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High-Intensity-Beam Applications

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Comparison of Cyclotron and Linacs for High-Intensity-Beam Applications

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Abstract

This is the final report of a two-year, Laboratory Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). The goals of this project were three-fold: (1) to understand the current level of cyclotron design expertise especially in the areas of space-charge modeling and simulation codes, (2) to develop a better understanding of the capabilities and limitations of circular machines, especially in the area of current limits, and (3) to stay abreast of the developments at other institutions in the area of high-current circular machines. These goals were partially met especially in the area of code development for the application of linac codes to motion of ions in a circular orbit. We were also able to continue our interactions with the other institutions working in this area.

Background and Research Objectives

Currently, two types of high-intensity sub-GeV ion accelerators are in use as drivers for accelerator-driven neutron sources. The first of these is the Los Alamos Neutron Science Center (LANSCE) linac, which accelerates up to 1 mA of ions to an energy of 800 MeV. (Presently, only a maximum of 70-80 μ A of the 800-MeV proton beam is used for neutron production. The full 1 mA of the accelerated beam is, however, available for delivery to a neutron production target). The second class of accelerator in use is the cyclotron at the Paul Scherrer Institute (PSI) where, again $\sim$ 1 mA of proton beam is accelerated to 600 MeV and used for neutron production. Projects involving transmutation of nuclear waste and energy production using accelerators as drivers, for example, require substantially larger beam powers than either the linacs or the existing cyclotron devices are producing. Substantially higher-power linacs have already been designed. Since cyclotrons use rf power more efficiently by making multiple turns through the rf cavities, there may be a substantial cost advantage over linacs. Our investigation focuses on the

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possibility of designing a GeV cyclotron that can accelerate beam currents of 10 mA or more.

Importance to LANL's Science and Technology Base and National R&D Needs

Los Alamos National Laboratory (LANL) has an abiding interest in accelerator technology. In addition to the short, pulsed spallation source (SPSS) program based on the LANSCE linac, two other projects currently in varying stages of development are the acceleration production of tritium (APT) and the transmutation of waste (ATW). The first of these two uses a high power linac. The other could also use a linac but if a cyclotron-based system could be shown to be cost effective, then we would have developed an alternative. It may be useful to develop a better understanding of the capabilities and limitations of circular machines.

Scientific Approach and Accomplishments

The accelerator community has had many years of experience with linac design codes such as PARMILA. As the linac projects evolved, requiring higher beam currents for linacs, this design and simulation code has undergone numerous modifications concerning its ability to handle space-charge effects. The design codes for circular machines, on the other hand, have not had to deal with high currents, and thus are limited in their ability to properly include effects due to space charge forces for machines with higher beam currents. We are seeking ways to use the linac codes, with their already built-in capabilities for space-charge calculations, for applications to circular machines. In order to be able to do so, we need to modify the linac codes. For these modifications to be credible, one must apply the modified linac codes to an existing cyclotron to benchmark the operation. PSI's 590-MeV, 1-mA cyclotron is an excellent candidate for this benchmarking. There is actual operational data available through years of use. The modified linac codes could be used to compare the simulated results with actual data. This would enhance the confidence in the use of linac codes for simulating the high-current behavior of cyclotrons.

The PARMILA code is an established, well-documented code for calculating phase and radial motion in linear accelerators. Can this code with appropriate modifications be used to track particles in a ring cyclotron? The simplest approach is to "straighten" the dynamics of particle motion into a series of straight sections (each representing one orbit) followed by focusing magnets and rf acceleration gaps. This method has been considered resulting in a technical note (Attachment I) describing the modifications the PARMILA code might need to accomplish this calculation with a suggestion on how to handle the

space-charge calculation. The applicability of such a modified linac code to the PSI cyclotron is discussed in the attachment.

One major concern for the design and operation of a high-intensity cyclotron device is the role played by space-charge forces, since these determine the limiting current and the energy spread. The energy spread in turn determines the losses at extraction. The losses throughout the overall cycle of operation must be kept to a minimum to insure maintainability of the facility without the need for remote handling. An internal technical note (Attachment II) describes the modeling for high-intensity isochronous cyclotrons.

In considering how one should include space-charge forces in an isochronous cyclotron, one should take into account how the vacuum chamber walls affect the distribution of the fields of a point charge. Specifically, how does the field in the median plane drop off with distance from the point charge? This fall off is important because it determines the number of turns one should include in the calculation of the median plane space-charge field. This is described in an internal note (Attachment III).

