Strong Motion Duration and Earthquake Magnitude Relationships

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Strong Motion Duration and Earthquake Magnitude Relationships

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DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
1. Introduction

Earthquake duration is the total time of ground shaking from the arrival of seismic waves until the return to ambient conditions. Much of this time is at relatively low shaking levels which have little effect on seismic structural response and on earthquake damage potential. As a result, a parameter termed "strong motion duration" has been defined by a number of investigators to be used for the purpose of evaluating seismic response and assessing the potential for structural damage due to earthquakes. Figure 1 shows an example of an accelerogram, and the differences between the strong motion duration and the total duration.
For design, the duration of the design basis earthquake generally is not an issue since for response spectrum, equivalent static, and even time history methods of analyses, only the peaks of the response are used in developing design seismic forces. However, the duration of strong earthquake motion is a significant parameter for earthquake damage potential when considering low cycle fatigue, soil liquefaction, soil settlement, and inelastic structural response. In addition, for one method of generating synthetic earthquake time histories, it is necessary to establish a time history envelope function consistent with the strong motion duration of the potential earthquakes governing the design/evaluation level seismic loadings.

This report presents methods for determining strong motion duration and a time history envelope function appropriate for various evaluation purposes, for earthquake magnitude and distance, and for site soil properties. There are numerous definitions of strong motion duration (Refs. 3, 5, 6, 7, 8, 10, 16, and 17). For most of these definitions, empirical studies have been completed which relate duration to earthquake magnitude and distance and to site soil properties. Each of these definitions recognizes that only the portion of an earthquake record which has sufficiently high acceleration amplitude, energy content, or some other parameters significantly affects seismic response. Studies have been performed which indicate that the portion of an earthquake record in which the power (average rate of energy input) is maximum correlates most closely with potential damage to stiff nuclear power plant structures (Ref. 8). Hence, this report will concentrate on energy based strong motion duration definitions.

2. Background

Earthquakes are initiated by rupture and slippage along geologic faults. As a result, body waves are emitted outward from the source of the earthquake (zone of energy release). Body waves are of two kinds: P (primary, longitudinal, or compressional) and S (secondary, transverse, or shear). Particle motion for P-waves is back and forth along the direction of wave travel. Particle motion for S-waves is back and forth perpendicular to the direction of wave travel. Body waves generally arrive at a surface location along a nearly vertical wave path. This is because they originate from the earthquake source at depth and travel horizontally through deep, highly competent material and then vertically upward. When body waves reach the ground surface they originate surface waves which travel parallel to the surface. There are many kinds of surface waves. Of greatest interest in earthquake engineering are L (Love) and R (Rayleigh) waves. Particle motion in L-waves is in a horizontal plane perpendicular to the direction of wave travel. In R-waves of a given frequency, particle motion is elliptical in vertical planes.

At an instrument location, P-waves are first to arrive, followed by S-waves and finally surface waves. As a result, earthquake acceleration records may be viewed as having three segments. The first segment represents the P-wave arrivals which are characterized by low amplitude. The second segment is mainly associated with direct S-wave arrivals which have relatively high amplitudes and high frequencies. The duration of the direct S-wave segment is mainly controlled by the duration of rupture at the causative fault. There is continuation of P-wave arrival during the second segment of the motion. The last segment is closely related to the surface wave presence or to the delayed indirect body wave arrivals and has low frequencies and amplitudes ranging from low to moderately high, depending on the local soil conditions, site topography, and the distance from the zone of energy release along the fault. The end of the direct S-wave arrivals and the beginning of the surface wave arrivals are difficult to distinguish clearly and typically there is overlap of the second and third parts of earthquake records. Of the three segments, the first contains P-waves only; the second contains P and S; and the third has some P and mostly S, L, and R-waves.
Earthquake duration increases with increasing earthquake magnitude. When an earthquake is of a larger magnitude, the dimensions of fault rupture is larger. Since the dislocation velocity does not change significantly with magnitude, the duration of fault rupture, which is closely related to the duration of the earthquake record, generally increases with increasing earthquake magnitude.

Earthquake duration also increases with increasing distance from the recording site to the zone of energy release of the causative earthquake. This phenomenon is due to the presence of slowly propagating surface waves arriving from long distance as well as of late time indirect body wave arrivals due to increased numbers of refractions, reflections, and scatterings of body waves over the longer travel path. Earthquake duration also is greater at soil sites than at rock sites. Earthquake records on soil sites have an additional long period portion not seen in rock records. This long period portion is associated with the difference in dynamic characteristics between hard and soft soils (high impedance ratio).

The generation of synthetic time-histories to match broad banded design spectra may follow two approaches. One approach is to use an actual earthquake time history which produces a response spectrum shape close to the required response spectrum. The Fourier phase spectrum from this time history may be retained and the Fourier amplitudes adjusted, frequency by frequency, until the resulting response spectrum closely envelopes the required response spectrum. The other approach uses a random Fourier phase spectrum and a deterministic time-envelope function. A typical time-envelope function as shown in Figure 2 consists of three portions: \( t_r \) is the rise time, \( t_m \) is the time of maximum power and is that portion which most closely corresponds to the strong motion duration, and \( t_d \) is the decay time. In Philippacopoulos, 1989, Kennedy states that the rise and decay time durations are relatively unimportant but should typically be taken to be about 1/7 and 5/7 of \( t_m \), respectively.

