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THE TYPE OF A DIFFUSE PINCH

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On the possible equilibria in a configuration of the type of a diffuse pinch

by

D. Ryutov and R. Siemon

Consider an axisymmetric equilibrium in a configuration where the current density j has only r and z components, and the magnetic field, accordingly, has only the ϕ component. Such configurations are of interest for magnetized target fusion (MTF) [1]: they include a simple diffuse Z pinch configuration (Fig.1) and a MAGO configuration, Fig 2. Both can be, in principle, imploded by conducting shells to create a plasma with fusion-grade parameters.

Fig. 1. Diffuse Z pinch with axial (vertical) current and poloidal magnetic field B_ϕ . Current flows in and out of conducting boundary at top and bottom, and plasma pressure can be supported by conducting boundary in radial direction.

Fig. 2. MAGO configuration with a current-carrying internal shaped conducting boundary.

To be of interest for fusion, these configurations have to provide MHD equilibria acceptable from the viewpoint of confinement requirements. In the present note we analyze possible equilibria and show that only equilibria where the plasma pressure is a function of a radial coordinate (no axial dependence) are possible. A framework for such an analysis is outlined, e.g., in Shafranov's survey in "Reviews of Plasma Physics" [2]. In

an arbitrary geometry the analysis may be quite cumbersome. What we show here is, that in the geometry of the type of an axisymmetric Z pinch equilibrium analysis is reduced to a set of simple algebraic relations, and allows one to come to very robust and reliable conclusions with regard to the possible equilibria.

The equation for MHD equilibrium is:

$$\nabla p = \frac{1}{c} \mathbf{j} \times \mathbf{B}. \quad (1)$$

The current streamlines are planar curves in the r - z plane and, because of the axisymmetry of the problem, form a set of axisymmetric nested "current surfaces." Equation (1) shows that the projection of the pressure gradient on a current streamline is zero, or that the pressure is constant over each current surface.

The axial current enclosed by some current surface is, obviously, constant. Therefore, the magnetic field (which, as we have already mentioned, has only an azimuthal component), is equal to

$$B_\phi = \frac{2I}{rc} \quad (2)$$

where r is a distance from the axis to a point on a current surface.

Fig. 3. Possible open and closed current surfaces in the diffuse Z pinch geometry.

In principle, there may exist two types of the current surfaces, the open and the closed ones (Fig. 3). We will show that, in fact, only current surfaces of the first type are possible. On the open current surfaces the pressure is constant from one material wall to another.

Consider first the open current surfaces. Take two infinitesimally close surfaces, as shown in Fig. 4. Take now two points, 1 and 2, and consider constraints imposed by the current continuity and equilibrium. Denote the distances between the current surfaces at these two points as a_1 and a_2 . Both are counted along the normal in the corresponding point. Both are infinitesimal. The current conservation equation reads:

$$j_1 a_1 r_1 = j_2 a_2 r_2 \quad (3)$$

Fig. 4. Open and infinitesimally close current surfaces with a variable radius along z .

The force balance equation (1) shows that the pressure does not vary along the current surface, so that the absolute value of the pressure gradient is equal to the normal derivative across the current surface. We also note that, as the pressure is constant over the current surface, so is the pressure difference between the two neighboring surfaces,

$$\delta p_1 = \delta p_2 \equiv \delta p. \quad (4)$$

This means that the absolute value of the pressure gradient in the points 1 and 2 is $\delta p / a_1$ and $\delta p / a_2$, respectively. The absolute value of the r.h.s. of Eq.(1) is just jB_ϕ/c , because the current is perpendicular to the magnetic field. Collecting these observations together, one can write the force balance equations in the points 1 and 2 as:

$$\frac{\delta p}{a_1} = \frac{j_1 B_{\phi 1}}{c}, \quad \frac{\delta p}{a_2} = \frac{j_2 B_{\phi 2}}{c}. \quad (5)$$

Eq. (5) shows that

$$\delta p = \frac{j_1 a_1 B_{\phi 1}}{c} = \frac{j_2 a_2 B_{\phi 2}}{c}. \quad (6)$$

Substituting Eq. (2), one finds:

$$\delta p = \frac{2I_1 j_1 a_1 r_1}{r_1^2 c^2} = \frac{2I_2 j_2 a_2 r_2}{r_2^2 c^2}. \quad (7)$$

Here we have reached a key point of the whole analysis: Eq. (3) shows that the product jar does not change along the current surface, $jar = \text{const}$. Therefore, Eq. (7) means that

$$r_1 = r_2, \quad (8)$$

i.e., that the open current surfaces are straight cylinders. This is a standard pinch equilibrium, where the current flows along the axis, and the plasma pressure depends only upon the radius. An alternative to this conclusion might be that $\delta p = 0$, i.e., the pressure is constant not only along the current surfaces but also across them (in other words, it is just

uniform). But this is nothing more than a degenerate limit of the previous system, with a zero current density and a uniform pressure.

Consider now a possible situation where there exists a domain of closed current surfaces (Fig. 5). Taking two infinitesimally close current surfaces and reproducing the previous analysis for two points, 1 and 2, one again arrives at relationship (7), from which it follows that $\delta p = 0$ (clearly, the closed current surfaces cannot be cylindrical, and Eq. (8) cannot hold for them). In other words, in the area of the closed current surfaces the pressure must be constant, meaning also that there is no current in this zone (actually, no

current surfaces). However, the "blob" of a currentless plasma cannot be immersed into the current carrying plasma, where the current surfaces must be straight coaxial cylinders (there would be a current discontinuity on the boundary).

Accordingly, we conclude that the only allowed configuration is that of a purely axial current and pressure uniform along the axis.

If one creates the initial state that does not belong to the class of the aforementioned equilibria, plasma motions set in that tend to bring the plasma to an "allowed" equilibrium. The plasma dynamics is governed by an equation

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \frac{1}{c} \mathbf{j} \times \mathbf{B} . \quad (9)$$

The current-continuity constraint, $\nabla \cdot \mathbf{j} = 0$ is maintained by the polarization current related to the plasma inertia (the l.h.s. of Eq. (9)). We will discuss the properties of this equation in a future memo.

In the quasi-steady-state problem with heat sources (which may be of any nature, including the $p dV$ work), the pressure distribution will be governed by the transport equations, which, generally speaking, do not produce the "allowed" equilibria. Therefore, one can expect that in this case convective cells will be formed. Conditions under which one can expect formation of a steady-state convection will also be discussed in a future memo.

Fig. 5. Closed and infinitesimally close current surfaces.

One can think of a situation where the pressure is constant along the axis over the whole length of the device, except narrow regions near the end-walls. Then, it is probable that the convection will occur only in the close proximity of the wall. What will be its effects, in particular, with regard to bringing impurities into the hotter plasma, is to be studied.

Note that there are no similar constraints in the wall-confined field-reversed configurations, where only the azimuthal current is present and the current continuity condition is satisfied automatically.

References

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2. V.D. Shafranov. In: "Reviews of Plasma Physics" (M.A. Leontovich - Ed.), v.2, p. 103, Consultants Bureau, NY 1966.