The Superconducting Super Collider

The Clustered Interaction Region Option for the SSC

"THE CLUSTERED INTERACTION REGION STUDY GROUP"

September 30, 1985

APPROVED FOR RELEASE OR
PUBLICATION - O.R. PATENT GROUP
BY...*[signature]*.....DATE *11/3/85*

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED *[signature]*

MASTER

OUTLINE

1. Introduction and Conclusions
 2. Advantages of Clustering IR's for Conventional Facilities
 3. Theoretical Consideration of Optics Issues
 4. Chromatic Properties of Test Lattices
 5. Lattice Designs Including Utility Insertions
 6. Background and Test Beam Calculations
 7. Beam-beam Effects
- Appendix 1. Comparison of Clusters with Two and Three Interaction Regions.
A. Wrulich
- Appendix 2. More Details about the Coherent and Incoherent Beam-beam Models.
M. Furman and S. Peggs

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

1. Introduction and Conclusions

The SSC design so far^{1,2,3,4} has concentrated mainly on having the interaction regions (IR) more or less evenly distributed around the ring. The option of having clustered IR's has been studied in a so-far separate effort by a group at the CDG following earlier studies.⁵ Members of the study group are:

- A. Chao (SSC)
- E. Courant (BNL)
- M. Furman (SSC)
- A. Garren (SSC)
- D. Groom (SSC)
- D. Johnson (FNAL)
- B. Leemann (SSC)
- P. Limon (SSC)
- D. Neuffer (TAC)
- S. Peggs (SSC)
- L. Schachinger (SSC)
- K. Steffen (DESY)
- W. Swanson (LBL)
- R. Talman (SSC/Cornell)
- T. Toohig (SSC)
- A. Wrulich (SSC/LBL)

The study group held a brief meeting on 4/25/85 to list the issues to be addressed. Four areas were identified: (a) the test lattices and more realistic lattices with utility insertions, (b) optics studies on the test lattice including tracking, (c) background, test beam and radiation issues, and (d) beam-beam issues. The aim was not to provide a final design of the SSC with clustered IR's but to make an attempt to compare distributed and clustered options on the identified issues and recommend to the CDG on which option to carry along as the non-site specific conceptual design model.

After an interim meeting on 6/6/85, a workshop was held at LBL on 9/4-6/85 to continue the studies and to summarize the results in preparation for this report. The four issues mentioned above are discussed in some detail in the following chapters. In addition, a chapter is included to discuss the advantages of IR clustering from the conventional facilities point of view. By its nature, this report is to be regarded as interim.

In all issues studied, the group has not found any serious difficulties in the clustered IR's as compared with distributed IR's. To illustrate this point, in the area of chromatic aberrations which was initially expected to be disadvantageous to the clustered case in general, the group has found that the clustered case is quite comparable to the distributed case, being more advantageous in some features and disadvantageous in some others. In fact, as will be discussed in Section 3.3, one of the features that favors clustering suggests that clustering IR's by pairs may offer the best over-all chromatic behavior although the differences are not overwhelming.

Since the clustered IR and the distributed IR options are quite comparable in the accelerator physics issues studied, and since there are potential conventional facilities gains to be made by clustering the IR's, the group recommends that clustered IR's be taken as the current non-site specific conceptual design model. If, at a later time, an accelerator physics, accelerator systems or conventional facilities issue arises to the disadvantage of clustering IR's, the distributed case is still a reliable backup.

2. Advantages of Clustering IR's for Conventional Facilities

In the Reference Designs Study¹ the lattice provided for six IR's uniformly distributed around the circumference of the Collider ring. Of these, four would be perhaps implemented as part of the initial project with two left for later development. While this distributed arrangement may be convenient from the viewpoint of lattice design, there may be overriding reasons from the standpoint of construction costs and operations for preferring to cluster the IR's in three groups of two, two groups of three or a four and a two⁵. Since these are all variants on the same theme, only the case of two three's will be discussed in this section. A possible arrangement is shown in Figure 1 where the IR's are indicated by asterisks.

It is expected that the refrigerator stations will be unmanned remote facilities in conformity with the standard practice in the cryogenic industry, and, on a smaller scale, with the operation of the distributed satellite system of the Tevatron. By clustering the IR's, particularly if one of the clusters is contiguous to the Campus, the need for support facilities is radically reduced. The distances involved within a cluster are comparable to the distances for the experimental areas at Fermilab. Using that as an existence proof it means that stockrooms, shops, maintenance and support services, cafeteria, etc. can be common. The higher concentration of people also means that these can be more extensive and more adequately manned than would be the case with six isolated areas. It is particularly attractive in the case of the cluster adjacent to the injector to locate the IR's on the campus side of the injector. This maximizes the sharing of facilities and utilities. It also gives these areas maximum accessibility to the test beams at the HEB.

From the point of view of environment and safety there are a number of advantages to clustering. Substantial water supply and waste disposal facilities are only required at two locations in the dual-cluster model. Emergency response of fire and guard services is concentrated in these major locations leading to reasonable response times and enhanced plant security.

In a model where the IR's would be contiguous to one another, external beams and test beams could be added without a new land acquisition, leading to enhanced physics capability of the facility.^{6,7,8}

Probably the most important result of clustering the IR's would be in the enhancement of physics productivity. Besides the accessibility of the library facilities, the cluster makes available to the experiment large pools of technical support in Physics Research Equipment Pool, Controls groups, mechanical support, and large computer resources. In addition, the interaction among the physicists involved in the various experiments and with the laboratory staff is facilitated.

The cost savings in the initial construction can be quantified approximately in terms of fewer roads, reduced scope of facilities, etc., but there are more important benefits that are not very easily quantifiable, such as improved fire protection, more adequate technical support.

3. Theoretical Consideration of Optics Issues

3.1 Ground rules for the comparison between distributed and clustered IR's.

In dealing with the optics issues, certain rules were adopted for this comparison. They are listed here and discussed in the next sections. The presence of IR's degrades the chromatic behavior of the storage ring. This makes it essential to anticipate the actual momentum aperture which will be required. We have adopted the following as ground rules:

- i) Operation with $B^* = 1.0\text{m}$ should be possible for all energies of 1.0 TeV and above.
- ii) For fractional momentum offset in the range $-.001 < \delta < .001$, the fractional variation of beta-function at the interaction points, $\Delta B^*/B^*$ should not exceed $\pm 10\%$.
- iii) For the same range of δ the machine tunes should deviate by no more than $\pm .005$.
- iv) Operation will never be required outside the range $-.0015 < \delta < .0015$.

3.2 Super-periodicity and its breaking by magnet errors.

In going from a lattice with distributed IR's to one with clustered IR's the super-periodicity will normally be reduced. This potentially increases the number of damaging resonances which restrict the region of satisfactory operation and may reduce the ultimate performance. But random magnet errors also tend to reduce the super-periodicity. If this symmetry breaking already negates any benefit of higher super-periodicity there would seem to be little advantage to retaining the super-periodicity of the idealized machine. Early

tracking studies such as those of B. Leemann, Douglas and Forest⁹ suggested that this was in fact the case for the SSC. The effect of "turning on" magnet errors was to greatly reduce the momentum aperture. Resonances, previously absent, are thought to be responsible for this. Theoretical estimates by Peggs, Furman and Chao¹⁰ of accumulated orbit errors also suggest that the super-periodicity of distributed IR designs is largely impaired by random magnet errors.

On the other hand, in the long run, there are reasons why magnet errors should not be regarded as the single decisive factor in assessing the potential loss of performance in going to a clustered design. As the design becomes more sophisticated the magnet errors will be reduced and methods will probably be found to restore the super-periodicity to the extent required. That is to say, methods will be found to compensate for the more important errors.

In this report calculations which make full allowance for the presently anticipated random magnet errors are contained in the body of the report while results describing the more idealized machine are contained in Appendix 1, due to A. Wrulich.

