A Computational Tool for Comparison of Kinematic Mechanisms and Commonly Used Kinematic Models

K. Hollerbach
A.M. Hollister
R.L. Van Vorhis

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INTRODUCTION

Accurate, reliable, and reproducible methods to measure the movements of human joints have been elusive. Currently, three-dimensional recording methods are used to track the motion of one segment relative to another as the joint moves. Six parameters describe the moving segment's location and orientation relative to the reference segment: three translations (x, y, and z) and three rotations (yaw, pitch and roll) in the reference frame. The raw data can be difficult to interpret. For this reason, several methods have been developed to measure the motion of human joints and to describe the resulting data. For example, instant helical axes or screw deviation axes (Kinzell et al., 1972), the Joint Coordinate System of Grood and Suntay (1983), and the Euler angle method have been used to describe the movements of bones relative to each other. None of these methods takes into account the physical kinematic mechanism producing the joint motion. More recently, Lupichuk (1995) has developed an algorithm to find, for an arbitrary revolute, the axis' position and orientation in three-dimensional space. Each of these methods has advantages and disadvantages in analyzing joint kinematics. The authors have developed software to provide a means of comparing these methods for arbitrary, single degree of freedom, kinematic mechanisms. Our objective is to demonstrate the software and to show how it can be used to compare the results from the different kinematic models as they are applied to specific kinematic mechanisms.

IMPLEMENTATION

The authors have developed software to calculate the forward kinematics for several different kinematic mechanisms. This software was used to generate motion paths for a body moving about a revolute, a revolute that translated in the motion plane as a function of rotation (evolute), and for a motion path about 2 revolutes. Data from these paths were then used as input to inverse kinematics software developed by the authors to calculate screw axes, Joint Coordinates, and Euler angles. The results were compared to the known axis' positions and orientations. In addition, an algorithm was applied to the path data to calculate the true axis' positions and orientations. A comparative analysis was performed on the inverse kinematics calculations for a revolute that was aligned with a global reference axis, an arbitrary revolute, an evolute, and a two revolute mechanism.

The transformations for rotation about the kinematic mechanisms as well as the inverse kinematics calculations were performed with programs written with LabView 3.0/4.0 (National Instruments, Austin, TX) on a Macintosh Quadra 840 AV (Apple Computer Corp., Cupertino, CA). The programs allowed changes to be made in the orientation of the axes, the axes' positions relative to the reference coordinate frame, and the moving point's positions relative to the axes themselves. The software calculated the rotations and displacements for 180° of rotation about each axis. Kinematic analyses were performed and displayed as a function of rotational angle.

Kinematic Mechanisms

The simplest kinematic mechanism that was analyzed is a single degree of freedom revolute that is aligned with (parallel to) one of the global reference coordinate axes. For the purposes of analysis, this axis was taken to be the global z axis. Movement about this revolute axis, therefore, produces a circular trajectory in the global x-y plane, at some z value determined
by the initial position (z value) of the point relative to the revolute axis and also by the distance along the z axis between the origin of the global coordinate frame and the z=0 position on the revolute axis.

Rotations of a body moving about an arbitrary revolute in a reference frame are determined by the revolute's α and β angles of offset from the reference frame and the θ angle of rotation about the arbitrary axis (Fig. 1). The body's displacements are affected by the rotations and by the vector, d, from the reference frame origin to the point on the axis about which the body rotates and r, the vector from the axis to the body.

An evolute is a kinematic mechanism with two distinct but mathematically dependent motions: rotation about a revolute and translation of the revolute in the motion plane as a function of degree of rotation. The trajectory of the body about the evolute is in a plane perpendicular to the axis, as in the case of a true revolute. However, the body's trajectory is not a circular path within that plane, because the evolute itself is also translated in the plane of motion. The evolute's translation is a function of the angle of rotation. Hence, the evolute is a single degree of freedom mechanism.

**Forward Kinematics**

For the revolute axes, including both the z axis rotation and the arbitrary axis rotation, the total transformation describing the kinematics of a point [P] moving to a new position [P*] about an arbitrary axis (Rogers and Adams, 1990) is a 4x4 matrix equation, \( [P*] = [P][M] \), where [P] and [P*] are the augmented x-y-z initial and final positions of the point, and [M] is the transformation for rotation about the arbitrary axis:

\[
[M] = [T][R_\alpha][R_\beta][R_\theta][R_\alpha]^{-1}[T]^{-1}
\]

This set of transformations translates the point on the axis to the origin ([T]), rotates the axis by an amount corresponding to the α and β offsets so that the arbitrary revolute is coincident with the z coordinate ([R_\alpha] and [R_\beta], respectively) and then rotates ([R_\theta]) by θ, the actual rotation about the arbitrary revolute (Fig. 1). The reference frame is then derotated ([R_\alpha]^{-1} and [R_\beta]^{-1}) and detranslated ([T]^{-1}) to return the point to its new position in the reference frame. In the z axis rotation, the α and β angles both equal zero, and the corresponding rotational matrices are equal to the identity matrix. The software allows the user to generate trajectories as a function of rotational angle (varied here from 0 to \(360^\circ\)) and to specify the interval of the rotational angle as well as the angles of offset of the arbitrary axis relative to the reference z axis. In addition, the user can vary the position of the axis relative to the reference frame origin (vector d) and the initial position of the point or
body relative to the axis (\(r\)). The software then generates the x-y-z trajectories, in the reference coordinate frame, of the initial point when it is rotated about the defined arbitrary axis, at the specified intervals, throughout the specified range of motion. These data are then used in the kinematic calculations in the same way that data obtained experimentally by a camera based marker system would be analyzed.

