TITLE: COOLING TOWER AND PLUME MODELING FOR SATELLITE REMOTE SENSING APPLICATIONS

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SUBMITTED TO: To be handed out to interested parties.

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Abstract

It is often useful in nonproliferation studies to be able to remotely estimate the power generated by a power plant. Such information is indirectly available through an examination of the power dissipated by the plant. Power dissipation is generally accomplished either by transferring the excess heat generated into the atmosphere or into bodies of water. It is the former method with which we are exclusively concerned in this report. We discuss in this report the difficulties associated with such a task. In particular, we primarily address the remote detection of the temperature associated with the condensed water plume emitted from the cooling tower. We find that the effective emissivity of the plume is of fundamental importance for this task. Having examined the dependence of the plume emissivity in several IR bands and with varying liquid water content and droplet size distributions, we conclude that the plume emissivity, and consequently the plume brightness temperature, is dependent upon not only the liquid water content and band, but also upon the droplet size distribution.

Finally, we discuss models dependent upon a detailed point-by-point description of the hydrodynamics and thermodynamics of the plume dynamics and those based upon spatially integrated models. We describe in detail a new integral model, the LANL Plume Model, which accounts for the evolution of the droplet size distribution. Some typical results obtained from this model are discussed.

1 Introduction

The objective of anti-proliferation activities is to be able to detect the possible production of materials that might be used in the creation of nuclear weapons. As part of this effort, it is important to be able to monitor power generation of nuclear power plants. Since all power generating plants are not 100% efficient, they must also dissipate heat. The amount of heat that must be dissipated is generally expected to be proportional to the amount of power generated. Power plants typically use water to carry the heat away from the reactor vessel. This water is commonly cooled either by cooling ponds or cooling towers. From a remote sensing point of view, it is the cooling ponds or exit gases from the cooling towers which are the most directly observable measurements of the power generation. In this report we will focus exclusively on the potential value of remotely observing the gases exiting from a wet cooling tower for predicting the power dissipation in the cooling tower.

Ambient air is drawn into a wet cooling tower where it exchanges heat with the atomized warm cooling water. The heat lost by the cooling water goes to both heating the air and water vapor and vaporizing the liquid water contained within the ambient air and the cooling water itself. The amount of heat which is lost by the circulating cooling water is gained by the ambient air; consequently, the total heat gained by the ambient air is directly related to the heat dissipated by the cooling tower. However, this heat exchange is not merely reflected in the temperature change of the air, but also in the heat required to vaporize liquid water.

From a remote sensing perspective, it is the radiative properties of the cooling tower gases which are of greatest interest. As will be demonstrated later in this report, it is the condensed water phase which most determines the radiative properties of the plume. For this reason and because the water content of the plume is important in determining the heat dissipation, both the liquid and water vapor content of the air must be tracked carefully.

The warm, moist air exiting from the cooling tower interacts with the ambient air in a complex fashion. Ambient air is entrained with the exiting air through shear induced turbulence, especially close to the cooling tower exit orifice. The entrained ambient air cools the plume exiting from the cooling tower. The air is also cooled by adiabatic expansion, thermal diffusion and radiative losses. As a consequence of the falling plume temperature, the saturation increases and water vapor condenses, thereby heating the air. Further from the cooling tower, entrainment continues and the diffusion of the plume becomes important. The plume continues to cool, retarded somewhat by condensation. Eventually, the air is no longer supersaturated, the droplets begin to evaporate and the excess water is diffusively dispersed.

The extent to which plume properties and processes must be accurately modelled depends, in part, upon the spatial resolution at which the data is obtained. If the properties of the plume change significantly over the distance which the plume is resolved, the derived properties of the plume will not directly reflect the exit plume conditions. Consequently, it is only with the aid of plume models that exit conditions might reasonably be extracted. Plume models can additionally be used to investigate the sensitivity of the plume properties to initial conditions, which will only be approximately known. In this document we will begin to address these issues.

2 What We Need to Know

Our stated objective is to determine the power dissipated in a wet cooling tower. Heat is transferred from the cooling water to the ambient air and the make-up water. The make-up water is added to the cooling tower to compensate for the loss of cooling water due to evaporation. Heat is also lost from the cooling water by evaporation. The air stream carries away the water evaporated from the cooling water as both water vapor and condensed water droplets. Consequently, the enthalpy of the air stream is increased due to both heating and the infusion of water.

Neglecting the heat transfer associated with the make-up water, the heat lost by the cooling water is taken up by the air. With this approximation, we can express the rate of heat dissipation in the cooling tower, $\dot{Q}$, as

$$\dot{Q} = (H_{out} - H_{in})WA,$$

where W is the vertical velocity of the air stream, A is the effective area of the cooling tower, and H , the total enthalpy of the air per unit volume, is defined by

$$H = c_{pa}\rho_a T + c_{pv}\rho_v T + L\rho_v.$$

Here c_{pa} and c_{pv} are the specific heats at constant pressure for dry air and water vapor, respectively; ρ_a and ρ_v are the mass densities of dry air and water vapor; and L is the latent heat of vaporization. Because the relative amount of water vapor compared to that of air is small, H can also be written as

$$H = \rho_a h = \rho_a ((c_{pa} + c_{pv}q_v)T + Lq_v),$$

where q_v is the mass mixing ratio of water vapor.

In order to determine the rate of heat dissipation, $\dot{Q}$, we must determine W , A , T_{in} , T_{out} , and the inlet and outlet air mass densities and water vapor mixing ratios. Because of the small changes in temperature, the inlet and output air mass densities are nearly identical. Using the ideal gas law, the inlet air density can be determined knowing the air pressure and inlet air temperature. If we assume that the outlet air is saturated, we can determine the outlet water vapor mixing ratio from the outlet temperature. Knowing the relative humidity of the inlet air and the inlet air temperature, the inlet water vapor mixing ratio is fixed. The enthalpy change of the air is relatively insensitive to the inlet air relative humidity since the cool unsaturated inlet air contains a relatively small amount of water vapor. In figure (1) is plotted the change of enthalpy of the air due to passage through the cooling tower under the assumption that the inlet air temperature is 295 degrees Kelvin and that the outlet air is saturated at a temperature of 315 degrees Kelvin. What is plotted is the dependence of the relative change of enthalpy on the inlet relative humidity. The maximum change in enthalpy occurs when the inlet relative humidity is zero. An examination of figure (1), indicates that knowing the relative humidity to within about 10% will result in an relative uncertainty of less than 5%, which is probably adequate in most cases.

The two additional parameters required are the outlet air velocity, W , and the effective area of the cooling tower. We will assume that the effective area can be provided from either the satellite data or other sources. Since this parameter does not change with time, lacking its value implies that only relative values of heat dissipation can be determined.

The vertical movement of the air inside the cooling tower is primarily due the buoyancy induced by the heating of the air. However, the flow of the air is impeded by the cooling tower; thereby, significantly reducing its value. Following Popendiek and Eishen (1992), there are pressure losses encountered at the entrance and exit of the cooling tower, on passage through the tower packing, where much of the heat exchange takes place, and passage through the cylindrical portion of the tower. In order to determine W various properties of the cooling tower must be known. Assuming that these can be determined or approximated, the air velocity is primarily a function of the inlet and outlet air densities (Popendiek and Eishen, 1992).

Regarding the ambient air, we require knowledge of its temperature, pressure, and relative humidity. These values can be obtained somewhat from nearby meteorological measurements. Assuming that the outlet air is saturated, we require in addition the outlet air temperature. It is the air temperature, therefore, that is the primary variable to be determined from satellite measurements. This conclusion is, however, based upon our assumption that the plume exiting from the cooling tower is saturated. We will want to test this hypothesis later, especially as regards the subsequent behavior of the plume.

3 What We See

The properties of the plume which are remotely detected are band dependent. In the visible bands the primary source of radiation is the sun. In these bands it is the scattered solar radiation which is detected. In the lower IR bands, when the wavelength is less than about 4 microns, radiation is supplied from both the sun and terrestrial sources. The solar radiation will contribute to the detected radiation through scattering. The terrestrial source of radiation consists of emittance from the atmosphere, earth's surface, and the plume itself. The plume will not only emit radiation but will also scatter radiation from all sources, including sources from within itself.

The plume temperature is in the neighborhood of 300 degrees Kelvin. At these temperatures the peak of the black body radiation lies above 9 microns. Since we are primarily concerned with the plume temperature, we will focus our attention upon bands in the mid-IR, above about 3 microns. For the sake of simplicity, at wavelengths in the solar overlap region, we will assume that any measurements of the plume occur at night when contamination from solar reflection can be neglected.

The visible bands, though they do not inform us directly of the plume temperature, still have some value. Assuming that the total scattering optical depth of the atmosphere is sufficiently less than one, solar scattering from the plume gives direct evidence of the location of plume scatterers. Molecular scattering within the plume is of a Rayleigh nature. This means that the scattering optical depth is inversely proportional to λ^4 . Standard Rayleigh scattering calculations show that for wavelengths near 0.55 microns the scattering optical depth per kilometer is on the order of 10^{-2} . An examination of the Shettle and Fenn aerosol data (1979) indicates that the scattering optical depth per kilometer at 0.55 microns is, even for a visibility of 2 kilometers, less than 2.0. The scattering optical depth of water vapor is somewhat complicated by the tendency of water vapor to coagulate into multiple molecules and to condense on dust. Using the result listed in Iqbal (1983), the scattering optical depth for one centimeter of precipitable water $k_{ws\lambda}$ is given by

$$k_{ws\lambda} = 0.008635 \frac{1}{\lambda^2}.$$

The amount of water vapor in air at 320 degrees Kelvin is less than 100 gm/m^3 , which is equivalent to 10 centimeters of precipitable water. Hence, the scattering optical depth per kilometer is less than about 0.285 at 0.55 microns.

The remaining contribution to scattering from the plume is that from water droplets. The Mie scattering results at a wavelength of 0.55 microns are shown in figure (2). The value of the scattering optical depth per kilometer at each wavelength is obtained under the assumption that the droplet size distribution is a delta function at the given radius and that the density of liquid water is 10^{-6} gm/cm^3 , which is a typical density in the cooling tower plume. For such a single size distribution, the optical depth per kilometer is given by

$$\frac{0.75 \rho Q_e}{\rho_l r} 10^9,$$

where ρ and ρ_l are the density of water in a volume of air and within the water droplet, respectively, Q_e is the extinction efficiency factor and r is the droplet size in microns.

Droplets size distributions discharged from cooling towers typically display a bimodal characteristic. The smaller sized distribution is composed chiefly of droplets from about 1 to 10 microns, while that for the larger sizes is generally greater than about 50 microns. (Sauvageot, 1989). The smaller sizes are due to condensation of water vapor onto aerosol nuclei. The larger sizes are due to salts emitted from the cooling tower and are referred to as drift droplets. The relative amount of drift droplets is generally less than ten times smaller than that for the condensation droplets.

Neglecting the influence of the large size drift droplets, we expect that the droplet size contained within the plume will be between about 0.1 to 20 microns. Consequently, for all droplet sizes of interest, the scattering optical depth for water droplets is about two orders of magnitude larger than that for any other plume constituent. The trend indicated in figure (2) indicates that the scattering optical depth for the drift droplets with sizes greater than about 50 microns are at least an order of magnitude smaller than that for droplet sizes in the .1 to 20 micron range.

The size of the plume is on the order of the cooling tower exit orifice. This is especially true for distances within a few diameters from the cooling tower. The orifice size is certainly less than or equal to about 100 meters, and much less than that for mechanical draft cooling towers. Consequently, in order that the scattering optical depth be at least one, plume constituents with scattering optical depths per kilometer less than 10 will not be visible. We conclude, therefore, that in the visible bands it is primarily the location of the droplets with sizes from about 0.1 to 20 microns that the solar scattered signal tracks. It is worth noting that the ground resolution in the visible is superior to that in higher bands; therefore, satellite imagery in the visible informs us of the concentration of these droplet sizes at a significantly higher spatial resolution than the IR.

3.1 Radiative Transfer with Plume

For the sake of simplicity let us examine the radiation transfer in the mid-IR at night. In particular, we are primarily interested in bands above 3.5 microns. The scattering optical depth per kilometer from molecules, water vapor and aerosols decreases with wavelength; consequently, their scattering optical depths are smaller than those listed above. The scattering optical depth per kilometer for Rayleigh scattering at 3.5 microns is on the order of 6×10^{-6} ; while for water vapor it is less than 7×10^{-4} ; and for aerosols it is less than about 0.4 for 2 kilometer visibility, but only about a third of that for 10 kilometer visibility. As a consequence, we will assume that molecular and aerosol scattering in the mid-IR and above can be neglected.