3.3 Chromaticity correction.

To avoid unnecessarily large tune spreads and to suppress collective instability it is assumed that the machine chromaticity will be adjusted from its naturally large negative value to a slightly positive value, using sextupoles. This is a global requirement which can easily be satisfied in either distributed or clustered designs. Typically the total chromaticity introduced by the low beta intersection regions is comparable with the total chromaticity from the arcs, but since the desired correction is global, the disposition of the IR's around the ring does not matter.

A distinction between clustered and distributed designs develops when the concept of "local chromaticity" is introduced. Here this term will be used in a very general sense to include any local deviations of the particle orbits or lattice optics which are ascribable to the momentum offset δ . A possible disadvantage of clustered IR's is that chromatic effects from adjacent IR's will accumulate unfavorably. There are schemes such as that of Steffen and Garren¹¹ for the local correction of chromaticity in clustered designs. Data on the optical quality of such designs will be presented though little work has gone into further developing this option so far. According to Appendix 1 this option can be regarded as chromatically inferior in an ideal machine. In the presence of magnet errors this conclusion is not clear.

There is one chromatic effect which can only be controlled by local compensation. The betatron phase advance from one side to the other of an IR without chromatic correction is momentum dependent and this will accumulate in clustered IR's. This will lead to a local phase modulation due to the synchrotron oscillations which is, however, not expected to be serious. The beam-beam interaction already has a more important modulation due to the finite bunch length.

More attention will be paid to global correction schemes. The issues thought to be important can be explained by presenting a clustering scheme due to Neuffer¹² and elaborated by Peggs.¹³ This will exhibit an important sense in which paired IR's are optically superior to individual (i.e. distributed) IR's. It assumes that the momentum dependence $B(\delta)$ is of prime importance.

It follows directly from formula 4.51 of Courant and Snyder¹⁴ that the quantity $\beta^{-1}d\beta/d\delta$ satisfies the approximate equation

$$\frac{d^2}{d\phi^2} \left(\frac{1}{\beta} \frac{d\beta}{d\delta} \right) + 4 \left(\frac{1}{\beta} \frac{d\beta}{d\delta} \right) = 2\beta^2 k(s) \quad (1)$$

where ϕ is the betatron phase (horizontal or vertical) and $k(s)$ is the quadrupole strength at the longitudinal positions. In words the function $\beta^{-1}d\beta/d\delta$ satisfies the harmonic oscillator equation with ϕ as independent variable. The factor 4 shows that this function oscillates at twice the rate of the betatron oscillations. The rhs of Eq. (1) shows that such "beta-waves" are launched primarily from quadrupoles located at points in the lattice where β is large. This corresponds to the well known fact that the chromatic effects of low beta sections come predominantly from the triplets adjacent to the IR. Sextupoles would be represented by another term in Eq. (1) which is essential to chromaticity correction but which will not affect the present discussion.

The beta-waves launched from two successive and identical IR's which are separated in tune by $\Delta Q=1/4$ (or any odd multiple of $1/4$) will cancel outside of the clusters, as illustrated in Fig. 2. The beta function at the interaction point, β^* , is also almost independent of δ . To the extent the beta-wave is dominated by the triplet, and to the extent the tune advance from triplet to IR is $1/4$ (both excellent approximations) both IR's lie at nodes of the beta-waves. In the region between IR's a beta-wave is unavoidable. If the triplets on either side of one IR are identical they interfere constructively. If they have reversed polarity (as they do in all cases studied) they do not cancel perfectly owing to the β^2 factor in Eq.(1).

Existing schemes for adjusting "sextupole families" such as those of Autin and Verdier,¹⁵ of Zyngier¹⁶ and of Montague¹⁷ have been based on Eq. (1) but such schemes become superfluous when the beta-waves from the IR pair cancel as they do here. We are unaware of any previous application of this cancellation.

The results of Appendix 1 show that paired IR's are highly favored, although the same comment about magnet errors that has been made previously weakens this conclusion.

While the cancellation described favors clusters containing an even number of IR's, it is based on the assumption that $dB/d\delta$ plays the essential role in chromatic aberrations. In Section 4 when we make the comparison between clustered and distributed cases neither test lattice has this $dB/d\delta$ cancellation feature. The test lattice for clustered case has 2 clusters, each having 3 IR's. In this configuration a tune advance ΔQ of $1/6$ causes no beta-wave to be launched into the regular arcs. Tracking and optical data for this configuration is presented below as is data for the case in which $\Delta Q = 1/4$. However, we do offer in Section 5 a few lattice examples that take advantage of the $dB/d\delta$ cancellation between IR's.

There is one other chromaticity correction issue which has been raised but not yet resolved having to do with "non-interleaved" sextupoles. Such schemes have superior on-momentum apertures but inferior off-momentum properties. Though some data on this is presented we feel that this issue is largely peripheral to the relative merits of clustered and distributed IR's.

3.4 Connection between $d\beta/d\delta$ and tolerance to misalignments.

Though it is not necessarily the most important reason for controlling $d\beta/d\delta$, sensitivity to misalignment of quadrupoles (particularly those adjacent to the IR's) can set a quantitative limit on what is acceptable. The following formula, due to Piwinski,¹⁸ gives the dispersion D_i at point in the lattice in terms of β_i and the deflection $\Delta x/f$ due to the displacement Δx of the quadrupole of focal length f situated there.

$$D_i = \frac{\beta_i \Delta x / f}{2 \sin \pi Q} \left[\frac{\pi \xi}{\sin \pi Q} + \cos \pi Q \left(\frac{1}{\beta} \frac{d\beta}{d\delta} \right) \right] \quad (2)$$

The first term, containing the chromaticity ξ , will be assumed to be negligible. This formula can be used to relate the tolerable misalignment error in the IR quads to $d\beta/d\delta$, by demanding that the beam enlargement at the crossing point due to momentum spread be small compared to its natural size. Requiring that $D^*\delta$ be less than 7×10^{-6} m and taking β to be 4000m, $\beta^{-1} d\beta/d\delta$ to be 100 in accordance with ground rule ii), one obtains the requirement

$$\Delta x \ll 0.2 \text{ mm}$$

with the value 2×10^{-4} used for δ . This supports the quantitative value which has been used in the ground rule, as such precision is about what is difficult to achieve.

4. Chromatic Properties of Test Lattices

In this section, we present the study results on a few test lattices with clustered IR's. These are called test lattices because they don't have utilities, beam separations, etc., and they are useful because they can be (a) obtained quickly and yet contain the important optical features, and (b) compared directly with distributed case for which we have ample results with test lattices.

There are many variables which can potentially affect the relative quality of different IR configurations. The chart in Fig. 3 includes most of the cases which have been studied with our best present understanding of random magnet errors included.

Dependence of β^* on momentum offset δ for different configurations are shown in Figures 4, 5, and 6. The shaded band, same in every figure for comparison, indicates typical behavior for distributed IR's. According to the ground rules, cases in which curves leave the shaded box through the top or bottom are unacceptable. Figure 7 shows the momentum dependence of the tunes Q_x and Q_y . The shaded boxes have the same significance as before.

Figure 8 compares the dynamic apertures of different lattices using the tracking program PATRICIA. Tracking has been performed only on the test lattices with 3 IR's in each cluster. Particles with given δ are launched from an IP with the initial conditions $x_0 = y_0$; $\dot{x}_0 = \dot{y}_0 = 0$. The dynamic aperture as shown is determined by

$$a^* = \sqrt{x_0^2 + y_0^2} \quad , \quad (3)$$

where x_0, y_0 are the maximum initial amplitudes for which the particle remained stable for a tracking period of 400 turns. The only magnetic imperfections included are the random errors for the dipole magnets.³

The base line for this comparison is the dynamic aperture of the D1 test lattice (6 Tesla bend field, 60° phase advance per cell) with distributed IR's and an interleaved sextupole correction as shown in Figure 8A. The chromatic properties of this lattice are represented by the shaded area in Figure 4(a). The solid circles in Figure 8B indicate the dynamic aperture for a D1 test lattice with clustered IR's and an interleaved, global sextupole correction scheme, whose chromatic properties are shown in Figure 6(c).