Finally, the effects due to the presence of the longitudinal space charge electric field of the beam was examined. This field acts on particles at the leading and trailing edges of the beam, increasing the energy of the particles in the lead, and decreasing the energy of those in the trail. This increased energy spread leads to an increase in the radial extent of the beam. Incorporation of an inductive wall in the regions of the cyclotron in which the beam drifts can compensate for the longitudinal space charge electric field. A simplified analysis is described in an internal note (Attachment IV).

As part of this project we have kept abreast of developments in the area of high-current circular machine ideas [1-3]. We have worked closely with the Yerevan Physics Institute, which has submitted a proposal for ISTP funding.

Attachments

- I. Cooper, R., "Thoughts on Using the PARMILA Code to Simulate the PSI Ring Cyclotron," LANSCE-1 note, pages 1-8 (1997).
- II. Cooper, R., "Modeling High-Intensity Isochronous Cyclotrons for Neutron Production," LANSCE-1 note, pages 1-7 (1997).
- III. Cooper, R., "Point Charge Between Parallel Plates," LANSCE-1 note, pages 1-4 (1997).
- IV. Cooper, R., "Compensating the Longitudinal Space-Charge Forces in Isochronous Cyclotrons," LANSCE-1 note, (1997).

References

- [1] Babayan, A., et. al., "Design of Proof-of-Concept Technology Demonstration for Accelerator-Driven Subcritical Nuclear Reactor," Proposal submitted to ISTP for funding (1996).
- [2] Tumanian, A., et. al., "The Coaxial-Ring Cyclotron as a Powerful Multipurpose Proton Accelerator," Yerevan Institute Technical note (1997).
- [3] Ruggiero, S., "A Superconducting Linac as the Driver of the Energy Amplifier," BNL-63527, (1996).

Thoughts on Using the PARMILA Code to Simulate the PSI Ring Cyclotron

R. K. Cooper, Amparo Corporation

The PARMILA code is an established, documented code for calculating phase and radial motion in linear accelerators. The question arises, "Can this code be used to track particles in a ring cyclotron such as the 590 MeV machine at the Paul Scherrer Institute (PSI)?" The first impulse is to "straighten" the machine into a series of straight sections, followed by bending magnets and acceleration gaps. In this note I outline what modifications to the PARMILA code might need to be made to accomplish this calculation, finishing with a suggestion on how to handle the space-charge calculation.

As shown in Figs. 1 and 2, the PSI ring cyclotron has eight bending magnets and four RF cavities, with a number of drift regions between the bending magnets. A short digression follows here to provide an understanding of the functioning of the ring cyclotron.

The classical cyclotron is a device in which particles circulate with angular velocity ω_c in a uniform magnetic field of strength B , each particle receiving an energy kick twice per turn from a constant-frequency RF voltage with angular frequency ω_c . The angular frequency of revolution of the particle is given by

$$\omega_c = \frac{qB}{\gamma m}, \quad (1)$$

and is a constant if γ is constant. But since γ is the total particle energy divided by the rest-mass energy, and since the aim of the cyclotron is to increase the particle energy, it is clear that, in order to maintain synchronism with the constant-frequency RF system, the magnetic field strength acting on the particles with energy γmc^2 must be proportional to γ .

Now the radius at which a particle of energy γmc^2 circulates is

$$r = \frac{[B\rho]}{B(r)} = \frac{p/q}{B(r)} = \frac{\gamma\beta mc/q}{\gamma m\omega_c/q} = \frac{\beta}{\omega_c/c} = \frac{\sqrt{1 - \frac{1}{\gamma^2}}}{\omega_c/c}, \quad (2)$$

where $[B\rho]$ is the magnetic rigidity of the particles. From this last equation we obtain the relation

$$\frac{1}{\gamma^2} = 1 - \frac{\omega_c^2 r^2}{c^2}. \quad (3)$$

Figure 1: The 590 MeV ring cyclotron at the Paul Scherrer Institute.

Figure 2: A photograph of the 590 MeV ring cyclotron.

Figure 3: A magnetic field decreasing in strength with radius provides a restoring force to particles above or below the median plane, while a magnetic field increasing with radius forces such particles further from the midplane.

Thus the magnetic field strength as a function of radius required to keep the circulation angular frequency of the particles constant is

$$B(r) = B_0 \gamma = \frac{B_0}{\sqrt{1 - \frac{\omega_z^2 r^2}{c^2}}}, \quad (4)$$

where B_0 is the central field strength. The problem with this scheme, however, is that particles above or below the median plane will be forced away from this plane, as shown in Fig. 3, and ultimately they will be lost from the machine.

Slice the
magnet

Pull the
slices apart

Figure 4: By providing drift spaces it is possible to allow for vertical focusing.