However, just as in the visible bands, scattering cannot be neglected from water droplets. Since we expect plume cross-sections to be on the order of tens of meters, scattering optical depths per kilometer greater than 100 will result in plume scattering optical depths significantly greater than one. An examination of figures (3) through (5) indicate that certainly within the expected droplet sizes scattering cannot be neglected. In these figures we have considered scattering within three bands, 3.5-4.0 microns, 8.0-8.8 microns, and 10.0-11.5 microns. The importance of scattering apparently decreases with wavelength. In all cases, it is the droplets in the range of about 1 to 20 microns that most significantly affect the scattering properties of the plume.

Because the computation of three dimensional transport is costly and time consuming, we want to develop a simplified one dimensional model of the plume, atmosphere and surface. If scattering were completely absent, radiance received at the detector from a given view direction would be due to the emission and transmission through the atmosphere from volume and surface elements only within the line of sight in that view direction. However, when scattering is present, scattering from volume and surface elements outside of the line of sight can contribute to the radiance received at the detector. For example, when the radiative surface properties are inhomogeneous, scattering from the atmosphere results in the well known adjacency effect.

In the mid-IR atmospheric scattering is negligible, but scattering within the plume is not. This means that plume volumes not located within the line of sight will affect the radiance emitted into the view direction. Because the temperature and radiative properties of the plume vary as a function of position, it will not be true that radiative properties of the plume are strictly a function of only one variable, e.g., the distance normal to the axis of the plume. Properties of the plume outside of the region defined by the line of sight will influence the radiance received at the detector. The effect, in a manner similar to that of the atmospheric point spread function, is to reduce the spatial resolution of the detector. To fully describe the radiative transfer in this environment would require a three dimensional transport code. We might, for example, attempt to define a transfer function through the plume as a function of absolute position along the plume and a coordinate relative to that location. Such an effort is, however, beyond the scope of this document.

We will determine the optical properties of a section of plume by considering a narrow slice in the direction of the line of sight, thereby, neglecting the influence, through the scattering process, of plume slices outside of the line of sight. The effective optical depth τ_e of the plume will simply be described by $\tau/|\Omega_v \cdot \hat{n}_p|$, where τ is the optical depth perpendicular to the plume axis, Ω_v is the view direction, and $\hat{n}_p$ is the unit vector normal to the plume axis.

In a one dimensional approximation to the plume, the effect of scattering within the plume can be taken into account by the use of the reflection and transmission functions for the plume (Liou, 1980). Consider a plume layer of given optical properties and optical depth τ_p embedded within a vacuum both above and below the layer. If I_0 is the radiance incident upon the top of the plume layer, the diffuse radiance reflected, I_r , and transmitted, I_t , through the layer are given by (Liou, 1980)

$$I_r(0, \mu, \phi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 R(\mu, \phi; \mu', \phi') I_0(-\mu', \phi') \mu' d\mu' d\phi'$$

$$I_t(\tau_p, -\mu, \phi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 T(\mu, \phi; \mu', \phi') I_0(-\mu', \phi') \mu' d\mu' d\phi',$$

where R and T are the reflection and transmission functions, respectively. In both cases, μ is the cosine of the angle between the vertical and the direction of the radiance. μ takes on values between 0 and 1. The optical depth is zero at the top of the layer and τ_p at the bottom.

If the incident radiation at the top of the plume layer is monodirectional, the reflected radiance is given by $R(\mu_0, \phi_0; \mu, \phi)S_0$, where S_0 is the incident flux and (μ_0, ϕ_0) is the direction of the incident radiation. Likewise, the transmitted radiation is given by $T(\mu_0, \phi_0; \mu, \phi)S_0 + \exp(-\tau_p/\mu_0)S_0/\mu_0\delta(\Omega - \Omega_0)$, where the expression $\delta(\Omega - \Omega_0)$ is a delta function in the direction of the incident radiation.

The total transmission through the plume layer is the sum of the directly transmitted radiance and the diffuse radiance. The direct transmittance is $I_0(-\mu_0, \phi_0)\exp(-\tau_p/\mu_0)$. Hence, the total transmitted radiance I_T is given by

$$I_T(\tau_p, -\mu, \phi) = I_0(-\mu_0, \phi_0)\exp(-\tau_p/\mu_0) + I_i(\tau_p, -\mu, \phi).$$

Molecular and atmospheric scattering is a function of angle between the incident and scattered radiation; consequently, it is a function of the dot product of the incident and scattered directions. In particular, it is a function of the difference between the azimuthal directions of the incident and scattered radiation. If the solution to the radiation field is divided into an unscattered and scattered field, the coefficients of the resulting transport equation for the scattered field is a function of the difference in azimuth between the monodirectional incident radiance and azimuth of the scattered field. If, in addition, the reflection boundary condition at the earth's surface is also a function of the difference in the azimuthal directions between the incident and reflected radiation, then the equation satisfied by the difference between the radiance at different azimuths, but identical zenith angles, will be satisfied by a radiance of zero. Consequently, the radiation field for a monodirectional source, resulting from scattering which is only dependent upon the relative scattering angle, is a function of the relative azimuth, and not upon the absolute azimuth. It is clear, therefore, that the azimuthal dependence of both the transmission and reflectance functions are only a function of the difference between the azimuthal angles.

For layers whose optical properties are invariant under reflection through a plane located within the layer, the reflection and transmission functions are invariant under changes of μ and μ' to $-\mu$ and $-\mu'$; and for ϕ and ϕ' to $\phi + \pi$ and $\phi' + \pi$. However, since the reflection and transmission functions depend upon $\phi - \phi'$, the azimuthal dependence is unaffected. Consequently, for such layers, $T_T(\mu, \phi; \mu', \phi') = T_B(-\mu, \phi; -\mu', \phi')$, where T_T and T_B are the transmission functions for transmission from the top and bottom, respectively. An analogous relationship holds for reflection from the top and the bottom of such symmetric layers. We will assume that plume layers possess this symmetry property.

The plume layer not only scatters and absorbs incident radiation, it also emits radiation. The cumulative effect of this emission will be described by an effective emissivity. The effective emissivity is determined by considering the emission from the plume layer embedded within a vacuum both above and below the plume layer. The radiances at the top and bottom of such a layer can be written formally as

$$\begin{aligned} I(0, \mu, \phi) &= \int_{-\tau_p/2}^{\tau_p/2} e^{-(\tau_p/2-\tau)/\mu} \omega(\tau_p/2-\tau) P(\tau_p/2-\tau, \mu, \phi; \mu', \phi') I(\tau_p/2-\tau, \mu', \phi') d\mu' d\phi' \frac{d\tau}{\mu} \\ &\quad + \int_{-\tau_p/2}^{\tau_p/2} (1 - \omega(\tau_p/2-\tau)) B_\nu(\tau_p/2-\tau) e^{-(\tau_p/2-\tau)/\mu} \frac{d\tau}{\mu} \\ I(\tau, -\mu, \phi) &= \int_{-\tau_p/2}^{\tau_p/2} e^{-(\tau_p/2+\tau)/\mu} \omega(\tau_p/2+\tau) P(\tau_p/2+\tau, \mu, \phi; \mu', \phi') I(\tau_p/2+\tau, \mu', \phi') d\mu' d\phi' \frac{d\tau}{\mu} \\ &\quad + \int_{-\tau_p/2}^{\tau_p/2} (1 - \omega(\tau_p/2+\tau)) B_\nu(\tau_p/2+\tau) e^{-(\tau_p/2+\tau)/\mu} \frac{d\tau}{\mu} \end{aligned}$$

The former expression is the value of the upgoing radiance at the top of the plume layer; and the latter expression is the downgoing radiance at the bottom of the plume layer. As above, the values of μ takes on values between 0 and 1. ω and P are the position dependent single scattering albedo and phase function, respectively. The last term in both expressions represents the direct contribution from the direct emission at a depth τ within the layer. If the plume layer's emission and optical properties are symmetric about $\tau_p/2$, $B_\nu(\tau_p/2+\tau) = B_\nu(\tau_p/2-\tau)$, and the direct contribution from the plume emission are equal for both the top and bottom radiances

The formal expressions for the radiances at the top and bottom of the layers can be solved iteratively by the use of the source terms $B_\nu(\tau_p/2 - \tau)e^{-(\tau_p/2 - \tau)/\mu}$ and $B_\nu(\tau_p/2 + \tau)e^{-(\tau_p/2 + \tau)/\mu}$ for the radiances at the top and the bottom, respectively. But because of the assumed symmetry the iterated source terms will also possess the same symmetry. As a consequence, if, in addition, the scattering and absorption properties of the plume layer are symmetric about $\tau/2$,

$$I(0, \mu, \phi) = I(\tau_p - \mu, \phi)$$

The contributions from the direct emission within the plume can be expressed for both the radiance at the top and at the bottom in terms of an effective plume temperature. Let us define an effective plume temperature by

$$B_\nu(\bar{T}_p) \equiv \frac{\int_0^{\tau_p} B_\nu(\tau) e^{-(\tau_p - \tau)/\mu} d\tau}{\int_0^{\tau_p} e^{-(\tau_p - \tau)/\mu} d\tau}$$

Because of the assumed symmetry with respect to $\tau_p/2$, the effective plume temperature will be the same for radiances both at the top and the bottom of the plume layer as can be seen by substituting $\tau_p - \tau$ for τ in the above definition.

Let us now define an effective emissivity as

$$I(0, \mu, \phi) = \epsilon_p(0, \mu, \phi) B_\nu(\bar{T}_p)$$

and similarly for the bottom of the plume layer

$$I(\tau_p, -\mu, \phi) = \epsilon_p(\tau_p, -\mu, \phi) B_\nu(\bar{T}_p)$$

However, since $I(0, \mu, \phi) = I(\tau_p, -\mu, \phi)$, we conclude that

$$\epsilon_p(0, \mu, \phi) = \epsilon_p(\tau_p, -\mu, \phi)$$

In the absence of a solar source, the radiation field will be azimuthally independent; consequently,

$$\epsilon_p(0, \mu) = \epsilon_p(\tau_p, -\mu)$$

Now consider a surface located directly below the plume layer with wavelength dependent emissivity ϵ and temperature T_s , thereby treating any intervening space between the plume layer and the earth's surface as a vacuum. Assume that the earth's surface is a Lambertian scattering surface with wavelength dependent scattering albedo $(1 - \epsilon)/\pi$. Let τ_a and τ_p be the atmospheric and plume optical depths, respectively. Lacking any solar radiation, the resulting radiation field will be azimuthally independent. With this in mind, define the spherical albedo, $\bar{F}$, (Liou, 1980) as

$$\bar{F} = \frac{1}{\pi} \int_0^{2\pi} \int_0^\pi \int_0^\infty R(\mu, \phi; \mu'; \phi') \mu' d\mu' d\phi' d\mu d\phi$$

and the transmission of the plume layer, $t(\mu)$, (Liou, 1980) as

$$t(\mu) = \frac{1}{\pi} \int_0^{2\pi} \int_0^\pi T(\mu, \phi; \mu'; \phi') \mu' d\mu' d\phi'$$

where we have used the dependence of the transmission function on the relative azimuth, $\phi - \phi'$.

The radiance incident upon the top of the plume layer is, in the absence of solar radiation, due to the emittance from the atmosphere. This irradiance is given by

$$\int_0^{\tau_a} B_\nu(T_a) e^{-(\tau_a - \tau)/\mu} \frac{d\tau}{\mu}$$

where T_a is the temperature of the atmosphere as a function of optical depth τ within the atmosphere and B_ν is the Planck function. In a manner similar to that used to define an effective plume temperature, we define a downgoing effective atmospheric temperature T_a^D as

$$B_\nu(T_a^D) \equiv \frac{\int_0^{\tau_a} B_\nu(T_a) e^{-(\tau_a - \tau)/\mu} d\tau}{\int_0^{\tau_a} e^{-(\tau_a - \tau)/\mu} d\tau}$$

where T_a^D is the temperature of the atmosphere as a function of optical depth τ within the atmosphere and B_ν is the Planck function. In a manner similar to that used to define an effective plume temperature, we define a downgoing effective atmospheric temperature T_a^D as

It is worth noting that this effective atmospheric temperature will be different for the radiance at the top of the atmosphere and at the bottom. This difference is a consequence of the lack of symmetry of the atmosphere about $\tau_a/2$. It is for this reason that we must define an effective downgoing and upgoing effective atmospheric temperature.

Neglecting all atmospheric scattering, the angle-independent upgoing radiance from the earth's surface I_p is given by the sum of the emission from the surface and the emission from the atmosphere. The emission from the surface is given by $B_\nu(T_p)$ and the emission from the atmosphere is given by $B_\nu(T_a)$. The average emissivity $\bar{\epsilon}_p$ is given by

$$\bar{\epsilon}_p = \frac{1}{\pi} \int_0^\pi \epsilon_p(\mu) \mu d\mu$$

where the average emissivity $\bar{\epsilon}_p$ is given by

$$I_p(\mu) = I_0 \gamma(\mu) + \bar{\epsilon}_p B_\nu(T_p) + \epsilon B_\nu(T_a) \gamma(\mu) + 2 B_\nu(T_a) \int_0^\pi (1 - e^{-\tau_a \mu'}) R(\mu, \mu') \mu' d\mu'$$

and, finally, the radiance I_{top} at the top of the atmosphere is given by

$$I_{top}(\mu) = I_p \exp(-\tau_a \mu) + B_\nu(T_a) (1 - e^{-\tau_a \mu})$$

where T_a is the effective upgoing atmospheric temperature. When the single scattering albedo is zero throughout the plume, the reflection and transmission functions both are zero; consequently, the radiance field in a non-scattering plume is easily derived from the result listed above.