The dependence of the dynamic aperture on the random number seed is indicated by additional data points at $\delta = -.15\%$, 0% , and $+.15\%$ obtained for 5 different random seeds.

The effect of a local sextupole correction scheme on the dynamic aperture is addressed in Figure 8C, for the lattice whose chromatic properties are shown in Figure 6(a).

It is obvious that on the basis of the dynamic aperture criterion all three lattices investigated in Figure 8 are quite equivalent.

Included on the chart in Fig. 3 are notations regarding the acceptability of the configuration according to the ground rules. Also there are notations indicating potential problems.

From Figures 4 to 8 one can make the following comments:

- i) In clusters of 3 or 4 (or more) IR's behavior at the inner and outer IR's can be very different. Figures 4(c) and 4(d).
- ii) The local non-interleaved scheme of reference 11 needs revision if it is to become acceptable. Figures 5(a) and 7.

- iii) The local interleaved correction scheme seems to be just as good as the distributed scheme. Figures 6(a) and 6(b). From Fig. 8 it can be seen that the dynamic aperture is also acceptable. This is in the presence of chromaticity correction sextupoles which are very strong. This illustrates the importance of magnet errors as has been discussed in the previous section. When this configuration is analyzed for a machine having ideal elements in Appendix 1, its dynamic aperture is found to be inferior.
- iv) The case with an odd number of IR's per cluster and $\mu = \frac{1}{6}$ is not acceptable according to the ground rules.

5. Lattice Designs Including Utility Insertions

Lattices discussed so far have not included utility insertions. Although we have not decided on a final detailed lattice design with clustered IR's, in this section we will present a few examples that show various designs of clustering the crossing and utility insertions. In Fig. 9 some of the possible dispositions for the SSC are shown schematically. In every case there are 6 IR's (labeled by empty or filled square boxes), and at least 2 utility regions (labeled U). The utility regions are required for injection and for beam abort lines for beams traveling in both directions. In the later designs the utility inserts and the IR inserts have been taken to have the same length and the same total angle of bend after inclusion of dispersion suppressors. This gives ring closure and a regular geometric figure for the ring.

The top plan, labeled (6x1), has previously been called the distributed case but now includes utility insertions which reduce the periodicity as compared to the test lattice case. The notation labeling clusters of 2 or 3 or 4 IR's should be self-explanatory.

Some considerations differentiating these options follow. As discussed in section 3.2 there continue to be reasons to strive for high super-periodicity. Hence the $(2,2,2)_a$ and $(2,2,2)_b$ designs having 3 pairs of IR's have been given one more utility section than is needed for injection and extraction. Even so, when the U's are clustered tightly with the X's as in case (a), one of the abort lines may give unacceptable radiation at the adjacent experimental area. If so, this will force the utilities to be separated as in case (b).

The next level of clustering are the (3,3) cases which have super-periodicity of 2. They have the same radiation problem which can be solved in the same way. Alternatively, one abort can be moved as in (3,3)_b.

Cases with one cluster of 2 and one cluster of 4 are shown at the bottom. Though they have the lowest periodicity they are free of the above mentioned blemishes. They both have very satisfactory clustering with 2 experiments and all specialized accelerator systems sharing a single cluster. In (2,4)_a it is to be expected that the outer IR's in the cluster of 4 will have somewhat different properties than the other 4. In (2,4)_a one of these just barely meets the ground rules introduced above. An attractive start-up phase would have these configured just like the two utility sections. This would restore the super-periodicity. Even at turn-on, these areas would perhaps be available for "high-beta" experiments. The inserts for such experiments and for utilities are very similar.

Configuration (2,4)_b has all the advantages mentioned so far plus one more. Though the chromatic effects of the utility sections are less important than those of the low beta sections they are not negligible. By pairing these inserts some of these effects can be canceled as described above. This configuration would permit injection of both beams from a monopolar booster, which has protons circulating only in one direction. The two proton beams in this case would be injected separately into the two utility straights, which are separated by a distance comparable to the booster diameter. A transfer line comparable in bending and length to 1/4 of the booster would be needed.

Chromatic data for these lattices are shown in Figures 10-12. Separable curves are plotted for the different IR's. All options are satisfactory at this level. Configurations $(3,3)_a$ and $(3,3)_b$ have not yet been analyzed in this way. The $(2,4)$ options are noticeably superior in their dependence of tune on δ . As expected from the discussion above, configuration $(2,4)_b$ is clearly the best as far as these chromatic considerations are concerned. As will be discussed in section 7.4, the $(2, 4)$ and the $(2, 2, 2)$ configurations offer the best resonance behavior as far as the incoherent beam-beam effects are concerned.

On the other hand, $(2,4)$ arrangement has a potential drawback in that the coherent beam-beam effects are to its disadvantage, as will be discussed in Section 7.2. Also, tracking investigations have not been performed with it. The study group, as mentioned in Section 1, recommends that the clustered option be adopted for the non-site specific conceptual design model, but is not ready to recommend which of the $(3,3)$, $(4,2)$ or $(2,2,2)$ arrangements is most highly recommended.

6. Background and Test Beam Calculations

Since in the clustered lattices interaction regions are 3 km apart instead of 10 km, without shielding any background caused by interactions at the nearest IR will be worse. The question addressed here is whether any additional shielding is necessary for the clustered case.

There are three potential sources of background. The first is elastically scattered protons from one IR which shower in the triplet in the next IR. It turns out that the rate for this process is miniscule. The second is diffracted protons from one IR which either hit the next IR triplet or the beam pipe somewhere upstream causing spray to hit the detector in the next IR. This is potentially a problem, but scraping between IR's in a large dispersion region can cure it. The third background source is inelasticities. Most of these particles hit the immediate IR triplet or are swept out by the beam separation dipoles before they enter the arcs. Those small-angle particles with momentum near the beam momentum could also be scraped where the dispersion is large. Both the diffracted protons and small-angle on-momentum inelasticities are sources of background for distributed and clustered IR's, so that shielding will be necessary in both cases.

6.1 Elastic Scattering and Diffractive Processes

For clustered IR's there was some concern that a proton which underwent an elastic or diffractive interaction at one IR would hit one of the IR quads just before the next IP, showering into the detector there. This was investigated using the RDS cross sections and test lattices in the elastic case, and a lattice with vertical beam separation in the diffractive case.

For the elastic case the cross section assumed in the RDS is 30 mb, 1/4 of the total cross section, and the rate of elastic scatters is approximately 1 per crossing, or 30 MHz. The angular dependence, again from the RDS¹, is exponential in $-t$, or in the square of the scattering angle,

$$\frac{d^2\sigma}{d\theta_x d\theta_y} = 1.6 \times 10^{11} \text{ mb } \frac{1}{2\pi} e^{-\frac{\theta_x^2}{2\theta_0^2}} e^{-\frac{\theta_y^2}{2\theta_0^2}} \quad (4)$$

Where $\theta_0 = 9 \mu\text{r}$. Note in figure 13 that this is the cross section for $-t \leq 0.5 \text{ Gev}^2$. The angular divergence of the beam at the interaction point is $7 \mu\text{r}$ at 20 Tev, and 95 percent of the beam is inside $30 \mu\text{r}$.

For $\beta^* = 1\text{m}$, an angular kick at the IP is transformed into a displacement downstream through

$$x_2 = \beta_2 \cos\Delta\theta x_1 + \beta_2 \sin\Delta\theta x_1' \quad (5)$$

The maximum displacement is reached from an angular kick at the IP where β is large, and the phase advance is such that $\sin \Delta\theta = 1$. These conditions hold in the IR triplet, where β_2 is approximately 4000 m and the phase advance is $\pi/2$. At this point, the displacement is

$$x_2 = \beta_2 x_1' \quad (6)$$

The kick needed to produce a 1 cm displacement at the triplet is $160 \mu\text{r}$, or $-t = 10\text{Gev}^2$. From figure 13, the cross section at such large $-t$ is negligible.

