed CC production (leaving a very low energy muon or electron in the final state):

$$\nu_\mu N \rightarrow \mu^- c X, \quad (6.5)$$

$$\bar{\nu}_e N \rightarrow e^+ b X. \quad (6.6)$$

These can be reduced by requiring the total energy to be less than a certain fraction of the beam energy. Charged current b production may be accompanied by a $\bar{c}$, and this information can be used as well. One must have very good charge identification and vertexing, however, since an FCNC produced b will nearly always decay to c .

- Backgrounds from allowed NC processes

$$\nu_\mu N \rightarrow \nu_\mu c \bar{c} X, \quad (6.7)$$

$$\nu_\mu N \rightarrow \nu_\mu b \bar{b} X, \quad (6.8)$$

in which one of the pair of heavy quarks decays too quickly or decays into an unfavorable final state to be picked up by a vertex detector.

6.3.2 Generic Four-Fermion Operators

Neutrino-nucleon processes at low momentum transfer are sensitive to generic four-fermion contact terms produced by the high energy neutral current interactions. Other, flavor changing couplings, are well constrained by the processes like $\pi \rightarrow e\nu$. These four-fermion interactions can be generated in a variety of new physics scenarios. Examples include, but not limited to, supersymmetric theories with $\mathcal{R}$ -parity non-conservation, new vector bosons (like Z' which appears in many superstring-motivated models), models with TeV-scale gravity, or quark compositeness models. For instance, low energy remnant of a generic high energy electron-quark neutral current interaction can be represented by

$$\begin{aligned} \mathcal{L}_{NC} = \sum_q & \left[\eta_{LL}^{eq} (\bar{e}_L \gamma_\mu e_L) (\bar{q}_L \gamma^\mu q_L) + \eta_{RR}^{eq} (\bar{e}_R \gamma_\mu e_R) (\bar{q}_R \gamma^\mu q_R) \right. \\ & \left. + \eta_{LR}^{eq} (\bar{e}_L \gamma_\mu e_L) (\bar{q}_R \gamma^\mu q_R) + \eta_{RL}^{eq} (\bar{e}_R \gamma_\mu e_R) (\bar{q}_L \gamma^\mu q_L) \right]. \quad (6.9) \end{aligned}$$

A similar equation can be written for direct neutrino-quark interactions. One can use $SU(2)$ symmetry to relate ν and e couplings

$$\begin{aligned} \eta_{LL}^{\nu u} &= \eta_{LL}^{ed}, \\ \eta_{LL}^{\nu d} &= \eta_{LL}^{eu}, \\ \eta_{LR}^{\nu u} &= \eta_{LR}^{eu}, \\ \eta_{LR}^{\nu d} &= \eta_{LR}^{ed}, \end{aligned}$$

so that νN interactions can be used to constrain η 's of the Lagrangian of Eq. (6.9). A particular example of a high-energy model that leads to the low-energy Lagrangian of this type is provided by $\mathcal{R}$ SUSY

$$\begin{aligned} \mathcal{L}_{\mathcal{R}} = \lambda'_{ijk} & \left(\bar{e}_L^i \bar{d}_R^k u_L^j + \bar{u}_L^j \bar{d}_R^k e_L^i + \bar{d}_R^{k*} \bar{e}_L^i u_L^j \right. \\ & \left. - \bar{\nu}_L^i \bar{d}_R^k d_L^j - \bar{d}_L^j \bar{d}_R^k \nu_L^i - \bar{d}_R^{k*} \nu_L^i d_L^j \right) + \text{h.c.} \quad (6.10) \end{aligned}$$

At low values of transferred momenta one can integrate out heavy $\bar{e}_L^i$ and $\bar{d}_L^j$ to rewrite the Lagrangian in terms of local four-fermion interactions

$$\begin{aligned} \mathcal{L}_{ed} &= \frac{(\lambda'_{1j1})^2}{m_{\bar{u}_L^j}^2} \bar{e}_L d_R \bar{d}_R e_L + \frac{(\lambda'_{1j1})^2}{m_{\bar{d}_L^j}^2} \bar{\nu}_L d_R \bar{d}_R \nu_L \\ &= \left(-\frac{(\lambda'_{1j1})^2}{2m_{\bar{u}_L^j}^2} \bar{e}_L \gamma^\mu e_L - \frac{(\lambda'_{1j1})^2}{2m_{\bar{d}_L^j}^2} \bar{\nu}_L \gamma^\mu \nu_L \right) \bar{d}_R \gamma_\mu d_R. \quad (6.11) \end{aligned}$$

Assuming that the squarks of first two generations are degenerate and imposing $SU(2)$ symmetry constraints,

$$\eta_{LR}^{ed} = -\frac{(\lambda'_{1j1})^2}{2m_{\bar{u}_L^j}^2} = -\frac{(\lambda'_{1j1})^2}{2m_{\bar{d}_L^j}^2} = \eta_{LR}^{\nu d} \quad (6.12)$$

Indeed, the best constraint on this coupling, $\eta_{LR}^{ed} < 0.07_{-0.24}^{+0.24}$ comes from the analysis of neutrino nucleon scattering experiments [109]. The data from ν MC should complement new constraints on the new physics contributions which will be available from new Tevatron and DESY analyses.

6.3.3 Heavy Neutral Lepton Mixing

High intensity neutrino beams should provide an excellent opportunity to significantly improve already existing bounds on the new physics states that are conventionally studied in neutrino experiment. One such opportunity is the ability to search for the existence of neutral heavy leptons [110]. In several models [111, 112], neutral heavy leptons are considered heavy isosinglets that interact and decay by mixing with their lighter neutrino counterparts. The high intensity neutrino beam created by the muon collider provides an ideal setting to search for neutral heavy leptons with a mass below 100 MeV.

It is postulated that neutral heavy leptons (L_0) could be produced from muon decay when one of the neutrinos from the decay mixes with its heavy, isosinglet partner. The expression for the number of neutral heavy leptons produced in a muon beam is given below:

$$N_{L_0} = N_\nu \times Br\left(\frac{\mu \rightarrow L_0 \nu e}{\mu \rightarrow \nu \nu e}\right) \times \epsilon \times e^{-L/\gamma c\tau} \times Br\left(\frac{L_0 \rightarrow \text{detectable}}{L_0 \rightarrow \text{total}}\right) \times (1 - e^{-\delta l/\gamma c\tau}) \quad (6.13)$$

Here N_ν is the number of neutrinos produced from muon decay, $Br(\mu \rightarrow L_0 \nu e / \mu \rightarrow \mu \nu e)$ is the branching ratio of muons decaying into neutral heavy leptons versus ordinary muon decay, L is the distance from the beam-line to the detector, δl is the length of the detector, ϵ is the combined detector and geometric efficiency, τ is the L_0 lifetime, and $Br(L_0 \rightarrow \text{detectable} / L_0 \rightarrow \text{total})$ is the branching ratio for the neutral heavy lepton decaying via a detectable channel (mainly, $L_0 \rightarrow \nu e e$).

Note here that the muon has two possible ways of producing L_0 's:

$$\mu^- \rightarrow L_0 + \bar{\nu}_e + e^- \quad (6.14)$$

$$\mu^- \rightarrow \nu_\mu + \bar{L}_0 + e^- \quad (6.15)$$

The branching ratio for each of these reactions is given by:

$$Br(\mu \rightarrow L_0 \bar{\nu}_e e / \mu \rightarrow \nu_\mu \nu_e e) = |U_{\mu L}|^2 (1 - 8x_m^2 + 8x_m^6 - x_m^8 + 12x_m^4 \ln x_m^2) \quad (6.16)$$

$$Br(\mu \rightarrow \nu_\mu \bar{L}_0 e / \mu \rightarrow \nu_\mu \nu_e e) = |U_{eL}|^2 (1 - 8x_m^2 + 8x_m^6 - x_m^8 + 12x_m^4 \ln x_m^2) \quad (6.17)$$

Here x_m is defined as m_{L_0}/m_μ and $|U_{(\mu,e)L}|^2$ is defined as the coupling constant between the specific type of neutrino and the neutral heavy lepton.

Once produced, a neutral heavy lepton of such low mass will either decay via three neutrinos ($L_0 \rightarrow \nu\nu\nu$), or via the channels:

$$L_0 \rightarrow \nu_\mu + e^+ + e^- \quad (6.18)$$

$$L_0 \rightarrow \nu_e + e^+ + e^- \quad (6.19)$$

The first decay mode involves only the neutral current, whereas the second contains a mixture of both neutral and charged current interactions. The branching ratios for decay process (6.18) and (6.19) have been calculated and can be found in [113]. Note that the number of L_0 's detectable at the muon collider depends roughly on U^4 .

Using the above model, one can estimate the number of neutral heavy leptons produced at the muon collider that then decay to two electrons and a neutrino. In testing the sensitivity of L_0 production at the muon collider, we make a few underlying assumptions. We assume that the muon collider utilizes a pure, unpolarized muon beam (containing 10^{20} muons/year) with straight sections such that 25 percent of the muons will decay to neutrinos pointing towards the detector. We assume that the detector is 3 meters in diameter and 30 meters in length, with enough resolution to detect the e^+e^- vertex from the L_0 decay. We also assume that the background is negligible. These parameters are adopted from the current L_0 search that is in progress at NuTeV.

The sensitivity of the detector has been calculated for a number of different muon energies and detector distances. The plots below show limits for the coupling constants as a function of L_0 mass. One achieves the best limits from using relatively low energy muon beams. This is a significant improvement over previous neutral heavy lepton searches, where limits do not reach below 6.0×10^{-6} in the low mass region.

The muon collider may prove to be an ideal location to continue the search for neutral heavy leptons. The high intensity neutrino beam allows for a neutral heavy lepton search to be sensitive to coupling constants in the low mass region. In addition, such a neutral heavy lepton program could easily interface with an already existing neutrino detector which utilizes the muon beam. It is also clear, however, that a neutral heavy lepton search would receive the most benefit at lower muon energies, and thus would yield best results at the earlier stages of the muon collider program.