The distribution of radiance at the top of the plume layer can be computed by summing the contributions from the following two transfer problems:

1. Place a plume layer with the desired temperature distribution over a surface at temperature T_p and emissivity ϵ . Above the plume layer place a vacuum.
2. Place a plume layer with all emission absent ($T_p = 0$) over a surface with reflectance $1 - \epsilon$. Determine the emission from the atmosphere that is incident upon the plume layer from another run. Modtran will suffice for this calculation with a surface emissivity of one and temperature of zero. Let the resulting radiance field at the plume height be incident upon this plume layer.

The sum of the resultant upgoing radiances at the top of the plume layer is the desired radiance field I_p . The radiance I_{top} can be additionally obtained by determining the upgoing radiance at the top of the atmosphere by running Modtran again with a zero reflectance and emission from the surface. From this calculation the atmospheric transmittance is simultaneously determined. Summing the product of the atmospheric transmittance and the radiation field at the top of the plume I_p and the result from the Modtran run, we obtain the total radiance I_{top} at the top of the atmosphere.

Similar results can also be obtained by determining the downgoing and upgoing effective atmospheric temperature and using that result, in conjunction with the wavelength dependent atmospheric transmittance, to compute the total atmospheric emittance at the top of the plume and the top of the atmosphere.

3.1.1 Effective Plume Emissivity

The radiance I_{top} incident upon the detector is a complex function of atmospheric, plume, and surface emission and scattering properties. The surface emissivity for water is sufficiently close to one for all bands of interest that reflection from its surface can be neglected. The emissivity for vegetated surfaces is generally greater than about 0.95 in those bands of interest. However, the emissivity of various soils and mineral surfaces can vary significantly from 1 throughout the mid-IR, as can painted surfaces (The IR Handbook, 1978). Both because the ground emissivity is often close to one and because much of the reflected radiation has to pass through the plume twice or nearly twice, we expect the contribution of the surface reflected radiance to be small. Neglecting, for purposes of demonstration, the contribution from surface reflection, we observe that the radiance received at the detector is a function of the surface temperature, radiative properties and temperature of the plume, and an atmospheric contribution. Assuming that a night-time temperature retrieval algorithm accounts for the direct emission from the atmosphere into the detector, the brightness temperature measured by the detector will still include reflectance of the atmospheric emission from the plume, emission from the surface and its transmission through the plume, and the plume emission. We are only interested in the latter quantity. For plumes of sufficient optical depth, we expect the surface emission to be of small consequence. The plume temperature will in all cases be significantly larger than the atmospheric temperature. We might expect, therefore, that the reflected atmospheric emission is, by comparison with the plume emission, small. The received radiance at the detector is, as a consequence, primarily most sensitive to $\epsilon_p B_\nu(\bar{T}_p)$.

As a consequence of this quite general analysis, it is clear that an understanding of the effective plume emissivity is essential in deriving the plume temperature from the radiance received at the detector. Let us consider a homogeneous plume layer at temperature T_p with single scattering albedo ω and asymmetry parameter g . The asymmetry parameter is the average cosine of the scattering phase function. A Henyey-Greenstein phase function is only a function of the asymmetry parameter. Such a phase function does not well represent the actual phase function for the droplets in both the forward and backward scattering directions. However, such a representation is frequently used for its convenience (e.g., Shettle and Fenn, 1979). We shall do likewise.

Knowing the droplet size distribution and the spatial variation of this distribution, we can determine with reasonable accuracy the optical properties of the plume. However, the details of the droplet size distribution will not be known, although model calculations can aid us in identifying a specific distribution. As a result, we must expect that this uncertainty in the droplet size distribution will yield an uncertainty in the emissivity, and consequently, an uncertainty in the temperature inversion. Figures (6) and (7) were produced under the assumption that the uncertainty in the temperature inversion be less than or equal to 0.5 degrees Kelvin. Note that these calculations were made with the Planck distribution at a given wavelength and not with the wavelength-integrated Planck Distribution. The results indicated in these figures demonstrate that the plume emissivity should be known to within a few percent. Apparently, the sensitivity of the temperature inversion is somewhat reduced for the 3.5-4 micron bands over that for the bands closer to the peak of the blackbody distribution associated with the plume temperature.

An examination of figures (8) through (13) depict the variation of ω and g with droplet size at various wavelengths. Our expectation is that the droplet size will be in the range from about 1 to 20 microns. These figures indicate that there is considerable variation in the values of these parameters. The closer the asymmetry parameter is to one, the more the scattering is in the forward direction. A value of zero corresponds to isotropic scattering. If the wavelength is much larger than the radius of the scatterer, the scattering is more isotropic. When the wavelength is much smaller than the radius, geometric optic effects dominate and the scattering tends to be more in the forward direction. When the radius is comparable to wavelength, resonance effects can dominate, producing oscillations in the optical properties. All of these effects can be observed in figures (8) through (13).

In table 1 is listed the average plume emissivity $\bar{\epsilon}_p$ for various values of optical depth τ_p , single scattering albedo ω , and asymmetry parameter g . The values chosen were intended to somewhat cover the range of values expected for each band.

In all cases, the value of the average emissivity increases with optical depth. We can see the reason for this

behavior by examining the effective plume emissivity ϵ_p for a homogeneous layer. We obtain the formal solution

$$\epsilon_p(\mu) = (1 - \omega)(1 - e^{-\tau_p/\mu}) + \omega \int_0^{\tau_p} \int_{-1}^1 P(\mu', \mu) \frac{I(\tau, \mu')}{B_\nu(T_p)} e^{-\tau/\mu} \frac{d\mu' d\tau}{\mu},$$

where P is the droplet scattering phase function integrated over all azimuths, and μ takes on values from 0 to 1. Since the value of the integral is positive definite, $\epsilon_p(\mu)$ is always greater than $(1 - \omega)(1 - e^{-\tau_p/\mu})$, which is the value we would obtain for the effective emissivity if scattering were neglected. If we now take the derivative of this expression with respect τ_p , we obtain

$$\frac{d\epsilon_p(\mu)}{d\tau_p} = e^{-\tau_p/\mu} \left(\frac{1 - \omega}{\mu} + \omega \int_{-1}^1 P(\mu', \mu) \frac{I(\tau_p, \mu')}{B_\nu(T_p)} \frac{d\mu'}{\mu} \right),$$

which demonstrates two important properties of the effective emissivity. First, ϵ_p , as well as $\bar{\epsilon}_p$, both increase with increasing optical depth since the derivative $d\epsilon_p/d\tau$ is positive definite. Secondly, what is possibly more important for our application, the rate of change of the effective emissivity with optical depth decreases with increasing optical depth. Consequently, for large optical depths, the effective emissivity tends towards an asymptotic value. This behavior is clearly demonstrated in Table 1.

The rate at which the effective emissivity approaches its asymptotic value is clearly dependent upon more than the plume optical depth. In the 3.5-4 micron band, the rate of approach is much slower than that for the other two bands. For this band, since the value of the single scattering albedo is close to one, the contribution to the effective emissivity due to direct emission is smaller than in the other bands. However, the contribution due to scattering is significantly larger. Increasing the optical depth, increases the amount of material emitting radiation. When absorption is large, increasing the optical depth has little effect since the additional emission is simply absorbed. However, when absorption is small, the increased emission can add to the emission at the top of the layer, albeit through multiple scattering. When the optical depth gets sufficiently large, thereby increasing the number of scattering and absorption opportunities, the increased emission is absorbed and the emissivity no longer increases.

Two additional characteristics of the average emissivity can be gleaned from Table 1. First, that average effective emissivity decreases with increasing single scattering albedo; and secondly, that it increases with increasing asymmetry parameter. Holding the optical depth constant, while increasing the single scattering albedo, has the effect of reducing the relative amount of absorption while increasing the amount of scattering. Reducing the amount of absorption, by Kirchhoff's law, reduces the amount of emission. Since the number of emitters is held constant, the emissivity decreases. Indeed, if the single scattering albedo is 1, nothing is absorbed, nothing emitted, and the emissivity is zero. Increasing g , while holding other variables constant, has the effect of more efficiently directing radiation toward the top and bottom of the layer, i.e., radiation finds it way through the layer through a smaller number of scatterings.

An examination of Table 1 indicates that unless we are able to constrain the range of parameter values more than those listed, we will not be able to know the emissivity to within a few percent. One hopeful sign is evidenced in the two higher bands: the variation in the emissivity is considerably less sensitive to variation in the radiative properties than in the 3.5-4 micron band. This feature is apparently due to the increased importance of absorption in the two higher bands. Unfortunately, as figures (6) and (7) indicate, the required uncertainty in the emissivity is smaller in these bands than in the lower band.

In the calculations for Table 1, we have, in order to more easily examine the character of the average emissivity, decoupled the optical depth, the single scattering parameter, and the asymmetry parameter from the underlying physics defined by the droplet size distribution. In Table 2 we have with the aid of specified droplet size distributions determined through Mie calculations the values of these three parameters. In all cases the density of liquid water was assumed equal to 1 gm/m^3 and the plume thickness was assumed to be 30 meters. The thickness of such a plume would be below average for a natural draft cooling tower, but probably about average for a mechanical draft cooling tower.

In Table 2, the calculations were made using a modified gamma distribution (Liou, 1992) $f(\alpha, r_m, r)$ for the

droplet size distribution, which is defined by

$$f(\alpha, r_m, r) = \frac{\alpha^\alpha}{\Gamma(\alpha + 1)} \frac{1}{r_m} \left(\frac{r}{r_m} \right)^\alpha e^{-\alpha r/r_m}$$

where Γ is the gamma function. α defines the halfwidth of the size distribution about its maximum value at r_m . However, the modified gamma distribution is not symmetric about the maximum except for large values of α . For example, for $\alpha = 30$, the half values are associated with radii of $0.8r_m$ and $1.23r_m$, whereas, for $\alpha = 1$, they are associated with radii of $0.23r_m$ and $2.68r_m$. r_m is also associated with the average radius $\langle r \rangle$ through the relationship

$$\langle r \rangle = \frac{\alpha + 1}{\alpha} r_m.$$

Let's first examine Table 2 for the variation in optical depth with the width of the distribution. These results are readily understood from an examination of figure (14) and figure (16), describing the extinction optical depth per kilometer as a function of the droplet size. In order to obtain the optical depth for a collection of droplets from these results for a single size droplet, the results depicted must be weighted with the probability of a droplet with a radius r having a given mass. That probability $P_M(r)$ is given by

$$P_M(r) = \frac{r^3 f(r) dr}{\int_0^\infty r^3 f(r) dr}$$

As a consequence, larger radii have a larger weight than smaller radii. For a modified gamma distribution, the maximum value of P_M is located at a radius of $(1 + 3/\alpha)r_m$. So for smaller α the maximum is shifted more from r_m than for narrower distributions.

For example, in the 3.5-4 micron band for a value of $r_m = 1$ micron, the optical depth decreases as the width of the size distribution decreases. Figure (14) indicates that the maximum extinction is obtained from radii near 2 microns. So that by narrowing the distribution about 1 micron, fewer droplets near 2 microns are included and the total extinction decreases. In the same band for a value of $r_m = 5$ microns, the opposite trend is observed as the size distribution is narrowed. In this case, broadening the size distribution, because of the r^3 weighting, especially includes the extinction associated with radii larger than 5 microns, where the extinction is significantly smaller. As a consequence, broadening the distribution lowers the optical depth. The general shift in the 3.5-4 micron band to smaller optical depths as the peak of the distribution shifts to larger radii, is due to shifting further away from the peak of the extinction at 2 microns.

The results in the 10-11.5 micron band are considerably less sensitive to droplet size distribution. In the 3.5-4 micron band, despite the fixed amount of total water, the optical depth varies from about 3.4 to about 25, while the variation in the 10-11.5 micron band is from about 3.2 to about 4.7. To understand the reasons for this difference it is necessary that we examine somewhat the fundamentals of the scattering process.