![Figure 2: Duration Envelope Function](image-url)
3. Strong Motion Duration Definitions by Various Investigators

There are a number of definitions of strong motion duration which have been developed by many investigators. Each of these definitions is based upon the fact that the damage potential of an earthquake is a function of the energy of the earthquake, and that the majority of the total energy associated with any earthquake is contained in portions of the earthquake time history which is much shorter in time than the total duration. Probably the most common strong motion duration definition is that given by Trifunac-Brady (1975) who proposed a definition of strong motion duration based on a percentage of cumulative energy. It is defined as:

\[ T_{95} - T_{90} = T_{95} - T_{90} \]  

(1)

where \( T_{95} \) and \( T_{90} \) represent the times at which 95% and 5%, respectively, of the cumulative energy are reached. Thus, this duration includes 90% of the total cumulative energy. The cumulative energy \( E(t) \) of an acceleration time history \( A(t) \) at time, \( t \), is given by:

\[ E(t) = \int_{0}^{t} A^2(t) \, dt \]  

(2)

Alternative definitions of strong motion duration have been developed because it was judged that, for stiff structures (2 to 10 hz) such as those in nuclear power plants, Eq. 1 provides too long of an estimate of strong motion duration for many records. Many records contain a long tail of oscillatory ground motion with lesser acceleration at the end of the record which continues to input energy but at a substantially lesser rate than the earlier portion of the record. These definitions of strong motion duration are denoted, \( T_D \), from NUREG/CR-5347 (Philippacopoulos, 1989) and, \( T_D' \), from NUREG/CR-3805 (Kennedy, et. al., 1984) as described below.

\[ T_D = T_{75} - T_{95} \]  

(3)

where \( T_{75} \) and \( T_{95} \) are the times at which 75% and 5, respectively, of the cumulative energy are reached. Thus, this duration includes 70% of the total cumulative energy.

NUREG/CR-3805 attempted to define a duration estimate which is directly related to damage for stiff engineered structures such as shear wall structures typically found in nuclear power plant sites. The study was conducted using the twelve earthquake time histories shown in Table 1. The time of maximum response for both elastic and nonlinear structure representative of degrading stiffness and degrading strength shear walls was considered. Four shear wall structure models with different initial elastic natural frequencies were used. Using the records shown in Table 1, the time of maximum displacement response for elastic response, and for two levels of nonlinear response was determined. The upper bound of maximum response was noted for the complete set of shear wall structure models and was compared to various times associated with percentages of the total cumulative energy. This study concluded that an upper bound on the time of maximum response, \( T_u \), can be reasonably approximated by:

\[ T_u = \max \left( \frac{t_{95}}{t_{95}}, \frac{t_{90}}{t_{90}} \right) \]  

(4)
where: \( T_{0.75} \) is the time at which 75\% of the cumulative energy contained in the accelerogram has been reached.

\( T_{\text{0.05}} \) is the time associated with the first zero crossing of the accelerogram following the maximum positive or negative acceleration.

and that the corresponding strong motion duration is then given by:

\[
T_{\text{D}'} = T_{\text{m}} - T_{0.05}
\]  

(5)

This definition is slightly different than \( T_{\text{D}} \), Eq. 3, recommended in NUREG/CR-5347 (Philippopoulos, 1989). This definition may sometimes yield values of strong motion duration greater than those given by Eq. 3. When the peak ground acceleration occurs later in time than \( T_{0.75} \), \( T_{\text{D}'} \) will be greater than \( T_{\text{D}} \). This is seen in Table 1 for both the Olympia, 1949, and Pacoima Dam, 1971 records. In other cases, the two definitions yield the same duration value.

### Table 1
Comparison of Duration Measures for Selected Input Accelerograms

<table>
<thead>
<tr>
<th>Earthquake Record (Component)</th>
<th>( T_{\text{D}} )</th>
<th>( T_{\text{D}'} )</th>
<th>( T_{\text{m}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taft, Kern. Co., 1952, (S69E) [M=7.7]</td>
<td>10.3</td>
<td>10.3</td>
<td>28.1</td>
</tr>
<tr>
<td>Olympia, WA., 1949 (N86E) [M=7.0]</td>
<td>14.8</td>
<td>15.6</td>
<td>17.3</td>
</tr>
<tr>
<td>El Centro Array No. 12, Imperial Valley, 1979 (140) [M=6.9]</td>
<td>9.6</td>
<td>9.6</td>
<td>18.6</td>
</tr>
<tr>
<td>El Centro Array No. 5, Imperial Valley, 1979, (140) [M=6.9]</td>
<td>3.4</td>
<td>3.4</td>
<td>8.2</td>
</tr>
<tr>
<td>Pacoima Dam, San Fernando, 1971 (S14W) [M=6.6]</td>
<td>5.6</td>
<td>6.1</td>
<td>7.4</td>
</tr>
<tr>
<td>Hollywood Storage PE Lot, San Fernando, 1971 (N90E) [M=6.6]</td>
<td>5.4</td>
<td>5.4</td>
<td>11.7</td>
</tr>
<tr>
<td>Artificial, (R.G. 1.60)</td>
<td>9.4</td>
<td>9.4</td>
<td>13.0</td>
</tr>
<tr>
<td>UCSB Goleta, Santa Barbara, 1978 (180) [M=5.1]</td>
<td>3.0</td>
<td>3.0</td>
<td>9.7</td>
</tr>
<tr>
<td>Gilroy Array No. 2, Coyote Lake, 1979, (050) [M=5.7]</td>
<td>2.2</td>
<td>2.2</td>
<td>7.5</td>
</tr>
<tr>
<td>Cholame .Array No. 2, Parkfield, 1966, (N65E) [M=5.6]</td>
<td>1.4</td>
<td>1.4</td>
<td>9.2</td>
</tr>
<tr>
<td>Gavilan College, Hollister, 1974 (S67W) [M=5.2]</td>
<td>1.1</td>
<td>1.1</td>
<td>1.6</td>
</tr>
<tr>
<td>Melendy Ranch barn, Bear Valley, 1972 (N29W) [M=4.7]</td>
<td>0.8</td>
<td>0.8</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Magnitude, \( M = M_s \) from 5.9 to 8.0; \( M_s \) below 5.9

Unlike the NUREG/CR-5347 definition of strong motion duration, \( T_{\text{D}} \), most of the other proposed definitions for duration commonly used in engineering practice do not attempt to relate the definition of duration to damage. Most definitions for duration are expressed in terms of some percentage of the total energy contained in the accelerogram as measured by the Arias intensity (Arias, 1970). Other definitions in use are based on the time between the first exceedance of some PGA level (typically 0.05g) and the last exceedance of this level, or upon the time at which the acceleration exceeds some set level.