Figure 6.1: Plots for $|U_{\mu L}|^2$, $|U_{e L}|^2$, and $|U_{\mu L} U_{e L}|$ limits as a function of L_0 mass for one year of running. The solid, dashed, and dotted lines represent muon collider energies of 10, 20, and 250 GeV respectively. The plots on the left show limits for $L = 12 m$, and the plots on the right show limits for $L = 1 km$.

6.4 Conclusion

Rare processes which could be studied with high intensity neutrino beams at a muon collider could probe, both directly and indirectly, a significant region of energy starting from fractions of a GeV all the way up to the TeV scale. While some of the regions of this vast parameter space will be probed in other experiments, ν MCs could provide important complementary information (low energy QCD studies, FCNC, contact interactions), or even substantially improve current bounds on the parameters of new physics models (heavy neutral lepton searches).

Chapter 7

Heavy Flavor Physics

7.1 Introduction

ν MCs will be rather impressive charm factories, with between 2×10^8 and 2×10^9 well-reconstructed charm events in a total event sample of 10^{10} events – depending on the ν MC energy – as shown in figure 1.2 of chapter 1. Several species of charmed hadrons should be produced, with measured [73] relative production fractions for the more common charmed hadrons of:

$$D^0 : D^+ : D_s^+ : \Lambda_C^+ = 0.60 : 0.20 : 0.10 : 0.10. \quad (7.1)$$

Also, the Σ_C^{++} and Σ_C^+ are expected [114] to have comparable production cross sections to Λ_C^+ and the charmed-strange baryons Ξ_C^+ and Ξ_C^0 should be produced at levels down by a factor of a few and Ω_c even more.

The ratios of equation 7.1 are relatively independent of the neutrino energy for energies above 10 GeV and are for production from neutrinos; the corresponding antiparticles containing anticharm should be produced in similar ratios – though in different numbers and kinematical regimes – from antineutrinos. The large asymmetry between D^0 and D^+ production is due to the prevalence of D^* production with its preference to decay into D^0 .

As well as having good all-around event reconstruction, there are two distinct and important experimental advantages that any ν MCs will have over all other types of charm facilities:

1. reconstruction of the charm decay vertex that should be superior to that at any collider experiment, particularly for the reconstruction of the challenging 1-prong charm decays. Figure 7.1 [9] gives a schematic comparison between the charm vertexing capabilities of the detector of appendix B and the current best vertexing detector in a collider experiment.

2. a uniquely pure and efficient tag of whether the production flavor is charm or anti-charm, from the 100% correlated sign of the primary lepton from the interaction:

$$\begin{aligned}\nu q &\rightarrow l^- c \\ \bar{\nu} \bar{q} &\rightarrow l^+ \bar{c}.\end{aligned}\tag{7.2}$$

This chapter discusses some interesting areas for charm physics at ν MC's where these experimental capabilities should be important. (Note that the measurements of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ from inclusive charm production have already been covered in chapter 4.) The theoretical interest of each measurement will be discussed and brief summaries of the expected experimental techniques and sensitivities at ν MCs will be included. The expected relative strengths and weaknesses of ν MC's compared to other future charm facilities will sometimes also be touched on. However, detailed numerical predictions for measurement precisions await more extensive feasibility studies than have been performed for this write-up.

There are also possibilities for B physics with site-filling multi-TeV muon colliders, where $b\bar{b}$ production in neutral current interactions should be at the level of 10^{-3} of the total cross section. Associated production of $b\bar{c}$ and $c\bar{b}$ can also be studied, however the relevant production cross-sections are suppressed by approximately two orders of magnitude compared to the $b\bar{b}$ production cross section.

7.2 Theoretical Motivation for Charm Physics

It is clear that from the point of view of the standard model electroweak physics, charm decays represent a decidedly dull affair. First, the relevant CKM parameters are reasonably well known, for the smallness of $|V_{cb}|$ and $|V_{ub}|$ constrains V_{cs} and V_{cd} very tightly through three-family unitarity (see Chapter 4). Second, $D^0 - \bar{D}^0$ oscillations proceed slowly. Third, CP asymmetries are small due to the fact that both decaying and final state particles contain quarks of only first two generations. Finally, rare charm decay rates are tiny and, again, dominated by the long-distance effects.

These apparent vices can however be turned into virtues. Since the weak dynamics apparently hold no secrets, one can employ charm decays as a laboratory to study QCD in the interface of perturbative and nonperturbative dynamics. Also, precisely because the SM promises us no drama in charm decays, one can conduct searches for $D^0 - \bar{D}^0$ oscillations, CP violation and rare charm decays as probes for New Physics with almost no background from the SM.

A) vertexing at MURINE

B) best collider vertexing so far: (SLD detector, same scale)

Figure 7.1: Conceptual illustration of the vertex tagging superiority at ν MCs over that with collider experiment geometries. (Figure reproduced from reference [9].) Neutrino targets such as those discussed in appendix B would have a vertex plane of CCD pixel detectors spaced at intervals of approximately one millimeter. For comparison, the VXD3 vertexing detector at the SLD [81] experiment, which is universally regarded as the best existing vertex detector in a collider experiment, has its two innermost CCD tracking planes at 2.8 cm and 3.8 cm from the interaction point (IP). A cartoon of a 1-prong D^+ decay has been drawn to illustrate the advantages of closely spaced vertex detectors. For clarity of illustration, the kink deflection angle has been drawn much larger than would be typical. The 2 cm distance to decay for the D^+ charmed meson corresponds to the average boosted lifetime for a 120 GeV D^+ .

7.3 Probing Strong Interactions through Charm Decays

Improved measurements of charm decays are needed for phenomenological and theoretical reasons even in the *absence* of new physics:

- To improve the data base needed for analyzing B decays one needs more

precise measurements of the *absolute* branching ratios of charm hadrons.

- Measuring the leptonic decay rates $D_{(s)} \rightarrow l\nu$ are required for determining the meson decay constants. These decay constants give us quantitative insight into the dynamics of heavy-light bound state systems and can be used for tuning the lattice QCD methods and a more reliable evaluation of $B^0 - \bar{B}^0$ oscillations.
- More precise studies of *inclusive* semileptonic D , D_s , Λ_c , etc. decays would provide us with valuable novel insights into the inner workings of QCD and at the same time sharpen our tools for a quantitative treatment of B decays.

7.3.1 Absolute Charm Branching Ratios

As the discussion about the charm content in the final state of B decays illustrates, a significant bottle neck in the detailed analysis of beauty decays of the $b \rightarrow c$ type is currently caused by the uncertainties in the absolute branching ratios of charm hadrons, in particular of D_s , Λ_c and Ξ_c . ν MCs should be well suited to obtaining these branching ratios, as we now discuss. This information will be useful even if it is obtained only after the next generation of B experiments have accumulated their samples.

As what is typically the less difficult part of the measurements, the expected excellent RID and event reconstruction at ν MCs should give good capabilities for determining relative branching ratios for each hadron. The more difficult task of obtaining the production normalization factors to convert these to absolute branching ratios should then be achieved by fitting the experimental decay length distributions in a procedure that was studied for the planned COSMOS (E803) neutrino experiment at Fermilab.

The COSMOS technique [115] was to fit normalization factors to the several known decay exponentials – one for each charmed hadron species – in the observed neutral and charged distributions for the variable $x = d/p$, with d the charmed hadron distance to the decay vertex and p it's reconstructed momentum. It is helpful that the exponential decay constants in this variable are well separated for both the charged and neutral hadron distributions:

$$\begin{aligned} x(D^+) &= 170 \mu\text{m}/\text{GeV}/c \\ x(D_S^+) &= 71 \mu\text{m}/\text{GeV}/c \\ x(\Xi_C^+) &= 43 \mu\text{m}/\text{GeV}/c \\ x(\Lambda_C^+) &= 27 \mu\text{m}/\text{GeV}/c, \end{aligned} \tag{7.3}$$

and

$$\begin{aligned} x(D^0) &= 67 \mu\text{m}/\text{GeV}/c \\ x(\Xi_C^0) &= 12 \mu\text{m}/\text{GeV}/c \\ x(\Omega_C^0) &= 7 \mu\text{m}/\text{GeV}/c \end{aligned} \tag{7.4}$$

Additional helpful information is also available from the RID in the detector. In particular, the presence of a proton in the final state will reliably indicate the decay of a baryon rather than a meson.

To test the method for the COSMOS environment, exponential fits were performed [115] for simulated decay length distributions from approximately 14 000 reconstructed D^+ , D_s^+ and λ_C^+ charm decays. The fitted statistical uncertainties for the three species were 3.4%, 12% and 5.4%, respectively. These simulations show that statistical uncertainties would be negligible for such a fit at a ν MC, which would have several orders of magnitude more events. The uncertainties in the charm production rates would instead be dominated by uncertainties in modeling the level of vertexing inefficiencies. Hopefully, these could uncertainties could also be made small due to the favorable vertexing geometry shown in figure 7.1 and the considerable potential for using the data itself to estimate the inefficiencies.

Another area where ν MC can be expected to make significant or even unique contributions is the analysis of final states that contain *more* than one neutral hadron like

$$D^0 \rightarrow \pi^+\pi^-\pi^0\pi^0; D^+ \rightarrow \pi^+\pi^0\pi^0; D_s^+ \rightarrow \pi^+\pi^0\eta \text{ etc.}; \quad (7.5)$$

even all neutral final states like

$$D^0 \rightarrow 2\pi^0, 3\pi^0 \quad (7.6)$$

might become observable. Such channels are rather elusive for the usual e^+e^- annihilation and photoproduction experiments. Filling in these ‘white spots’ in the map of charm decays would close or at least narrow the gap between exclusive and inclusive decays and thus can provide us with important lessons on how quark-hadron duality is realized in subclasses of total decays. For example: a quark based description leads to the prediction that the (Cabibbo suppressed) inclusive rates driven by $c \rightarrow s\bar{u}u$ and $c \rightarrow d\bar{d}u$ should practically coincide since $m_d, m_s \ll m_c$. Yet exclusive channels like $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ do not at all follow this expectation! Duality suggests that a (near) equality will emerge for $\Gamma(D \rightarrow K\bar{K} + \pi's)$ vs. $\Gamma(D \rightarrow \pi's)$. Testing this expectation requires the measurement of final states with neutrals.