In the implementation of the evolute mechanism, at each interval the new position of the axis is calculated and is used in the calculation of the body's position in the next interval. The body's trajectory, therefore, depends on its rotation relative to the axis and also on the axis' movement relative to the reference frame. The user can specify the function defining the movement of the evolute relative to the reference frame (although that movement is restricted to movement in the fixed plane that is perpendicular to the axis). The output of the program is an x-y-z trajectory of a point or body moving about the axis, calculated in the reference frame.

A single degree of freedom mechanism can also exist in which a body rotates about two axes, but where rotation about one is coupled with rotation about the other. Mathematically, the system may be represented by a single variable (the angle of rotation of the independent axis). Mechanically, however, two separate axes may be used to effect the motion. The simulation describes this behavior by building a two joint system from any of the single degree of freedom models and linking them by allowing the user to specify a relationship between the two angles of rotation.

**Kinematic Models**

A screw axis is a fixed, arbitrary axis, where a point rotates about the axis, specified by some rotational angle, and simultaneously translates along the axis. In a fixed-pitch screw, there is a constant relationship between translation of the point along a fixed axis and the rotational angle. With instant screw axis definitions, the screw parameters are determined solely by the current trajectory interval. For an entire trajectory, therefore, the screw axis' position, orientation, and pitch may vary. In our software implementation, screw deviation axes' positions, orientations, rotational angles, and translations were calculated according to the interval method outlined by Kinzell et al. (1972). With the rotational and translational elements of the arbitrary axis known, the x, y, z trajectories of 4 non-coplanar points were calculated. Accuracy of interval methods such as screw deviation axes are affected by step size. For this reason, calculations were performed at 10° intervals.

The Joint Coordinate System uses coordinate axes that are not orthogonal. The first coordinate (\(e_1\)) is fixed relative to the proximal bone and usually represents flexion-extension (FE) movements. A second coordinate (\(e_3\)) is fixed relative to the distal bone and is used to represent internal-external (IE) motions. A third coordinate (\(e_2\)), a function of the other two, moves and is used to represent abduction-adduction (AA) movements. Our software implementation is based on Grood and Suntay's method (1983).

Euler angles are rotations of the reference frame. Rotations of a body are determined by rotating the reference frame to match the body's orientation. Euler angles can be used to describe any motion, but will give different results for the same motion depending on the relative positions and orientations of the body and reference coordinates. The Euler rotations and displacements can be recorded as changes in the six individual components. When the z-x-z Euler convention is used, the Euler angle transformation for rotations of a body in space is:

\[
R = \begin{bmatrix}
\cos \phi_e & -\sin \phi_e & 0 \\
\sin \phi_e & \cos \phi_e & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \theta_e & 0 & \sin \theta_e \\
0 & 1 & 0 \\
-\sin \theta_e & 0 & \cos \theta_e
\end{bmatrix}
\begin{bmatrix}
\cos \varphi_e & -\sin \varphi_e & 0 \\
\sin \varphi_e & \cos \varphi_e & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The axis finder algorithm was developed using aggregate optimization to determine an arbitrary revolute's location from x, y, z, yaw, pitch, and roll data (Lupichuk, 1995). First, the plane of motion is determined, using the Gauss' Method of Least Squares. Next, the center of
motion is located within that plane. Finally, axis parameters are adjusted and closeness of fit analyses completed. As a result, the axis location and orientation can be determined with some degree of accuracy.

DISCUSSION

Four different kinematic mechanisms were chosen for a comparative demonstration of the analysis software, although any single degree of freedom mechanism can be implemented by the user. The forward kinematics module for each kinematic mechanism generated a trajectory of the body moving about the axis in 3-D space. These trajectories represent a noiseless simulation of the type of data that would be obtained in an experimental, camera based marker system, except that the mechanism is known to the user. Noise can be added to the system by the user, as desired. Once the trajectories are generated, they are analyzed with kinematic models that were chosen from the biomechanics literature and implemented in our software. Final analysis of the results can demonstrate the relationship between the models' predictions of motion and the kinematic mechanism generating the motion (Fig. 2).

![Kinematic Mechanism Diagram]

Figure 2: For each kinematic mechanism that is simulated, a forward kinematics module generates a trajectory in 3-D space. Trajectories are analyzed using several kinematic models.

The software demonstration will show how the user can define and alter the kinematic mechanism that generates a body's trajectory in three-dimensional space. The effects of changing the mechanism's parameters on the trajectory are easily observed. Comparing the results of several kinematic models on trajectories thus created intuitively demonstrates the significance of using the appropriate kinematic model to describe a particular mechanism. Quantitative comparisons showing error measures are graphically portrayed in the demonstration.

In addition to using these computational tools to analyze single degree of freedom mechanisms, the user can link together single degree of freedom modules and examine multi-link systems such as the human leg in gait analyses. The authors will demonstrate a general purpose tool that will help the user gain an intuitive understanding of the relationship between mechanism and analysis results.

REFERENCES


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