BIOGRAPHY
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ABSTRACT
We are sometimes presented with data with serious flaws, like saturation, over-range, zero shifts, and impulsive noise, including much of the available pyrotechnic data. Obviously, these data should not be used if at all possible. However, we are sometimes forced to use these data as the only data available. A method to salvage these data using wavelets is discussed. The results must be accepted with the understanding that the answers are credible, not necessarily correct. None of the methods will recover information lost due to saturation and over-range with the subsequent nonlinear behavior of the data acquisition system. The results are illustrated using analytical examples and flawed pyrotechnic data.

KEYWORDS
pyrotechnic, shock, wavelets, shock response spectrum

INTRODUCTION
We are sometimes presented with data with minor flaws caused by aliasing and more serious flaws, like overloads, zero shifts, and impulsive noise (spikes or dropouts), including much of the available pyrotechnic data. Obviously, the later data should not be used if at all possible. When presented with flawed data, the best procedure is to reject the data and collect a more valid data set. Other authors have shown the risks involved if the data are accepted. We are sometimes forced to use this data as the only data available and it is not possible to gather more data. The data may have been gathered as a one-time experiment that cannot be repeated, or additional experiments would be prohibitively expensive. In this paper we will be discussing acceleration data, typically pyrotechnic data, but the methods could be applied to other data sets.

The shock response spectrum (SRS) is a common tool for the specification of pyrotechnic environments. If the flawed data are used, serious errors in the shock response spectrum result. These errors are usually the prediction of much higher responses at the low frequencies than the true environment justifies. For this reason the flawed data must be corrected if the data is used in a specification.

The minor errors caused by aliasing can also lead to serious errors in the numerically integrated velocity and displacement, which can cause problems with waveform reproduction programs on shaker systems. Earlier papers showed how some of the data can be salvaged using a parametric form for the corrections. The purpose of this paper is to show that wavelets can also be used to salvage the data.

Using these methods requires judgment, and the results must be accepted with the understanding that the answers are credible, not necessarily correct. None of the methods will recover information lost due to overloads or nonlinearities of the data acquisition system. The best that can be accomplished is the recovery of data after the data acquisition system has recovered from the overload. The methods require assumptions on the characteristics of a credible data set and a model of the corrections. The authors described a series of corrections which could be made to correct data with minor flaws. In the investigation of some data with more serious flaws it was found that these tools did not work very well. This paper will discuss a new correction technique based in wavelets.

CORRECTION WITH WAVELETS
Wavelets are a new method (about 10 years old) for analyzing data. Unfortunately space will permit only a
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brief description of wavelets. A good introduction is contained in an article by Strang and a thorough discussion is contained in a book by Strang and Nguyen. Wavelets are a powerful new way to look at transient data. Basically a function is described as a combination of basis functions (wavelets)

\[ f(t) = \sum_{jk} b_{jk} w_{jk}(t) \]  

(1)

An inverse exists: if you know the coefficients \( b_{jk} \), you can reconstruct the original waveform. A typical wavelet is compressed in time and shifted

\[ w_{jk}(t) = 2^{j/2} w(2^j t - k) \]  

(2)

Normalized wavelet on \([k \Delta t, (k + N) \Delta t]\)

The wavelet, \( w \), often has a quite complicated shape, but is completely described by a pair of filters: a high pass filter and a low pass filter. A number of useful wavelets have been defined, and one of the problems of a user is to pick the most appropriate wavelet for a particular application.

The wavelets typically have compact support. Simply, this means that the wavelets are nonzero over a short finite period of time.

In practice the continuous form of the wavelet, Eq. (1), is seldom used, we almost always are dealing with finite waveforms sampled at discrete times.

\[ C(a, b) = C(j, k) = \sum_{n \in \mathbb{Z}} f_n w_{j,k}(n) \]  

\[ \Delta t = 1, a = 2^j, b = k 2^j, j \in \mathbb{N}, k \in \mathbb{Z} \]  

(3)

The advantage of wavelets is that they provide a description of a waveform that is localized in both time and frequency. Although, users of wavelets don't like to talk about frequency, they use the term level instead. This should prove useful for the task of this paper, since the flaws in the data are typically localized in time and frequency. Some of the wavelets are members of a family of wavelets, identified by the order number. In general, the higher order wavelets provide better frequency localization at the expense of time localization. The wavelet chosen for this paper was the Daubechies wavelet of order 3, abbreviated as the db3 wavelet in the MATLAB Wavelet Toolbox. The basic wavelet is plotted as Figure 1.