Experimental studies of multi-body decays with more than one neutral meson in the final state (in particular, Dalitz plot analyses) also allow us to have different handles on the studies of direct CP-violation in D-decays in and beyond the standard model [116] as well as the dynamics of hadronic resonances governing these transitions (see, e.g. E791 analysis [117]).

7.3.2 $D_s, D^+ \rightarrow \mu^+\nu, \tau^+\nu$

The primary goal behind measuring leptonic decays $D_s \rightarrow l^+\nu$ or its Cabibbo suppressed versions $D^+ \rightarrow l^+\nu$ with $l = \mu, \tau$ is driven by the desire to extract the decay constants f_{D_s} and f_D . These quantities are important probes of heavy meson wave functions. In addition, these decay constants have been extracted

from Monte Carlo simulations of QCD on the lattice with a predicted ~ 20 % accuracy for their absolute values and ~ 10 % for their ratio at present and better in the future. For proper evaluation one wants to have data of corresponding accuracy.

Currently, the branching ratios for $D_s \rightarrow l\nu$ transitions have been measured by the CLEO collaboration with large uncertainties, $Br(D_s \rightarrow \mu\nu) = 4.0^{+2.2}_{-2.0} \times 10^{-3}$ and $Br(D_s \rightarrow \tau\nu) = (7 \pm 4) \times 10^{-2}$. No measurement is currently available for other D mesons (there is an upper bound on $Br(D^+ \rightarrow \mu\nu) < 7.2 \times 10^{-2}$). This can be explained by the $\lambda = 0.2$ CKM suppression factor for the D^+ leptonic decays relative to those of the D_s .

Once the absolute value of f_D or f_{D_s} was known experimentally with ~ 10 % accuracy or better, one would feel more confident about extrapolating to the decay constants in the B system, f_B and f_{B_s} , which are crucial quantities for a quantitative understanding of $B^0 - \bar{B}^0$ oscillations and the extraction of V_{td} from them.

Observing and measuring these transitions has always represented a highly nontrivial experimental challenge (and much more so for $D^+ \rightarrow l^+\nu$), so the potentially exceptional performance for observing 1-prong D decays at ν MC's could allow them to make a significant contribution here even down the line. As a secondary goal one might even perform a detailed comparison of the rates for $D \rightarrow \mu\nu$ and $D \rightarrow \tau\nu$ as a probe for new physics in the form of a non-minimal Higgs sector; for charged Higgs exchanges would affect the latter much more than the former.

7.3.3 Inclusive Charm Hadron Decays

Heavy quark expansions (HQE) allow the treatment of inclusive heavy flavor decays, including their nonperturbative aspects [118, 119, 120, 121, 122]. In addition to total decay widths other central quantities are inclusive semileptonic branching ratios and decay spectra for the different meson and baryon species. These techniques are at the basis of some of the most reliable methods for extracting $|V_{cb}|$ and $|V_{ub}|$ in B decays. Obviously one wants to cross check these methods in a system where the CKM parameters are known, namely the charm system, by testing how precisely $|V_{cs}|$ and $|V_{cd}|$ can be extracted from semileptonic charm decays. In addition one can extract the size of the matrix elements of four-fermion operators that are of direct relevance in *beauty* decays and at the same time provide important calibration points for lattice simulations of QCD.

No data of sufficient detail are available. The B factories (CLEO, BaBar and Belle) will significantly improve the situation, but might not achieve the desired experimental accuracy. Furthermore it turns out that comparing neutrino with charged lepton spectra in semileptonic decays provides us with particularly probing insights.

One has to keep the following in mind. Since the expansion parameter is μ_{had}/m_c with $\mu_{had} \sim 0.7 - 1$ GeV, one has to allow for uncalculated higher order contributions to modify the results significantly in charm decays. To

have a handle on this complication, one has to be able to perform detailed comparisons of the lepton spectra separately in D^0 , D^+ , D_s and Λ_c decays, which should be possible at ν MC's.

FOCUS and SELEX data will presumably yield precise lifetimes for $\Xi_c^{0,+}$ baryons, but quite possibly not for the Ω_c . The latter is presumably the shortest lived hadron in the single charm sector with $\tau(\Omega_c) < 10^{-13}$ sec; due to its different spin structure its lifetime is affected by different matrix elements than for the other baryons. It is also quite unclear whether next generation experiments like LHC-B and BTeV can measure such a short lifetime with good accuracy. A ν MC thus could make a relevant measurement that would serve as an a posteriori calibration of some theoretical tools. Furthermore a whole new spectroscopy could be entered, namely that of baryons carrying two units of charm: $[ccq]$.

In principle, radiative inclusive (and exclusive) decays can also be studied. The predicted branching ratio for the short-distance contribution is tiny, $Br(c \rightarrow u\gamma) = (4.21 - 7.94) \times 10^{-12}$ [123], the two-loop QCD corrections can bring it up to 5×10^{-8} [124]. This could have made it a sensitive probe of new physics as these processes occur in the standard model only at one loop. Unfortunately, the problem is that the long-distance effects can actually completely dominate the this decay enhancing it up to $\sim 10^{-5}$. These effects cannot be estimated model-independently.

7.4 Searches for New Physics in Charm Decays

7.4.1 $D^0 - \bar{D}^0$ Oscillations

The phenomenon of meson-antimeson mixing has been studied both experimentally and theoretically for a long time as it provides an extremely sensitive test of the Standard Model (SM) as well as its various possible extensions. This is especially true for $D^0 - \bar{D}^0$ mixing, as already indicated.

To study such oscillations one has to tag separately the flavor of the produced and of the decaying meson. A unique method is available at ν MCs to tag the production sign of the charm quark, from the charge of the primary lepton from CC interactions. This should easily be the cleanest and most efficient tag but it can be checked by a more conventional method, which involves production of the charged D^* mesons and studies of the decay chain $D^{*\pm} \rightarrow D^0(\bar{D}^0)\pi^\pm$ [125, 126]. Anti-correlation studies of the charge of π and decay products of D would reveal whether mixing took place.

For the charmed mesons with tagged production flavor, $D^0 - \bar{D}^0$ oscillations are most cleanly probed through 'wrong-sign' semileptonic decays with the branching ratios:

$$\tau_D = \frac{\Gamma(D^0 \rightarrow l^- X)}{\Gamma(D^0 \rightarrow l^+ X)} \simeq \frac{1}{2} (x_D^2 + y_D^2) \quad , \quad x_D = \frac{\Delta m_D}{\Gamma_D} \quad , \quad y_D = \frac{\Delta \Gamma_D}{2\Gamma_D} \quad (7.7)$$

Alternatively one can tag the flavor of the final state through charged kaons;

mistags that happen due to doubly Cabibbo suppressed decays can in principle be eliminated, as discussed below.

The most recent experimental limits, which are from fixed target experiments at FNAL and from CLEO at CESR and combine tagging through ‘wrong’ sign leptons and kaons, read:

$$\tau_D \leq 5 \cdot 10^{-4}, \text{ 95\% C.L.; CLEO[127]} \quad (7.8)$$

$$-0.04 \leq y_D \leq 0.06, \text{ 90\% C.L.; E791[128]} \quad (7.9)$$

$$-0.058 \leq y'_D \leq 0.01, \text{ 95\% C.L.; CLEO[127]} \quad (7.10)$$

where

$$y'_D \equiv y_D \cos \delta_{K\pi} - x_D \sin \delta_{K\pi} \quad (7.11)$$

with $\delta_{K\pi}$ denoting the strong phase shift between $D^0 \rightarrow K^+\pi^-$ and $\bar{D}^0 \rightarrow K^+\pi^-$;

$$y_D = 0.0342 \pm 0.0139 \pm 0.0074 \text{ FOCUS[129]} \quad (7.12)$$

The B factories at Cornell, SLAC and KEK will refine the search for $D^0 - \bar{D}^0$ oscillations. It is expected to reach a sensitivity of $\tau_D \sim \text{few} \times 10^{-4}$ [130].

While the SM undoubtedly predicts slow $D^0 - \bar{D}^0$ oscillations – $x_D, y_D \ll 1$ – there is considerable uncertainty in the numerical predictions: a *conservative* Standard Model bound is given by [131]

$$\tau_D|_{SM} < 10^{-4} \hat{=} y_D, x_D|_{SM} \leq 10^{-2} \quad (7.13)$$

Bolder predictions have been made that x_D, y_D cannot exceed 10^{-3} [132, 131] and therefore $\tau_D \leq 10^{-6}$ with the SM. On the other hand New Physics in the form of various supersymmetric models [133] (including SUSY models with quark-squark alignment which actually requires Δm_D close to the current experimental bound) [134], models with singlet up quarks [135], various leptoquark models [136], and multiscalar models with [137] and without [138] natural flavor conservation could enhance x_D to and actually even above the present bound

$$x_D|_{NP} \sim 0.1, \quad (7.14)$$

without violating any other limit while leaving y_D unaffected. Any experimental effort in lowering the current limit on Δm_D is essential in determining the available parameter space for many possible extensions of the SM!