Essentially all elastic scatters are inside the dynamic aperture by a similar argument. From tracking studies we know that the dynamic aperture is close to 0.4 mm at the interaction point. This corresponds to a displacement at the IR triplet of

$$x_{TR} = 4000 \times .4mm = 2.5cm$$

A displacement this large is almost never reached by single elastic scattering.

The single diffractive dissociation cross section has approximately the same t dependence as the elastic cross section, so displacements due to angular kicks at the IP are negligible. The total diffractive cross section is comparable to that for elastic scattering, around 30 mb. However we now have the added complication that scattered protons have lost momentum. The x dependence of the diffractive cross section assumed in the RDS is

$$\frac{d\sigma}{dx} \propto (1-x)^{-1} \quad (7)$$

where x is the ratio of the diffracted proton's momentum to the beam momentum. Note that in the small region of $\Delta p/p$ with which we are concerned the cross section is essentially flat in momentum.

Diffracted protons will have different trajectories from beam protons due to the difference in momentum and angle. The difference in trajectories will be greatest where the dispersion is large.

$$x_2 = \sqrt{\beta_2} \sin \Delta\theta x_1' + \eta_x \frac{\Delta p}{p} \quad (8)$$

In the lattice studied, there are seven quadrupoles where the dispersion reaches the maximum value of 4 m between IR's in a cluster. The phase advances from the previous IP to these quads are 1.25, 1.4, 1.6, 1.75, 1.9, 2.1, and 2.25 (in units of 2π). The effect of an angular kick at the IP on the displacement is maximum at 1.25, 1.75, and 2.25, so most of the diffracted protons will be lost at the first few of these quads.

The displacement from a 40 μ r kick (down by 4 orders of magnitude in the cross section) at the IP is 1mm here ($\beta = 350$ m), but the displacement from the dispersion is 1.5cm for $\Delta p/p = 0.00375$. Diffracted protons with $\Delta p/p$ greater than 0.00375 account for about half of the single diffraction cross section. So about half of the diffracted protons will stay in the machine and the other half will hit the beam pipe in the quadrupoles in the phase trombone where the dispersion is large. In the lattice studied here, the first such quadrupole is 3 km from the next IP, and the last is 1 km from the next IP. It may be necessary to add shielding in a large dispersion region between IR's. The impact of these particles on the detector in the next interaction region needs further study, but note that the same study is needed for whether the IR's are clustered or distributed.

6.2 Inelastic Processes

A numerical integration was performed to calculate the energy carried by charged mesons leaving a round bore in the IR triplet. The RDS cross section was used:

$$x_R \frac{d^3N}{dx_R dp_\perp^2} = \sum_{\text{mesons}} A_i (1-x_r)^{n_i} \left[\frac{C_m}{(1+p_\perp/p_0)^m} \right] \quad (9)$$

C_m is chosen so that the integral over $p_\perp^2$ is unity, and m varies from 6.5 to 9.0, depending on the pseudorapidity.

If the total cross section is 120 mb and the luminosity is $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, 768 W goes into scattered particles. We take 3/4 of this to be inelastic, including single and double diffraction. Half goes each way, and of this 52% goes into charged mesons in each direction due to inelastic scattering. The total charged multiplicity (in both directions) is 48.4^{19} .

Charged particles were tracked through the IR triplet, and the point where they left a 1.5 cm beam pipe was found. Since the nuclear interaction length in the coil or nearby metal is about 15 cm this is within a meter of where the energy is actually deposited. A 1 cm radius collimator was inserted just before the first quadrupole in the triplet. Figure 14 shows the differential and figure 15 the integral distribution of the energy deposition. Of the 150 W produced at the IP, 86 W leaves the triplet. Most of the low energy particles will be swept out in the beam separation dipoles. Some high energy particles will remain propagating down the beam pipe. As is the case for the diffracted protons, a high dispersion region with a collimator will be needed between IR's to clean up the beam. This issue needs to be dealt with whether the IR's are clustered or distributed.

6.3 Test Beams in a Clustered IR Lattice

The questions of test beams for equipment calibration and of secondary particle beams for fixed target physics were examined by various participants in the Snowmass summer study, 1984. G. Dugan studied the possibility of charged secondary hadron beams from the interaction regions (Proceedings, p.456). C.T. Murphy studied the providing calibration beams for the large detectors in the "garages" (proceedings, p.468). D. Neuffer examined the possibility of beams from gas jets in the machine (Proceedings, p.479). The conclusions from these studies were that such beams are feasible. They argue for placing the detector assembly areas on the outside of the ring to take advantage of these possibilities. These calculations were based on an early clustered IR lattice due to D. Johnson (Figure 16a). The separation between IR's in this lattice is 2.1km, while the offset between the projection from

one IR and the midpoint of the next IR is 28m. The angular offset between the beam at one IR and the succeeding one is 26.4mrad. N. Gelfand and A. VanGinneken examined the question of muon backgrounds produced in an IR from the preceding one. They calculated that there would be <500 muons/sec over a 12mx12m detector from this source. Their preliminary conclusions were that a 2km separation with 28m offset looks quite safe. (See rapporteur view-graphs.) This lattice had two clusters of three IR's. In the summer study the locations of the utility straight sections were not explicitly treated. In particular the possible background in some IR's from placement of the injection and abort between IR's was not treated.

Figure 16b details a lattice due to A. Garren which explicitly includes the utility straight sections and has a high order of symmetry. In this lattice, as indicated, the separation between the IR's is 3.6km with an angular offset of 126.mrad leading to a separation of 222m between the center point of one IR and the projection from the preceding one. This requires a substantial bend and a longer beam line to get test beams from one IR to the "garage" of the succeeding one. By the same token it probably is a more favorable configuration for dedicated secondary beam areas. The land area requirements projected in the Siting Parameters Document would have to be adjusted to accommodate this possibility.

7. Beam-beam Effects

7.1 Outline of the Beam-beam Studies Performed

The beam-beam interaction in the SSC will cause effects which can be classified in three ways. A beam-beam effect can be i) coherent or incoherent, ii) due to short range or long range collisions, iii) with normal or 'pac-man' collision sequences.

Incoherent studies of the beam-beam interaction presume that a single test particle interacts on each turn with several bunches in the other beam. Coherent studies assume that each bunch in one beam interacts as a whole during the collisions with the other beam, in (at least) one of a set of possible multipole modes. The simplest possible behaviour is the dipole mode, when the bunches respond linearly to each other, like rigid structureless macroparticles. These dipole modes are nonetheless quite complicated, since each bunch in one beam affects every bunch in the other beam, albeit indirectly, and since there will be of order 10,000 bunches in the SSC.

The individual beam-beam interactions themselves will occur not only at short range, in the centers of the six interaction regions, but also at long range in the wings of the IRs, before the beams are completely separated. Thus, bunches separated longitudinally by 15 metres will suffer one collision every 7.5 metres upto a distance of (say) 90 metres from the interaction point, with a total of 24 long range collisions in each IR. Nominally the beams are separated at each interaction by an almost constant amount of about 7 beam sigmas at 20 TeV, assuming a crossing angle θ^* of 50 μ rad, a normalized emittance of 10^{-6} metres, and a collision β^* of 1.0 metre.

If each beam is full of uniformly spaced bunches, all particles or bunches will see a 'normal' sequence of collisions. In practice, however, there will be at least one particle-free abort gap of as much as several microseconds duration in both beams, in order that the abort kickers may ramp without massive quenching of the superconducting magnets. Bunches near the edge of an abort gap will therefore see an irregular 'pac-man' sequence of collisions²⁰. It is conjectured that these bunches will be especially prone to poor lifetime and loss, causing the abort gap to 'eat' its way through the populated bunches, much like the protagonist in the video game 'Pac-Man'.