Esteva and Rosenblueth (1964) defined the strong motion duration as the duration of an equivalent ground motion with uniform intensity per unit time. They proposed one of the first empirical relationships for predicting duration for a given magnitude and site to source distance.
Bolt (1973) defined "bracketed duration" of a record, as the elapsed time between the first and last acceleration excursions greater than a given level. He proposed the use of 0.05 or 0.1g acceleration levels. This definition requires that the absolute values of the acceleration of a record exceed some level. Therefore, records having a peak acceleration smaller than 0.05g have zero duration.

Housner (1965) proposed a conservative relationship from which to estimate the upper bound duration of ground motion. This upper bound estimate was based on enveloping the strong motion durations of 16 horizontal accelerograms from pre-1957 west coast U.S. earthquakes. Strong motion duration was not defined definitively in Housner (1965); however, Trifunac (1975) reports that results from Bolt (1973) for the bracketed duration of acceleration greater than 0.05 g yield essentially the same durations as reported in Housner (1965).

Hisado and Ando (1976) defined the duration of ground motion to be the total time from the beginning of the record to the time when the amplitude of the wave becomes equal to one-tenth of the peak acceleration.

Vanmarcke and Lai (1980) proposed a definition of strong motion duration, $S_o$, as the time in which the total energy of a ground motion record is distributed uniformly at constant average power. By this definition, $S_o$ is given in terms of the total cumulative energy, $E_m$ (determined from Eq. 2 integrated over the entire duration), maximum ground acceleration, $p_g a$, and the predominant period of the strong motion phase of the earthquake motion, $T_o$ (commonly ranges from 0.2 to 0.6 seconds, determined by counting zero crossings of a record) as:

$$S_o = \left[ 5.42 - 2 \ln T_o + 2 \ln \left( \frac{E_m}{p_g a^2} \right) \right] \frac{E_m}{p_g a^2}$$  (6)

McCann and Shah (1980) defined the strong motion phase of an accelerogram as that part of the time history that exhibits a "consistent" root mean square (rms) or power level. They further proposed that the intensity of strong ground shaking could be realistically represented by a statistical average such as the root mean square, and the duration over which the rms persists. A cumulative root mean square function forms the basis for McCann and Shah's duration definition. For a discretized accelerogram containing $m$ points, the rms is determined for each point $n$, where $n$ varies from 0 to $m$:

$$r m s_n = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \alpha_i^2}$$  (7)

The beginning of the strong motion is obtained by forming the cumulative rms function of the time reversed accelerogram and noting the time at which the cumulative rms function takes on a negative slope. The end of the strong ground motion is obtained in a similar manner using the original time history of the accelerogram.

Theofanopulos-Watabe (1989) proposed a definition of strong motion duration based upon the distribution of wave energy over the time history, as opposed to the cumulative energy distribution defined by Eq. 2. The following function of time was proposed:

$$E(t) = I(t + dt) - I(t)$$  (8)
where $dt$ is the time increment and $I(t)$ is the Arias intensity function (Eq. 2 $\times (\pi/2g)$) with integration limits from 0 to $t$.

This function represents the energy added at each time increment of the earthquake motion. The weighted by $E(t)$ average value ($\mu$) of $t$ was proposed as follows:

$$\mu = \frac{\sum_{i=1}^{N} u \cdot dt \cdot E(u \cdot dt)}{\sum_{i=1}^{N} E(u \cdot dt)}$$  (9)

The weighted by $E(t)$ standard deviation ($\sigma$) of $t$ was proposed as follows:

$$\sigma = \left[ \frac{\sum_{i=1}^{N} (u \cdot dt - \mu) E(u \cdot dt)}{\sum_{i=1}^{N} E(u \cdot dt)} \right]^{1/2}$$  (10)

where $N$ is the total number of data points, $N = T_m / dt$ and $T_m$ is the total duration of the record.

The end of the strong motion proposed by Theofanopulos-Watabe (1989) is obtained as:

$$T_z = \mu + \sigma$$  (11)

The strong motion duration is the difference between the time at which a threshold value of $E(t)$ is exceeded and $T_z$, the end of strong motion. Theofanopulos-Watabe took this threshold value as equal to the value of $E(T)$ at the end of the strong motion interval $E(T_z)$.

NUREG/CR-3805 (Ref. 8) concluded that the portion of an earthquake record in which the power (rate of energy input) is maximum correlates most closely with potential damage to stiff nuclear power plant structures. The Trifunac-Brady, NUREG/CR-5347, Esteva-Rosenblueth, Vanmarcke-Lai, McCann-Shah, and Theofanopulos-Watabe strong motion duration definitions are all based on the energy content of an earthquake record. Furthermore, all of these definitions except Trifunac-Brady attempt to represent the time associated with maximum power. These energy based strong motion duration definitions are considered in the remainder of this report.

4. Empirical Strong Motion Duration Relationships

Studies of past earthquake time histories have been performed to develop relationships between earthquake magnitude and strong motion duration using many of the strong motion definitions described above. Theofanopulos and Watabe (1989) conducted extensive empirical studies using the Trifunac-Brady, Theofanopulos-Watabe, and McCann-Shah strong motion duration definitions. Theofanopulos and Watabe developed the following empirical equation relating strong motion duration, $D$ (seconds), earthquake magnitude, $M$, distance to causative fault, $R$ (km), and site soil conditions, $S$ (0, 1, and 2 for hard, intermediate, and soft soil, respectively):

$$D = a + b e^{cM} + dR + eS$$  (12)