The cleanest way to probe for $D^0 - \bar{D}^0$ oscillations is to analyze the time evolution of transitions into ‘wrong-sign’ leptons:

$$\Gamma(D^0(t) \rightarrow l^+ X) \propto e^{-t/\tau_D} x_D^2 \left(\frac{t}{\tau_D} \right)^2. \quad (7.15)$$

Here we have invoked the $\Delta Q = -\Delta C$ rule of the SM which makes oscillations the only source for wrong-sign leptons. Yet since one is embarking on a search for New Physics, one should allow for a violation of this rule:

$$\Gamma(D^0(t) \rightarrow l^+ X) \propto e^{-t/\tau_D} \times$$

$$\left[\left(1 - y_D \frac{t}{\tau_D} \right) |\hat{\rho}_{wrong}|^2 + \left(\frac{x_D}{2} \right)^2 \left(\frac{t}{\tau_D} \right)^2 + y_D \frac{t}{\tau_D} \operatorname{Re} \frac{p}{q} \hat{\rho}_{wrong} \right] \quad (7.16)$$

where

$$\hat{\rho}_{wrong} \equiv \frac{T(D^0 \rightarrow l^- X)}{T(D^0 \rightarrow l^+ X)} \quad (7.17)$$

denotes the ratio of $\Delta C = \Delta Q$ to $\Delta C = -\Delta Q$ amplitudes; the oscillating functions multiplying the usual e^{-t/τ_D} term have been expanded in powers of the proper time t since $x_D, y_D \ll 1$. The $\Delta C = \Delta Q$ term has no t dependence beyond that of e^{-t/τ_D} , the pure oscillation term has a t^2 dependence, while the interference between the two generates a term linear in t .

Such a complication certainly arises in $D^0 \rightarrow K^+ \pi^-$ due to the doubly Cabibbo suppressed transition (DCST) which “mocks” the signal for $D\bar{D}$ mixing as a direct decay $D^0 \rightarrow K^+ \pi^-$ with a branching ratio $Br(D^0 \rightarrow K^+ \pi^-) = (2.8 \pm 0.9) \pm 10^{-4}$ [139]:

$$\begin{aligned} \Gamma(D^0(t) \rightarrow K^+ \pi^-) &\propto e^{-\Gamma_D t} \operatorname{tg}^4 \theta_C |\hat{\rho}_{K\pi}|^2 \\ &\times \left[1 - \frac{1}{2} \Delta \Gamma_D t + \frac{(\Delta m_D t)^2}{4 \operatorname{tg}^4 \theta_C |\hat{\rho}_{K\pi}|^2} + \frac{\Delta \Gamma_D t}{2 \operatorname{tg}^2 \theta_C |\hat{\rho}_{K\pi}|} \operatorname{Re} \left(\frac{p}{q} \frac{\hat{\rho}_{K\pi}}{|\hat{\rho}_{K\pi}|} \right) \right. \\ &\quad \left. - \frac{\Delta m_D t}{\operatorname{tg}^2 \theta_C |\hat{\rho}_{K\pi}|} \operatorname{Im} \left(\frac{p}{q} \frac{\hat{\rho}_{K\pi}}{|\hat{\rho}_{K\pi}|} \right) \right] \end{aligned} \quad (7.18)$$

where

$$\operatorname{tg}^2 \theta_C \cdot \hat{\rho}_{K\pi} \equiv \frac{T(D^0 \rightarrow K^+ \pi^-)}{T(D^0 \rightarrow K^- \pi^+)}. \quad (7.19)$$

One can also search for lifetime differences in certain well-chosen channels. With CP invariance holding (at least) to good approximation, CP eigenstates can be treated as mass eigenstates. While $D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$ will then exhibit Γ_+ , $D^0 \rightarrow K_S \Phi, K_S \omega, K_S \rho, K_S \eta$ etc. will be controlled by Γ_- , where $\Gamma_+ [\Gamma_-]$ denotes the width for the CP even [odd] state and

$$\Delta \Gamma = \Gamma_+ - \Gamma_- . \quad (7.20)$$

Furthermore the width for $D^0 \rightarrow K^- \pi^+$ is approximately given by the algebraic mean $\frac{1}{2}(\Gamma_+ + \Gamma_-)$.

7.4.2 CP Violation in D Decays

There is a wide field that can be discussed in close qualitative analogy to B decays.

CP Asymmetries involving Oscillations

CP asymmetries that necessarily involve $D^0 - \bar{D}^0$ oscillations can arise in final states that are CP eigenstates like K^+K^- or $\pi^+\pi^-$:

$$\begin{aligned}\Gamma(D^0(t) \rightarrow K^+K^-) &\propto e^{-\Gamma_D t} \left(1 + (\sin \Delta m_D t) \cdot \text{Im} \frac{q}{p} \bar{\rho}_{K^+K^-} \right) \simeq \\ &\simeq e^{-\Gamma_D t} \left(1 + \frac{\Delta m_D t}{\Gamma_D} \cdot \frac{t}{\tau_D} \cdot \text{Im} \frac{q}{p} \bar{\rho}_{K^+K^-} \right)\end{aligned}\quad (7.21)$$

With $x_D|_{SM} \leq 10^{-2}$ and $\text{Im} \frac{q}{p} \bar{\rho}_{K^+K^-}|_{KM} \sim \mathcal{O}(10^{-3})$ one arrives at an asymmetry of around 10^{-5} , i.e. for all practical purposes zero, since it presents the product of two very small numbers. Yet with New Physics one conceivably has $x_D|_{NP} \leq 0.1$, $\text{Im} \frac{q}{p} \bar{\rho}_{K^+K^-}|_{NP} \sim \mathcal{O}(10^{-1})$ leading to an asymmetry that could be as large as of order 1%. Likewise one should compare the doubly Cabibbo suppressed transitions $D^0 \rightarrow K^+\pi^-$, see Eq.(7.18), and $\bar{D}^0 \rightarrow K^-\pi^+$:

$$\begin{aligned}\Gamma(\bar{D}^0(t) \rightarrow K^-\pi^+) &\propto e^{-\Gamma_D t} \text{tg}^4 \theta_C |\hat{\rho}_{K\pi}|^2 \\ &\times \left[1 - \frac{1}{2} \Delta \Gamma_D t + \frac{(\Delta m_D t)^2}{4 \text{tg}^4 \theta_C |\hat{\rho}_{K\pi}|^2} + \frac{\Delta \Gamma_D t}{2 \text{tg}^2 \theta_C |\hat{\rho}_{K\pi}|} \text{Re} \left(\frac{p}{q} \frac{\hat{\rho}_{K\pi}}{|\hat{\rho}_{K\pi}|} \right) \right. \\ &\quad \left. + \frac{\Delta m_D t}{\text{tg}^2 \theta_C |\hat{\rho}_{K\pi}|} \text{Im} \left(\frac{p}{q} \frac{\hat{\rho}_{K\pi}}{|\hat{\rho}_{K\pi}|} \right) \right]\end{aligned}\quad (7.22)$$

where

$$\text{tg}^2 \theta_C \cdot \hat{\rho}_{K\pi} \equiv \frac{T(\bar{D}^0 \rightarrow K^-\pi^+)}{T(\bar{D}^0 \rightarrow K^+\pi^-)}.\quad (7.23)$$

In such new physics scenarios one would expect a considerably enhanced asymmetry that could be as large as $1\%/\text{tg}^2 \theta_C \sim 20\%$ – at the cost of smaller statistics. Hoping for an asymmetry of several percent is more realistic, though. Effects of that size would unequivocally signal the intervention of new physics! One should note that these guestimates are based on $x_D \simeq 10^{-2}$ which would correspond to $\tau_D \simeq 10^{-4}$. That means the following: even if oscillations have not been found on the $\tau_D = 10^{-4}$ level in semileptonic D^0 decays, a CP asymmetry of several percent (or conceivably ten percent) could be encountered in $D^0 \rightarrow K^+\pi^-$!

Direct CP Violation

There are two types of effects:

- differences between partial rates for CP conjugate transitions

$$A_{CP} = \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})} ; \quad (7.24)$$

- asymmetries in the final state distributions like Dalitz plot populations etc.

Strong final state interactions play an important part in both cases: in the former case they have to induce the phase shifts that are essential to make a difference observable; in the latter they can very significantly affect the observable asymmetry. The existence of resonances in the neighborhood of the charmed meson mass is proof that hadron dynamics is active in this energy region and will affect weak decays of charmed particles.

The good news are that whenever there are CP violating weak phases one can count on final state interactions to make them observable. The bad news are that interpreting a signal as evidence for new physics will pose a highly nontrivial theoretical challenge.

The SM with the KM ansatz can induce direct CP asymmetries only in Cabibbo suppressed, but not Cabibbo favored channels. Model-dependent estimates usually predict direct CP asymmetries to be of the order of 10^{-3} and exceptionally at the 10^{-2} level [140]. Lattice calculation hold out the realistic hope to greatly improve our understanding of final state interactions in charm decays over the next ten years. Furthermore a measurement of the branching ratios of all related channels – in particular also those with neutral hadrons in the final state as sketched above – would enable us to constrain the strong phase shifts quite significantly. A ν MC will have a significant advantage in this respect!

7.4.3 T Odd Correlations in Λ_c Decays

One special feature of ν MC is represented by the production of Λ_c in CC and NC reactions:

$$\nu N \rightarrow \nu \Lambda_c X \text{ or } \mu \Lambda_c X \quad (7.25)$$

This allows novel studies of various Λ_c form factors. Yet even more intriguing and promising would be a detailed analysis of the final state in its semileptonic decays

$$\Lambda_c^+ \rightarrow l^+ \nu \Lambda \quad (7.26)$$

With the parent c quark being left-handed one expects the Λ_c to emerge in a highly polarized state. The usual valence quark description actually suggests

that the Λ_c polarization is completely carried by its c quark; i.e., a left-handed c quark fragments into a left-handed Λ_c . Yet even with unpolarized Λ_c one can form a T odd correlation

$$C_{T=-} \equiv \langle \vec{\sigma}_\Lambda \cdot (\vec{p}_\Lambda \times \vec{p}_l) \rangle \quad (7.27)$$

connecting spin and momentum of the hyperon with the lepton momentum.

In a general experimental process, observing a nonvanishing value for a T odd correlation does not automatically establish that T (and CP) invariance is violated since in general final state interactions could fake such an effect. However, this problem does not occur for Eq.(7.26) since it cannot be affected by either strong or electromagnetic final state interactions! This is analogous to the well known situation in $K^+ \rightarrow \mu^+ \nu \pi^0$ (vs. $K_L \rightarrow \mu^+ \nu \pi^-$). Like there the KM ansatz cannot generate an observable effect here, yet certain new physics scenarios can. An effect of order 10^{-2} is not inconceivable, in particular of the channel $\Lambda_c \rightarrow \tau \nu \Lambda$ could be studied [141].

