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Title Growth Rate in Terms of $Z_\perp$

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No invention subject matter is described therein and may be released for distribution outside the laboratory.

Steve Brumley
Authorized Signature

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Date

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U. S. DEPARTMENT OF ENERGY
CHICAGO OPERATIONS OFFICE

REQUEST FOR PATENT CLEARANCE FOR RELEASE OF UNCLASSIFIED DOCUMENTS

DOCUMENT NO. SSCL-508

2 1	Smith
MB	Desk

TO: CHIEF, OFFICE OF PATENT COUNSEL

FROM: SSC Laboratory

ADDRESS: Technical Info + Pubs Dept.
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1. Document Identification and Proposed Disposition
Growth Rate in Terms of $Z_{\perp}$

2. Contract No.:
DE-AC35-89ER 40486

☑ 3. Return of document is necessary.

4. In order to meet a publication schedule or submission deadline, patent clearance by 9/20/91 would be desirable.

☐ 5. This document discloses no possibly patentable subject matter.

☐ 6. This document describes an invention reported as Contractor Docket No. ______; DOE Case No. ______.

☐ 7. An invention is disclosed for the first time on page(s) ______.

8. Remarks:

Signed: Steve Brumley mw Date: 9-10-91

TO: INITIATOR OF REQUEST

FROM: CHIEF, OFFICE OF PATENT COUNSEL

☒ 9. No patent objection to above-identified release.

☐ 10. Please defer release until advised.

☒ 11. Document returned herewith.

Signed: Bradley [illegible] Date: 9/13/91
OFFICE OF PATENT COUNSEL

DOE-CH 380 (Rev. 1-83)

1. Growth Rate in Terms of $Z_\perp$

The most dangerous regime for the multibunch transverse resistive instability is the lowest (allowed) frequencies, where the fields diffuse significantly through the conducting pipe (or liner). In this regime, the actual beam structure consisting of discrete bunches spaced $\Delta z \sim 5$ meters ~~apart~~ apart can be treated as a continuum. The slowly-varying EM fields responsible for the ~~coupling the bunch motions~~ (out of phase) transverse forces are proportional to $I_b Y_b$, where

$$I_b = \langle I \rangle$$

is the average beam current, and Y_b is the coherent transverse displacement of the beam particles. ~~We define the coupling~~ ~~this coupling~~ For a given beam environment, we characterize the transverse coherent force by the transverse interaction impedance, defined by[1]

$$\bar{F}_{\perp}(z,t) = e\left(\bar{E} + \bar{v}\times\bar{B}\right)_{\perp}$$

$$= -\frac{j e I_b Y_b}{2\pi R} Z_{\perp} \qquad (1)$$

assuming the dependence $\exp j(\omega t - kz)$ for the displacement ~~of~~ (Y_b) and ~~the~~ fields, where $2\pi R$ is the ring circumference and z is the axial coordinate (in the direction of proton motion) (we use electrical engineering notation, ~~as~~ preferred to keep impedance definitions in their traditional form).

If we describe the transverse focusing system by an average beta function (β_{ave}), we can write the following for the coherent transverse motion of the beam particles:

$$\frac{1}{c^2}\frac{d^2 Y_b}{dt^2} + k_\beta^2 Y_b = \frac{F_\perp}{\gamma m_0 c^2} \qquad (2)$$

Here, $k_\beta = (\beta_{ave})^{-1}$ and $d/dt = \partial/\partial t + v\,\partial/\partial z$. With the assumed z, t ~~dependence~~ dependence, we have the following dispersion equation, obtained by substituting Eq(1) into Eq(2):

$$\left(\frac{\omega}{c} - k - k_\beta\right)\left(\frac{\omega}{c} - k + k_\beta\right) = \frac{j e Z_\perp I_b}{2\pi R \gamma m_0 c^2} \qquad (3)$$

The unstable root has $\frac{\omega}{c} \approx k - k_\beta$, and the approximate solution for ω assuming the coherent forces are ~~so~~ small (i.e. – assuming we don't have disastrous growth rates) is ~~[illegible]~~

$$\omega = (k - k_\beta)c + j\frac{e Z_\perp I_b}{4\pi R \gamma m_0 c k_\beta} \qquad (4)$$

In this closed system, the axial wavenumber is quantized as $k(2\pi R) = 2\pi n$, and the betatron tune is defined by $k_\beta(2\pi R) = 2\pi \nu$. Introducing the particle revolution frequency $\omega_0 = c/R$, we can write ~~[illegible]~~ the following expression for ω/ω_0.

$$\frac{\omega}{\omega_0} = n - \nu + j \frac{I_b}{I_0} \frac{Z_\perp}{Z_0} \frac{\beta_{ave}}{\gamma} \qquad (5)$$

Here we define $I_0 = 4\pi m_0 c / \mu_0 e$, $Z_0 = (\mu_0/\varepsilon_0)^{1/2}$, and we replace $1/k_\beta$ by β_{ave}. The (real) frequency of the mode is given by the usual expression,

$$\mathrm{Re}\,\omega = (n-\nu)\,\omega_0 \qquad (6)$$

while the growth rate, $\omega_g = -\mathrm{Im}\,\omega$, is

$$\frac{\omega_g}{\omega_0} = \frac{I_b}{I_0} \frac{\beta_{ave}}{\gamma Z_0} \mathrm{Re}(Z_\perp) \qquad (7)$$

The expression given here for the growth rate agrees with that given by Eq(4.5-25) in the "Blue Book" (Ref 1). To make the connection, note that the average current is $I_b = M N_B e / \tau_0$,

where $\tau_0 = 2\pi/\omega_0$ is the revolution period, and

the classical radius of the proton is

$r_p = ec/I_0 = e^2 \mu_0 / 4\pi m_0$.

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