The way around this vertical instability is to slice the magnet into pie-shaped pieces and then to separate the pieces so that there are drift spaces between the magnets as shown in Fig.4. Then in the drift spaces we can put vertical focusing elements. What is done in practice is to curve the edges of the pie-shaped pieces so that the particles being accelerated enter the magnetic field at an angle. The fringe field of the magnet acts to push the particles toward the median plane, with a focal length given by

$$\frac{1}{f} = \frac{B}{[B\rho]} \tan \psi, \quad (5)$$

where ψ is the angle at which the particle enters the magnet. This *edge focusing* works both upon entering the field and upon exiting the field. An

inspection of Fig. 1 shows that it is necessary to increase the value of w as the energy of the particle increases, i.e., moves to larger radii.

The measure of the rate of increase or decrease of the strength of the magnetic field that proves to be most useful is called the *field index*, given the symbol n and defined as

$$n = -\frac{R}{B} \frac{\partial B}{\partial R}. \quad (6)$$

Note that n is positive for a magnetic field which becomes weaker with radius.

Given the above introduction, then, we can start to imagine tracking particles through a ring cyclotron. Let us start by considering a drift space between magnet sectors. Aside from the space-charge forces which we will discuss later, we can treat the drift almost as we would treat any other drift space. Because the length of this drift depends on the turn number of the reference orbit, there will be a first-order correction to the length of the drift. That is, $L_n = L_{n,0} + L'x$, where x is measured from the reference orbit. The time of flight through the drift will be $t_n = L_n/v = L_n/v_0 + \delta v = (L_n/v_0)(1 - \delta v/v_0) = (L_n/v_0)[1 - (1/\gamma^2)\delta p/p_0]$.

Then we encounter the edge of a sector magnet. Even though the edge angle is a function of position, it is sufficient to use simply the matrix for an edge because to first order in small quantities only the vertical offset of a particle influences the change in vertical motion. That is, the expression for the change in the slope of the orbit in the vertical direction $\Delta p_z/p = B/[B\rho]_0 \tan \psi_0 z (1 - \delta p/p)(1 - \sec^2 \psi_0 x)$ reduces simply to $\Delta p_z/p = B/[B\rho]_0 \tan \psi_0 z$ to first order in small quantities.

In the magnet the equations of motion (x is horizontal, measured from the reference orbit, and z is vertical, measured from the midplane) are

$$\frac{d^2}{ds^2}x + \frac{1-n}{\rho^2}x = \frac{1}{\rho} \frac{\delta p}{p} \quad (7)$$

$$\frac{d^2}{ds^2}z + \frac{n}{\rho^2}z = 0 \quad (8)$$

For a magnetic field given by Eq. (4) the field index n has the value $n = -\beta^2\gamma^2$, which ranges between -0.167 and -1.69 for protons in the range from 75 to 600 MeV. As the field index n is negative, the z motion will

be exponential in character, while the horizontal motion is oscillatory. The solution of the equations of motion are then

$$x(s) = a_1 \cos(\sqrt{1-n}s/\rho) + a_2 \sin(\sqrt{1-n}s/\rho) + \frac{\rho \delta p/p}{1-n}, \quad (9)$$

and

$$z(s) = a_3 \cosh(\sqrt{-n}s/\rho) + a_4 \sinh(\sqrt{-n}s/\rho) \quad (10)$$

In terms of the initial conditions upon entry into the magnetic sector, these solutions yield the transfer matrix for the horizontal motion, where $\theta = \Delta s/\rho$

$$\begin{pmatrix} x \\ x' \\ \frac{\delta p}{p} \end{pmatrix}_{\text{out}} = \begin{pmatrix} \cos(\sqrt{1-n}\theta) & \frac{\rho}{\sqrt{1-n}} \sin(\sqrt{1-n}\theta) & \frac{\rho}{1-n} (1 - \cos(\sqrt{1-n}\theta)) \\ -\frac{\sqrt{1-n}}{\rho} \sin(\sqrt{1-n}\theta) & \cos(\sqrt{1-n}\theta) & \frac{1}{\sqrt{1-n}} \sin(\sqrt{1-n}\theta) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \frac{\delta p}{p} \end{pmatrix}_{\text{in}} \quad (11)$$

while for the vertical motion we have

$$\begin{pmatrix} y \\ y' \end{pmatrix}_{\text{out}} = \begin{pmatrix} \cosh(\sqrt{-n}\theta) & \frac{\rho}{\sqrt{-n}} \sinh(\sqrt{-n}\theta) \\ \frac{\sqrt{-n}}{\rho} \sinh(\sqrt{-n}\theta) & \cosh(\sqrt{-n}\theta) \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_{\text{in}} \quad (12)$$

For purposes of tracking it is probably best to use the transfer matrices such as those used in TRANSPORT which include the longitudinal separation of a given particle from the reference particle. That is, one should track six-component vectors $(x, x', y, y', \delta p/p, \delta l)$. This longitudinal separation is required to give the correct energy impulse to a particle as it passes through the accelerating cavity.