$a$, $b$, $c$, $d$, and $e$ are empirical constants as given in Table 2. The distance to causative fault, $R$, is defined as the shortest distance between the site and the surface projection of the fault. Hard soil or rock is defined to be that with shear wave velocity of 3500 fps (1100 mps) or greater. Soft soil sites are characterized by deep alluvium (depth greater than 16m) or otherwise soft sedimentary deposits. Intermediate sites are those which can be classified as neither hard nor soft. Note that Dobry, 1978 has also developed empirical relations for the Trifunac-Brady definition but the
Theofanopulos-Watabe work is more extensive and more recent. Esteva and Rosenblueth had developed one of the earliest magnitude-duration relations in 1964. The relation which was independent of soil conditions fits into the format of Eq. 12 with the constants given in Table 2. Vanmarcke and Lai studied 140 earthquake records of which 118 were from soil sites. They developed the following relations for their definition of duration, \( S_o \):

\[
S_o = 4.4(M - 4.2) \quad \text{for all distances}
\]

\[
S_o = 0.115(R + 30) \quad \text{for all magnitudes}
\]

### Table 2

**Empirical Constants for Strong Motion Duration Equation**

<table>
<thead>
<tr>
<th>Strong Motion Duration Definition</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trifunac-Brady</td>
<td>2.2Cl</td>
<td>0.02489</td>
<td>0.860</td>
<td>0.05335</td>
<td>2.883</td>
</tr>
<tr>
<td>Theofanopulos-Watabe</td>
<td>-13.230</td>
<td>4.36900</td>
<td>0.253</td>
<td>0.03673</td>
<td>2.121</td>
</tr>
<tr>
<td>McCann-Shah</td>
<td>-2.707</td>
<td>0.02811</td>
<td>0.757</td>
<td>0.03290</td>
<td>1.224</td>
</tr>
<tr>
<td>Esteva-Rosenblueth</td>
<td>0</td>
<td>0.02</td>
<td>0.74</td>
<td>0.3</td>
<td>-</td>
</tr>
</tbody>
</table>

Strong motion duration vs. magnitude relations for all the energy based definitions except that from NUREG/CR-5347 (not available) are shown in Figure 3. The strong motion values shown in the figure are for a rock site located in the near field of the causative earthquake (15 km from the earthquake source) with the exception of Esteva-Rosenblueth which is independent of soil conditions and Vanmarcke-Lai which is independent of both distance and soil conditions. Figure 3 demonstrates that strong motion duration increases with increasing earthquake magnitude.

The Esteva-Rosenblueth, Vanmarcke-Lai, McCann-Shah, and Theofanopulos-Watabe strong motion duration definitions are based on the time of maximum power and, thus, would be appropriate to use for the design/evaluation of stiff nuclear power plant structures. The Trifunac-Brady definition is expected to give durations longer than the time associated with maximum power including most of the earthquake record energy content (90%). Earthquake magnitudes governing design earthquake ground motion generally range from a lower bound magnitude of 5 - 5.5 to an upper bound magnitude of 7.5 - 8. From Figure 3 and considering durations by the Esteva-Rosenblueth, Vanmarcke-Lai, McCann-Shah, and Theofanopulos-Watabe definitions, it may be seen that for the magnitude 5 - 5.5 range, durations range from about 3 to 5 seconds and, for the magnitude 7.5 - 8 range, durations range from about 10 to 20 seconds. Note that durations by the Vanmarcke-Lai and Theofanopulos-Watabe definitions agree closely.

To illustrate the effects of distance and soil conditions on magnitude, strong motion duration vs. earthquake magnitude by the Trifunac-Brady and Theofanopulos-Watabe definitions are plotted in Figures 4 and 5 respectively. Each figure includes rock and soft soil conditions and near-field (15 km) and distant (50 km) earthquake locations. Figures 4 and 5 demonstrate that, by either definition of strong motion duration, the soil conditions have a significant effect on duration and the distance has a less significant effect. The effect of increased distance from 15 km to 50 km is that strong motion duration increased by about 1.5 seconds at all magnitudes for both definitions. The
effect of going from a rock site to a soft soil site was an increase in strong motion duration at all magnitudes of about 6 seconds for the Trifunac-Brady definition and of about 4 seconds for the Theofanopulos-Watabe definition. These figures demonstrate that duration increases with increasing distance and that duration for rock sites is lower than that for soil sites. The qualitative reasons for these observations were discussed previously in Section 2.

Figure 3 Strong Motion Duration vs. Earthquake Magnitude for Energy-Based Duration Definitions (Rock site, 15 km)

Empirical studies of the NUREG/CR-5347 duration definition, $T_D$, which includes 70% of the cumulative energy (Eq. 3) are not available as they are for Trifunac-Brady, Theofanopulos-Watabe, McCann-Shah, Vanmarcke-Lai, and Esteva-Rosenblueth. Values of strong motion duration, $T_D$, computed in accordance with Eq. 3 are available for the records summarized in Table 1. In addition, $T_D$ values have been computed for records from the 1989 Loma Prieta earthquake and other earthquakes as reported in Geomatrix, 1991. The $T_D$ values from Table 1 and from Geomatrix, 1991 are plotted as a function of magnitude in Figure 3. Also plotted on this figure are the strong motion duration vs magnitude relations for the other definitions which attempt to use the time associated with maximum power (rock site at 15km, where applicable). Even though the earthquake data are from a variety of soil conditions and distances, durations from this data tend to be somewhat lower than from the empirically developed relations (with the exception of McCann-Shah). It should be noted that the Loma Prieta data at Magnitude 7 is anomalously low due to the particular fault rupture characteristics of this earthquake. In addition, these are much fewer data points than were used to establish the relations in Eqs. 12 and 13 and Table 2. Therefore, there is insufficient data available to establish a reliable relation for $T_D$ as a function of earthquake magnitude. An approach for establishing such a relationship will be proposed in Section 6.
Figure 4 Trifunac-Brady Strong Motion Duration vs. Magnitude

Figure 5 Theofanopulos-Watabe Strong Motion Duration vs. Magnitude
Duration Requirements for Seismic Analyses of Nuclear Power Plants

Section 3.7.1. (II.1 b) of NUREG-0800 provides the minimum strong motion duration requirements for linear structural analyses, using site-independent response spectra such as Regulatory Guide 1.60. The strong motion duration requirements in NUREG-0800 are as follows:

1. The total time duration should be between 10 and 25 seconds.
2. The corresponding stationary phase strong-motion duration should be between 6 seconds and 15 seconds.
3. If site-specific information indicates duration estimates outside these bounds, the site-specific values should be used.

