FINAL REPORT

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Title: Plasma Transport in the Stochastic Fields at the
Tokamak Edge

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Purpose:

The purpose of this project is to calculate the contribution of field line diffusion to particle diffusion in the stochastic magnetic field at the tokamak edge.

Approach:

We use the approach of quasi magnetic surfaces. If the magnetic field line makes sufficiently large number of toroidal transits before suffering large radial excursion, then the method of quasi magnetic surface is valid for this problem. This method has three components: determination of particle drift trajectories, a model for magnetic field configuration, and determination of field line diffusion.

Determination of Field Line Diffusion:

The Hamiltonian for the field line is the toroidal flux $\psi(\chi, \theta, \phi)$. θ and ϕ are the poloidal and toroidal angles, and χ is the poloidal flux. Hamilton's equations of motion for the field line are:

$$\frac{d\chi}{d\theta} = \frac{\partial \psi}{\partial \phi} \quad (1)$$

$$\frac{d\phi}{d\theta} = -\frac{\partial \psi}{\partial \chi} \quad (2)$$

The field line Hamiltonian is $\psi(\chi, \theta, \phi) = \psi_0(\chi) + \psi_1(\chi, \theta, \phi)$ where the first term represents the unperturbed magnetic surfaces, and the second term the resistive perturbation. For sufficiently large two or more Fourier modes in the perturbation, surfaces break and field line becomes stochastic. Let $\chi_0 = \chi(\theta_0)$ and $\phi_0 = \phi(\theta_0)$ be the initial conditions for a field line. Then the intersection of this line with the Poincare plane $\phi = \text{constant}$ will have coordinates (χ_i, θ_i) , $i=1, \dots, N$. Let $x_i = \sqrt{\chi_i} \sin(2\pi\theta_i)$, $y_i = \sqrt{\chi_i} \cos(2\pi\theta_i)$. The points (x_i, y_i) define a closed curve in xy plane. The area A_1 determined by this curve gives the χ coordinate for the quasi surface. Let l_A be the distance along the field line at the end of N toroidal circuits. After following field line for a distance much greater than l_A , we can repeat the above procedure and determine the area A_2 . Field line diffusion coefficient is given by

$$D_F = \frac{(\Delta\chi)^2}{2l} \quad (3)$$

where l is the distance along the field line.

Determination of Plasma Particle Diffusion:

For this we use the Monte Carlo method developed by Punjabi and Boozer [1]. In this Monte Carlo method, the drift Hamiltonian in Boozer coordinates is given by

$$H(\theta, P_\theta, \phi, P_\phi) = \frac{1}{2} v_{||}^2 + \mu B + q\Phi. \quad (4)$$

Here P_θ and P_ϕ are canonical momenta, $v_{||}$ is the guiding center velocity parallel to magnetic field, B is magnetic field, μ is magnetic moment. Canonical momenta are given by

$$P_\theta = \frac{I v_{||}}{B} + 4\pi^2 \text{sign}(q) \psi \quad (5)$$

and

$$P_\phi = \frac{g v_{||}}{B} + 4\pi^2 \text{sign}(q) \chi. \quad (6)$$

Here I and g are toroidal and poloidal currents. Then the Hamilton's equations of motion for particle drift trajectories are:

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{\partial H}{\partial P_\theta} \\ \frac{d\phi}{dt} &= \frac{\partial H}{\partial P_\phi} \\ \frac{dP_\theta}{dt} &= -\frac{\partial H}{\partial \theta} \\ \frac{dP_\phi}{dt} &= -\frac{\partial H}{\partial \phi} \end{aligned}$$

The Lorentz collision operator is represented by the random sequence

$$\lambda_n = \lambda_0(1 - v_d \tau) \pm \sqrt{[(1 - \lambda_0^2) v_d \tau]}.$$

where $\lambda = v_{\parallel}/v$, ν_d is the ion-ion deflection frequency, $t = nr$ with $\nu_d r \ll 1$. If j th plasma particle is located at position χ_0 at time $t=0$, and at χ_j at time t , then the diffusion coefficient for that particle is given by

$$D_j = \frac{1}{2} \frac{(\chi_j - \chi_0)^2}{t_0}.$$

Then the estimate of D from J particles is

$$D = \frac{1}{J} \sum_{j=1}^J D_j.$$

Usually t_0 is taken to be one collision time or larger.

Equations of motion are integrated using a fourth order Runge-Kutta method. The size of time step is determined by the level of accuracy desired on energy conservation. We usually require this to be one part in one billion.