The RF accelerating gap in a ring cyclotron is unlike that in any linear accelerator. The resonator is a rectangular box with the accelerating gap in the broad face of the box. This construction gives an accelerating electric field which can have a marked radial distribution. In treating off-axis particle motion the $x-z$ distribution will have to be taken into account as well as the turn-by-turn variation.

The space-charge calculation will not be as straightforward as calculating the electric and magnetic fields of a simple beam. Because the beam returns to the vicinity it occupied on the previous turn and will return again to this same vicinity on the next turn, one needs to think about the past, present,

and the future. But since we are only interested in the steady state, there is some hope for a reasonable calculation. I propose that in order to calculate the space charge forces, that particles be initially tracked completely ignoring space charge. Then the turn-by-turn distribution can be taken as a first approximation to the distribution of charge in the machine in the steady state, and this distribution can be used to calculate the electric fields of the beam. These fields can then be used to recalculate the motion of the particles throughout the machine. I suggest that the approach to the steady-state solution be gradual, that is, after the first tracking establishes a charge distribution, that the fields calculated from that distribution be given only, say, one-tenth the strength appropriate to the actual beam charge. Then these fields, used to perform a second tracking, can be increased in strength tracking by tracking until the full strength is used and a self-consistent solution is achieved.

Modeling High-Intensity Isochronous Cyclotrons for Neutron Production

Richard K. Cooper, Amparo Corporation

September 3, 1997

1 Introduction

There has been renewed interest in the use of isochronous cyclotrons for applications that require continuous intense (10 mA) beams of medium-energy (1 GeV) protons [1,2]. A workshop held in Santa Fe in December, 1995 studied the feasibility of successful design, construction, and operation of such cyclotrons, with the conclusion that there was no fundamental obstacle to such success [3]. There arose a number of concerns, however, requiring further study. Key among these concerns is the role played by space-charge forces, since these determine the limiting current and the energy spread. The energy spread in turn determines losses at extraction. Losses throughout the overall cycle of operation must be kept to an absolute minimum to insure maintainability of the facility without the need for remote handling. It is the object of this proposal to evaluate the effects of space-charge forces on the beam-loss problem in isochronous cyclotrons, specifically one designed to achieve 10 MW operation (see Fig. 1). The well-documented 590 MeV ring at the Paul Scherrer Institute (see Fig. 2) will serve to benchmark these calculations. We expect to determine whether halo formation will lead to unacceptably high levels of beam loss, and whether longitudinal space-charge-induced energy spread can be held to levels consistent with clean extraction. The following table shows a comparison of the 590 MeV PSI ring and the proposed 1 GeV ring.

specification	1.5mA/590MeV	10mA/1GeV
Cyclotron		
magnet	8 sectors	12 sectors
extraction radius	4.45m	5.67m
RF system	4 cavities	8 cavities
frequency	50.6.MHz	44.2 MHz
peak voltage	730 kV	1000 kV
avg. E -gain/turn	2.4 MeV	6.3 MeV
Beam parameters		
number of turns	215	140
dR/dn at extraction	4.3-8.0mm	5.6-10.4mm
beam width	5mm	6.5mm
normalized acceptance	2.5π mm mrad	5π mm mrad
phase width	10°	15° - 30°

2 Approach

In an isochronous cyclotron the circumference of the equilibrium orbit as a function of energy must satisfy the condition

$$\frac{C}{v} = T_0 = \text{constant} = \frac{2\pi}{\omega_0} . \quad (1)$$

The circumference may be simply a circle resulting from a magnetic field $B(r)$ which is chosen to satisfy this condition, namely

$$\begin{aligned} \frac{2\pi\rho}{v} &= \frac{2\pi}{\omega_0} , \\ \frac{[B\rho]}{B(r)v} &= \frac{1}{\omega_0} , \\ \frac{p/e}{B(r)v} &= \frac{1}{\omega_0} , \end{aligned}$$

from which one obtains, using $p = \gamma\beta m_p c$,

$$B(r) = \gamma\omega_0 m_p / e = \gamma B_0 , \quad (2)$$

a CYCLOTRON for 10 MW BEAM POWER

Figure 1: A ring designed to deliver 10 mA at 1 GeV.