The strong motion duration definition used for commercial nuclear power plants is one that includes the time of maximum power. Duration is defined in this manner because it was felt that Trifunac-Brady and other similar “long” definitions of duration were overly conservative. It has been shown that there is a portion of many records which can extend for a long period of time in which the last 20 to 25% of the earthquake energy is contained but which is a low acceleration levels and which will have very little effect of the seismic response of nuclear power plant type structures. Furthermore, in NUREG/CR-5347 (Philippacopoulos, 1989) Kennedy gives two reasons for not using artificial time histories with strong motion duration in excess of 15 seconds, or total durations longer than 25 seconds. These are:

1. The high frequency power can be concentrated near the start of the record with the low frequency power concentrated near the end of the record. In this way, the high and low frequency modes of a 5% or more damped structure will not combine
because the high frequency response is damped out before the low frequency response becomes strong. Thus, combined response can be severely unconservatively biased.

2. If random phasing is assumed for all Fourier harmonics, then modes have an increased probability of coming into essentially worst-case phasing (absolute sum combination) at some time as strong motion durations are increased to very long times. Thus, combined responses can be severely overestimated when excessively long strong motion durations are used.

The SRP provisions primarily apply to the time duration envelope parameters as shown in Figure 2 which are used to develop artificial accelerograms. For this purpose, a "short" duration definition such as that which from NUREG/CR-5347 (70% on the cumulative energy of the record) is most appropriate as neither such a definition nor the time duration envelope function include a long tail of low acceleration input. This provides a reasonable definition of effective strong motion duration for stiff structures (2 to 10 hz). Such "short" duration definitions include NUREG/CR-5347, Theofanopulos-Watabe, McCann-Shah, Vanmarcke-Lai, and Esteva-Rosenblueth. Studies which have correlated strong motion duration defined in this manner with earthquake characteristics such as magnitude, distance from earthquake source to the site, and site soil conditions have been performed for all definitions except that from NUREG/CR-5347 (Eq. 3). However, the means of relating these definitions to the parameters of the time envelope function (Figure 2) have not been performed. In the following section, an approach for estimating strong motion duration corresponding to 70% of the cumulative energy and of estimating duration envelope function parameters as a function of earthquake and site characteristics is presented.

6. Approach for Estimating NUREG/CR-5347 Duration and Duration Envelope Parameters as a Function of Magnitude

The Trifunac-Brady, Vanmarcke and Lai, and NUREG/CR-5347 strong motion duration definitions are all based on relatively simple expressions related to the energy content of an earthquake record as given by Eq. 2. In addition, it is assumed that the parameters defining the duration envelope function shown in Figure 2 can be related to the energy content. Also, it is useful that there are empirical studies relating duration by the Trifunac-Brady and Vanmarcke and Lai definitions to earthquake magnitude and other characteristics. Using the information listed above, an approach for estimating NUREG/CR-5347 duration and duration envelope parameters as a function of magnitude will be developed in this section. This duration estimation approach is based on the following assumptions:

1. Consider the duration envelope function as shown in Figure 2. Assume this to be an acceleration vs. time function from which a parameter proportional to cumulative earthquake energy by utilizing Eq. 2 can be calculated.

2. For typical duration envelope functions [i.e., time of maximum amplitude, \( t_m \); \( t_r = (1/7) t_m \); \( t_d = (5/7) t_m \); \( E_{50\%}, E_{75\%}, E_{90\%}, E_{95\%}, t_{0.05}, t_{0.75}, t_{0.95}, T_D \) and \( T_{T-H} \) (i.e., strong motion durations corresponding to 70 and 90% of cumulative energy, respectively), and \( S_2 \) (Vanmarcke-Lai duration) will be computed. It is assumed that these computed values are proportional to corresponding values for actual earthquake records.
3. The Vanmarcke-Lai and Trifunac-Brady strong motion duration definitions are evaluated in this section for earthquake records idealized by the duration envelope function. The empirical duration-magnitude relation for $S_o$ (Vanmarcke-Lai) is independent of distance and site conditions (Eq. 13a). The empirical duration-magnitude relation for $T_{TB}$ (Trifunac-Brady) depends on both distance and site conditions (Eq. 12). When $T_{TB}$ is evaluated from the duration envelope function, near field (15km) and rock site conditions are assumed.

4. The duration envelope function as shown in Figure 2 is only appropriate for earthquake records which do not have long tails of low acceleration ground motion oscillations. Dobry, et.al., 1978 and others suggest that these long tails are caused by amplification of the soil of multi-path body wave arrivals and by surface wave effects. These phenomena are most significant at soil sites and/or when the earthquake source is very distant from the site. Assume that earthquake records measured in the near field of the earthquake source are predominantly the direct shear wave arrival segment with a minimum of surface wave input and contributions due to refractions, reflections, scatterings, and high impedance ratio effects. As a result, a rock site and near field (15km) conditions are judged most appropriate for relating $T_{TB}$ to the envelope function.

5. Assume that a relation between the NUREG/CR-5347 duration, $T_D$, and earthquake magnitude can be estimated from the Trifunac-Brady duration, $T_{TB}$ vs. magnitude empirical relation by using the near-field (15km), rock site relation and scaling this relation by a scale factor, SF, which is the ratio of $T_D/T_{TB}$ as determined from the duration envelope calculations (Item 2 above).

6. Assume that $T_{TB}$, $T_D$, and $S_o$ can all be related to the time of maximum amplitude, $t_m$ from the duration envelope calculations (Item 2 above). From these relations, $t_m$ may be estimated as a function of earthquake magnitude.