7.5 B Physics

An extrapolation of figure 1.2 indicates that TeV-scale muon colliders and above should produce enough B's to have some modest capabilities for B physics. Depending on the ν MC energy, a total event sample of 10^{10} interactions should include of order 10^7 B's produced in pairs from neutral current interactions:

$$\nu N \rightarrow \nu b \bar{b} X. \quad (7.28)$$

This sample size is comparable to the yearly event rate in e^+e^- B factories.

Charged current production of B's is heavily suppressed by small CKM matrix elements connecting b quarks to lighter flavors. The most copious CC production mode at the TeV energy scale will be associated production with charm:

$$\nu N \rightarrow l^- \bar{b} c X \quad (7.29)$$

and

$$\bar{\nu} N \rightarrow l^+ b \bar{c} X, \quad (7.30)$$

which will be down by about one-and-a-half orders of magnitude from the production mechanism of equation 7.28. For studies of B oscillations, however, the smaller statistics will be compensated by the very efficient and pure tag of the production sign of the b quark. Extremely precise vertex reconstruction will be the other compensation for the smaller statistics relative to the e^+e^- and, particularly, hadronic B factories.

It appears there would be at least one area where ν MC has the potential to make a decisive contribution to B physics: when extracting the angle ϕ_1 of the KM triangle – often also referred to as α – from the CP asymmetry in $B_d \rightarrow$

$\pi^+\pi^-$, one has to deal with a serious ‘pollution’ problem from so-called Penguin contributions. The impact of the latter could be determined and thus eliminated if one could measure the branching ratio for $B_d \rightarrow \pi^0\pi^0$ which is expected to be of order 10^{-6} . However it is widely thought that such a measurement is feasible neither at the e^+e^- nor the hadronic B factories [130]. The possibility for performing this measurement at very high energy ν MCs has been discussed elsewhere [9]. Further study is needed to assess whether ν MCs might in fact have any significant potential for B physics even after the B factories have run.

7.6 Summary

The research program at a ν MC is likely to improve our knowledge and understanding of charm decays quite significantly even ten years from now:

- It would fill out many white spots on our map of D , Λ_c , Ξ_c and Ω_c decays by measuring many new relative as well as absolute branching ratios, including for final states with multi-neutrals.
- It would allow measuring D^+ , $D_s^+ \rightarrow \mu\nu, \tau\nu$ in a very clean environment.
- It could probe for $D^0 - \bar{D}^0$ oscillations and for CP asymmetries involving them with superbly clean systematics. It would significantly lower the sensitivity that can be obtained at B factories for such phenomena.
- It would enable us to search for direct CP asymmetries in many different channels and at the same time provide us with information that could help us in properly interpreting a signal.

Chapter 8

Summary

This book has presented a preliminary overview of the emerging potential for neutrino physics at future muon colliders and dedicated neutrino factories. The discussion of long baseline studies of neutrino oscillations is complementary to that presented in another study [6], while high-rate studies of neutrino interactions are presented at a level of depth that, while still superficial, nevertheless goes beyond that of previous discussions.

The beams from ν MCs have the potential for vast improvement over today's conventional neutrino beams from pion decays in several respects: they are much more intense and with a much smaller transverse extent, and the beam spectrum is precisely predictable and is a completely pure 2-component mixture: $(\nu_\mu \bar{\nu}_e)$ beams from decays in parent μ^- beams and $(\bar{\nu}_\mu \nu_e)$ beams from μ^+ parents.

The advantage for long baseline (LB) neutrino oscillation experiments has been seen to rest mainly in the total neutrino beam flux, predictable spectra and the potential to exploit the first intense high energy source of electron neutrinos. As was discussed in section 1.2, LB experiments at ν MCs should require only modest extrapolations in detector technology from today's accelerator-based LB experiments. The experimental boundary conditions are very different for the high rate (HR) neutrino experiments discussed in the remainder of the book, where many of the HR analyses were seen to either become feasible or continue to benefit from unprecedented event statistics approaching of order 10^9 to 10^{10} precisely constructed DIS events – i.e. at least three orders of magnitude larger than any of the data samples collected using today's much larger and cruder neutrino targets. Chapter 1 shows that ν MCs might achieve such event samples using compact fully-active tracking neutrino targets backed by high-rate, high-performance detectors and placed as close as is practical to the neutrino source in order to fully exploit the beam intensity and uniquely small transverse extent close to production.

ν MCs show great potential for advancing experimental elementary particle physics areas that are traditional to neutrino physics as well as in unique new ways that are beyond the capabilities of the current generation of neutrino

experiments. Such a breadth of physics reach should complement or improve upon many other analyses in diverse areas of collider and fixed target physics.

The most important analysis topics might include:

- substantially extending the reach of accelerator-based experiments to study neutrino oscillations
- the only realistic opportunity, in any physics process, to determine the detailed quark-by-quark structure of the nucleon
- with polarized targets, additionally map out the quark-by-quark spin structure of the nucleon and, perhaps, determine the gluon contribution to the nucleon's spin
- some of the most precise measurements and tests of perturbative QCD
- some of the most precise tests of the electroweak theory through measurements of $\sin^2 \theta_W$ with fractional uncertainties approaching the 10^{-4} level
- measurements of the CKM quark mixing matrix that will be interesting for lower energy ν MCs ($|V_{cd}|$ and $|V_{cs}|$) and will become extremely important ($|V_{ub}|$ and $|V_{cb}|$) at higher energies
- a new realm to search for exotic physics processes
- as a bonus outside neutrino physics, a charm factory with unique capabilities.

The expected experimental capabilities of ν MCs are so novel and impressive that the topics presented in this book must surely represent no more than a limited first attempt to understand their full potential for neutrino physics.

Appendix A

Neutrino Beams at Muon Colliders

This appendix derives some of the the equations and results presented in section A.1, discusses in detail the different design options for neutrino production and explains the design constraint due to neutrino radiation

A.1 Neutrino Spectra, Beam Intensities and Event Rates at Experiments

This section presents useful but approximate formulae for neutrino spectra and event rates. These formulae are relevant when the neutrino source is a monoenergetic, unpolarized muon beam of negligible transverse size. As discussed below, these approximations will usually be reasonable. Further, the angular divergence of the muon beam will usually be too small to contribute to the neutrino beam divergence, as discussed in section A.3 on beam design. Where this is not the case, the additional beam divergence will be characterized by the parameter $\delta\theta$, as defined in subsection A.1.1. The derivations of the formulae presented in this section have been relegated to appendix A.2.

A.1.1 Overview of neutrino beam characteristics

Muon storage rings produce very intense, pure 2 component beams whose spectra can be relatively accurately predicted using simple analytic formulae. The muon decays:

$$\begin{aligned}\mu^- &\rightarrow \nu_\mu + \bar{\nu}_e + e^-, \\ \mu^+ &\rightarrow \bar{\nu}_\mu + \nu_e + e^+\end{aligned}\tag{A.1}$$

As yet another nice feature for systematic studies, it will probably be fairly straightforward at all types of muon storage rings (muSR's) to periodically swap the muon sign circulating in a given direction. (Essentially, this is done by swapping the polarity of the magnet power supply leads and injecting each beam sign in the opposite direction.)

Muon colliders may well run with polarizations at collision of order 20% and, for example, some possibilities for making use of this polarization for oscillation physics are discussed in reference [5]. However, the energy spectra of the neutrino event samples are likely to average out to the unpolarized distribution, either because of spin precessions during each muon fill or from periodically flipping the muon polarization at injection into the muSR. For simplicity, our discussion from now on will deal with unpolarized beams.

It is expected that the angular divergence of the neutrino beam will often be dominated by the p_t given to the neutrinos at decay, rather than from the angular divergence of the muon beam. To set the scale, the maximum p_t given to the neutrinos from muon decays is 53 MeV/c. The muon beam spread in p_t will often be considerably less than this and, since the p_t contributions add in quadrature, the p_t given to the neutrinos from the muon decays will dominate for these muSR scenarios. (The different muSR scenarios are discussed in section A.3.)

For cases where the muon beam divergence should not be neglected, we will assume that the neutrino beam divergence can still be simply parameterized by an equivalent angular spread, $\delta\theta$. From equation 1.9, it is sensible to define $\delta\theta \equiv 1/\gamma_\mu$ for the neutrinos from a divergenceless muon beam. Then the scaling factor $\gamma \cdot \delta\theta$ can be used in equations that need to describe the effect of muon beam divergences, with a scaling factor of unity corresponding to the neutrino beam from a divergenceless muon beam.

A.2 Calculations of the Neutrino Beam Parameters

It will be seen in this section that simple, approximate quantitative expressions can be straightforwardly obtained for the flux spectra of the characteristic two-component beams produced at ν MCs. These simple predictions are useful for analytic estimates of experimental event rates and for fast Monte Carlo simulations of potential experiments. For many ν MC designs, the spectra will depend largely only on the muon beam energy and the total number of muon decays in the production straight section. More complicated beams may be parameterized by the addition of an angular divergence parameter, $\delta\theta$.