where $B_0 \equiv m_p \omega_0 / e$. This increase in field strength with increasing energy (and thus increasing radius) leads to unstable particle motion in the vertical direction (perpendicular to the median plane), and thus some vertical focussing is required. This focussing is provided by the fringe fields of the magnets shown in Figs. 1 and 2, which occasion the introduction of drift spaces between the magnet sectors. The focal length of the fringe field is, for the vertical motion,

$$f_{\text{edge}} = \frac{[B\rho]}{B \tan \psi} \quad (3)$$

where $[B\rho]$ is the magnetic rigidity ($= p/e$) of the proton and ψ is the angle of entry into the magnet. Given reasonable separation of the equilibrium orbits turn by turn, then, a tracking program would be built as follows:

1. For each particle in the particle distribution, track the values of x , x' , $\frac{dx}{p}$, y , y' and ϕ . This last quantity is the longitudinal separation of the given particle and the reference particle.
2. Track particles through the drift regions (of varying, tabulated lengths) just as one would track in any drift space.
3. At the edge (entry or exit) of a magnet, apply the edge-focusing thin lens transformations¹ $\Delta y' = -y/f_{\text{edge}}$ and $\Delta x' = -x/f_{\text{edge}}$.
4. Track particles through the bending magnet using the combined-function magnet matrix including the path-length difference term.
5. Track particles through the rf gaps using the transformation

Quantity	Gap height=0	Gap height=2a
ΔW	$\cos \phi V_0 T$	$\cos \phi V_0 T \frac{\cosh K_0 a}{\cosh K_0 a}$
Δp_x	$-\sin \phi \frac{\omega}{\gamma + v^2} x V_0 T$	$\sin \phi \frac{\omega}{\gamma + v^2} V_0 T \frac{1}{K_0} \frac{\sinh K_0 a}{\cosh K_0 a}$

where V_0 is the cavity voltage, T is the usual transit time factor, and $K_0 = \omega_H / \gamma c$.

6. Calculate space-charge electric fields acting on the particles and apply these forces as kicks in the middle of each drift region. (See more detail following.)

¹In the $\Delta y'$ equation a correction factor proportional to the magnet gap height divided by the square of the bending radius has been omitted for clarity of presentation.

Figure 2:
5

The PSI separated-sector cyclotron. This machine accelerates protons from 72 to 590 MeV. (1) Beam line from 72-MeV injector cyclotron, (2) magnetic inflector channel, (3) electrostatic inflector, (4) one of eight sector magnets, (5) set of correcting (trim and harmonic) coils wound on a magnet pole, (6) one of four 50-MHz accelerating cavities, (7) coaxial rf power transmission line, (8) 150-MHz "flat-top" cavity, (9) beam diagnostic probe (radial drive), (10) electrostatic deflector, (11) magnet to focus the deflected beam, (12) septum magnet, (13) extracted beam line (590 MeV).

Figure 3: The effectiveness of a point charge between parallel conducting plates separated by a distance $2w$. The plot is of $r^2 E_r$ plotted vs r/w . If there were no conducting plates above and below the charge this plot would be a horizontal straight line at ordinate value 1.

Point Charge Between Parallel Plates

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In considering how one should include space-charge forces in an isochronous cyclotron, one should take into account how the vacuum chamber walls affect the distribution of the fields of a point charge. Specifically, how does the field in the median plane fall off with distance from the point charge? This fall off is important because it determines how many turns one should include in the calculation of the median plane space-charge field.

We start with a point charge located midway between parallel conducting plates separated by a distance $2w$. We take the origin of coordinates at the location of the charge, with the z direction perpendicular to the plane of the plates. We use circular-cylinder coordinates r, θ, z and imagine that the charge is distributed as a surface charge density on the surface of a small cylinder of radius a extending from $-w$ to w in z . We shall subsequently let the radius a go to zero. That is, we shall write the surface charge density on the surface of a cylinder of radius a as

$$\sigma(a, z) = \frac{q}{2\pi a} \delta(z). \quad (1)$$

Notice that the integral $\int_{-w}^w \sigma(a, z) 2\pi a dz = q$.

We write the electrostatic potential as a solution of Laplace's equation in two regions, namely $r > a$ and $r < a$. For $r < a$ we must have a solution that vanishes at $z = \pm w$ and that is finite at $r = 0$. Thus we write

$$\phi_{<}(r, z) = \sum_{j=1, \text{ odd}}^{\infty} a_j \cos \frac{j\pi z}{2w} I_0 \left(\frac{j\pi r}{2w} \right) / I_0 \left(\frac{j\pi a}{2w} \right). \quad (2)$$

while for the region $r > a$ we need a solution that goes to zero as r goes to infinity, so we write

$$\phi_{>}(r, z) = \sum_{j=1, \text{ odd}}^{\infty} a_j \cos \frac{j\pi z}{2w} K_0 \left(\frac{j\pi r}{2w} \right) / K_0 \left(\frac{j\pi a}{2w} \right). \quad (3)$$

Note that the potential forms have been chosen such that the potential is continuous at $r = a$.