Mie scattering theory informs us that the scattering and absorption properties of droplets are a function of the refractive index and the ratio of the droplet size r to the wavelength of the incident radiation λ (Van de Hulst, 1957). When the value of r/λ is much less than one the scattering efficiency factor Q_{sc} and the absorption efficiency factor Q_{ab} , are given by (Van de Hulst, 1957)

$$Q_{sc} = \frac{8}{3} x^2 \frac{|m^2 - 1|^2}{|m^2 + 2|^2}$$

$$Q_{ab} = -4x \Im \frac{m^2 - 1}{m^2 + 2},$$

where $x = 2\pi r/\lambda$, m is the complex refractive index, and $\Im$ means the imaginary part. Since the imaginary part of the refractive index is negative, Q_{ab} , is positive. The extinction efficiency factor $Q_{ext} = Q_{sc} + Q_{ab}$. Clearly, the efficiency factors and their associated optical depths tend to zero as the value of x tends to zero.

The total absorption optical depth τ_{ab} , is given by

$$\tau_{ab} = Nz \int Q_{ab} \pi r^2 f(r) dr,$$

where N is the number density and z is the path length; consequently, for small droplets τ_{ab} is proportional to the total amount of water along the path since, in this case, τ_{ab} is proportional to $Nz \int r^3 f(r) dr$, which is proportional to the mass per unit area. When scattering efficiency is small compared to the absorption efficiency, the total absorption optical depth is approximately equal to the optical depth. Such is the case for radiative extinction through molecular constituents in the IR.

When the wavelength is much smaller than the droplet radius, the extinction paradox dictates that the extinction efficiency factor be 2, or that the extinction cross section is equal to twice the geometric cross section of the droplet, i.e., $2\pi r^2$.

With these results, we can describe the behavior of the optical depth per kilometer as a function of droplet size. When the single scattering albedo is not small and radii are significantly smaller than the wavelength, the optical depth per kilometer falls off sharply as the droplet size decreases. Keeping in mind that the optical depth per kilometer is given by

$$\frac{0.75\rho Q_e}{\rho_1 r} 10^9,$$

if the single scattering albedo is small, the optical depth per kilometer will remain constant as the radius decreases. For droplets much larger than the wavelength, optical depth per kilometer will fall off inversely with increasing droplet size. So, in general, as the value of r/λ increases from zero, the optical depth per unit length increases; and, beginning at values significantly greater than one, the optical depth per unit length decreases inversely with the droplet size for increasing values of r/λ .

In between these two extremes, resonant effects occur. In this regime the optical depth per kilometer can be larger or smaller than its asymptotic limit for large radius. For values of r/λ greater than about 1/5 to values significantly greater than one, resonant effects influence the scattering properties of the droplets. The degree which resonant affects dominate are influenced not only by the ratio of r/λ , but also by the amount of absorption within the droplet. Significant absorption dampens the constructive and destructive interference of waves within the droplet, thereby, reducing the resonant affects. In the 3.5-4 micron band the imaginary part of the refractive index is about ten times smaller than it is in the 10-11.5 band. In the visible bands the imaginary part of the refractive index is effectively zero. It is for this reason that resonant effects can be detected in the extinction curves for 0.55 μm and the 3.5-4 micron band, but are greatly reduced in the 8-8.8 and 10-11.5 micron bands. In all cases, however, resonance is evident by the maxima that occur for r/λ similar to one.

In the 10-11.5 micron band because of significant absorption, resonance effects are nearly absent and for small radii the optical depths per kilometer are nearly constant. This is especially true for wavelengths near 11.5 microns, where the imaginary part of the refractive index is about three times larger than it is at 10 microns. Because of the reduced resonance effect and because the real part of the refractive index is about 15% smaller, the optical depth per kilometer is considerably smaller in the 10-11.5 micron band than in the 3.5-4 micron band. In addition, in the 10-11.5 micron band r_m/λ is less than one in all cases, except the last case listed, where r_m equals 20 microns; whereas, in the 3.5-4 micron band the value of r_m/λ is greater than one in all cases, except the first one. This implies that resonance effects are not fully expressed in much of the 10-11.5 micron band; whereas, in the 3.5-4 micron band they are fully expressed. It is for this reason that we have included the $r_m = 20$ micron result in Table 2 in the 10-11.5 micron band. Resonance effects are small in this band, so that any resonance effects present are not sufficient to offset the decrease in optical depth which occurs for radii larger than the wavelength. As a consequence, the 20 micron result shows dramatic decreases in the optical depth over that for smaller values of r_m .

Addressing now the variation of the average effective plume emissivity, we note that in both Table 1 and Table 2 the variation is considerable. Both results imply that the variation is considerably greater in the 3.5-4 micron band than in the higher bands. This result is largely due to the influence of resonance effects in the 3.5-4 micron band and to the relatively small sizes of r/λ in the higher bands.

Of additional interest in Table 2 is the quantity $\bar{\epsilon}'_p$. $\bar{\epsilon}'_p$ is the effective emissivity that is obtained when a surface at temperature T_s is added below the plume layer. In Table 2 we have used a temperature of 300 degrees

Kelvin and a surface emissivity of one. In this case no reflection occurs at the surface and from equation (1)

$$\bar{\epsilon}'_p = \bar{\epsilon}_p + \frac{B_\nu(T_s)}{B_\nu(T_p)} \int_0^1 \gamma(\mu) \mu d\mu$$

Hence, $\bar{\epsilon}'_p$ is always larger than $\bar{\epsilon}_p$. $\bar{\epsilon}'_p$ is of interest because, not knowing the surface temperature, it is the emissivity which is most directly observed. As the optical depth tends to infinity, $\gamma(\mu)$ tends to zero and $\bar{\epsilon}'_p$ tends to $\bar{\epsilon}_p$. The optical depth at which the emission from the surface no longer affects the emissivity is band dependent. From Table 1 we conclude that for the 3.5-4 micron band this critical optical depth is on the order of 30, for the 8-8.8 micron band it is on the order of 10, and for the 10-11.5 micron band it is on the order of 5.

Regarding the critical optical depth, let us consider the specific results listed in Table 2. In the 10-11.5 micron band all optical depths, except for those associated with $r_m = 20$, are close to the critical optical depth; hence, the difference between $\bar{\epsilon}'_p$ and $\bar{\epsilon}_p$ is small. For the $r_m = 20$ cases, the optical depth is considerably below the critical optical depth and the differences between $\bar{\epsilon}_p$ and $\bar{\epsilon}'_p$ is significant. However, in the 3.5-4 micron band, the optical depths can be considerably below the critical optical depth. Consequently, in this band, unless the droplet size is small, knowledge of the surface temperature and its emissivity will be important. Of course, for thicker plumes the influence of the surface will be diminished over that indicated here.

In practice, it is not the flux that is remotely sensed, but the angular dependent radiance. Consequently, it is the angular dependent emissivity $\epsilon_p(\mu)$ which is of particular relevance for remotely sensed analysis and not the integrated emissivity $\bar{\epsilon}_p$. In figure (17) is depicted the angular dependence of the effective emissivity ϵ_p as a function of the cosine of the view direction relative to the vertical. We display here the results for four cases listed in Table 2. The band associated with each result is indicated by the wavelength λ , as is the value of mean radius r_m and halfwidth α . This not being in any way an exhaustive study, the central point is that variations in the effective emissivity on the order of 10% can be obtained for different view directions. As in the case of the integrated average emissivity $\bar{\epsilon}_p$, the situation is somewhat better in the 10-11.5 micron band over that for the lower 3.5-4.0 micron band.

Finally, we should keep in mind that these results are for horizontal plume layers. The view direction should, therefore, be properly thought of as the angle between the normal to the plume axis and the view direction. Real plumes both rise due to buoyancy effects and are driven downwind by horizontal surface winds, producing, thereby, a plume axis direction which lies between the vertical and horizontal. In addition, the angle which the plume axis makes with the vertical is position dependent. Very near the cooling tower exit it will be very nearly vertical, whereas near the maximum vertical extent of the plume it will be horizontal. However, for all but the smallest of horizontal wind speeds, the plume will be generally aligned in the horizontal direction. Nevertheless, the angular dependence of the plume emissivity complicates the interpretation of remotely sensed data.

4 Plume Model

The purpose of a plume model is to sufficiently duplicate the properties of actual plumes that we can deduce the ability of a given detection system to determine the temperature distribution of the plume. There are two distinct approaches that have been developed in describing plume physics. One is a detailed description of the hydrodynamic flow, including turbulence; and the second are integral models, which consider sections of the plume as a whole.

The basic equations required to describe the steady state plume behavior are given by

$$\nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla P - \rho g \hat{z} - \nabla \cdot (\overline{\rho \vec{u} \vec{u}'})$$

$$\nabla(\rho \vec{u}) = 0$$

$$\nabla(\rho \vec{u}(w_v + w_L)) = 0$$

$$(c_p + w_v c_{pv} + w_L c_w) \nabla \cdot (\rho \vec{u} T) = \vec{u} \cdot \nabla P - L \nabla(\rho \vec{u} w_v) + \nabla \cdot \vec{F} - c_p \nabla \cdot (\overline{\rho \vec{u} T'})$$

These expressions represent the conservation of momentum, mass, total water and internal energy, respectively. $\vec{u}$ is the fluid velocity; ρ the density; P the pressure; and g the acceleration due to gravity. The Reynolds stress $\overline{u'u'}$ is explicitly included in order to justify the absence of viscosity effects. The turbulent Reynolds number is defined by $R_T \equiv u'D/\nu$, where u' is the large scale turbulent velocity, D is the diameter of cooling tower, and ν is the kinematic viscosity of air ($\sim 0.15\text{cm}^2/\text{sec}$). R_T is expected to be in the tens of thousands for typical flows. Consequently, viscosity is dominated by turbulent viscosity over that of molecular viscosity everywhere except where the flow velocities or scale lengths are significantly smaller than that of the free flow, e.g., very near the walls of the cooling tower. Aside from the boundary layer effects at the walls of the cooling tower, we expect the plume flow to be free of any boundary layer effects. Except in rare cases, we don't expect the boundary layer at the surface of the earth to have any effect upon the free convective flow of the plume.

In equation (4), w_v is the water vapor mass mixing ratio, defined by $w_v \equiv \rho_v/\rho$, where ρ_v is the density of water vapor. w_L is the liquid water mixing ratio; c_p and c_{pv} are the specific heat at constant pressure for air and water vapor, respectively. c_w is the specific heat of liquid water and T is the temperature in degrees Kelvin. We have explicitly included the turbulent transport of thermal energy, $\overline{u'T'}$, in order to indicate that turbulent transport of heat dominates over molecular diffusion of heat. Again, the reason for this is that turbulent Reynolds number R_T is much greater than one and the Prandtl number, ν/κ , is similar to one for air. Both here and in the momentum equation, we have neglected turbulent density fluctuations because the turbulent mach number is much smaller than one (Bradshaw, 1977).

The quantity $\vec{F}$ in the equation of internal energy conservation represents the radiative energy absorbed. The amount of energy absorbed by a given volume Q_R is given by

$$Q_R = \int (I_\nu^{\text{in}} - I_\nu^{\text{out}}) \hat{\Omega} \cdot \hat{n} d\Omega d\nu dA,$$

where I_ν^{in} and I_ν^{out} are the incoming and outgoing radiance, respectively; $\hat{\Omega}$ is a unit vector in the direction of outgoing radiative transport; and $\hat{n}$ is an outgoing unit vector directed normal to the volume surface. Integration takes place over all frequencies ν , but especially, in our case, over the entire IR band from about 1 to 100 microns. Defining

$$\vec{F} = \int (I_\nu^{\text{in}} - I_\nu^{\text{out}}) \hat{\Omega} d\Omega d\nu,$$

where $\hat{\Omega}$ is in the direction of the outgoing radiance, we obtain that

$$Q_R = \int \vec{F} \cdot \hat{n} dA = \int \nabla \cdot \vec{F} dV$$

Consequently, for infinitesimal volumes, the radiation absorbed per unit mass is $\nabla \cdot \vec{F}/\rho$.

There are two major components to the flow. First, there is the bulk flow due to the local meteorology; second, there is the flow of the warm, moist plume. In what follows, we will assume that the plume does not affect the gross meteorological flow. The gross meteorological effects will be associated with local horizontal winds and with the vertical motion of air.

There are two general causes of vertical air motion: convective and turbulent transport. Convective transport occurs because of temperature differences; and, thus tends to reduce those differences. However, temperature differences can be maintained under certain conditions. In particular, if in the adiabatic lifting of a parcel of air the resulting density is less than that of the ambient air, the parcel will continue to rise. However, if it is heavier, it will oscillate about the displacement at the Brunt-Vaisala frequency. The former situation is described as unstable, whereas, the latter is referred to as stable. The rate at which the temperature decreases with height is called the lapse rate. If the lapse rate of the ambient air is less than that produced by an adiabatic lifting of an air parcel, the air is stable. If the converse is true, the air is unstable. In stable or neutrally stable air, vertical motions will be negligible, despite the presence of temperature gradients.