A.2.1 Calculations of the Neutrino Energy Spectra

The energy and angle distribution for muon neutrinos (from μ^- decays) or antineutrinos (from μ^+ decays) is given by [142]

$$\frac{d^2 N_{\nu_\mu}}{dx d\cos\theta'} = x^2[(3 - 2x) \pm (1 - 2x) \cos\theta'], \quad (\text{A.2})$$

where x is the fraction of the maximum neutrino energy in the muon rest frame: $E'_\nu = \frac{x \cdot m_\mu \cdot c^2}{2}$, θ' is the angle between the neutrino and the muon spin direction in the muon rest frame and the $+$ ($-$) sign refers to neutrinos (antineutrinos). The corresponding distribution for electron-type neutrinos is:

$$\frac{d^2 N_{\nu_e}}{dx d\cos\theta'} = 6x^2(1 - x)(1 \pm \cos\theta'), \quad (\text{A.3})$$

This provides the distributions for a muon beam with average polarization P along the beam direction:

$$\frac{d^2 N_{\nu_\mu}}{dx d\cos\theta'} = x^2[(3 - 2x) \pm P(1 - 2x) \cos\theta'] \quad (\text{A.4})$$

and

$$\frac{d^2 N_{\nu_e}}{dx d\cos\theta'} = 6x^2(1 - x)(1 \pm P \cos\theta'). \quad (\text{A.5})$$

Neglecting the polarization removes the dependence on $\cos\theta'$, giving:

$$\frac{dN_{\nu_\mu}}{dx} \equiv f^\mu(x) = 6x^2 - 4x^3 \quad (\text{A.6})$$

for muon-type neutrinos or anti-neutrinos and

$$\frac{dN_{\nu_e}}{dx} \equiv f^e(x) = 12x^2 - 12x^3 \quad (\text{A.7})$$

for electron-type neutrinos or anti-neutrinos. That is to say, $f^i(x)$ gives the energy probability distribution in the muon rest frame for the production of neutrino type i , where $i = \mu$ or e .

On applying a Lorentz transformation to boost into the lab frame, it is found that the neutrino energy distribution at any polar angle is just scaled by a relativistic boost factor depending on that angle:

$$E_\nu = x \frac{E_\mu}{2} (1 + \beta \cos\theta'), \quad (\text{A.8})$$

where $\beta = 1$ to a very good approximation and each angle in the muon rest frame, θ' corresponds to an angle θ in the laboratory frame according to:

$$\sin\theta = \frac{\sin\theta'}{\gamma(1 + \beta \cos\theta')}. \quad (\text{A.9})$$

(The specific value $\theta' = \frac{\pi}{2}$ reproduces the information of equation 1.9 that the forward hemisphere in the muon rest frame is boosted into a cone of half angle $\theta_\nu = 1/\gamma$.)

It follows that the distributions $f^i(x)$, which were defined for the muon rest frame, still describe the energy distributions in the laboratory frame with x now reinterpreted as the fraction of the maximum neutrino energy, E_ν^{max} for that beam energy and polar angle, where

$$E_\nu^{max}(E_\mu, \cos \theta') = E_\mu(1 + \cos \theta')/2. \quad (\text{A.10})$$

It further follows trivially that, at any position in the beam, both the mean energy as a fraction of the maximum energy and the fractional rms energy spread have the same values as in the muon rest frame. Explicitly:

$$\frac{\langle E_\nu \rangle}{E_\nu^{max}}|_{\text{beam}} = \langle x \rangle,$$

and

$$\frac{\sigma_{E_\nu}}{E_\nu^{max}}|_{\text{beam}} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

for both electron and muon neutrinos, where " $\langle \rangle$ " denotes the integral of the enclosed quantity over $f^i(x)$. Using the explicit forms for the f^i s gives $\frac{\langle E_\nu \rangle}{E_\nu^{max}}|_{\text{beam}} = 60\%$ for electron (anti)neutrinos and 70% for muon (anti)neutrinos; $\frac{\sigma_{E_\nu}}{E_\nu^{max}}|_{\text{beam}} = 20\%$ for electron (anti)neutrinos and 21% for muon (anti)neutrinos.

The corresponding expressions for the *event sample* will be weighted by a further power of x to account for the neutrino cross section being proportional to the neutrino energy. That is:

$$\frac{\langle E_\nu \rangle}{E_\nu^{max}}|_{\text{events}} = \frac{\langle x^2 \rangle}{\langle x \rangle}.$$

and

$$\frac{\sigma_{E_\nu}}{E_\nu^{max}}|_{\text{events}} = \sqrt{\frac{\langle x^3 \rangle}{\langle x \rangle} - \left(\frac{\langle x^2 \rangle}{\langle x \rangle}\right)^2}$$

Again, the numerical values are $\frac{\langle E_\nu \rangle}{E_\nu^{max}}|_{\text{events}} = 76\%$ for electron (anti)neutrinos and 67% for muon (anti)neutrinos; $\frac{\sigma_{E_\nu}}{E_\nu^{max}}|_{\text{events}} = 18\%$ for both electron and muon (anti)neutrinos.

A.2.2 calculations for short baseline detectors

With the definition of equation 1.10, the numerical value for the event size parameter N^{sb} is easily seen to be given by the expression:

$$N^{sb} = N_\mu^{ss}[yr^{-1}] \times N_{Avo} \times \frac{1}{2} \int_0^1 dy' \sum_{\nu \text{ type}, i=1,2} \int_0^1 dx f^i(x) \sigma^i(E_\nu)[cm^2], \quad (\text{A.11})$$

as explained in the following paragraphs.

The factor N_μ^{ss} is the number of muon decays in the straight section per year. For a storage ring,

$$N_\mu^{ss}[yr^{-1}] = f^{ss} \times N_\mu[yr^{-1}] \times t_D[\gamma\tau_\mu], \quad (A.12)$$

where N_μ is the total number of muons per year, f^{ss} is the length of the straight section as a fraction of the collider circumference, $f^{ss} = s/C$, and the time to beam dump, $t_D[\gamma\tau_\mu]$, is expressed in units of the boosted muon lifetime and allows for the possibility of dumping the beam instead of holding it until all the muons decay.

The variable $y' \equiv \cos \theta'$ gives the polar angle in the muon rest frame corresponding to a given position in the target. For simplicity, the target thickness has been assumed to be spherically symmetric about the beam axis and the target has been assumed to subtend the boosted forward hemisphere of the neutrino beam – that is, $y'_{min} = 0$. More generally, the integration limits would run from y'_{min} to 1, where y'_{min} corresponds to the maximum beam angle subtended by the target.

Avagadro's number, $N_{Avo} = 6.02 \times 10^{23}$, is needed to convert from the neutrino cross section per nucleon, σ^i , to the cross section per gram-centimeter-squared of target.

The summation in equation A.11 is over the 2 neutrino types produced by the sign of muon traveling in the considered direction, i.e., either $\bar{\nu}_e$ and ν_μ for μ^- decays or $\bar{\nu}_\mu$ and ν_e for μ^+ decays (equation A.1).

Because neutrino cross-sections are almost linear with energy in the TeV range they can be expressed as

$$\sigma^i(E_\nu)[cm^2] \simeq \sigma_R^i[cm^2/TeV] \times E_\nu[TeV],$$

where the “reduced cross section”, σ_R^i , can be approximated as a constant over a fairly large energy range. This allows the replacement:

$$\sum_{\nu \text{ type}, i=1,2} \int_0^1 dx f^i(x) \sigma^i(E_\nu)[cm^2] \rightarrow \sum_{\nu \text{ type}, i=1,2} \sigma_R^i[cm^2/TeV] \times \langle x \rangle^i \quad (A.13)$$

where E_ν^{max} is defined by equation A.8 as

$$E_\nu^{max}(E_\mu, \cos \theta') = E_\mu(1 + \cos \theta')/2. \quad (A.14)$$

and $\langle x \rangle^i$ is defined by

$$\langle x \rangle^i \equiv \int_0^1 dx f^i(x) \cdot x. \quad (A.15)$$

The calculation of the numerical value of equation A.13 is summarized in table A.1. The summed contributions for μ^- are seen to be only 4.3 percent higher than for μ^+ , so it is sensible to use the average contribution, $0.948 \times 10^{-35} cm^2/TeV$, for either sign.

Putting it all together gives the numerical expression for N^{sb} of equation 1.11.

Table A.1: Contributions to the cross section from the different types of neutrino interactions. The reduced cross-section and product are in units of $10^{-35}\text{cm}^2/\text{TeV}$. Their numerical values, summed over CC and NC interactions, are given in reference [7] with σ_R^i specified for 100 GeV neutrinos on an isoscalar target.

int., i	σ_R^i	$\langle x \rangle^i$	$\sigma_R^i \cdot \langle x \rangle^i$
ν_μ	0.948	0.700	0.664
$\bar{\nu}_e$	0.505	0.600	0.303
SUM			0.967
$\bar{\nu}_\mu$	0.505	0.700	0.354
ν_e	0.948	0.600	0.569
SUM			0.922

A.2.3 calculations for long baseline detectors

The derivation of the numerical value for N^{lb} , as defined in equation 1.12 can either proceed by taking the limit of the N^{sb} derivation as $\cos\theta'_{\text{min}} \rightarrow 1$ or, more directly, by noting that the number of neutrinos crossing a unit area in the beam center and at baseline distance L is:

$$\nu/\text{area} = \frac{N_\mu^{\text{ss}}}{L^2} \times \frac{\gamma^2}{\pi}, \quad (\text{A.16})$$

where the factor γ^2/π is easily shown to be the fraction of neutrinos decaying in the forward direction per unit solid angle after being boosted from isotropic decays in the muon rest frame into the laboratory frame.

This gives an equation analogous to equation A.11:

$$N^{\text{lb}}[\text{events}/\text{yr}/(\text{kg} \cdot \text{km}^{-2})] = 6.02 \times 10^{16} \times N_\mu^{\text{ss}}[\text{yr}^{-1}] \times \frac{\gamma^2}{\pi} \times \sum_{\nu \text{ type}, i=1,2} \int_0^1 dx f^i(x) \sigma^i(E_\nu[\text{cm}^2]), \quad (\text{A.17})$$

where the numerical factor, 6.02×10^{16} , incorporates Avagadro's number, a factor of 10^3 to convert between kilograms and grams and a factor of 10^{-10} to convert between square centimeters and square kilometers.

Numerical calculations completely analogous to those for equation A.11 then give the numerical expression:

$$N^{\text{lb}}[\text{events}/\text{yr}/(\text{kg} \cdot \text{km}^{-2})] = 1.62 \times 10^{-20} \times N_\mu^{\text{ss}}[\text{yr}^{-1}] \times (E_\mu[\text{GeV}])^3.$$

This formula can clearly then be extended to include the parameter $\delta\theta$ for large muon beam divergences:

$$N^{\text{lb}}[\text{events}/\text{yr}/(\text{kg} \cdot \text{km}^{-2})] = \frac{1.62 \times 10^{-20} \times N_\mu^{\text{ss}}[\text{yr}^{-1}] \times (E_\mu[\text{GeV}])^3}{(\gamma \cdot \delta\theta)^2},$$

which is equation 1.13.