At the surface $r = a$ the radial electric field must be discontinuous in the amount of the surface charge density divided by ϵ_0 . That is,

$$E_{r>}(r = a, z) - E_{r<}(r = a, z) = \frac{\sigma}{\epsilon_0}, \quad (4)$$

which is, in terms of the potentials,

$$-\left. \frac{\partial \phi_{>}(r, z)}{\partial r} \right|_{r=a} - \left. \frac{\partial \phi_{<}(r, z)}{\partial r} \right|_{r=a} = \frac{q}{2\pi\epsilon_0 a} \delta(z). \quad (5)$$

Using the expressions for the potentials assumed above, we have

$$-\sum_j a_j \frac{j\pi}{2w} \cos \frac{j\pi z}{2w} \left[\frac{K'_{0a}}{K_{0a}} - \frac{I'_{0a}}{I_{0a}} \right] = \frac{q}{2\pi\epsilon_0 a} \delta(z), \quad (6)$$

where the subscript a on the Bessel functions is to indicate that the argument is $j\pi a/2w$, and the prime means derivative with respect to the total argument. We now multiply this last equation by $\cos(k\pi z/2w)$ and integrate over z from $-w$ to w . The cosine functions are orthogonal, yielding

$$\int_{-w}^w \cos(k\pi z/2w) \cos(j\pi z/2w) dz = w\delta_{kj}. \quad (7)$$

The Bessel functions satisfy the Wronskian relationship

$$I'_0(x)K_0(x) - K'_0(x)I_0(x) = \frac{1}{x}. \quad (8)$$

allowing us to write the result of the integration as

$$a_k \frac{k\pi}{2w} \left(\frac{-2w/k\pi a}{K_{0a}I_{0a}} \right) w = \frac{q}{2\pi\epsilon_0 a}. \quad (9)$$

We thus arrive at the expression for the expansion coefficients a_j

$$a_j = \frac{q}{2\pi\epsilon_0 w} K_0 \left(\frac{j\pi a}{2w} \right) I_0 \left(\frac{j\pi a}{2w} \right). \quad (10)$$

Putting this expression into Eq. (3) for the potential outside the radius $r = a$ gives us the potential written as

$$\phi_{>}(r, z) = \frac{q}{2\pi\epsilon_0 w} \sum_{j=1, \text{ odd}}^{\infty} I_0 \left(\frac{j\pi a}{2w} \right) \cos \frac{j\pi z}{2w} K_0 \left(\frac{j\pi r}{2w} \right). \quad (11)$$

At this point we can let a go to zero, with the result that the I_0 term goes to unity, so that our final expression for the potential of a point charge midway between two conducting parallel plates separated by a distance $2w$ is

$$\phi_{>}(r, z) = \frac{q}{2\pi\epsilon_0 w} \sum_{j=1, \text{ odd}}^{\infty} \cos \frac{j\pi z}{2w} K_0 \left(\frac{j\pi r}{2w} \right) . \quad (12)$$

We can recover the free-space potential by letting w go to infinity, and converting the summation in this last equation to an integral. We do this conversion by letting $x = j\pi/2w$, so that $\Delta x = \Delta j\pi/2w = \pi/w$, since in the summation $\Delta j = 2$. Thus we write, in the limit as w goes to infinity so that $\Delta x \rightarrow dx$,

$$\phi_{>}(r, z) = \frac{q}{2\pi^2\epsilon_0} \int_0^{\infty} \cos zx K_0(rx) dx . \quad (13)$$

From the integral tables of Ryzhik and Gradshteyn, Eq. 14 of Sec. 6.671, we have

$$\int_0^{\infty} \cos zx K_0(rx) dx = \frac{\pi}{2\sqrt{r^2 + z^2}} . \quad (14)$$

Putting this result into the previous equation gives us the free-space potential of a point charge

$$\phi_{>} = \frac{q}{4\pi\epsilon_0} \frac{1}{R} , \quad (15)$$

where R is the distance from the point charge, $R = \sqrt{r^2 + z^2}$.

Let us now examine the radial electric field in the median plane, i.e., the plane $z = 0$. If the distance w were infinite, this value would be simply given by

$$E_r(r, z = 0) = \frac{q}{4\pi\epsilon_0 r^2} \quad \text{free space} . \quad (16)$$

If we differentiate the potential as given by Eq. (12) to get an expression for the radial component of the electric field we obtain

$$E_r(r, z) = -\frac{q}{2\pi\epsilon_0 w} \sum_{j=1, \text{ odd}}^{\infty} \frac{j\pi}{2w} \cos \frac{j\pi z}{2w} K'_0 \left(\frac{j\pi r}{2w} \right) , \quad (17)$$

so that, in the plane $z = 0$,

$$E_r(r, z = 0) = \frac{q}{4\pi\epsilon_0 w^2} \sum_{j=1, \text{ odd}}^{\infty} j K_1 \left(\frac{j\pi r}{2w} \right) . \quad (18)$$

This last expression is easily evaluated: a plot of $r^2 E_r$ would be a straight line with value $q/4\pi\epsilon_0$ if the free-space potential were used. We plot here this quantity calculated as a function of r/w . The plot shows that at a distance of $r = 4w$ the effective charge is 5 per cent of the original value.