Determination of transport coefficients in stochastic fields:

As shown above, we can determine $(\chi_i, \theta_i, \phi_i)$, $i=1, \dots, N$ which define the initial quasi surface with area A_1 . We use these coordinates $(\chi_i, \theta_i, \phi_i)$, $i=1, \dots, N$ as initial conditions for N plasma particles. The remaining particle coordinates $v_{\parallel,i}$ are chosen randomly. We follow the trajectories of these N particles for time t_0 . Let $(\chi_j, \theta_j, \phi_j)$ be the end points of these trajectories. We use $(\chi_j, \theta_j, \phi_j)$ as the initial conditions for the equations of motion for magnetic field line trajectories. We follow these field lines for sufficient number of toroidal circuits. The positions of the points in Poincare plane give us the area A_j of the quasi surface for j th particle. then the diffusion coefficient for the j th particle is

$$D_j = \frac{\Delta \chi_j}{2t}$$

where

$$\Delta \chi_j = (A_j - A_0).$$

Repeating this procedure for N particles gives us $D(E, \chi)$ for plasma particles in the stochastic fields. A comparison of field line diffusion coefficient $D_F(\chi)$ with particle diffusion coefficient $D(E, \chi)$ gives us the contribution of field diffusion to particle diffusion in stochastic fields.

Codes:

We have developed three different codes for this project. These codes are developed from previous codes called TKMKCHnn and FIELDnn on NERSC Cray computers. These codes were transferred from Cray to Sun Station at Hampton. Major modifications and additions were made to these codes. Code DPFIELD18.f integrates the equations of motion for the field line and calculates A_1 and outputs initial conditions for plasma particles. These initial conditions are input to code DPTKMK1.f which integrates particle trajectories. Final positions of particle are determined by this code. These final positions are input to code DPFIELD19.f which integrates field line trajectories and calculates areas A_j . All these codes are run on Sun Station.

Results:

For tokamaks, $I \ll g$, and we take $I=0$, and $g=1/\epsilon$ where ϵ is the inverse aspect ratio. We choose parabolic profile for the safety factor. We take $B_0=5$ KG, $a=9$ cm, $R_0=50$ cm, $q_0=1$, $q_a=3$, and $V_{loop}=0$ V. For resistive perturbations, we choose two modes $\{(m,n)\}=\{(2,1), (3,2)\}$ which are located in the outer region. We choose the initial conditions for the radial location of 64 electrons exactly between the resonant surfaces for these two modes. Poloidal and toroidal positions are chosen at random. Initial pitch of the particles are also chosen at random. In the absence of collisions, the results are tabulated below. A_{mm} is the amplitude of the resistive perturbation. For the results here this amplitude is same for various modes. D is the average value of the particle diffusion coefficient, σ is the standard error of the mean for D , D_F is the diffusion coefficient for the field line. All results are in dimensionless units which are same as in reference 1. Magnetic flux is normalized by $\pi a^2 B_0$, lengths are normalized by the minor radius a , and time is normalized by the cyclotron time. B_0 is the strength of toroidal field at the magnetic axis.

$A_{mm}(X10^6)$	$D(X10^8)$	$\sigma(\%)$	$D_F(10^8)$
1	3.0373	21	.3513
2.5	3.0340	21	.3511
5	3.0302	21	.3486
7.5	3.0288	21	.3506
10	3.0203	21	.3489
25	2.9864	21	.3460
50	1.1532	45	.0919
100	2443.8	20	.0960
1000	14402.	17	.1140

These initial results show that $D/D_T \approx \text{constant} \approx 8.66$ for $A_{mn} < 2.5 \times 10^{-5}$. For $A_{mn} > 2.5 \times 10^{-5}$, we see that the standard error in D is large, and results for $A_{mn} > 2.5 \times 10^{-5}$ are not reliable. To obtain reliable results, we need to modify the codes to increase the sample size of the plasma particles. Also we should turn on velocity space scattering. This does not require additional work. Thus the next step in the project should be to simulate a larger particle sample with pitch angle scattering.

Future directions:

In the future we would like to excite the resistive perturbations with more than two modes in the tokamak edge with larger sample size for particles with pitch angle scattering and energy scattering as well. We would like to compare our results with the results from the method of maps [2]. Finally we would like to combine the method of maps with the method of quasi magnetic surfaces to get a better prediction of the heat load on divertor plates of ITER and TBX.

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References:

1. Punjabi A., Boozer A., Lam M., Kim H. and Burke K., J Plasma Phys., 44, 403-430 (1990)
2. Punjabi A., Verma A. and Boozer A., Phys. Rev. Let., 69, 3322-3325 (1992)