A.3 Design Options for Neutrino Production

This section enlarges on the three main design options for producing the production straight section: dedicated muon storage rings, recirculating linacs and collider rings themselves. Section A.4 discusses a serious design constraint on the production straight sections: the need to limit the neutrino radiation hazard.

A.3.1 Neutrino beams from recirculating linacs

The acceleration of muon bunches to collider energies takes place in recirculating rings with long straight sections of rf cavities that may be considered as sources for neutrino beams. The neutrino energy spectrum produced at these linacs ramps up from that produced at the injection energy for the muons up to the beam spectrum for collider energy muons. The integrated neutrino event rate can be obtained by summing equation 1.11 or 1.11 over each of the beam passes through the linac.

The fraction of the total muons in the collider that contribute to the neutrino beam is likely to be a couple of percent of the total muons in the collider ring since those decaying in the linacs might be of order 10% of the total and the linac might occupy roughly 25% of the acceleration ring.

A.3.2 Neutrino beams from long straight sections in a muon collider ring

Long straight sections in the collider ring are the final option that has been considered [3, 4] for producing neutrino beams.

Muon collider rings naturally require one or more long straight sections to prepare the beams for collision. However, this is also the one place around the ring where the angular divergence of the muon beam is likely to be large enough to significantly spread out the neutrino beam.

This is likely to be the straight section of choice only at very high energy colliders where the angular divergence will be small anyway and other long straight sections will be discouraged by neutrino radiation concerns (see the following section).

The racetrack geometry used to close muon collider rings with only one collision point makes it natural to include this utility straight section opposite the colliding straight section. This is probably the best choice for experimental beams at all but very high energy muon colliders.

A couple of percent to a few percent of the total muons will contribute to the neutrino beam in either of the two back-to-back beam directions aligned with the straight section, according to the fractional length of the straight section (of order several percent – see table 1.2.2) and noting that this fraction must be divided by two relative to the recirculating linac option because only one sign of muons will contribute to each beam.

This is similar to the number of contributing muons expected in the linac scenario of the preceding section. However, the event rate at the ν MC is likely to

be somewhat higher because all of the muons decay at the full collider energy and the event rate has a cubic dependence with the muon beam energy (equation 1.11 or 1.11).

A possible option is to deliberately lay out the geometry of muon collider complexes such that both accelerator ring and collider ring neutrino beams contribute neutrino flux at the location chosen for a ν MC.

A.3.3 Neutrino beams from dedicated muon storage rings

An alternative realization for neutrino beams that is beginning to attract considerable interest is *dedicated* muSR that will use muon collider technology to produce and store large muon currents for neutrino experiments, particularly LB experiments to study neutrino oscillations. (Such dedicated muSR are sometimes regarded as a cheaper intermediate step leading to muon colliders.)

For financial reasons, dedicated muSR are likely to be generally smaller in size than many of the options for muon collider rings, producing the relatively low energy neutrino beams that are mostly suited for neutrino oscillation experiments.

The muSR would have one or, possibly, two fractionally long straight sections to produce the neutrino beams and the ring orientation and tilt would be chosen to point the straight section at desirable sites for neutrino targets.

There are two possible options for obtaining the muon beams for such muSR:

1. parasitically use the muon beams at a muon collider complex when they are not being utilized for collider physics
2. a dedicated complex including a source to produce and accelerate the beams for the muSR.

The first option would be a relatively cheap add-on to a muon collider complex. The one or more muon colliders in a complex would certainly have priority use of the muon beams and would likely use all the muon beam when running. Instead, the ring would have use of the beam during collider shutdowns for upgrades etc.

The second option is usually discussed as an early opportunity for neutrino oscillation physics using muon collider technology but before muon colliders themselves are built. It would be advantageous if such a muSR was itself upgradable to a muon collider. The neutrino beams from these muSR will have similar compositions and intensities to those from muon colliders but they are likely to differ in three respects:

1. the muon beam energy will be lower than expected for muon colliders: less than 50 GeV for dedicated muSR as opposed to 50 GeV or above for muon colliders.
2. the muon bunch structure will likely be composed of many mini-bunches in a bunch train spanning a large fraction of the ring circumference, rather than a small number of large bunches at a muon collider. The neutrino

bunch structure will mirror the muon bunch structure, with possible consequences for event triggering and background rejection in both SB and LB detectors.

3. the muSR will be fully optimized to maximize the fractional length of the decay straight section. It might occupy approximately 30% of the ring circumference as opposed to perhaps 10% or less for muon colliders. This has two consequences: (i) for the experimental event rate, it will help to compensate for the lower neutrino cross-section at lower energies, and (ii) the target for SB detectors may need a larger transverse extent to subtend the relatively more distant neutrino production at the upstream end of the straight section.

With this understanding, the discussion of ν MCs can therefore be taken to include the experiments at dedicated muSR's. The role of dedicated muSR's will mainly be discussed in chapter 2, on neutrino oscillations. Apart from this, we will not explicitly discuss the lower energy beams from muSR's in much detail since the design parameters and ideas for this nascent option are in a state of flux and the topic is likely to be covered in more detail soon once the parameters for these machines and experiments are better understood.

A.4 Neutrino radiation constraints

A serious and unexpected problem that has arisen for high energy muon colliders is the potential radiation hazard posed by neutrinos emitted from muon decays in the collider ring [3, 143]. This section characterizes the problem, gives a numerical estimate of the dose downstream from the production straight section and describes several strategies to minimize the hazard.

The neutrinos from muon decays produce a “radiation disk” in the plane of the ring, and the potential radiation hazard results from the showers of ionizing particles from occasional neutrino interactions in the soil and other objects bathed by this disk. Although the neutrino cross-section is tiny, this is greatly compensated by the enormous number of tightly collimated high energy neutrinos produced at the collider ring.

The radiation intensity will generally be very uneven around the radiation disk, with more intense radiation hot-spots downstream from straight sections in the collider ring. Clearly, the beam intensity will rise in proportion to the length of the straight section and ν MCs will want to sit in the most intense neutrino beam available at the muon collider complex. Hence, the long straight section used for neutrino experiments may well produce the worst potential radiation hazard at the muon collider complex and care must be taken to minimize this problem.

Numerical estimates for the dose downstream from a straight section can be easily obtained using derivations similar to those used for equations 1.11 and 1.13 with some additional reasonable assumptions. The predicted dose at

distance L from the straight section is [143]:

$$D^{ss}[Sv] \simeq 1.1 \times 10^{-18} \times \frac{f^{ss} \times N_\mu \times (E_\mu[TeV])^4}{(L[km])^2 (\gamma \cdot \delta\theta)^2}. \quad (A.18)$$

with notation as for equations 1.11 and 1.13. For comparison, the U.S. federal off-site radiation limit is 1 mSv/year, which is of the same order of magnitude as the typical background radiation from natural causes (typically 0.4 to 4 mSv/yr [10]). This equation is not intended to be accurate at much better than an order of magnitude level and is deliberately conservative, i.e. it may well overestimate the radiation levels, particularly at very high energies [143]. Because of the energy dependence, the radiation levels rapidly become a serious design constraint for colliders at the TeV scale and above.

Equation A.18 has the same angular dilution factor - $1/(\gamma \cdot \delta\theta)^2$ - as equation 1.13 describing the event rate at LB detectors, but it is notable that this dilution factor is absent in equation 1.11 for SB experiments. This means that it will probably be advantageous at higher energies to use either use straight sections where the muon beam naturally has a large angular divergence. It has even been proposed to artificially sweep the beam about its equilibrium position. An increase in beam divergence would mean extending the width of the short baseline targets to collect most of the neutrinos, but the resulting target width will presumably still be reasonable due to the increasingly collimated beams at higher energies.

Clearly, it would be helpful if the direction of the neutrino production straight section is chosen such that public exposure is minimized and other strategies include judicious choices in the azimuthal direction and tilt of the production straight section.

For even higher potential doses, the possibility exists to use radiation enclosures along the strip of land where the radiation cone exists the Earth. Ultimately, muon colliders at CoM energies of beyond a few TeV will anyway probably have to be constructed at isolated sites where the public would not be exposed to the radiation from the neutrino disk and straight sections.

A.5 MC Scenarios Used for the Calculation of Neutrino Event Rates

The 20 GeV scenario is assumed to be a dedicated muSR for neutrino oscillation physics and its specifications are within the range of design options discussed for this energy. The 50 GeV, 500 GeV and 5 TeV scenarios use collider ring parameter sets similar to those from reference [144]. The 175 GeV collider ring, with parameters that interpolate between those of the 50 GeV and 500 GeV rings, is similar to that specified in [145]. The 5 TeV parameter set corresponds to a hypothesized second generation muon collider built at a remote site to control the neutrino radiation hazard. This collider scenario is obviously more speculative than the others.

Although these scenarios consider a muon collider ring, the discussions in subsections A.3.1 and A.3.2 indicate that the neutrino beams from recirculating linacs should lead to similar rates. Hence, table 1.2.2 should also give realistic order-of-magnitude event rates when the neutrino source is a recirculating linac.

The length of the production straight section gives the largest uncertainty in estimating the event size parameter, N^{sb} , for the short baseline detector, while an additional large uncertainty in the corresponding parameter for long baselines, N^{lb} , comes from the estimation of N^{lb} , the angular divergence of the muon beam. (The high rate detector is assumed to subtend the boosted forward hemisphere of the neutrino beam, regardless of its angular divergence, so it is insensitive to the angular divergence of the muon beam.) Because of the additional uncertainty, it is clear that N^{lb} is much more difficult to estimate than N^{sb} .