Figure 1 Daubechies 3 wavelet function, psi

The procedure used for this paper is almost the inverse of de-noising of signals as described in the Wavelet Toolbox User's Guide. In this case the "noise" is the shock we want to keep. The "signal" is the data correction we want to remove. For this paper, we used a level 5 correction. We decomposed the signal using 5 levels of decomposition. Each level of decomposition decomposes the signal into two bands, with a high pass filter and a low pass filter. The output of the high pass filter is called the detail, and the output of the low pass filter is called the approximation. The approximation and the detail are decimated by two and the process is repeated for the approximation for the next level. The detail is not processed again. Remarkably, the desired wavelet coefficients are the decimated detail. At first glance it would appear the decimation of the detail would cause serious aliasing problems, but this is not the case. The two filters are carefully constructed such that the information lost in the decimation of the detail is retained in the approximation. Exact reconstruction is possible from the approximation and details.

Since each level of approximation has about half the bandwidth of the previous level, the upper frequency limit of the nth level of approximation is about \( f_s / 2^{n+1} \), where \( f_s \) is the sampling frequency. Thus the level chosen provides frequency localization. In this case the approximation contains the low frequencies and the detail contains the high frequencies. We also need localization in time, since the locations in time of the errors in the original waveform are not known. The wavelet transform inherently provides the localization in time.

To summarize, the corrected acceleration was reconstructed from the n levels of details (crudely the highest frequency components). The correction was reconstructed from the last level of approximation (roughly the lowest
frequencies). The sum of the correction (the approximation) and the corrected waveform (the details) will be the original waveform.

RESULTS
The first two examples will look at flaws associated with the numerical integration of waveforms with minor aliasing errors.

The first example is the sum of a 100 Hz and a 4000 Hz exponentially decaying sinusoid samples at 12,000 samples/second from Smallwood[3]. The analytical acceleration, velocity, and displacement are presented as Fig. 2. Figure 3 shows the waveform integrated with a rectangular rule. The velocity waveform which shows an offset and the displacement clearly are in error, which is caused by aliasing[3]. The intent of the correction is to change the acceleration in such a manner that the numerically integrated velocity and displacement are near the analytical waveforms. The correction and the corrected waveforms are shown as Figs. 4 and 5. As can be seen the corrections to the acceleration are small, about 5 parts per thousand, and isolated to the first hundredth of a second. The resulting corrected velocity and displacement are nearly correct.

The second example is also from Smallwood[3]. The waveform is the sum of three exponentially decaying sinusoids, 100, 3500, and 4000 Hz, sampled at 12,000 samples/second. A delay between the 3500 and 4000 Hz sinusoids makes this a more severe test of the correction methods. The analytical waveforms (the correct result), the numerically integrated waveform (showing the numerical integration errors), the correction using wavelets, and the corrected waveforms are shown as Figs. 6-9. For this example the corrected acceleration is very similar to the analytical waveform. The corrections to the acceleration are about 3 parts per ten thousand. The corrected velocity and displacement show minor errors.

The third and fourth examples are from Smallwood and Cap[4]. These waveforms are two pyrotechnic shocks with more serious flaws. The sample rate was 200,000 samples/second. For these waveforms a credible result will be acceleration, velocity and displacement waveforms which are oscillatory. The initial and final values of the acceleration, velocity and displacement should all be near zero. The previous paper gave credible corrections for these waveforms using a parametric filter. The measured acceleration and the numerically integrated velocity and displacement are shown as Figs. 10 and 13. The velocity of x shows erroneous behavior at large times, which causes large displacements as time increases. The velocity of h shows an almost step change in velocity. The waveform corrections are shown as Figs. 11 and 14. The corrected waveforms are shown as Figs. 12 and 15. The bandwidth of the correction is approximately 200,0002 or 3 kHz. The corrected waveforms are credible for the acceleration, velocity, and displacement. The magnitudes of the corrections (a significant part of the magnitude of the original waveform) raise significant questions about the validity of the corrections. These corrections are larger than the corrections from the previous paper. However in defense of the wavelet technique, the peak magnitude of the acceleration is not changed very much. The effects on the shock response spectrum (SRS) are shown in Fig. 16. The effects on the SRS are very similar to the effects on the SRS from the previous paper. As can be seen the SRS is essentially unchanged except at the low frequencies. The SRS of both x and h are, now similar at low frequencies, a credible result.

To summarize, the magnitude of the corrections are larger and have more high frequency content than the corrections from the previous paper, but the results on the SRS are very much the same.