Duration envelope function calculations using the assumptions listed above are presented in the Appendix to this report. The results of these calculations for maximum energy content, $E_m$, time at which 5%, 75%, and 95% of the total cumulative energy, $t_{0.05}$, $t_{0.75}$, $t_{0.95}$, Trifunac-Brady duration, $T_{TB}$, NUREG/CR-5347 duration, $T_D$, and Vanmarcke-Lai duration, $S_o$, are:

$$E_m = A^2_m (t_m / 3) + A^2_m (t_m) + A^2_m (t_d / 3) = 1.286 A^2_m t_m$$

<table>
<thead>
<tr>
<th>$t_{0.05}$</th>
<th>0.159$t_m$</th>
<th>$T_{TB}$</th>
<th>1.235$t_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{0.75}$</td>
<td>1.059$t_m$</td>
<td>$T_D$</td>
<td>0.900$t_m$</td>
</tr>
<tr>
<td>$t_{0.95}$</td>
<td>1.394$t_m$</td>
<td>$S_o$</td>
<td>1.286$t_m$</td>
</tr>
</tbody>
</table>

The values summarized above are illustrated in Figure 7. For the typical duration envelope function [i.e., time of maximum amplitude, $t_m$; $t_d = (1/7) t_m$; $t_4 = (5/7) t_m$], $t_{0.05}$ and $t_{0.75}$ fall within the maximum amplitude region and $t_{0.95}$ falls within the decay region. $T_D$ is significantly shorter than both $T_{TB}$ and $S_o$, where $T_D$ is shorter than $t_m$ and $S_o$ is longer than $t_m$. From the relations tabulated above, it may be shown that $T_D = 0.7S_o$, which is reasonable since $T_D$ includes 70% of the total energy and $S_o$ includes 100% of the energy.
Using the values given above, the duration vs. magnitude relation given by Eq. 12 (with Trifunac-Brady coefficients, $R = 15\text{km}$, and $S = 0$ [rock]) can be modified by scale factor, $SF$, to provide the following NUREG/CR-5347 duration vs. magnitude relationship:

$$T_D = SF\left(a + b e^{\alpha M} + d(15) + e(0)\right)$$

$$T_D = SF(2.201 + 0.02489 e^{0.8506M} + 0.05335(15))$$

$$T_D = SF(3.001 + 0.02489 e^{0.8506M})$$

Based on the information summarized above and developed in the Appendix, the scale factor, $SF$, to be used in Eq. 14 is $T_D/T_{TB} = 0.900/1.235 = 0.729$. Therefore, a candidate prediction equation for $T_D$ is:

$$T_D = 2.187 + 0.01814 e^{0.8506M}$$

Alternatively, $T_D$ as a function of earthquake magnitude may be estimated from the Vanmarcke-Lai relation (Eq. 13) using that $T_D = 0.7S_o$. This results in:

$$T_D = 3.08(M - 4.2)$$

Duration values from Eqs. 15 and 16 as a function of earthquake magnitude are plotted in Figure 8 along with values from the limited actual earthquake data for which durations by the NUREG/CR-5347 definition have been computed. Equations 15 and 16 produce estimates for $T_D$ which are in close agreement except in the 7.5 to 8 magnitude range where the Eq. 15 values are larger. Eq. 15 is based on the Trifunac-Brady empirical relation which rises more rapidly at higher magnitudes than any of the other relations. These equations produce duration values which
are generally above the available data. Equations 15 and 16 result in duration-magnitude relations which are generally similar to relations for other duration definitions which correspond to the time of maximum power with the exception of the McCann-Shah definition which is significantly lower than the other definitions (compare with Fig. 6).

![Diagram of duration-magnitude relation](image)

**Figure 8 NUREG/CR-5347 Duration-Magnitude Relation Estimated from Trifunac-Brady (Eq. 15) and from Vanmarcke-Lai (Eq. 16)**

Based on Duration Envelope Calculation

Also using the values given above, \( t_m \), for the duration envelope function is equal to 
\[
(1/0.900)T_D = 1.111T_D \quad \text{and} \quad (1/1.286)S_o = 0.778S_o.
\]
From these expressions, \( t_m \) as a function of earthquake magnitude can be estimated from Eqs. 15 (NUREG/CR-5347) and 13 (Vanmarcke-Lai) as follows:

\[
t_m = 2.430 + 0.02016e^{0.660M}
\]  
(17)  
\[
t_m = 3.422(M - 4.2)
\]  
(18)

Time of maximum amplitude, \( t_m \), for a duration envelope function as shown in Figures 2 and 7, as determined from Eqs. 17 and 18 are plotted in Figure 9. Figure 9 illustrates that \( t_m \) determined from either the NUREG/CR-5347 (Eq. 16) or Vanmarcke-Lai (Eq. 17) strong motion duration definitions are very similar except at very high magnitudes. Using either relationship to obtain \( t_m \), rise time and decay time, \( t_r \) and \( t_d \), are then determined as recommended by Kennedy in Philippacopoulos, 1989, to be about 1/7 and 5/7 of \( t_m \), respectively.
7. **Recommended Strong Motion Duration Relationship**

In Section 6, equations have been developed which relate strong motion duration defined to be the time over which the power of an earthquake record is near its maximum in accordance with NUREG/CR-5347 with magnitude of the causative earthquake. This definition of strong motion duration, $T_D$, begins when the cumulative energy of the earthquake record is at 5% of the total energy and ends when the cumulative energy is at 75% of the total energy. This definition is judged to be most closely correlated to potential seismic damage to stiff, nuclear power plant type structures.