Air and heat can also be carried aloft by turbulent velocities generated by wind shears (particularly near the surface) and by turbulent convection. A measure of the relative importance of the destruction of turbulence

by the stable stratification of the atmosphere to the generation of turbulence by wind shears is described by the Richardson number Ri . For our purposes, it suffices to know that in a stable or neutral atmosphere Ri is greater than or equal to zero. In such cases, turbulence decays rapidly; consequently, vertical motions are negligible. Apparently, for mildly unstable air and small positive Richardson numbers, vertical motions can also be neglected. We will assume that such stable situations prevail.

The motion of horizontal air interacts with the cooling tower. At very slow speeds the flow around the tower remains laminar and attached to the tower. However, at larger speeds the flow becomes turbulent, generating a wake of ever increasing size as the wind speed increases. The flow around a cylinder becomes turbulent at Reynolds numbers greater than about 2000 (Van Dyke, 1982). This indicates that even for small cooling towers the flow will be turbulent for velocities greater than about 1 cm/sec. For typical horizontal wind speeds, the wake region is at least several cooling tower diameters long. In the wake region pressure is reduced over the ambient region, consequently there is vertical force tending to drive fluid toward the ground. As a consequence, in almost all situations horizontal winds will have a significant effect upon the plume behavior.

In order to investigate these issues in greater detail, divide the properties of the flow into that which would exist in the absence of the thermal plume and the remaining part, which we associate with the plume itself.

The former will be associated with a subscript 'a' and the latter with the subscript 'p'. Using this notation, we have, at any time t and position $\mathbf{r}$:

$$\begin{aligned}\mathbf{u} &= \mathbf{u}_a + \mathbf{u}_p, \\ \rho &= \rho_a + \rho_p, \\ P &= P_a + P_p\end{aligned}$$

Using this division of fluid properties in the equation of momentum conservation, we obtain

$$\nabla \cdot (\rho_a \mathbf{u}_a \mathbf{u}_a) = -\nabla P'_a - \nabla \cdot (\rho_a \mathbf{u}_a' \mathbf{u}_a'),$$

where P'_a is the excess pressure over that of the hydrostatic pressure, where $dP/dz = -\rho g$. As above, we have neglected fluctuations in density, as we expect turbulent Mach numbers to be very small.

Similarity solutions for this gross flow can be found under the assumption that derivatives in the direction of mean wind direction are much smaller than derivatives transverse to the mean wind direction (Tennekes and Lumley, 1972). The similarity variables are the mean wind direction and scaled variables perpendicular to the mean flow. Consequently, the structure of the fluid variables at a given down wind distance from the cooling tower are invariant, although the magnitude of those variables will decrease with down wind distance. Such a situation will prevail at distances not close to the cooling tower. By assuming that the fluid is incompressible, it can be shown that, except for the mean wind speed, all fluid velocities have a Gaussian distribution in the transverse direction. For further details refer to the book by Tennekes and Lumley (1972). Although the similarity region begins at least several cooling tower diameters down stream, many integral models use this result to describe the behavior of the plume over its entire length.

The equation of momentum conservation describing the behavior of the plume is given by

$$\nabla \cdot (\rho_p \mathbf{u}_p \mathbf{u}_p) = -\nabla \cdot (\rho_a (\mathbf{u}_p \mathbf{u}_a + \mathbf{u}_a \mathbf{u}_p)) - \nabla P_p - \rho_p g_z.$$

Much experimental and theoretical work has been done investigating the properties of jets of all kinds, especially round and planar jets (Chen and Rodi, 1980; Rodi, 1982). However, as far as I can tell, no extensive theoretical work has been done investigating the analytic solutions for jets in a cross flow. And certainly no work has been done for moist, thermal plumes in a cross flow. In most theoretical studies, therefore, $\mathbf{u}_a$ is set to zero. We see from this momentum equation that the gross meteorological flow $\mathbf{u}_a$ serves to advect plume properties in the direction of $\mathbf{u}_a$. It therefore serves to mix the plume with the ambient air. Experimental studies have been made on dry plumes in a cross flow, and we will discuss their results shortly.

Despite the shortcomings of other studies there is much that can be learned from them. The parameters of particular interest for these flows are (Rodi, 1980)

1. The Reynolds number $R_e \equiv \frac{u_a D}{\nu}$ which describes the relative importance of turbulent diffusion and viscosity over that of molecular diffusion and viscosity.

2. The momentum flux of the jet $M \equiv \int \rho U^2 dA$, where the integration is performed over the plume cross-section. In many cases M can be treated as nearly constant.
3. The weight deficit $W \equiv \int (\rho_a - \rho) dA$
4. The densimetric Froude number $F \equiv \frac{\rho U_0^2}{g D (\rho_a - \rho)}$, which is a measure of the relative importance of inertial to buoyant forces.
5. The stratification parameter $S \equiv -\frac{g}{\rho_{a0}} d\rho_a/dz$, where ρ_{a0} is the density at the jet orifice.

Additionally, for moist plumes water vapor and liquid water fluxes could also be defined (Csanady, 1970).

For vertical buoyant plumes with an initial vertical velocity, the plume can be divided into three regions. In the initial region, referred to as the zone of flow establishment (ZFE), vortices which are shed from the jet opening induce entrainment of the ambient air into the rising plume. Close to the centerline of the plume there is a core of unmixed plume gases. Due to entrainment, the size of the central core decreases with distance from the jet orifice. The distance at which the central core disappears marks the end of the ZFE and the beginning of the zone of established flow (ZEF) (Rodi, 1980; Chen and Nikitopoulos, 1979). In the ZFE region, the plume behavior is dominated by inertial effects; i.e., the initial momentum given the plume dominates over buoyant effects. For this reason, the plume initially acts like a non-buoyant plume. In the far field, well beyond the ZFE region, buoyancy effects dominate over the inertial effects. As the plume rises, the density of the warm plume increases due to entrainment; consequently, buoyancy forces decrease. Nevertheless, when buoyancy forces become negligible, no matter what the initial momentum of the plume, the plume will cease to rise. This is true even for plumes associated with large Froude numbers. Between these two regions, lies an intermediate region where the plume behavior is more complicated. The beginning of the intermediate zone depends largely upon the initial Froude number. Obviously, the larger the Froude number and the smaller the buoyancy effects, the larger the region where the plume is dominated by inertial effects. For cooling towers, we expect Froude numbers to be positive but not much greater than one. The plume flow becomes self-similar at the end of the ZFE region. For expected Froude numbers, the self-similar region will begin after about 6 diameters (Rodi, 1980; Schatzmann, 1978, Chen and Nikitopoulos, 1979).

In the self-similar region, the cross-sectional properties of the flow have a Gaussian functional form (Rodi, 1980). The self-similar region is somewhat different in the inertial region of the plume than in the far field buoyant region. The intermediate region cannot be exactly self-similar (Rodi, 1980). Nevertheless, we expect that a Gaussian distribution should be a reasonable approximation. Evidently, throughout the self-similar region we can expect the width of the plume to grow linearly. Rodi (1980) also comments that in stratified media, where the temperature is a function of height, self-similarity cannot be fully realized. In addition, since the Boussinesq approximation is assumed, further considerations are necessary when the difference in density between the plume and the ambient air is considerable. Such a concern has also been raised by others in regard to thermal plumes (Schatzmann, 1979; Schatzmann and Policastro, 1984a).

As mentioned above, theoretical work on buoyant plumes in cross flows (aside from integral models) appears to be lacking. However, some experimental studies have been performed. Close to the outlet, inertial effects dominate and the influence of the cross flow is negligible. Far from the orifice the plume is bent over and the cross flow dominates the horizontal flow. Buoyancy effects, as for zero cross flow plumes, are also important. The relative effect of the cross flow depends upon the ratio of the cross flow velocity to the characteristic plume velocities, the exiting plume velocity and rate of rise due to buoyancy. The relative effect of buoyancy is contained within the expression $(U_{p0}/U_0)^2(1/F)$, where U_{p0} is the initial plume velocity and U_0 is the cross flow velocity. If this expression is greater than one, there are three regions, corresponding to a region where the plume initial momentum dominates, followed by a region where buoyancy dominates over inertial effects, and finally followed by a region where buoyancy and cross flow effects dominate. On the other hand, if this expression is less than one, the sequence is from inertially dominated to cross flow and inertially dominated, and finally to a region where cross flow and buoyancy effects dominate (Rodi, 1980). However, the division between these regions only makes sense when the initial plume velocity is much greater than the cross flow velocity. Rodi (1980) believes that in this case the self-similar region would have to develop much further downstream,

perhaps tens of diameters downstream. It is for this reason that many investigators have made the bent over plume assumption, whereby the plume is assumed to attain the cross flow velocity immediately upon exiting from the jet orifice. Clearly, much additional work is required on cross flows in order to more fully understand their behavior.

4.1 Integral Plume Models

As we come to consider integral models, we will understand the relevance of many of the concepts presented above; and, as a consequence, the need for a more complete understanding of the underlying physics of moist, thermal plumes in a cross flow.

Integral models treat the cross-section of the plume as a whole. In most cases the plume is treated as locally cylindrical; hence, the plume properties are assumed to vary only as functions of the radial distance from the plume axis and the distance along the plume axis. It is these integral models which have been favored since Morton, Taylor and Turner (1956) first introduced the concept for vertical plumes. Subsequent models have incorporated the effects of wind, vapor within the plume, and more sophisticated turbulence effects (Morton, 1956; Csanady, 1971; Wigley and Slawson, 1971; and Hanna, 1976). More recently a five volume EPRI document (Carhart, et al., 1981) has compared a number of these models using an extensive data set and described its own improved model. This model was recently described, with some minor modifications, in a journal article by Carhart and Policastro (1991), where it was referred to as the ANL/UI plume model. A similar model, described by Schatzmann and Policastro (1984b), was found by Carhart and Policastro (1991) to fit their data set as well as the ANL/UI model.

The fundamental approach of integral models can be understood by returning to the set equations described in equation (4). Over a short length of the plume, consider it to be well represented by a cylindrical cross-section. Now integrate the equations of momentum, mass, total water, and internal energy over a cylindrical volume of length dS . We obtain

$$\begin{aligned} \frac{d}{dS} \int (\rho U \bar{u} + P' \hat{s}) dA &= \frac{d}{dS} (W \hat{z} - \int \rho \bar{U}' \bar{u}' dA) - \oint (P' \hat{r} + \rho \bar{V}'_e \bar{u}' dC + \oint \rho \bar{u} V_e dC \\ \frac{d}{dS} \int \rho U dA &= \oint \rho V_e dC \\ \frac{d}{dS} \int \rho U (w_v + w_L) dA &= \oint \rho V_e (w_v + w_L) dC \\ \frac{d}{dS} \int c_p \rho U T dA &= \frac{d}{dS} \int (\bar{F} \cdot \hat{s} - L \rho U w_v - \rho \bar{U}' T') dA \\ &+ \oint (\rho V_e (c_p T + L w_v + \bar{V}'_e T') + \bar{F} \cdot \hat{r}) dC + \int (\bar{u} \cdot P' - \rho g w) dA \end{aligned}$$

Here, dA is the differential cross-section of the plume; dC indicates contour integration about the circumference of the plume cross-section; P' is the deficit pressure over that of the hydrostatic pressure; W is the weight deficit, previously described; U and V are the velocities parallel and radial to the plume axis, respectively; V_e and V'_e are the radial mean and fluctuating velocities, respectively, entering the plume through its circumference and hence are directed in the direction opposite to that of V ; w is the velocity in the vertical direction; and $\hat{s}$ and $\hat{r}$ indicate unit vectors parallel and radial to the plume axis, respectively. I have implicitly assumed that the plume flow is azimuthally independent within a given cross-section; hence, the velocity V and the vector $\hat{r}$ are radial quantities, with $\hat{r}$ directed outward from the surface of the plume. However, it is easy to see how these results could be further generalized. In the expression governing internal energy conservation I have neglected the term $c_{pv} w_v + c_{wL} w_L$ for the sake of simplicity and because w_v and w_L being less than a few percent are generally negligible.