Figure 1: A plot of $r^2 E_r$ vs r for $w = 1$. The free-space value is represented by the value 1.

Compensating the Longitudinal Space-Charge Force in Isochronous Cyclotrons

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1 Introduction

A major problem with clean extraction of the beam in an isochronous cyclotron is the radial size of the beam, which is made greater by the presence of the longitudinal space charge electric field of the beam. This field acts on particles at the leading and trailing edges of the beam, increasing the energy of the particles in the lead and decreasing the energy of those in the tail. This increased energy spread leads to an increase in the radial extent of the beam. We propose here to compensate the longitudinal space charge electric field by incorporating an inductive wall in the regions of the cyclotron in which the beam drifts. (If there is no free drift space the inductive wall could be placed in the bending magnets, but at an increase in cost in providing the requisite bending field.) A simplified analysis follows. The idea of an inductive wall has previously been suggested for the compensation of longitudinal space charge in circular accelerators and storage rings[1],[2]. Recent experiments at LANL and KEK suggest that an inductive wall can indeed have salubrious effects.

2 Analysis

This analysis follows closely that of Albert Hofmann[3] for round beams in round pipes. The geometry chosen for analysis is two-dimensional. That is, the beam is represented by a sheet beam and the vacuum chamber walls extend to plus and minus infinity in z and y . The x axis is perpendicular to the median plane, and the beam moves in the z direction, with charge density $\rho(z - vt)$. For ease of analysis we imagine the beam to consist of two parallel sheets of surface charge density $\frac{1}{2}\sigma(z - vt)$ located at $x = \pm a$. The surface

Figure 1 The two-dimensional geometry analyzed.

charge density is related to the volume charge density by $\sigma = \rho 2a$. Assuming slow variations with z , the electric field $E_x(z, t) = \frac{1}{2}\sigma(z-vt)/\epsilon_0$ extends from $x = a$ to $x = b$, the height of the vacuum chamber wall. Additionally, there will be a y component of the magnetic field $H_y = K = v\sigma(z-vt)$ extending from $x = a$ to $x = b$. See the details in Fig. 1. The vacuum chamber wall is corrugated in a manner to be described subsequently, the key feature being that the electric field lines from the beam terminate at $x = \pm b$, while the magnetic field of the beam extends further, so that the beam moves in an inductive environment. This corrugated wall will support a nonzero average z component of the electric field, produced by the wall current.

Performing the line integral around a path which extends from $z = -l/2$ to $+l/2$ along the z axis, then up to $x = b$, back along $x = b$ to $z = -l/2$ and then returning to the starting point, we obtain

$$\oint \mathbf{E} \cdot d\mathbf{l} \approx E_{z, axis}l + E_x(z + l/2)(b - a) - E_{z, wall}l - E_x(z - l/2)(b - a). \quad (1)$$

Equating this result to the negative of the time rate of change of the flux through this path, $-d\text{Flux}/dt = (b - a)l\mu_0 v^2 \frac{1}{2} d\sigma/dz$, dividing through by l ,

and letting l go to zero gives us

$$E_{z, \text{axis}} - \frac{\partial E_x}{\partial z}(b-a) = E_{z, \text{wall}} + (b-a)\mu_0 v^2 \frac{1}{2} d\sigma/dz . \quad (2)$$

Now

$$\frac{\partial E_x}{\partial z} = \frac{1}{2\epsilon_0} \frac{\partial}{\partial z} \sigma(z-vt) ,$$

so that, using $\mu_0\epsilon_0 = 1/c^2$ we have

$$E_{z, \text{axis}} = E_{z, \text{wall}} - \frac{b-a}{\epsilon_0} \left(1 - \frac{v^2}{c^2}\right) \frac{1}{2} d\sigma/dz = E_{z, \text{wall}} - \frac{b-a}{2\epsilon_0\gamma^2} d\sigma/dz .$$

If the wall were perfectly conducting and perfectly smooth, this last term would be the electric field on axis, and would serve to give energy to particles in the head and take energy from particles in the tail. (Note that at the head of the beam $d\sigma/dz$ is negative.)