It should be noted that these equations are strongly based on a number of assumptions. An important assumption in developing Equation 15 is that $T_D$ is primarily due to the direct shear wave arrival segment of earthquake ground motion with little contribution from later time arrivals of surface waves and refracted, reflected, and scattered shear waves. It is further assumed that earthquake records from rock sites in the near field of the earthquake are due primarily to these direct shear wave arrivals. As a result of this latter assumption, empirical studies of strong motion duration by the Trifunac-Brady definition which includes 90% of the cumulative energy, $T_{TB}$, for rock sites in the near field (15km) have been used as one basis for estimating NUREG/CR-5347 duration, $T_D$, as a function of earthquake magnitude (Equation 15). The above assumptions lead to the conclusion that $T_D$ depends only on magnitude and not on distance to causative fault or on site soil properties. The other equation developed for estimating $T_D$ is based on scaling the empirical relation between the Vanmarcke-Lai definition of duration and magnitude. The Vanmarcke-Lai duration is the time over which the total energy is uniformly distributed. $T_D$ is the time over which 70% of the total energy is uniformly distributed such that scaling the Vanmarcke-Lai relation by 0.7 is the basis for the other equation (Equation 16). Another significant assumption in developing both of these duration estimation equations has been that cumulative energy estimates can be made from the duration envelope function.
For the evaluation of seismic response of stiff nuclear power plant type structures, Eqs. 15 and 16 provide reasonable methods of estimating strong motion duration, $T_D$. As shown in Figure 8, these equations agree closely except at very high earthquake magnitudes, they provide duration-magnitude relations that are similar to other energy based duration definitions, and they generally provide duration values larger than the limited data for which $T_D$ values have been computed. For evaluation purposes in which the total number of cycles are of greater importance even though they may be of lesser amplitude, the Trifunac-Brady duration, $T_{TB}$, may be determined as a function of earthquake magnitude, distance to causative fault, and site soil properties from Eq. 12 and Table 2 empirical constants. Eq. 12 and Table 2 also provides information to evaluate strong motion duration by the Theofanopulos-Watabe, McCann-Shah, and Esteva-Rosenblueth definitions if those definitions are judged to be appropriate for the specific evaluation purposes considered.

Equations 15 and 16 imply a great deal more accuracy than exists in the estimation of NUREG/CR-5347 strong motion duration, $T_D$, with magnitude. As a result, it is judged that the duration-magnitude relation given in Table 3 is appropriate for evaluation of stiff nuclear power plant structures. These duration values are intended to correspond to the definition of strong motion duration as given in Eq. 3 and designated $T_D$. The recommended values are provided for ranges of magnitudes and are about the average values from Eqs. 15 and 16. These values are appropriate as one means for selecting actual earthquake time histories to be used for seismic evaluation or for developing artificial input time histories by the approach in which the Fourier phase spectrum is retained and the Fourier amplitudes are adjusted to match the desired input response spectrum. Note that durations as defined and specified herein must be used in a consistent manner when evaluating PSD functions and Fourier amplitudes of earthquake time histories. The recommended $T_D$ vs. magnitude relation is illustrated in Figure 10 along with the earthquake data for which $T_D$ values have been computed, and the Eq. 15 and 16 values.

For the approach for generating synthetic earthquake time histories in which a random Fourier amplitude spectrum and a duration envelope function (Figures 2 and 7) is employed, recommended values for time of maximum power, $t_m$, for rise time, $t_r$, and for decay time, $t_d$, are provided in Table 4. The recommended values for $t_m$ were developed from the average values determined for Eqs. 17 and 18 as shown in the table. The recommended time of maximum power, $t_m$, values as a function of magnitude are illustrated in Figure 11 along with values from Eqs 17 and 18. Note that recommended values of $t_m$ are less than the upper bound of 15 seconds from the Standard Review Plan (SRP) at all magnitudes. The recommended values of $t_m$ are below the lower SRP bound of 6 seconds for magnitudes less than 6.
Table 3
T₆ vs. Earthquake Magnitude

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>T₆ from Eq. 15</th>
<th>T₆ from Eq. 16</th>
<th>Recommended Strong Motion Duration, T₆, (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5 - 8</td>
<td>16.7</td>
<td>10.9</td>
<td>14</td>
</tr>
<tr>
<td>7 - 7.5</td>
<td>11.7</td>
<td>9.4</td>
<td>11</td>
</tr>
<tr>
<td>6.5 - 7</td>
<td>8.3</td>
<td>7.9</td>
<td>8</td>
</tr>
<tr>
<td>6 - 6.5</td>
<td>6.2</td>
<td>6.3</td>
<td>6</td>
</tr>
<tr>
<td>5.5 - 6</td>
<td>4.8</td>
<td>4.8</td>
<td>5</td>
</tr>
<tr>
<td>5 - 5.5</td>
<td>3.9</td>
<td>3.2</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 10 Recommended T₆ vs. Magnitude
### Table 4
Duration Envelope Function Parameters vs. Earthquake Magnitude

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>t&lt;sub&gt;d&lt;/sub&gt; from Eq. 17</th>
<th>t&lt;sub&gt;d&lt;/sub&gt; from Eq. 18</th>
<th>Time of Maximum Amplitude, t&lt;sub&gt;m&lt;/sub&gt;</th>
<th>Rise Time, t&lt;sub&gt;r&lt;/sub&gt;</th>
<th>Decay Time, t&lt;sub&gt;y&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5 - 8</td>
<td>18.6</td>
<td>12.1</td>
<td>15</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>7 - 7.5</td>
<td>13.0</td>
<td>10.4</td>
<td>12</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>6.5 - 7</td>
<td>9.3</td>
<td>8.7</td>
<td>9</td>
<td>1.5</td>
<td>7</td>
</tr>
<tr>
<td>6 - 6.5</td>
<td>6.9</td>
<td>7.0</td>
<td>7</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>5.5 - 6</td>
<td>5.3</td>
<td>5.3</td>
<td>5</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>5 - 5.5</td>
<td>4.3</td>
<td>3.6</td>
<td>4</td>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

**Envelope Time of Maximum Amplitude**

- Equation 17
- Equation 18
- Recommended Values

**Figure 11 Recommended Duration Envelope Function**
Time of Maximum Amplitude, t<sub>m</sub>, vs. Magnitude
8. References


18. Trifunac, M.D., and Westermo B.D., "Dependence of the Duration of Strong Earthquake Ground Motion on Magnitude, Epicentral Distance, Geologic Conditions at the Recording Station and Frequency of Motion", University of Southern California, Department of Civil Engineering Report No. CE 76-02, November, 1976.