Having derived these integral expressions, the difficulty is in being able to estimate undetermined quantities and in the integration of the expressions themselves. Regarding integration, there are two methods which have

been generally used. The first, and simplest, is to assume that all quantities are constant over the cross-section and circumference of the plume. It is customary to assume in this case that the plume cross-section is circular. In this case, an equation for the plume radius $R(S)$ is derived from the conservation of mass. This was the technique used by all early integral models (Morton, Taylor and Turner, 1956; Morton, 1956; Csanady, 1971; Wigley and Slawson, 1971; and Hanna, 1976)

However, later models have adopted more sophisticated approaches. Their methods all utilize the results of similarity analysis of the plume structure. As mentioned above, the existence of a self-similar region has been verified both analytically and experimentally (Chen and Rodi, 1980), although its location for cross flows is uncertain. It is also known that the Gaussian similarity profiles are different for temperature and velocity (Rodi, 1982; Taulbee, 1987). Rodi (1982) derives that in the inertial region the temperature similarity profile should lie outside of the velocity profile for a stable atmosphere, although they may cross in the buoyant dominant region. The results in Taulbee (1987) appear to indicate that the velocity profile should lie outside of the temperature profile. One might assume, therefore, that the similarity profile for liquid water might also be different. Hanna (1976) came to similar conclusions based entirely upon observations of plume rise, the rate of plume radius change, and visible plume lengths. Following others, his conclusion was that the velocity profile must lie outside of the temperature profile, and the 'moisture' profile should lie within the other two. Making these assumptions, one still must determine the functional form of the fluid variables across the plume cross-section. Hanna and Carhart and Policastro (1991) assumed a top hat distribution. Others (Schatzmann, 1978; Hutter, 1980; and Schatzmann and Policastro, 1984b) have used Gaussian similarity profiles for all fluid quantities, the rate of decay being generally different for velocity, temperature, and moisture. One researcher (Hutter, 1980) has developed a model for dry plumes that recognizes that the rate of plume spread should be different in the vertical and lateral directions. Consequently, he assumes an elliptical shape for the plume cross-section. Regardless of the method of integration, the resulting equations are left with undetermined parameters, which are to be determined by comparison with experimental data.

The typical Boussinesq approximation is to neglect all density differences except those associated with buoyancy forces. Such an assumption is expected to be satisfactory for $(T_p - T_a)/T_a$ less than about 0.1 (Schatzmann and Policastro, 1984a). In general, we cannot expect this condition to be satisfied for all cooling tower situations. This means that for integral models we should differentiate between plume densities and ambient densities. Most of the analytic work on plume behavior has neglected this difference. Apparently, only plume integral methods have investigated this aspect of plume behavior (Schatzmann and Policastro, 1984a).

All of the plume models mentioned above include the effects of entrainment. Entrainment is the process whereby ambient air is included into the plume. The integral expressions all include terms involving the flux of ambient material into the plume at a velocity V_e . In order to obtain a model for the effects of entrainment, the entrainment rate μ is defined by

$$\mu = \frac{1}{m} \frac{dm}{ds},$$

where m is the mass of the plume contained within a given cross-sectional volume. Alternatively, we express the entrainment rate in terms of the radially directed velocity as

$$\mu = \frac{\oint \rho V_e dC}{\int \rho U dA}$$

Using the integral expression for mass conservation, we find

$$\frac{d}{dS} \text{Log} \left(\int \rho U dA \right) = \mu$$

The manner in which these expressions are incorporated into the integrated equations depends upon the functional form assumed for the fluid quantities in the plane transverse to the plume axis. For a top hat distribution, we obtain

$$\mu = \frac{2\rho_a V_e}{\bar{\rho}_p R U},$$

where $U = dS/dt$ is the velocity of the plume in the axial direction, and $\bar{\rho}_p$ is the plume density. In general, we expect μ to be a function of the distance along the plume axis.

Still to be determined to complete any entrainment model is the value of the entrainment velocity V_e . Entrainment is clearly due to shears generated at the interface between the plume and ambient air. Ultimately, for large Reynold's number flows, it attempts to model the flux of material into the plume by turbulent diffusion. From Prandtl mixing length arguments, it is reasonable to assume that this mixing is proportional to mean velocity differences. How well such methods work in the initial mixing region, where more organized vortex mixing occurs, and under low Reynold's number parts of the flow is uncertain. Carhart and Policastro (1991) have addressed this issue by developing a separate molecular diffusive model for mix near the end of plume rise, where fluid velocities are low. For a greater discussion on the uses and development of entrainment refer to Turner (1986).

The entrainment of ambient air into the plume is influenced by the bulk flow near the cooling tower. As previously described, this flow is turbulent, especially within the first ten cooling tower diameters. On the downwind side of the cooling tower, not only is a wake region generated, but, in addition, turbulent velocities are generated. The effect of the wake region can be viewed as generating a downward directed force on the plume. The turbulent velocities generated by the bulk flow increase turbulent diffusion of ambient fluid into the plume. Carhart and Policastro (1991) in their ANL/UI plume model treat both of these effects using results from similarity theory and experimental measurement. In addition, it has been observed that cross flow engenders a drag on jets which is in agreement with what would be expected for solid bodies of similar size and shape. As with solid bodies, the cross flow sheds vortices from the surface. Indeed, the vortices generated can be of such magnitude to bifurcate the plume flow (Rodi, 1980). Schatzmann (1978) has treated the drag on the plume induced by the cross flow by the use of a drag coefficient, which impedes the movement of the plume in the direction transverse to the plume's axis.

An additional assumption made by many of these integral models is the bent-over plume assumption (Wigley and Slawson, 1971; Csanady, 1971; Hanna, 1976; Carhart and Policastro, 1991). In this case, the horizontal velocity of the plume is assumed to be equal to the ambient horizontal velocity.

Of course, all of the integral models possess undetermined coefficients. Because of its extensive data base, the ANL/UI model has been fully tested and its parameters well established. However, these parameters have been established upon the measurement of characteristics that, for our purposes, are not ideal. The data set consists of the rise and 'shape' of actual cooling tower plumes and laboratory data of a large water flume. The former data set only indirectly measures the adequacy of the model; and the latter, while including important rates of dilution, do not include the important effects of condensation and evaporation. From the dilution measurements, however, Carhart and Policastro (1991) and Carhart et al. (1981), were able to conclude that the wake effects were significant.

All integral models suffer from the uncertainty associated with initial values and the mixing process close to the cooling tower exit. This is so for a number of reasons. The initial fluid flow is not self-similar in this region and the mix is dominated by organized vortex generation. In addition, the initial water vapor and liquid water mass fraction is not generally known. It is in this regard that more sophisticated cooling tower models are required.

4.2 LANL PLUME MODEL

The

The model chosen to describe the plume momentum flow is that described in detail by Carhart et al. (1981) and later by Carhart and Policastro (1991). This model includes wake and enhanced entrainment effects that are due to the flow around the cooling tower. It uses the bent-over plume assumption and they assume that the spatial decay rate of the velocity, temperature and moisture similarity profiles are different. The momentum radius is about 1.2 times larger than the temperature radius; and the moisture radius is about 0.51 times that of the temperature radius. Clearly, at the cooling tower exit orifice all three radii must coincide. Using a suggestion of Fan, they smoothly adjust the radii ratios until they asymptotically achieve the values stated above. The distance over which the asymptotic values are reached depends upon the ratio of the horizontal wind speed to

that of initial vertical plume speed, k ; explicitly

$$F(S) = \left(\frac{R_w(S)}{R(S)} \right)^2 = \lambda + (1 - \lambda) \exp(-2s/s_0)$$

$$G(S) = \left(\frac{R_m(S)}{R(S)} \right)^2 = \nu + (1 - \nu) \exp(-2s/s_0),$$

where

$$s_0 = D \exp(-3.3k)$$

and

$$\lambda = 0.51 \quad \nu = 1.2$$

R_w and R_m are the moisture and momentum radii. Note that for a vertical plume the length of the ZFE region, as defined by s_0 , is about 6.2 tower diameters; while for equal horizontal and plume wind speeds, it is only 0.23 cooling tower diameters. They use these radii to describe top hat similarity functional forms for the radial variability of velocity, temperature and moisture. The plume is assumed to possess a circular cross-section. All variables are assumed to be azimuthally independent.

With these assumptions, integration of the integral equations is possible:

$$-\frac{dT_p}{dt} = \frac{LF}{C_p} \frac{dX_p}{dt} + \frac{\gamma_d \rho'_a W}{\rho_p} + \mu V_e \left(\frac{\mu_w LF}{\mu C_p} (X_p - X_a) + (T_p - T_a) \right) - \frac{F}{C_p} Q_R$$

$$Q_R = \sigma(T_a^4 - T_p^4) \int \langle Q_{obs} \rangle \pi a^2 f(a) da$$

$$\mu = \frac{2\rho'_a V_e}{R\rho_p V}$$

$$\frac{d\sigma_L}{dt} = -\frac{dX_p}{dt} + \mu_w(X_a - X_p - \sigma_L)$$

$$\frac{dW}{dt} = \frac{g}{G} \left(\frac{T_p^* - T_a^*}{T_a^*} - F\sigma_L \right) - \mu_m W V$$

$$\mu_w = \mu + (1/F) \frac{dF}{dS}$$

$$\mu_m = \mu + (1/G) \frac{dG}{dS}$$

These equations are written in the Lagrangian frame moving with the fluid elements. The assumption is that the droplets move at the same speed as the air. We expect that for droplets smaller than those customarily associated with drift deposition (around 50 microns), such an assumption is valid. Assuming that the droplets move with the fluid, the relationship between position along the plume and the time is given by $Vdt = dS$, where V is the velocity along the plume length. Outside the ZFE region μ_w and μ_m are equal to the entrainment coefficient μ . X_p is the water vapor mixing ratio inside the plume, which is confined to a radius R_w . X_a is the ambient water vapor mixing ratio. ρ'_a is the ambient air density. T_a and T_p are the ambient air and plume temperatures in degrees Kelvin; and T_a^* and T_p^* are the virtual ambient and plume temperatures, defined by

$$P = (\rho_a R_a + \rho_v R_v) T = R_a (\rho_a + \rho_v) T^*$$

where R_a and R_v are the gas constants for air and water vapor. Solving for the virtual temperature, we obtain

$$T^* \equiv \frac{1 + w_v/\epsilon}{1 + w_v} T \sim \left(1 + \frac{1 - \epsilon}{\epsilon} w_v \right) T,$$

where ϵ is equal to the ratio the molecular weight of water to that of air. There is only an expression for the vertical velocity W and none for the horizontal velocity U because, by the bent-over plume assumption, the horizontal velocity is a constant and given by the ambient wind speed. The pressure throughout the plume is assumed to be identical with the ambient pressure.

Q_R is the energy added to the plume due to radiative transfer. This term does not exist in the original ANL/UI model and is approximated by assuming that the plume and the surrounding radiative environment can be treated as black bodies. The integral containing the term $\langle Q_{ab} \rangle$ represents the effective emissivity of the plume droplets which have been averaged over the droplet size distribution $f(a)$. An expression for $\langle Q_{ab} \rangle$ has been obtained from the work of Barrett (1985). In figure (18) is plotted the results of Barrett (1985) for $\langle Q_{ab} \rangle$ as a function of droplet radius. Additionally, the dotted line represents a fit to his results, which is given by

$$\langle Q_{ab} \rangle = 0.06569r - 0.004728r^2 + 0.0001107r^3$$

where r is the droplet radius expressed in microns. This expression holds for $r < 12$ microns. For $r > 12$ microns, the value of $\langle Q_{ab} \rangle$ is set to 0.302.

Treating Q_R as due to blackbody radiation at the plume and ambient atmospheric temperature is clearly a convenient approximation. In order to incorporate the effect of plume scattering and all sky radiation, the plume effective emissivity and effective atmospheric temperature should be used. The effective plume emissivity, as previously shown, is a complicated function of droplet size distribution. At present, this is the primary motivation for employing the simpler expression.

In order to complete the equation set, an equation for either the plume liquid water mixing ratio σ_L or plume water vapor mixing ratio X_p must be found. In the standard ANL/UI model, the plume moisture core is assumed to be saturated. Since the saturation water vapor mixing ratio is only a function of temperature and pressure, both of which are already known, the plume mixing ratio is determined from

$$\frac{dX_p}{dt} = \frac{\partial X_p^{sat}}{\partial T_p} \frac{dT_p}{dt} - \frac{\rho'_a g W}{V} \frac{\partial X_p^{sat}}{\partial P}$$

Because of the sensitivity of the plume effective emissivity to droplet size distribution, we have chosen to treat the plume liquid water mixing ratio σ_L in terms of the droplet size distribution

$$\sigma_L = \frac{4}{3} \frac{\pi \rho_w}{\rho} \int f(a)(a^3 - a_s^3) da$$

where ρ_w is the density of water and a_s is the radius that the dissolved salt would occupy were all the water removed. With this expression, we obtain

$$\frac{d\sigma_L}{dt} = \frac{4\pi\rho_w N}{\rho} \int f(a)a^2 \frac{da}{dt} da$$

We have neglected in this expression the entrainment of droplets or nucleation centers from the ambient air. As a consequence, the total time derivative of the integral of the droplet size distribution f is zero since this integral represents the total number of nucleation centers.

4.2.1 Droplet Model

Having decided to tract the droplet size distribution, we require a model to determine how the droplet size distribution changes along the length of the plume. Much of what has been used in the LANL Plume Model is taken from classic cloud models and, in particular, the classic work by Pruppacher and Klett (1978).