Coherent and incoherent studies, both in simulation and in theory, have focussed on finding the critical short range tune shift parameter ξ as a function of betatron tune Q , with various configurations of IRs and collision sequences. In contrast with the incoherent studies, the coherent studies have emphasized situations with many bunches per beam, i.e. with long range collisions, because the number of coherent azimuthal modes is proportional to the number of bunches. Coherent simulations are computation time limited, however, and so have only been made for the dipole mode, with a realistic number of about 24 long range collisions per interaction region, but with only about 100 or 200 bunches per beam. This reduction from about 10,000 bunches per beam is justified by the empirical observation that the critical short range tune shift parameter varies only slowly with the number of bunches per beam.

Furthermore, the coherent simulation results are in excellent agreement with the linear theory of the dipole mode, which is tractable for an arbitrary number of bunches per beam^{21,22}. So far the linear theory has only been applied to distributed lattices. Although the linear theory is not yet solved

for the general pac-man situation, simulations reported below show that the degradation of critical tune shift parameter with an increasing number or size of abort gaps is quite slow. It appears possible to extend the 'coherent, normal' theory beyond the dipole mode to higher multipole modes. Preliminary indications are that the dipole mode is indeed the most dangerous, as expected, and that the danger of higher modes decreases rapidly with their order. This can be understood by noting that the $1/R$ tail of the beam-beam kick is not as highly non-linear as the short range kick.

Short range incoherent beam-beam effects have been extensively studied, in theory and by simulation, because they are usually critical in limiting the performance of storage rings. Consequently, predictions based on short range theory and simulation are expected substantially correct in comparisons of distributed and clustered lattices. For proton storage rings the one-dimensional incoherent theory of modulation sideband overlap²³⁻²⁷ agrees very well with simulation results²⁸, in both distributed and clustered lattices. So far, however, the incoherent theory is fully quantitative only for round beam short range collisions, and incoherent simulations including long range interactions have yet to be performed.

The extension of the theory to include long range collision sequences should be straightforward, and is already semi-quantitatively understood. In particular, the part of the theory which describes how collision sequences interfere with each other to suppress or enhance beam-beam resonances carries over directly into the case of many long range interactions. Nonetheless, it is expected that the short range collisions will more or less dominate the incoherent effect, even after the inclusion of many long range collisions, as is also true for the coherent effect.

7.2 Coherent Effects

First results of the one-dimensional effect, in the coherent dipole approximation, for distributed IR's and uniform bunch population in the beams, have been presented previously²¹. Updated results are presented here for several beam configurations including distributed and clustered IR's, and gaps in the beam population (but not both simultaneously). The bunches are evenly spaced and may or may not populate the beams uniformly. Thus the two beams are not necessarily identical, since the number of gaps and their lengths may be different in the two beams. All the interaction regions are identical, however.

The configuration of the machine is described by the number of IR's, the number of bunches within an IR, the number of bunches within the arcs, and the position and size of the gaps. For a beam with no gaps

$$N_b = N_{ir} \cdot m + m'_1 + m'_2 + \dots \quad (10)$$

where N_b is the number of bunches per beam, N_{ir} is the number of IR's, m is the maximum number of bunches that can fit simultaneously within an IR and $m'_1, m'_2, \dots$ are the minimum number of bunches that can fit simultaneously within the arcs. More details about the coherent simulation model can be found in appendix 2. In the following, only the results are presented.

In each configuration presented here the largest value of the head on tune shift parameter ξ for which there is stability is plotted, as a function of Q , the betatron tune. The calculation is such that the period of all curves presented is (at most) 3 units in tune, and in all of them $N_{ir}=6$ and $m=13$.

Fig. 17(a) compares the stability limits between distributed and clustered IR's. The region above the curve is unstable. The solid line corresponds to distributed IR's with 20 bunches in each of the arcs; since there are 13 bunches in each of the IR's, $N_b=198$. The dashed line represents the results for the (3,3) clustering scheme in which the arcs are such that the number of bunches in them is $m'_i=(1,10,1,1,10,1)$, so that $N_b=102$. The long arcs are 10 times as long as the short ones, but the tunes of all 6 arcs are set to be equal. The dash-dotted line is similar except that $m'_i=(5,50,5,5,50,5)$, corresponding to $N_b=198$. or regions AB and EFG of the curves all three cases overlap within the accuracy of the calculation.

Fig. 17(b) shows the results simulating the 'pac-man' effect, when there are gaps in the beams, for the distributed IR case, when the "would-be" number of bunches in each arc is 20. The solid line describes the results with no gaps. The dashed line corresponds to the case in which there is one gap in each beam of length 13. The beam configuration in which the simulation is initially launched is such that the two gaps are about to "collide" at the center of one of the IR's. Wherever the dashed line is not shown, it coincides with the solid one within the accuracy of the plot. Results have been obtained for a few other gap configurations, e.g. when only one of the beams has a gap and when there are up to 6 gaps per beam. All these results are quite similar to the ones shown, and for the sake of simplicity they are not all exhibited (however, as the total gap size becomes substantial, i.e., about $N_b/2$, the stable region becomes larger and larger, as it should be expected).

Fig. 17(c) shows results for the (3,3) clustered IR case when the tune of the short arcs is kept fixed at integer+1/4. The tune of the 2 long arcs varies so that the tune of the ring is what is shown. The solid line corresponds to $m'_i=(1,10,1,1,10,1)$ and the dashed line to $m'_i=(5,50,5,5,50,5)$, i.e., $N_b=102$ and 198, respectively. The two lines coincide within plot accuracy everywhere except in the DE section.

Fig.17(d) shows results for the (4,2) clustered IR case when the tune of the short arcs is kept fixed at integer+1/4. Results are presented for the $m'_i=(1,1,1,10,1,10)$ case, with $N_b=102$ only. Note that additional coherent stopbands are created by this particular clustered IR arrangement.

Note that the results are quite insensitive to the number of bunches in the arcs m'_i . This feature is understood analytically for the distributed IR case with no gaps, and it remains true in the cases presented here. Thus it appears that a fairly realistic simulation of the SSC, with $N_b \approx 10,000$, does not require nearly as many bunches provided the number of bunches within the IR's is set at its realistic value $m=13$.

Since $\xi = 0.005$ represents the approximate operating value for the SSC, it is seen that the results pose little significant constraints on the choice of tune. More important for this study is the fact that there is little variation between the results for the distributed IR cases without gaps and those of clustered IR's with or without gaps. The qualitative behavior of the stability curves is similar in all but the (4,2) clustered IR case. The more detailed structure that appears in this case is due to the broken reflection symmetry in the ring structure. The additional stopbands make this case less favored as far as coherent beam-beam is concerned. This is in contrast with

chromatic aberration studies which favor it. Higher modes are expected to behave similarly, i.e., clustered IR cases will behave similarly to the corresponding distributed arrangements except for the (4,2) case.

7.3 Incoherent Effects

The 'overlap criterion' is an analytic test which is applied to a potential incoherent Beam-beam resonance situation, in order to predict the presence or absence of chaotic behaviour. A brief description of the key physical ingredients in the theory of modulation sidebands is included in appendix 2, and more detailed references are available elsewhere^{24-26,28}.

Without going into details, the overlap criterion for short range collisions is

$$G_N(\alpha) > (\pi q Q_s^3)^{1/4} / [4.f_N^{1/2} . \Sigma \xi] \quad : 0.0 \leq f_N \leq 1.0 \quad (11)$$

$$> 0.11 / f_N^{1/2} \quad \text{for nominal SSC parameters} \quad (12)$$

where $G_N(\alpha)$ is plotted in figure 18 for various values of N, the order of the resonance. The motion of a test particle is described in this expression by α , the (initial) amplitude of betatron motion measured in units of beam size, and q, the tune modulation amplitude, which occurs at a frequency Q_s , assumed to be the synchrotron frequency. Values of $q=0.002$ and $Q_s=0.005$ are assumed as nominal SSC parameters. It is important to notice that according to (11), when the suppression factor f_N is zero, chaos does not occur.