The fifth and sixth examples are a different pyrotechnic shock test sampled at 100,000 samples/second. These two waveforms also have serious flaws. These were separation shocks of a payload from a bus. Unfortunately, as is often the case, no credible measurements are available for comparison. The measurements will be called B13 and B31. B31 was located near one of the explosive bolts in the separation system. B13 was a few inches away. Other useful measurements were made further from the explosive bolt, but are not useful for this discussion.

The original data together with the numerically integrated velocity and displacement are shown Figs. 17 and 20. B31 shows a small zero shift in the acceleration and other possible errors which result in erroneous velocity and displacement waveforms. B13 shows an even larger zero shift in the acceleration.

The correction methods from the previous work[4] did not give pleasing results. An attempt was then made to correct the data with wavelets.

B13 was corrected with the wavelet transform as described the previous section. The correction and the corrected waveform are shown as Figs. 18 and 19. The bandwidth of the correction is about 1.6 kHz. The correction has a peak of a little over 300 g (about 1/2 of the peak amplitude of the original waveform). The correction is confined primarily to about 1 msec. The magnitude of
the correction raises serious questions about the correction. The corrected acceleration and velocity look credible. The displacement still appears to have a little too much low frequency oscillation.

The correction and the corrected waveforms for B31 are shown as Figs. 21 and 22. The magnitude of the acceleration correction is about 1/4 of the peak amplitude of the original waveform. A zero shift is clearly present in the original data and is also present in the correction.

The effects on the SRS are shown in Figs. 23 and 24. The SRS was calculated for the original waveform, and the wavelet corrected waveform. The effects of the corrections on the Fourier spectrum are shown as Figs. 25 and 26. As can be seen the corrections primarily affected the low frequencies.

CONCLUSIONS
Wavelets are a robust method to correct flawed acceleration waveforms. The corrections are easier to apply with fewer subjective choices than the methods of the previous paper. The method produced credible data sets for a larger set of examples than the previous paper. The data sets used in the examples of this paper cover the range from minor corrections to corrections which should probably not be used. If used with care, the corrections discussed can produce credible data sets from data with minor flaws.

Since the corrections can also produce credible data sets for seriously flawed data, it is important to preserve the original waveform, the correction waveform, as well as the corrected waveform. The magnitude, duration and frequency content of the correction can serve as a guide to the validity of the correction. If the magnitude of the correction is a substantial fraction of the magnitude of the original waveform the corrections should be viewed with great caution. The duration and frequency content of the correction should be consistent with a reasonable correction. The corrected data sets may underestimate the environment at high frequencies, but should result in reasonable estimates in the mid frequencies (1-10 kHz, for an SRS with an upper usable frequency of 10 kHz). The corrections prevent the gross overestimates of the SRS at the low frequencies (below 1 kHz) which are common in flawed pyrotechnic data. The intent is not to hide the data flaws. The corrected data sets should never be placed in a data bank without references to the original data set, and a clear explanation of the correction method used.

Figure 2 Waveform composed of the sum of a 100 and 4000 Hz exponentially decaying sinusoids

Figure 3 Waveform of Fig. 2 integrated with rectangular rule

Figure 4 The wavelet correction for the waveform of Fig. 3.
Figure 5 The corrected Fig. 3 waveform

100 3500 & 4000 Hz exponentially decaying sinusoids, SR=12000

Figure 6 A waveform composed of the sum of 100, 3500, and 4000 Hz exponentially decaying sinusoids

Figure 7 Waveform of Fig. 6 integrated with rectangular rule

Figure 8 The wavelet correction of waveform of Fig. 7

Figure 9 The wavelet corrected waveform of Fig. 7

Figure 10 The waveform called x, with the numerically integrated velocity and displacement
Figure 11 The wavelet correction to $x$

Figure 12 The wavelet corrected $x$

Figure 13 The waveform called $h$ with the numerically integrated velocity and displacement

Figure 14 The wavelet correction to $h$

Figure 15 The wavelet corrected $h$

Figure 16 The SRS of the original and corrected $x$ and $h$
Figure 17 B13 with numerically integrated velocity and displacement

Figure 18 The wavelet correction to B13

Figure 19 The wavelet corrected B13

Figure 20 B31 with the numerically integrated velocity and displacement

Figure 21 The wavelet correction to B31

Figure 22 The wavelet corrected B31
Figure 23 The SRS of the original and corrected B13

Figure 24 The SRS of the original and corrected B31

Figure 25 The FFT of the original and corrected B13

Figure 26 The FFT of the original and corrected B31

REFERENCES


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