20. Westermo B.D., and Trifunac, M.D., "Correlations of the Frequency Dependent Duration of Strong Earthquake Ground Motion with the Magnitude, Epicentral Distance, and the Depth of Sediments at the Recording Site", University of Southern California, Department of Civil Engineering Report No. CE 78-12, September 1978.

Appendix

Evaluation of Duration Parameters from Envelope Function

Consider a duration envelope function such as that shown in Figure 2 but with maximum acceleration amplitude of $A_m$ (Fig. 2 has maximum amplitude of unity) and where $t_r = (\frac{1}{7})t_m$ and $t_d = (\frac{5}{7})t_m$. For such a duration envelope function, it may be shown that the cumulative energy by Eq. 2, through time, $t$, is:

for $0 \leq t \leq t_r$,

$$E(t) = \frac{A_m^2}{3t_d^2} t^3 \quad \text{(A-1a)}$$

for $t_r \leq t \leq t_r + t_m$,

$$E(t) = E(t_r) + A_m^2 (t - t_r) \quad \text{(A-1b)}$$

for $t_r + t_m \leq t \leq t_r + t_m + t_d$,

$$E(t) = E(t_r + t_m) + A_m^2 \left( t' - \frac{t_r^2}{l_d} + \frac{t_d}{3t_d^2} \right) \quad \text{(A-1c)}$$

where $t' = t - (t_r + t_m)$

By these relations, the total cumulative energy for a record is:

$$E_m = E(t_r + t_m + t_d) = A_m^2 (t_r/3) + A_m^2 (t_m) + l_m^2 (t_d/3)$$

or

$$E_m = (9/7) A_m^2 t_m = 1.286 A_m^2 t_m \quad \text{(A-2)}$$

The cumulative energy at $t = t_r$ and $t = t_r + t_m$ and at 5%, 75%, and 95% of the total cumulative energy are:

$$E(t_r) = A_m^2 (1/7) t_m/3 = 0.048 A_m^2 t_m \quad \text{(A-3)}$$

$$E(t_r + t_m) = 0.048 A_m^2 t_m + A_m^2 t_m = 1.048 A_m^2 t_m \quad \text{(A-4)}$$

$$E_{5\%} = 0.05 (1.286) A_m^2 t_m = 0.064 A_m^2 t_m \quad \text{(A-5)}$$

$$E_{75\%} = 0.75 (1.286) A_m^2 t_m = 0.964 A_m^2 t_m \quad \text{(A-6)}$$

$$E_{95\%} = 0.95 (1.286) A_m^2 t_m = 1.221 A_m^2 t_m \quad \text{(A-7)}$$

Therefore, for this shape duration envelope function, $t_{0.05}$ and $t_{0.75}$ fall within the constant amplitude region and $t_{0.05}$ falls within the decay region. $t_{0.05}$, $t_{0.75}$, and $t_{0.95}$ are the times at which $E_{0.05}$, $E_{0.75}$, and $E_{0.95}$ are reached, respectively. Where $t_r = (1/7)t_m$, $t_r + t_m = 1.143t_m$, and $t_r + t_m + t_d = 1.143t_m + (5/7)t_m = 1.857t_m$, $t_{0.05}$, $t_{0.75}$, and $t_{0.95}$ may be calculated in the manner presented below.
From Equations A-1b, A-3, and A-5, \( t_{0.05} \) is:
\[
0.064 A_m^2 t_m = 0.048 A_m^2 t_m + A_m^2 (t_{0.05} - (1/7) t_m)
\]
\[
t_{0.05} = 0.064 t_m - 0.048 t_m + (1/7) t_m = 0.159 t_m
\]

From Equations A-1b, A-3, and A-6, \( t_{0.75} \) is:
\[
0.964 A_m^2 t_m = 0.048 A_m^2 t_m + A_m^2 (t_{0.75} - (1/7) t_m)
\]
\[
t_{0.75} = 0.964 t_m - 0.048 t_m + (1/7) t_m = 1.059 t_m
\]

From Equations A-1c, A-4, and A-7, \( t_{0.95} \) can be obtained from:
\[
1.221 A_m^2 t_m = 1.048 A_m^2 t_m + A_m^2 \left( t'_{0.95} - \frac{t'_{0.95}^2}{(5/7) t_m} + \frac{t'_{0.95}^3}{3(5/7) t_m^2} \right)
\]
\[
t'_{0.95} - 2.144 t_m t'_{0.95} + 1.531 t_m^2 t'_{0.95} - 0.265 t_m^3 = 0
\]

where \( t_{0.95} = t'_{0.95} + 1.143 t_m \)

The cubic equation for \( t'_{0.95} \) may be solved from relations given in mathematical handbooks or by trial and error to give:
\[
t'_{0.95} = 0.251 t_m
\]
\[
\therefore \quad t_{0.95} = 1.394 t_m
\]

From \( t_{0.05} \), \( t_{0.75} \), and \( t_{0.95} \), strong motion duration by the Trifunac-Brady definition, \( T_{T-B} \), and by the NUREG/CR-5347 definition, \( T_{D} \), are:
\[
T_{T-B} = 1.394 t_m - 0.159 t_m = 1.235 t_m
\]
\[
T_{D} = 1.059 t_m - 0.159 t_m = 0.900 t_m
\]

The Vanmarcke-Lai strong motion duration definition is the time over which the entire energy of the earthquake record, \( E_m \), is uniformly distributed. From Eq. A-2, this energy is \( 1.286 A_m^2 t_m \). The time over which this energy is uniformly distributed may be determined from Eq. A-1b, as \( E_m = A_m^2 S_o \). Setting these two expressions for \( E_m \) equal to each other gives \( S_o = 1.286 t_m \).