The temperature of a droplet changes both because of conduction, evaporation, and radiative exchange. The heat flux into the droplet is due to both heat conduction and radiative exchange. The conduction heat flux is given by $\vec{q} = k'_a \nabla T$; and that due to radiative exchange by $\vec{F}$, where $\vec{F}$ is the net flux into the droplet. The heat flux out of the droplet is due to the heat supplied by the droplet to vaporize liquid water at the surface and release it as water vapor flux into the surrounding air. This heat flux out of the droplet is given by $-Ldm/dt$,

where m is the mass of the droplet. We assume in a quasi-steady state that the heat flux into the droplet equals the heat flux out of the droplet. We neglect all work done by the system as negligible.

Integrating over the entire sphere of the droplet to obtain the total heat flux exchange, we find

$$4\pi a^2(F_r + k'_a dT/dr) = -Ldm/dt$$

where F_r is the component of $\vec{F}$ in the radial direction and dT/dr is evaluated at the droplet surface. We have assumed that $\vec{F}$ and dT/dr are constant over the entire surface of the sphere.

For times much greater than $a^2\rho_a c_p/(\pi k'_a)$, the diffusive influx of heat achieves a steady state value. This time is less than about 10^{-6} seconds for all droplets smaller than 100 microns. In this case, the value of dT/dr at the surface is found to be $(T_p - T_d)/a$ (Pruppacher and Klett, 1978).

From these considerations we conclude (Pruppacher and Klett, 1978) that the rate of change of droplet mass m is given by

$$4\pi k'_a a(T_p - T_d) + L \frac{dm}{dt} + Q_R = 0$$

where k'_a is the thermal conductivity, and

$$Q_R = 4\pi a^2 F_r$$

If we assume that the radiation field both inside and outside the droplet is well represented by that of a black body at their respective temperatures,

$$Q_R = 4\pi a^2 \epsilon \sigma (T_e^4 - T_d^4)$$

and

$$\epsilon = \frac{1}{4} < Q_{abs} >$$

The amount of radiation energy absorbed by the droplet Q_R has been added to Pruppacher and Klett's analysis. T_d is the droplet temperature, but it is unclear what the background temperature T_e should be. Barrett and Clement (1988) have also considered this term, but not for the case when a collection of droplets are contained within an ambient gaseous fluid. The thermal conductivity must be modified by very small droplets on the order of the mean free path, where assumptions about the continuity of surrounding water vapor break down. These corrections are included in k'_a (Pruppacher and Klett, 1978).

The diffusion of mass from the droplet is also of interest because it must depend upon the difference between the water vapor density far from the droplet and that immediately above the droplet's surface. This is of interest because it is the water vapor density far from the droplet ρ_v which is related to the integral equations given above. With this in mind, we have (Pruppacher and Klett, 1978)

$$\frac{dm}{dt} = 4\pi a D'_a (\rho_v - \rho_{va})$$

Here, ρ_{va} is the water vapor density immediately above the droplet surface. As above, D'_a includes corrections for kinetic effects. Using that $m = 4\pi \rho'_a a^3/3$ (where ρ'_a is the density of the water droplet with salt) and assuming that the ideal gas law describes the water vapor pressure, we obtain

$$a \frac{da}{dt} = \frac{D'_a}{R_v \rho'_a} \left(\frac{e_v}{T_p} - \frac{e_a}{T_d} \right),$$

where e_v and e_a are the vapor pressure far from the droplet and at the droplet surface. T_d here is temperature of the droplet, assumed to be in thermal equilibrium with the vapor film above the droplet's surface.

Using the two expressions for dm/dt , we obtain

$$T_d = T_p + \frac{\rho'_a L a da/dt}{k'_a} + \frac{Q_R}{4\pi a k'_a}$$

Consequently, the temperature of the droplet is different from that of its environment. Condensation ($da/dt > 0$) increases the temperature of the droplet due to release of thermal energy through the heat of vaporization; while,

as expected, the absorption of radiation also tends to increase the droplet temperature. Using this result in the expression for da/dt , we observe that, since T_d is a function of da/dt , the derived equation for da/dt is nonlinear in da/dt . With the expectation that the heat released by condensation is efficiently dissipated by conduction, we expect $T_d - T_p$ to be small (Pruppacher and Klett, 1978). In practice, it is found that for all situations of interest the value of $|(T_d - T_p)/T_p|$ is less than about 10^{-5} (Pruppacher and Klett, 1978). Consequently, it is this assumption that we will use in advancing our derivation.

We expect that Q_R will be a function of the temperature difference between the droplet temperature and some other temperature. There are three temperatures to consider, the plume temperature which immediately surrounds the droplet, the effective atmospheric temperature, and the surface temperature. The difference between the droplet temperature and the plume temperature is small; hence, the exchange of radiative energy between the plume and the droplet is negligible (Pruppacher and Klett, 1978). There remains the effective atmospheric and surface temperatures. These differences can be significant, 30 degrees C and more. However, because of the generally large plume optical depth, the amount of this radiation which penetrates into the core of the plume will be small.

One simple approach notes that the amount of radiation absorbed by the entire plume is $\bar{\epsilon}_p$. The amount of radiation absorbed by a small differential layer of thickness $d\tau$ is then $(d\bar{\epsilon}_p/d\tau)d\tau$. If the radiation from the sky is represented as $\epsilon_a B_\nu(T_a)$, where T_a is the effective atmospheric temperature and that from the surface by $\epsilon_s B_\nu(T_s)$, then the amount of radiation absorbed by a differential layer $dF/d\tau$ situated at τ' is approximately given by

$$\frac{dF}{d\tau} \sim \pi (\epsilon_a B_\nu(T_a) d\bar{\epsilon}_p(\tau')/d\tau + \epsilon_s B_\nu(T_s) d\bar{\epsilon}_p(\tau - \tau')/d\tau)$$

where the first derivative of $\bar{\epsilon}_p$ with respect to τ is evaluated at τ' and the second at $\tau - \tau'$, τ being the total optical depth of the plume. Since virtually all of the radiation absorbed by the plume is absorbed by the liquid water, we know that this absorption within a given layer must be absorbed by the water droplets. The main point to note about this model is that the radiation absorbed by the droplets, although still a function of droplet size, is independent of the plume temperature.

The saturation vapor pressure immediately above the droplet e_a is a complex function of the curvature of the droplet and the effects of dissolved salts. It is appropriate at this time to think of e_a as given by $e_{a,s}$, whereby, we emphasize that e_a is a function of droplet size a and salt concentration, s . We have (Pruppacher and Klett, 1978)

$$\frac{e_a}{e_{\infty,w}} = \frac{e_{a,s}}{e_{a,w}} \frac{e_{a,w}}{e_{\infty,w}}$$

where $e_{\infty,w}$ is saturation vapor pressure for a flat surface of pure water; $e_{a,w}$ is the saturation vapor pressure over a curved surface of salt dissolved in water. The first ratio is related to the activity of the water and is described by Raoult's law (Pruppacher and Klett, 1978). Curvature effects have generally negligible effects upon the activity. The second ratio is related to curvature effects and is described by Kelvin's law. Combining the two effects, we obtain

$$\frac{e_a}{e_{a,s}} = \exp \left(\frac{2\sigma_{s/a}}{R_v T \rho_w a} - \frac{\nu \Phi_s m_s M_w / M_s}{m_d - m_s} \right)$$

where $e_{a,s}$ is equal to $e_{\infty,w}$; $\sigma_{s/a}$ is the surface tension for a droplet of radius a ; m_s and m_d are the mass of salt and droplet, respectively; and M_w and M_s are the molecular weights of water and salt, respectively. ν is the number of molecules that the salt dissociates into; and Φ_s is referred to as the molal osmotic coefficient (Pruppacher and Klett, 1978).

$\sigma_{s/a}$ is given approximately by (Lee et al., 1980)

$$\sigma_{s/a} = \sigma_{w/a} + 20 \frac{m_s}{m_w}$$

where $\sigma_{w/a}$ is expressed in dynes/cm. However, except for extremely small droplets, $\sigma_{w/a}$ is dependent only upon temperature; and the fraction of salt is so small that it is generally assumed that $\sigma_{s/a}$ equals $\sigma_{w/a}$ (Pruppacher and Klett, 1978). Additionally, as the amount of salt tends to zero, the molal osmotic coefficient tends to one. For the sake of simplicity, this is the limit which is often used (Pruppacher and Klett, 1978).

Defining the supersaturation ratio S ,

and using the Clausius-Clapeyron equation for equilibrium between liquid and vapor, we obtain

$$\frac{e_a}{e_{sat}} = \exp \left[\frac{L(T_d - T_p)}{R_g T_d T_p} \right] \quad (1)$$

where R_g is equal to the gas constant divided by the molecular weight of water, we obtain

$$\frac{da}{dt} = \frac{S+1}{S} \frac{dS}{dt} \quad (2)$$

where $\frac{da}{dt}$ is the rate of change of droplet radius, $\frac{dS}{dt}$ is the rate of change of supersaturation, and S is the supersaturation ratio. Here e_a/e_{sat} is e_a/e_{sat} evaluated at the plume temperature T_p , e_{sat} which is the saturation vapor pressure for a flat surface, is also evaluated at the plume temperature.

Consider a situation where, for a given droplet size, that the rate of droplet growth is zero. Neglecting radiative cooling and heating, this occurs when e_a/e_{sat} equals s_0 , where $s_0 = S + 1$ is the supersaturation. If the droplet were to grow by incidental condensation and e_a/e_{sat} , being a function of droplet size, should also increase, then, for fixed supersaturation, the droplet growth rate would be negative, tending to, thereby, decrease the droplet size through evaporation. If, in addition, e_a/e_{sat} should decrease for decreasing droplet size due to incidental evaporation, the droplet is in stable equilibrium. Since e_a/e_{sat} is continuous and the second derivative is assumed to be nonzero, both conditions imply that the derivative of e_a/e_{sat} with respect to droplet size should be positive. On the other hand, if this derivative is negative for a given droplet size, incidental evaporation would generate a negative growth rate, which would, thereby, generate further evaporation, and incidental condensation would induce still further condensation. In this case, the droplet is in unstable equilibrium. Assuming that the supersaturation is constant is equivalent to assuming that the water droplets are located within an infinite volume. In practice, the continued condensation of water droplets is curtailed by a subsequent decrease in the supersaturation, until the ambient air is no longer supersaturated, and the continued evaporation of droplet water is halted when the droplet moves into the stable equilibrium region.

In general, e_a/e_{sat} increases with increasing droplet size, goes through a maximum, and decreases for increasing sizes (Pruppacher and Klett, 1978). The droplet size associated with the maximum value of e_a/e_{sat} is referred to as the critical droplet size, r_{crit} . For droplets larger than the critical droplet size, droplets are located within the unstable regime. For a given value of supersaturation, there are two values of droplet size which correspond to zero growth rate, one is located within the stable regime and the other within the unstable regime (Pruppacher and Klett, 1978). Droplets located within the unstable regime which are larger than that value associated with zero growth, are referred to as activated. These droplets experience unstable growth. If the supersaturation is larger than the maximum value of e_a/e_{sat} , all droplets are activated since the growth rate is positive for all droplet sizes, even those in the stable regime.

The coupling of the droplet size distribution to that of the gross plume properties takes place through the rate that the liquid water vapor mixing ratio changes with time or with distance along the length of the plume. Because the rate of change of the droplet radius is much greater than that of its integrated quantity, w_L , the time step is controlled by the droplet growth rate, especially, that for the smallest droplets.

Noting that the droplet growth rate is determined in part by the supersaturation, implies that we require an equation to determine the time rate of change of supersaturation. Such an equation is readily obtained (Pruppacher and Klett, 1978) and is given by

$$\frac{dS}{dt} = \frac{1}{X_{sat}} \frac{dX_p}{dt} - (1+S) \left(\frac{\partial \log(X_{sat})}{\partial T_p} \frac{dT_p}{dt} + \frac{\rho_a g W_a}{P} \right) \quad (3)$$

5 LANL IDL Code

We have developed an IDL code which implements the model described above. At present there are a number of elements of the model which have not been implemented. In particular, the wake effects and enhanced entrainment due turbulent mixing induced by the wake have not been implemented. In addition, the effects of radiative cooling have also not been implemented. The general procedure is to first determine, for fixed plume temperature and supersaturation, the new droplet sizes and droplet size distribution using fourth order Runge-Kutta; and, then, for a fixed droplet size distribution, compute the new values for the liquid water mixing ratio and the time rate of change of the liquid water mixing ratio. These results are then used in the integrated plume equations. The plume temperature, vertical velocity, and supersaturation are determined by using fourth order Runge-Kutta for fixed values of water vapor mixing ratio and time rate of change of the water vapor mixing ratio. The timestep is determined such that the droplet rate of growth equation remains stable.

