Patent Review/Release

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Title  Growth Rate in Terms of $L$

Author(s)  D. Briggs

No invention subject matter is described therein and may be released for distribution outside the laboratory.

Steve Brumley
Authorized Signature  9-24-91

Report Coordination

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TO: CHIEF, OFFICE OF PATENT COUNSEL

FROM: SSC Laboratory

ADDRESS: Technical Info & Publ Dept.
2550 Becklemade Ave
Dallas, Texas 75237

1. Document Identification and Proposed Disposition

2. Contract No.: DE-AC35-89ER40484

3. Return of document is necessary.

4. In order to meet a publication schedule or submission deadline, patent clearance by 9/20/91 would be desirable.

☐ 5. This document discloses no possibly patentable subject matter.

☐ 6. This document describes an invention reported as Contractor Docket No. ________________; DOE Case No. ________________.

☐ 7. An invention is disclosed for the first time on page(s) ____________.

8. Remarks:

Signed: Steve Bunley
Date: 9-10-91

TO: INITIATOR OF REQUEST

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☐ 9. No patent objection to above-identified release.

☐ 10. Please defer release until advised.


Signed: Bradley Bunley
Date: 9/13/91

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DOE-CH 380 (Rev. 1-83)
1. Growth Rate in Terms of $z_1$

The most dangerous regime for the multibunch transverse resistive instability is the lowest (allowed) frequencies, where the fields diffuse significantly through the conducting pipe (or liner). In this regime, the actual beam structure consisting of discrete bunches spread $\Delta z \approx 5$ meters apart can be treated as a continuum. The slowly varying $E_y$ fields responsible for coupling the fundamental transverse force are proportional to $I_b \frac{V_b}{V}$, where $I_b$

$$I_b = \langle I \rangle$$

is the average beam current, and $V_b$ is the transverse displacement of the beam particles. After defining the beam environment, we characterize the $\Delta z$ profile. For a given structure, the transverse coherent force by the transverse interaction impedance...
\[ F_L(z, t) = e \left( E + \delta x B \right) \]

\[ = -\frac{i e E_0 y}{2 \pi R} \frac{Z_L}{Z_{\pi}} \]

assuming the dependence \( \exp \left( i (w t - k z) \right) \) for the displacement \( E(x, t) \) and the field, where \( \omega \) is the angular frequency, \( k \) is the axial coordinate, the electric field rotating in a circumference (we use electrical engineering notation) and expressed to keep impedance definitions in their traditional form.

If we describe the transverse focusing system by an average beta function \( (\beta_{ave}) \) we can write the following for the transverse motion of the beam particles:

\[ \frac{1}{c^2} \frac{d^2 y}{dt^2} + \frac{\beta_{ave}^2}{\gamma} \frac{d y}{dt} = \frac{F_z}{\gamma m_0 c^2} \]  

Here, \( \beta_{ave} = (\beta_{ave})^{-1} \) and \( \gamma dt = \frac{2}{c^2} + 2 \beta_{ave}^2 \). With the assumed \( E, t \) dependence, we have the
following dispersion equation, obtained by substituting Eq. (1) into Eq. (2):

\[
\left( \frac{\omega}{c} - k + k_p \right) \left( \frac{\omega}{c} - k - k_p \right) = \frac{j e^2 I_b}{2 \pi R Y_m c^2}
\]

(3)

The unstable root has \( \omega = k - k_p \) and the approximate solution assuming the coherency forces are small (i.e., assuming we don't have dissipation growth rates) is achieved.

\[
\omega = (k - k_p) c = \frac{j e^2 I_b}{4 \pi R Y_m c k_p}
\]

(4)

In this closed system, the axial wavevector is quantized as \( k (2\pi R) = 2\pi m \), and the rotation time is defined by \( k_p (2\pi R) = 2\pi l \). Introducing the revolution frequency \( \omega_0 = c / R \), we can write the expression for \( \omega / \omega_0 \).
\[ \frac{\omega}{\omega_o} = n - \gamma + j \frac{I_b}{I_0} \frac{Z_1}{Z_o} \frac{\text{Par}}{Y} \]  

(5)

Here we define \( I_o = \frac{4\pi m_o c}{\omega_o} \text{NAVFA} \), \( Z_o = (\rho_o / \epsilon_o)^{1/2} \) and replace \( V_kp \) by \( \text{Par} \). The (real) frequency of the mode is given by the usual expression

\[ \text{Re}\,\omega = n - \gamma \, \omega_o \]  

(6)

while the growth rate \( \frac{\omega_g}{\omega_o} = -\text{Im}\,\omega \), is

\[ \frac{\omega_g}{\omega_o} = \frac{I_b}{I_0} \frac{\text{Par}}{Y Z_o} \text{Re}(\gamma) \]  

(7)

The expression given here for the growth rate agrees with that given by Eq.(4.5-75) in the "Blue Book" (Ref.1). To make the connection, note
\[ I_0 = MN e \rho / \alpha \]

where \( \alpha = 2 \pi / \omega_0 \) is the revolution period, and the classical radius of the proton is

\[ r = e c / I_0 = e^2 \mu_0 / 4 \pi m_0. \]

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