The sense of (11) is that if the (initial) amplitude of a trajectory is above a critical value α_c , the expression is true, and the amplitude of the test particle will fluctuate by large amounts, on the time scale of several synchrotron periods. However, chaos will only occur, roughly speaking, if the instantaneous tune is modulated across the resonance in question, that is, if the average tune of the particle is within $\pm q$ of the resonance. It is in this sense that $2q$ acquires the significance of the 'resonance (tune) width'.

As an example of the application of the overlap criterion and figure 18, the critical amplitude on a sixth order beam-beam resonance with $q=0.003$, (not 0.002), should be approximately $\alpha_c = 1.8$ if $f_N = 1$. Figure 19 shows the result of a numerical simulation of this situation, using the code EVOL₂₈, in a scan of the chaotic amplitude bounds versus the unperturbed tune, where the thick lines show the amplitudes outside which chaotic behaviour is not observed. The critical amplitude is measured as about 1.7, and the observed resonance (tune) width is only somewhat larger than $2q$, as shown by the thin lines. The simulation is in good agreement with the theoretically predicted behaviour.

In this simulation there were six head-on collisions of nominal strength, in a (3,3) clustered lattice with a total tune near $72 + 2/6$, and with a tune advance of $3 + 1/6$ between adjacent interaction regions. This configuration ensures that all the interactions interfere constructively for the sixth order resonance, to make $f_6 = 1$, while for the third order resonance $f_3 = 0$, since there was no mechanism for odd resonance excitation. With the exception of this suppression factor, the clustered or distributed configuration of an SSC lattice does not enter the overlap condition (11). Hence the evaluation

of f_N , which refers explicitly to the resonance order, is crucially relevant to a discussion of the effects of clustering on the incoherent beam-beam effect.

7.4. The Suppression of Incoherent Beam-beam Resonances in IR Clusters

The suppression factor for an even resonance resulting from head-on collisions can be shown to be

$$f_N = \frac{\sum_i (\xi_i e^{jN\phi(i)})}{\sum_i \xi_i} \quad (13)$$

where $\phi(i)$ is the betatron phase, and where the possibility of (complex) vector cancellation is explicit. It is clear that $f_6 = 1$ for the configuration of figure 19, since in that case $N \phi(i)$ advances by an integer multiple of 2π between two adjacent collisions, and the vectors in (13) are all perfectly collinear. If the tune advance between adjacent collisions in one cluster is now changed to $3 + 2/9$, the triad of vectors in each cluster will independently sum to zero length. Thus f_6 will be zero, no matter how the extra phase necessary to restore the total tune of the lattice to $72 + 2/6$ is distributed between the two long arcs which connect the clusters.

Figure 20 shows how the simulated critical amplitude varies as f_6 changes from 1.0 to 0.0, as a function of tune advance between IRs in one cluster. Figure 19 constitutes one data point in figure 20. There is good agreement between measurement (thick lines) and theory (thin lines), as described by (11) and (13). Also shown is the lack of perfect cancellation in a simple pac-man configuration, where one of the bunches in the opposing beam is missing, and there are only four collisions per turn.

Now it is assumed that each collision is of equal strength, and that the fractional tune advance between IR's in a cluster is fixed at 1/4 (or 3/4), which appears to be the optimal value for the best chromatic behaviour of a lattice. It will also be conservatively assumed that the phase advance between clusters is maximally bad for the given resonance under study, so that the vector contributions in (13) due to each cluster add colinearly. This is reasonable because while in principle it will be possible to measure and set the phase advance (within reason) between any two interaction regions, in practice the tolerance will naturally be tighter between IR's which are close together. These assumptions allow the behaviour of clusters to be studied individually, as a function of only N, the resonance order, and n, the number of IRs per cluster, since

$$f(N,n)_{\text{cluster}} = \left| \sum_{i=1,n} (e^{jNn\pi/2}) \right| / n \quad (14)$$

$$= \left| \sin (nN.\pi/4) / \sin (N.\pi/4) \right| / n \quad (15)$$

(Note that this expression remains true when long range collisions are included in the sum.)

n	2	4	N 6	8	10
1	1	1	1	1	1
2	0	1	0	1	0
3	1/3	1	1/3	1	1/3
4	0	1	0	1	0

Table A1 The suppression factor $f(N,n)_{\text{cluster}}$, on a resonance of order N for a cluster with n IRs.

Table A1 shows the behaviour of $f_{\text{cluster}}(N,n)$ for individual clusters of 1 (i.e. distributed), 2, 3 and 4 interaction regions. Combining clusters to find f_N for a complete lattice under the assumption of maximally bad inter-cluster phase advance, $f_N = f_{\text{cluster}}(N,3)$ for a (3,3) lattice, and $f_N = f(N,2)$ for both (2,2,2) and (4,2) lattices, since $f(N,2) = f(N,4)$, as shown in the Table.

These suppression factors in different lattices lead to the worst case critical amplitudes which are illustrated in Figure 21, for incoherent beam-beam resonances with nominal SSC parameters. A sufficient condition for satisfactory operation of the SSC on the tune plane diagonal is that a line segment of length 0.012, representing the tune spread of the beam, can be located between resonances at all amplitudes of interest. While this condition is somewhat restrictive for the distributed lattice, there is a noticeable improvement for (3,3) clustered lattices if there is concern for particles with $\alpha = 4.0$ amplitudes. There is a further improvement in going to a (2,2,2) or a (4,2) clustered lattice for amplitudes as low as $\alpha = 2.3$ for average tunes near a sixth order resonance.

7.5 Summary of Beam-beam Results

For nominal SSC parameters, there are expected to be no critical problems with the beam-beam effect in any of the lattices. Provided a finite set of resonances are avoided in the tune plane, the performance of the SSC will not be adversely affected. Closer comparisons of the relative merits of different lattices rest on the fraction of the tune plane which is available.

Coherent dipole effects of the beam-beam interaction have been studied extensively. The maximum allowed head-on tune shift parameter ranges from about 0.02 to 0.05, if important resonances are avoided. These numbers do not vary greatly between configurations with different lattices, different abort gaps, or with different numbers of long range collisions per interaction region. In all cases they are an order of magnitude larger than $\xi = 0.002$, the nominal SSC head-on tune shift parameter. In all cases except one, the coherent dipole instability occurs when the total betatron tune of the SSC approaches an integer, a value which will be avoided in any case.

The (4,2) clustered lattice results also exhibits instability at half-integer tunes - which will also have to be avoided anyway. This is because the 180° rotational symmetry of the other lattices, under which the same two bunches collide twice per turn under the same conditions, is broken. When coherent multipole modes are included, it is expected that the (4,2) lattice will contain more instability stopbands, just as in the dipole case. This is regarded as a potential disadvantage of the (4,2) lattice since the tune values will be somewhat more restricted, although these coherent resonances are expected to be weak.

Clusters with an even number of interaction regions allow more freedom in the tune plane under the influence of the incoherent beam-beam effect. (It is assumed that the fractional tune advance between adjacent clustered IRs has been fixed at 1/4 or 3/4.) Resonances of order $N = 4, 6, 8$ and 10 must be avoided in the distributed case. Under normal conditions sixth order and tenth order resonances are partially suppressed by going to three IRs per cluster, while they disappear with two or four IRs per cluster. Clustering does not affect fourth order or eighth order incoherent resonances.

In short, unless the SSC parameters deviate substantially from the assumed nominal values, the beam-beam effect is expected to be an important, but not a critical, phenomenon. There is some stabilization of incoherent beam-beam effects gained from clustering, especially with a (4,2) or a (2,2,2) clustered lattice, due to the improved control of the phase advances between collisions. Some of the incoherent gain with a (4,2) lattice is lost due to less stable coherent beam-beam effects.