Now the electric field along the wall will be given by

$$E_{z, \text{wall}} = L' \frac{dI_{\text{wall}}}{dt} = -L' \frac{dI_{\text{beam}}}{dt} ,$$

where L' is the inductance per unit length in the z direction. This inductance value will be calculated subsequently for a particular model of a wall. Putting this last relation into the previous equation gives us

$$E_{z, \text{axis}} = L' v^2 \frac{1}{2} d\sigma/dz - \frac{b-a}{2\epsilon_0\gamma^2} d\sigma/dz , \quad (3)$$

so that this inductive term can cancel the longitudinal space-charge term, at least at one energy.

3 An Inductive Wall

Let us take as a simple model of an inductive wall a conducting wall which consists of a plane located at $x = a$ periodically recessed to a depth d . The recesses are filled with a material of relative permeability μ_r . The recesses are of width w and the distance from one recess to the next is D . Figure 2 shows the detail. The z component of the electric field will be zero along that

Figure 2: A model of an inductive wall.

portion of the wall lying along $x = a$, and nonzero over the recess opening. Assuming that there is a current I_{wall} per meter of length in the y direction flowing along the $x = a$ portion and up and around the recess down again to the $x = a$ portion of the wall, there will be a magnetic field $H_y = -I_{\text{wall}}$ uniformly filling the recess. (This analysis will be valid when all dimensions are very small compared to any wavelength generated by the beam current.) Integrating the electric field across the recess opening and around the recess gives

$$\oint \mathbf{E} \cdot d\mathbf{l} = E_z w,$$

and by Faraday's law we must equate this to the negative of the time rate of change of the flux in the recess

$$E_z w = -w d \frac{dB_y}{dt} = \mu_r \mu_0 w d \frac{dI_{\text{wall}}}{dt} \quad (4)$$

The average value of the longitudinal electric field along the wall is therefore

$$E_{z,\text{avg}} = [E_{z,\text{recess}} w + 0(D - w)]/D = E_z w/D = \mu_r \mu_0 \frac{w}{D} d \frac{dI_{\text{wall}}}{dt}.$$

Finally, we take the wall current to be the negative of the beam current, and since

$$\frac{dI_{\text{beam}}}{dt} = -v \frac{dI_{\text{beam}}}{dz},$$

we have

$$\frac{dI_{\text{wall}}}{dt} = v \frac{dI_{\text{beam}}}{dz},$$

and the average value of the longitudinal electric field along the wall is

$$E_{z,\text{wall}} = \mu_r \mu_0 \frac{w}{D} dv \frac{dI_{\text{beam}}}{dz} = \mu_r \mu_0 \frac{w}{D} dv^2 \frac{1}{2} \frac{d\sigma}{dz}.$$

Comparing this expression with Eq. 3, we see that the inductance per unit length in the z direction is

$$L' = \mu_r \mu_0 \frac{w}{D} d.$$

Putting all this in Eq. 3 gives us the expression for the field on axis for this particular model of the vacuum chamber wall

$$E_{z,\text{axis}} = \mu_r \mu_0 \frac{w}{D} dv^2 \frac{1}{2} d\sigma/dz - \frac{b-a}{2\epsilon_0 \gamma^2} d\sigma/dz.$$

For complete cancellation of the space-charge longitudinal field by the inductive wall field we have

$$\mu_r \mu_0 \frac{w}{D} dv^2 = \frac{b-a}{\epsilon_0 \gamma^2},$$

or, solving for μ_r ,

$$\mu_r = \frac{b-a}{v^2 \mu_0 \epsilon_0 \gamma^2} \frac{D}{wd} = \frac{b-a}{\beta^2 \gamma^2} \frac{D}{wd}.$$

For a rough estimate of the relative permeability required, neglect a with respect to b , set $D = w$, and take d , the depth of the recess, equal to b . Then

$$\mu_r = \frac{1}{\beta^2 \gamma^2}.$$

The following table shows the value of μ_r as a function of proton kinetic energy.

E (MeV)	μ_r
50	9.2
100	4.4
200	2.1
500	0.74
1000	0.30

From this table it can be seen that for low proton energies, reasonable values of permeability will suffice to cancel the longitudinal space-charge field, while higher energies call for diamagnetic materials. There is, of course, no real need for diamagnetic materials; in a real machine only a fraction of the circumference would contain an inductive wall, and it is necessary for the inductive wall field to be stronger than the space-charge field by a factor of circumference/(circumference occupied by inductive wall), thus leading to a requirement which once again gives reasonable values for permeability.

4 Conclusion

If there can be found space in an isochronous cyclotron in which an inductive wall can be installed, it appears feasible to reduce the energy spread due to longitudinal space-charge electric fields.

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