The length of each of the straight sections for the collider scenarios was estimated as an educated guess from a specialist in the design of magnet lattices for colliders [146]. The specific choice of the best straight section in the lattice changes between the two lower energy scenarios and those at the higher energies. For the 50 GeV and 175 GeV rings the most appropriate straight section may well be the utility straight section opposite the ip where the beams are injected and scraped. In contrast, the scraping scenario is likely to change for the higher energy machines and current designs predict the longest straight sections to be close to, and on either side of, the final focus section around the ip. For the 50, 175 and 500 GeV designs it was estimated that these straight sections could be made as long as 12% of the ring circumference without significantly impacting on the operation of the ring for collider physics. For the 5 TeV design the corresponding fractional length was estimated at 3%.

In the straight section scenarios for each of the collider rings, the angular divergence of the muon beam would naturally be too small to significantly contribute to the angular divergence of the neutrino beam ($\gamma \cdot \delta\theta \simeq 1$). However, it is quite conceivable that the divergence of the 500 GeV beam would be artificially increased – perhaps by “sweeping” the beam – to bring the neutrino radiation downstream from the straight section to an acceptably low level. Without this, a non-tilted 500 GeV ring at a depth of 125 meters would give a maximum predicted neutrino dose of around 1.7 mSv/year, perhaps slightly above the U.S. federal off-site limit of 1 mSv/year. To avoid this scenario, table 1.2.2 has assumed a level of beam sweeping to increase the effective angular spread in both transverse dimensions by a factor of 10 ($\gamma \cdot \delta\theta = 10$), decreasing the neutrino radiation by a factor of 100 to what is, presumably an acceptable level. Of course, the GeV ring poses a negligible radiation hazard. The radiation levels from the production straight section at the 50 GeV and 175 GeV machines also appear acceptable: 0.006 mSv/year and 0.08 mSv/year for depths of 20 meters and 125 meters, respectively. The straight section for the 5 TeV ring is, of course, well over the limit, at 40 mSv/year. This is assumed to be OK for an isolated site.

Appendix B

A General Purpose Detector for DIS Neutrino Interactions

This appendix provides some details on the general purpose short baseline (SB) detector shown in figure 1.3 of chapter 1. A more detailed presentation of the detector will be given elsewhere [147].

The detector performance is an important factor in determining the potential of many SB analyses, as is clear from many discussions in this book. The specification of this detector design is also useful for illustrating the design considerations that might be shared by other DIS detector designs for ν MCs, including detailed particle-by-particle event reconstruction and fully active precision vertexing targets in place of the coarse-sampling calorimetric targets used in past and present high rate neutrino experiments.

B.1 Detector Overview

The detector layout comprises a narrow cylindrical neutrino target surrounded by a gas-based tracking sub-detector and backed by electromagnetic and hadronic calorimetry plus a muon range stack. A very attractive option for the neutrino target is a stack of order 1000 CCD vertexing detectors. This would give exceptional vertex reconstruction for short-lived final state hadrons containing a charm or beauty quark.

Overall, the detector would provide outstanding event reconstruction for both CC and NC interactions and so would be very well matched to the requirements for both precision and rare-process physics analyses with the expected very large event samples. The relativistically invariant quantities that are routinely extracted in DIS experiments are 1) Feynman x , the fraction of the nucleon momentum carried by the struck quark, 2) the inelasticity,

$y = E_{\text{hadronic}}/E_\nu$, which is related to the scattering angle of the neutrino in the neutrino-quark CoM frame, and 3) the momentum-transfer-squared, $Q^2 = 2M_{\text{proton}}E_\nu xy$. ν MCs will have the further capability of reconstructing the hadronic 4-vector, resulting in a much better characterization of each interaction.

The rest of this subsection contains a summary of the detector components and their expected performances. The reader is referred to reference [147] for further details and for discussion on several other aspects of the detector design: data acquisition (DAQ), event pile-up, beam-related backgrounds, evolution of detector design with beam energy, detector placement and shielding from primary beam muons.

B.2 CCD target

Besides providing the mass for neutrino interactions, the CCD target allows the precise reconstruction of the event topologies from charged tracks, including event-by-event vertex tagging of those events containing charm or beauty hadrons or tau leptons. CCD's give the best vertexing performance of any electronic tracking device: working CCD vertex detectors have demonstrated 3-dimensional hits with point resolutions of 3.5 microns [148].

As a reasonable illustrative design scenario, one can consider a row of 2000 planes of 300 micron thick CCD's stacked with equal spacing along the beam direction and with a transverse extent to approximately span the boosted forward hemisphere of the neutrino beam cone (see appendix A). For example, a 5 cm radius, which approximately matches the beam radius at 200 meters from production for a 500 GeV muon beam. For a 2 meter target length (i.e. twice the target length shown in figure 1.3 this corresponds to a 700 micron gap between detector planes, a 30 percent packing fraction and a mass per unit area of approximately 140 g.cm^{-2} . This would present up to 6.4 radiation lengths and 1.3 nuclear interaction lengths for an event at the upstream end of the target.

Such a vertexing geometry appears very promising for high-performance vertex tagging, as is illustrated in figure 7.1 of chapter 7. The gap between vertexing planes is shorter than the typical lifetimes of the interesting short-lived particles (typically of order a few millimeters, depending on the neutrino beam energy [147]). The extrapolation lengths to secondary vertices will be much shorter than is possible in collider experiments and the geometric acceptance inefficiency will be negligible for decays not occurring in the material of the CCD's themselves.

Estimated tagging efficiencies for the various vertex tags are given in table B.1. The tags and their estimated efficiencies are explained in reference [147].

As an additional attractive feature, the detector can presumably be constructed to allow multiple and/or interchangeable targets on a retractable target rack on rollers for different DIS physics analyses. Alternative target options are higher mass sampling tracking targets and hydrogen targets. Hydrogen targets allow studies of neutrino scattering off free protons (protium) and of quasi-free

vertex tag for c	50%
fully reconstr. c	10%
high eff. for b	70%
ultrapure b	20%
fully reconstr. b	1%

Table B.1: Tagging efficiencies for charm and beauty tags.

neutrons (using deuterium, on subtracting the proton contribution using the protium data).

B.3 Detector surrounding the target

A TPC tracker hermetically encloses the neutrino target, providing tracking, momentum analysis and PID through both dE/dx measurements and imaging of cherenkov radiation from the charged tracks. A detailed specification of the TPC parameters is given in [147]. The dimensions of the TPC are 5 m long x 4 m high x 4m wide. The TPC geometry is the same as used in, e.g., BNL experiment E910. A large dipole magnet produces a vertical 1.5 Tesla magnetic field transverse to the neutrino beam direction and aligned with an electric field. This geometry ensures good momentum reconstruction for all forward-going tracks. As a demonstration of feasibility, it is shown in reference [147] that the operating parameters of the TPC – read-out, gas mixture, electric field gradient and magnetic field strength – can be chosen to be similar to those of the ALEPH TPC [149]. Reference [147] also discusses such technical issues as the readout and gas mixture.

The option of a concave spherical mirror at the downstream end of the TPC could reflect and image the cherenkov light onto a plane of photodetectors at the upstream end of the target. There could well also be a small auxiliary mirror on axis to focus the light away from the area shaded by the neutrino target. The photodetectors could be either an array of conventional photomultipliers suitably shielded from the magnetic field or one of the several promising new photodetector technologies under development. The detection plane is at the focal length of the mirror – this is the same geometry as proposed for the large water cherenkov neutrino detector, AQUARICH.

The electromagnetic calorimeter (ECAL) would likely use a fully-active crystal array and, as an illustrative example, this could reasonably consist of an array of lead tungstate scintillating crystals read out by avalanche photodiodes, as in the CMS detector at the LHC. This choice is motivated by the excellent energy resolution of lead tungstate crystals for electromagnetic showers [150]:

$$\sigma(E)/E = 0.5\% \oplus 2\%/\sqrt{E(\text{GeV})} \quad (\text{B.1})$$

and their compactness: a radiation length of 0.89 cm and Moliere radius of 2.2 cm.

subdetector	rms uncertainty
CCD target:	$\sigma_{x,y}[\mu\text{m}] = 4.5 \oplus \frac{0.90}{\cos^{3/2}\theta \cdot p[\text{GeV}/c]}$ $\sigma_{p_t}[\text{MeV}/c] = 1.5 \cdot (p[\text{GeV}/c])^{1/4}$ $\sigma_p/p = \frac{0.047}{\sqrt{L^{CCD}[m]}} \oplus \frac{7.5 \times 10^{-6} \times p[\text{GeV}/c]}{(L^{CCD}[m])^{2.5}}$
TPC tracker:	$\sigma_p/p = \frac{0.0029}{\sqrt{L^{CCD}[m]}} \oplus \frac{0.0012 \times p[\text{GeV}/c]}{(L^{CCD}[m])^{2.5}}$
ECAL:	$\sigma_E/E = 0.5\% \oplus 2\%/\sqrt{E[\text{GeV}]}$
HCAL:	$\sigma_E/E \sim 65\%/\sqrt{E[\text{GeV}]}$

Table B.2: Approximate values for the uncertainties in some of the important resolution parameters of the high rate neutrino detector. The symbol “ $\oplus$ ” denotes addition in quadrature and the dimensioned quantities are to be evaluated in the units given in square brackets. It is assumed that charged tracks have length L^{CCD} in the CCD’s, determined by the vertex position and the distance to either interacting or exiting out the side of the CCD. Since the neutrino target is at the upstream end of the TPC, the assumed length, L^{TPC} , in the TPC will typically be close to the 5 meter length of the TPC.

As a possible technology for the hadron calorimeter (HCAL), metal-plus-scintillator sampling calorimeters are compact, well understood and offer very good energy resolution. Because of their relatively high densities, either iron or copper are both good choices for the metal absorber. Monte Carlo shower development studies would be required to optimize the calorimeter depth and segmentation.

An instrumented range stack behind the hadron calorimeter would identify muons by filtering out all the hadron tracks. This could well be magnetized iron, forming a muon spectrometer. The extremely precise momentum measurements already available from the CCD target and TPC would limit the value of those from such a spectrometer, but the improved acceptance and possibly better hadron background rejection might still justify the modest additional expense.

Table B.2 gives a summary of some of the detector performance resolutions that are derived or estimated in [147].

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