References

1. SSC Reference Design Studies (RDS), May 1984.
2. E.D. Courant, D.R. Douglas, A.A. Garren and E. Forest, "SSC Test Lattices," SSC-19 (1985).
3. SSC Aperture Task Force studies. See SSC-SR-1013 (1985) for recent results.
4. 'Realistic' Lattices Workshop, 5/29-6/4/85, LBL.
5. R. Diebold and D.E. Johnson, Proc. 1984 Summer Studies on Design and Utilization of the SSC, Snowmass, edited by R. Donaldson and J.G. Morfin, page 399.
6. A. DeRujula and R. Ruckl, "Neutrino and Muon Physics in the Collider Mode of Future Accelerators," ECFA 84/85, Proceedings of the ECFA-CERN Workshop, Lausanne 21-27 March 1984, Vol. II.
7. T. Toohig, "Interaction Regions in an SSC with Clustering," Fermilab, 8 Oct. 84.
8. C.T. Murphy, "Getting Calibration Beams to Detector Garages," Snowmass, 1984, p. 468.
9. B. Leemann, D. Douglas, and E. Forest, "Tracking the SSC Test Lattices." SSC-24 (1985).
10. S. Peggs, M. Furman, and A. Chao. SSC-20 (1985).
11. K. Steffen, DESY HERA 85-13, and A. Garren and K. Steffen, SSC-note "Toward an SSC Test Lattice Design With Two Achromatic Clusters of Intersection Regions." (Draft only.)
12. D. Neuffer, SSC Aperture Workshop Summary. SSC-21, SSC-TR-2001, p. 116.
13. S. Peggs, Clustered IR Study Group, 6 Jun. 1985.

14. E. Courant and H. Snyder, "Theory of the Alternating Gradient Synchrotron." Ann. of Phys. 3, 1-48 (1958).
15. B. Autin and A. Verdier, Report CERN-ISR-LTD.
16. H. Zyngier, Report LAL 77/35 (1977).
17. B. Montague, LEP Note 165, 30/7/1979.
18. Piwinski, Private communication with A. Wrulich.
19. D. Groom, SSC-N-37 (1985).
20. R. Diebold, Ann Arbor Workshop, P5, UM HE 84-1 (1984).
21. M. Furman, A. W. Chao, SSC-28 (1985), and references therein
22. E. Forest, M. Furman, SSC-32 (1985).
23. E. Courant, ISABELLE Project Technical Note 163 (1980).
24. L. Evans, CERN SPS/DI(MST)/Note/81-1 (1981).
25. L. Evans, J. Gareyte, CERN SPS/82-8 (DI-MST) (1982).
26. L. Evans, J. Gareyte, IEEE Trans. Nucl. Sci. NS-30, 2436 (1983).
27. S. Peggs, Ann Arbor Workshop, p56, p58, UM HE 84-1 (1984).
28. S. Peggs, CESR CBN 84-12 (1984).

Figure 1. A possible cluster layout.⁵

Figure 2. Chromatic behaviour of pairs of IR's having tune separation of $\frac{1}{4}$ (or an odd multiple of $\frac{1}{4}$). Schematic.

Figure 3. Chart of the lattices studied. The numbers refer to subsequent figure numbers.

Figure 4. Momentum dependence of β^* . 3 IR's per cluster. $\Delta Q \cong 1/6$. Interleaved sextupoles. Typical distributed results lie in the shaded bands.

Figure 5. Same as Fig. 4, but non-interleaved sextupoles and $\Delta Q \cong 1/4$.

Figure 6. Same as Fig. 4, but $\Delta Q = 1/4$.

Figure 7. Tunes Q_x and Q_y as a function of δ .

Figure 8. Comparison of dynamic aperture for three different lattices with random dipole errors based on PATRICIA tracking data. A: Lattice with distributed IR's and interleaved global sextupole correction. B: Lattice with clustered IR's and interleaved global sextupole correction. C: Lattice with clustered IR's and interleaved local sextupole correction.

Figure 9. Various possible SSC lattice configurations including utility sections for injection and abort lines.

Figure 10.

Figure 11.

Figure 12.

Figure 13. The differential elastic scattering cross section.

Figure 14. Differential energy deposition in quadrupoles in charged meson inelastic production.

Figure 15.

10-3-85
TET

Figure 16(a).

Possible clustered straight section layout.
DE JOHNSON - RD3-B

54

DR. 425m DISPERSION KILLER needed: Injection
 ϕ_0 485m PHASE ADJUST
 S.S. 850m 1/2 SEPARATOR SECTION
 I.R. 1856m
 Abort RP

Figure 16(b).

Figure 17(a).

Figure 17(b).

Figure 17(c).

Figure 17(d).

Figure 18. The function $G_N(\alpha)$ for various resonance orders, N .

Figure 19. Observed and Theoretical chaotic bounds for a (3, 3) clustered lattice with $q = 0.003$ and a local tune advanced of $3 + 1/6$ between IR.

XBL 859-11387

Figure 20. Observed and predicted critical amplitude as a function of adjacent IR phase advance in a (3, 3) lattice with $q = 0.003$.

Figure 21. Worst case incoherent beam-beam resonances for various cluster arrangements with tune advance integer + 1/4 between IR's.

Appendix 1. Comparison of Clusters with 2 and 3 Interaction Points*
W. Wrulich

For both types of cluster the following sextupole arrangements were compared with respect to dynamical aperture and variation of tunes

(Q_x, Q_z) and interaction beta values (β_x^*, β_z^*) with momentum:

- global correction scheme, i.e. 2 families of sextupoles with equal strengths for each family
- local interleaved correction
- local noninterleaved correction

For the two local correction schemes the sextupoles in the regular arcs were tuned to compensate the local cell chromaticity only. An optic with approximately $n + 1/4$ tune variation between two interaction points were used.

I. 3-IR Cluster (2 per Ring)

Figures A1 and A2 show the variation of tune with momentum for the sextupole distributions C3.G (global), C3.LI (local interleaved) and C3.LN (local noninterleaved).

For the two local correction schemes only sextupoles between the two outer interaction points were used (6 1/2 cells). Since the tune advance between 2 IP's is $n + 1/4$, the linear beta variation with momentum produced by sextupoles is locally compensated within the cluster.

To avoid large sextupole strengths, for the local interleaved compensation both drift spaces left and right from a cell quadrupole were equipped with sextupoles.

The variation of horizontal and vertical interaction beta with momentum for the inner and outer interaction points of the cluster are shown in Figures A3 to A6.

Figure A7 shows the variation of dynamical aperture (i.e. maximum horizontal amplitude for 100% emittance coupling) with momentum.

The tracking was performed for 4 different initial conditions $(x, x', z, z') = (x, 0, z, 0)$, $(-x, 0, z, 0)$, $(x, 0, 0, z')$, and $(-x, 0, 0, z')$ over 50 revolutions.

II. 2-IR Cluster (3 per Ring)

To achieve a cancellation of the linear beta beat produced by sextupoles, in the local correction schemes a corresponding number of cells outside the cluster was added for the local correction, so that the arrangement got the following structure:

For the local noninterleaved compensation one pair of sextupole was used in each section V and H.

Figures A8 and A9 show the variation of horizontal tune and β^* with momentum. Due to symmetry reasons both interaction points in the cluster show the same β -variation. Even tune and beta variation in both planes are equal, as shown in Figures A10 and A11. The slight difference between the two planes is a result of the difference in tune ($Q_x=85.265/Q_z=85.285$).

Figure A12 shows the variation of dynamical aperture with momentum. The symmetric behaviour of the global compensation (C2.G) is caused by a drastic

increase of the $2Q_x - 2Q_z$ driving term for positive momentum deviation.

Figure A13 shows the variation of 2 different tunes with momentum. For both cases the dynamical aperture is reduced for positive momentum deviation as shown in Figure A12. That means a change of the resonance driving term has caused the loss in stability and not a shift of the tune onto a resonance as a result of the momentum variation.

A motion of the tune away from the coupling resonance $2Q_x - 2Q_z$, should bring an increase in stable amplitude for positive momentum deviation.

Comparison of Different Aspects

1) Noninterleaved/Interleaved

For the noninterleaved distribution the dynamical aperture is extremely large for small energy deviations. Even for the tails of the energy distribution it is sufficiently large.

But orbit errors (i.e. focusing errors due to orbit displacements in the sextupoles), sextupole displacement errors and quadupole errors have not been taken into account yet. The noninterleaved distribution tend to be very sensitive to these errors.

On the other hand also the momentum dependence of tunes and interaction betas are intolerably large and it is difficult to compensate them (by introducing more sextupole families).

2) Global Compensation/Local Compensation

An acceptable sextupole distribution should provide

- large dynamical aperture
- small variation of tune with momentum
- small variation of β^* with momentum

All local compensations in the examples given above are clearly worse than the global distributions. For the specific examples there is no preference for the local sextupole compensation

3) 3-IR Cluster/2-IR Cluster

Structures with 2-IR cluster are clearly preferable compared to 3-IR clusters. The dynamical aperture is significantly larger and momentum dependence of tune and β^* are considerably weaker.

The maximum deviations of tune and $(\frac{\Delta\beta}{\beta} / \frac{\Delta\rho}{\rho})$ over the momentum range of ± 0.001 , and the minimum stable amplitude are compared in the following table for 2-IR and 3-IR clusters:

Parameters for the Range $-0.001 < \frac{\Delta\rho}{\rho} < .001$	3-IR	2-IR	
$ \Delta Q_x $ max	2.8×10^{-3}	1.8×10^{-3}	
$ \Delta Q_z $ max	2.9×10	1.7×10	
$\left \frac{\Delta\beta_x^*}{\beta_x} \frac{\Delta\rho}{\rho} \right $ max	10.2	2.8	[%]
$\left \frac{\Delta\beta_z^*}{\beta_z} \frac{\Delta\rho}{\rho} \right $ max	9.6	2.9	[%]
(A_x) min	3.47	6.35	[mm]

Figure A1.

(3IR) Horizontal Tune vs. Momentum

Figure A2. (3IR) Vertical Tune vs. Momentum

Figure A3. (3IR) β_{x^*} vs Momentum
(Inner Interaction Point)

Figure A4. (3IR) β_z^* vs Momentum
(Inner Interaction Point)

Figure A5. (3IR) β_x^* vs Momentum
(Outer Interaction Points)

Figure A6. (3IR) β_z^* vs. momentum
(Outer Interaction Points)

Figure A7. (3IR) Maximum Stable Horizontal Amplitude vs. Momentum

Figure A8. (2IR) Horizontal Tune vs. Momentum

Figure A-9. (2IR) β_{x^*} vs. Momentum

Figure A10. (2IR) Vertical Tune vs. Momentum

Figure A11. (2IR) β_x^* vs. Momentum

Figure A12. (2IR) Maximum Stable Horizontal Amplitude vs. Momentum

Figure A13. (2IR) Variation of Tune with Momentum

1. The Coherent Simulation Model

Here we describe in more detail the coherent beam-beam simulation. Each bunch is fully described by its transverse coordinate x and its slope x' . The bunches are assumed to interact only within the IR's, which are identical. A given bunch interacts every time it moves a distance $L/2$, where L is the interbunch spacing. In between two successive collisions it is drifted by a simple 2×2 drift matrix. In the arcs, it is transported by a 2×2 phase advance matrix from the end of an IR to the beginning of the next one.

There is one short range collision at the center of the IR plus several long range interactions away from the center. We assume these interactions between bunches to be kicks which are linearized as

$$\begin{aligned} \text{short range: } \Delta x' &= (1/f^*) \cdot \Delta x \\ \text{long range: } \Delta x' &= -(1/f^*) \cdot p \cdot \Delta x \end{aligned} \tag{A1}$$

where the parameter $p = 2 (\sigma^*/d)^2$. The distance d between the design trajectories at the collision point is determined by the beam configuration, crossing angle and interbunch distance. With the nominal values of $\theta = 50 \mu\text{rad}$, $L = 15 \text{ m}$, $\beta^* = 1 \text{ m}$, and $\sigma^* = 7 \mu\text{m}$ the parameter p takes on the value 7×10^{-4} for the long range collision nearest to the center of the IR. For the other long range collisions, p decreases inversely proportional to the square of the distance to the center. We start the tracking by assigning random values, within a certain range, to x and x' for all the bunches. After

each turn we evaluate the maximum amplitude x for all the bunches at the center of the IR's; if this exceeds $2\sigma^*$ we call the motion unstable. If no instability is found after a large number of turns, typically 100, we call the motion stable. By gradually increasing ξ we find its critical value for a given tune; from this we construct the stability boundary in the (ξ, Q) -plane for a given beam configuration.

2. Ingredients of the Modulation Sideband Overlap Theory of the Incoherent Beam-Beam Effect

In the nominal configuration of the SSC it is assumed that there are 6 collisions per turn of strength $\epsilon = 0.002$, with a total maximum tune shift of 0.012, since the average tune of a particle with a one dimensional betatron oscillation of amplitude α is

$$\begin{aligned} Q(\alpha) &\rightarrow Q_0 + \sum \xi & \text{as } \alpha \rightarrow 0 \\ &\rightarrow Q_0 + 4 \cdot \sum \xi \alpha / 2 & \text{as } \alpha \rightarrow \infty \end{aligned} \tag{A-2}$$

In this expression Q_0 is the unperturbed tune. This is the first ingredient of the incoherent beam-beam model.

The second ingredient of the model is that while equation (A-2) expresses the average tune of the particle, there is also a periodic tune modulation of amplitude q , at a frequency Q_s , which is usually taken to be the synchrotron oscillation frequency. Such a modulation arises, for example, from energy oscillations coupled through a finite linear chromaticity. (Other potential tune modulation sources include magnet power supply ripple, at dangerously low frequencies of around a few Hertz.) Thus, a test particle actually has an instantaneous tune on turn t of

$$Q(\alpha, t) = Q(\alpha) + q \cdot \cos(2\pi Q_s \cdot t) \quad (A-3)$$

Values of $q = 0.002$, and $Q_s = 0.005$ are nominal SSC parameters.

The third and final ingredient of the model, the location of Q_0 close to a rational fraction a/N , taken together with the previous two ingredients, creates a family of synchrotron sideband resonances at average tunes of

$$Q(\alpha) = a/N + b \cdot Q_s/N \quad (A-4)$$

where a and b are integers, and where N is the order of the beam-beam resonance. This means that particles with an appropriate initial amplitude and phase will circulate on one or more islands in phase space. However, if the calculated separation of these islands in phase space is less than their calculated width the 'overlap criterion' is met, and a particle will not be confined to a small region of phase space. Rather the motion of the particle will be chaotic, looking to the casual observer as if its motion is not due to the deterministic laws of a closed set of equations, but rather is influenced by an exterior source of noise.

It is important to note that the width of a sideband island (A-4) is very small for trajectories whose instantaneous tune is not modulated across the fundamental resonance. That is, since there is no similar decrease in the separation of neighboring sideband islands for such a trajectory, a chaotic particle will at least be limited in its range of accessible tunes to a band of amplitudes which is roughly given by the conditions

$$a/N - q < Q(\alpha) < a/N + q \quad (A-5)$$

This equation should be interpreted as stating that the 'resonance (tune) width' of a beam-beam resonance is simply $2q$, almost always a very small number, providing that the resonance is chaotic. If the resonance is not chaotic,

tic, its tune width is less than Q_s/N , typically smaller than $2q$ (for the SSC) by an order of magnitude. The narrow tune width of the chaotic region does not necessarily mean that the amplitude width is narrow, however, since the slope $dQ/d\alpha$ can also be very small, as shown by (A-2), especially if q and ξ are commensurate, and especially if α is large.