We have examined the effect of radiative cooling by considering a plume layer emitting blackbody radiation at a temperature of 315 K for a fixed droplet size distribution. The plume layer was embedded in a simplified atmospheric layer and lying over a surface emitting blackbody radiation at a temperature of 300 K. The atmosphere was assumed to radiate at a uniform temperature of 295 K, with an optical depth of 0.3. Scattering within the atmosphere was neglected. We used modified Gamma distribution with r_m set equal to 4 microns and the half width α to 6. The layer thickness was set at 60 meters and the density of water droplets set at $6gm/m^3$. With this distribution, droplet density and plume thickness, we then calculated the optical depth, asymmetry parameter, and single scattering albedo for all wavelengths from 3 to 100 microns. These results were subsequently used in a one dimensional radiative transfer code to determine the net flux entering the plume layer. Integrating over a section of the plume of length dS , we find that the change of temperature due to radiative cooling is given by

$$\frac{dT}{dt} = \frac{F_r}{\rho c_p Z}$$

where F_r is the total net flux into the plume, ρ is the plume density, and Z is the plume thickness. We find that the rate of temperature change is about -0.006 K/sec.

This time rate of change is intended to be somewhat representative of the what would be expected near the exit from the cooling tower. Further downstream the temperature of the plume decreases, while the atmospheric radiation should change little. Results of our calculations which neglect radiative cooling indicate that the time rate of temperature decrease due to all other processes is on the order of -0.5 K/sec. We conclude, therefore, that radiative cooling can be neglected unless the plume should be considerably hotter and the atmosphere considerably cooler. Further work is still required, however, in order to make a more definitive statement as to the importance of radiative cooling for cooling tower plumes.

A number of numerical issues were addressed in designing the code. First, as mentioned above, the time step is controlled by the droplet rate of growth. Let us express the droplet growth rate by

$$\frac{da}{dt} = \Phi(a)$$

If now we attempt to solve this equation by forward differencing, we obtain

$$a^{n+1} - a^n = \Phi(a^n)\Delta t,$$

where Δt is the timestep. This difference equation is stable if Δt is chosen such that

$$\left| 1 + \frac{d\Phi}{da}\Delta t \right| < 1$$

Since Δt is positive, this is only possible if $d\Phi/da$ is negative.

Generally, we expect that for activated droplets the growth rate will decrease monotonically with droplet size (Pruppacher and Klett, 1978). However, when the dependence of droplet temperature with respect to droplet size is taken into account, the situation is slightly more complicated. Figure (19) depicts a typical plot

of the growth rate with respect to droplet size. For a droplet size of approximately 0.02 microns the growth rate reaches a maximum positive value. For droplet sizes smaller than this, the growth rate falls rapidly through zero to negative values because the supersaturation is too small. For droplet sizes greater than about 0.21 microns, the droplets are activated. Note that in the region immediately larger than the critical radius, the slope of the droplet growth is positive. The region of positive slope is only a few tens of microns long, nevertheless, in this region we expect that instability might occur in our numerical solution. However, since the length of this unstable region is relatively small, we might have some reason to believe that its effect is small. Hence, except for droplet sizes less than about 0.02 microns, the only region which exhibits a numerically unstable region is located near the critical radius. It is in this region that we focus our concerns.

In order to understand the effect of this unstable region upon our solution method, let us model the region near those values of droplet size where the second derivative of the droplet growth rate is zero by

$$\Phi(a) = \alpha + \beta^2 a - a^3/3$$

In this case, the values of droplet size a where the first derivative is zero are located at $\pm\beta$, where β is a positive number. Letting the solution to $da/dt = \Phi(a)$, be given by $\bar{a} + \epsilon$, where ϵ is some small perturbation introduced by round off, then

$$\frac{d\epsilon}{dt} = \beta^2 - \epsilon^2$$

since $\bar{a}$ is assumed to satisfy the ODE exactly. The solution to this nonlinear equation is readily found to be

$$\bar{\epsilon}(t) = \beta \frac{\epsilon_0 + \beta \tanh(\beta t)}{\beta - \epsilon_0 \tanh(\beta t)}$$

where ϵ_0 is value of ϵ at $t=0$. Since ϵ_0 is assumed to be much less than β , the maximum value of ϵ is approximately equal to β . In the particular case under consideration, the value of β is no more than a few tenths of microns. In addition, we should keep in mind that, at least in this particular case, the droplet spends no more than about 1 second within this region of instability; hence, the growth of ϵ is actually almost one tenth of β . What this seems to imply is that droplet sizes near the critical radius are sensitive to noise; however, this error being carried into droplet sizes greater than about 1 micron will result in a small relative effect. We conclude that this difficulty can probably be endured; however, a more stable numerical method would be preferred. Unfortunately, the analysis of the solution to the droplet growth equation using a Runge-Kutte method, where multiple timesteps are employed in achieving the full timestep, do not affect the conclusions of the foregoing analysis.

The stability condition can also be approximately expressed as

$$\left| \frac{\Delta a_{j+1}^{n+1}}{\Delta a_{j+1}^n} \right| < 1$$

where

$$\Delta a_{j+1}^n \equiv a_{j+1}^n - a_j^n$$

This condition implies that stability is achieved for compression of the zones, but not on expansion.

The time step is controlled in such a way that one droplet size bin does not overstep that of another. This is directly analogous to a Courant time step control for hydrodynamics codes. Essentially it prevents the formation of bowties, or mesh volumes with negative values; i.e., we choose the timestep Δt such that

$$\frac{a_{j+1}^{n+1} - a_j^{n+1}}{a_{j+1}^n - a_j^n} < \frac{1}{2}$$

where a_j^n refers to the j -th droplet size at the n -th time. Such a constraint is required not only for the sake of mesh integrity, but also for stability.

Because the smallest activated droplet sizes grow at the greatest rate, there is a tendency with uniform droplet size bins to have poor resolution for these small sizes. One solution to this problem is to generate a mesh for which

$$a^{n+1} - a^n = \alpha(a^n - a^{n-1}),$$

from which we choose a special solution associated with

$$a^{n+1} = \alpha a^n$$

The solution of which is

$$a^n = a^0 \left(\frac{a^N}{a^0} \right)^{n/N}.$$

Since the spacing between the logarithm of the droplet size is uniformly spaced, we will refer to such spacing as logarithmic spacing.

We have assumed that the total number of nucleation centers is constant over time; hence, the total number of droplets does not change with time. Such an assumption is undoubtedly false since additional activated nuclei are entrained during the mixing process. Nevertheless, for the time being, we accept this assumption, with the implication that

$$\frac{d}{dt} \int f(a(t)) da = 0$$

This condition has been implemented in the code by requiring

$$f_j^{n+1} = f_j^n \frac{a_{j+1}^n - a_j^n}{a_{j+1}^{n+1} - a_j^{n+1}}$$

where f_j^n is the value of the droplet size distribution at the n -th timestep for the j -th droplet size

5.1 Some Results

Figures (20) through (27) depict some typical results. We have examined with these results the effect of tracking the droplet size distribution and the effect of the uncertainty in the initial supersaturation. The effect of tracking the droplet size distribution was examined by comparing the results of the LANL Plume Model with what we refer to as the Standard Model. As previously mentioned, in the Standard Model the plume moisture core is assumed to be saturated; hence, the plume water vapor mixing ratio is fixed by the plume temperature. By conservation of total water, any excess water is contained within the liquid water mixing ratio. Consequently in the Standard Model, computing the supersaturation is unnecessary since it is simply assumed to be zero.

In figures (20) through (23) show the results of the comparison between the LANL Plume Model and the Standard Model. These differences are indicated on the figures in a manner dependent upon the available labelling space. In all cases something like Standard or Std indicate results from the Standard Model and those labeled by Droplet refer to the LANL Plume Model. The differences between the plume temperature, water vapor mixing ratio, liquid water mixing ratio, and plume location are not large. Except for the liquid water mixing ratio, the effects are only apparent close to the cooling tower. The plume temperature in the LANL Plume Model is smaller than that for the Standard Model because in the Standard Model condensation is increased over that of the LANL Plume Model; consequently, the latent heat of evaporation released is greater for the Standard Model. This effect is a direct consequence of permitting the supersaturation to be greater than zero; hence, providing a sink for water vapor and forestalling condensation. The comparisons for the plume location are unlabeled because the results virtually lie on top of each other.

The differences between the LANL Plume Model and the Standard Model were judged to not be significant except in the case of the plume temperature, where the difference near the cooling tower was at most a few degrees Kelvin. However, when we come to examine the change of droplet size distribution obtained from the LANL model in figure (24), we see rather dramatic results. Figure (24) indicates that the droplet size distribution both narrows and shifts dramatically toward larger droplet sizes along the length of the plume. The average droplet

size shifts from 6 microns at $t = 0$ to about 17.5 microns at $t = 10$ s. In 10 seconds the plume droplets have moved, in this case, 50 meters downstream, which is about the pixel size available from an IR detector operating from a satellite platform. Beyond that distance the change in droplet size distribution is much slowed from that in the first 50 meters since the supersaturation, as indicated in figure (25), is dropping quickly. Nevertheless, over the next 200 meters of plume length, the average droplet size distribution increases to about 22 microns. As our previous study of plume emissivity indicates, such changes along the length of the plume translates into significant within band and band dependent changes in the effective plume emissivity.

Much more work regarding the sensitivity of these results to initial conditions must be done. However, if the results regarding sensitivity to changes in the initial supersaturation are any indication of what we might expect, we can have much hope in being able to extract the plume temperature from IR images. An examination of figures (26) and (27), indicate that changing the supersaturation from zero to 0.5% produce negligible differences. In fact, the differences are so small for all quantities that in order to depict the differences at all we had to depict the differences and not the absolute quantities.

6 Conclusion

We have described herein some of the problems associated with the extraction of plume temperature and its associated cooling tower dissipation from remotely sensed IR images of the plume. Much additional work needs to be done. In particular, although the ANL/UI Plume Model has been benchmarked with the use of cooling tower data, the character of that data set does not evaluate the model with regards to quantities of interest to us or remote sensing in general. No examination of plume temperature was performed. And, of course, no examination of droplet size distribution or plume emissivity was done. This study still needs to be done. In addition, cooling tower models in general use do not meet our needs since they do not track liquid water content, and certainly are unconcerned with evolution of droplet sizes.

Additionally, no work has been done in examining or perfecting an algorithm for the extraction of plume temperature from remotely sensed data. We imagine that such an algorithm would necessarily require knowledge and application of the effective plume emissivity. To this end, considerably more study of the character and parameterization of the plume emissivity is required.

Finally, we have no evidence to suggest that the integrated model presented herein or more complicated, detailed modeling will suffice for our purposes. Such questions can only begin to be answered by an examination of detailed data sets correlating measurements of cooling tower air and water inlet and outlet temperatures, ambient atmospheric conditions, and spatial measurements of plume temperature, liquid water and water vapor content, and droplet size distributions. Only after having developed adequate models, be they integrated or detailed models, can we effectively examine issues of sensitivity to initial conditions and the development of adequate retrieval algorithms.

REFERENCES

J.C. Barrett, The Optical Properties of Water Droplets in the Infrared, *J. Phys. D: Appl. Phys.*, vol 18, pp 753-764, 1985.

J.C. Barrett and C.F. Clement, Growth Rates for Liquid Drops, *J. Aerosol Sci.*, vol 19, pp 223-242, 1988.

P. Bradshaw, Compressible Turbulent Shear Layers, *Ann. Rev. Fluid Mech.*, vol 9, pp 33-54, 1977.

C.J. Chen and W. Rodi, Vertical Turbulent Buoyant Jets - A Review of Experimental Data, HMT, The Science and Applications of Heat and Mass Transfer, Reports, Reviews and Computer Programs, Volume 4, Pergamon Press Ltd., Oxford, England, 1980.

Ching Jen Chen and Constantinos P. Nikitopoulos, On the Near Field Characteristics of Axisymmetric Turbulent Buoyant Jets in a Uniform Environment, *Int. J. Heat Mass Transfer*, vol 22, pp 245-255.

G.T. Csanady, Bent-Over Vapor Plumes, *J. Appl. Met.*, vol. 10, pp 36-42, 1971.

R.A. Carhart, A.J. Policastro, S. Ziemer, K. Haake, and W. Dunn, Studies on Mathematical Models for Characterizing Plume and Drift Behavior from Cooling Towers, vol 2, Mathematical Model for Single-Source Cooling Tower Plume Dispersion, Electric Power Research Institute CS-1683, Palo Alto, CA, 1981.

R.A. Carhart and A.J. Policastro, A Second-Generation Model for Cooling Tower Plume Rise and Dispersion - I. Single Sources, *Atmos. Environ.*, vol 25A, pp 1559-1576, 1991.

Steven R. Hanna, Predicted and Observed Cooling Tower Plume Rise and Visible Plume Length at the John E. Amos Power Plant, *Atmos. Environ.*, vol 10, pp 1043-1052, 1976.

K. Hofer and K. Hutter, Turbulent Jet Diffusion in Stratified Quiescent Ambients, Part 1: Theory, *J. Non-Equilib. Thermodyn.*, vol 6, pp 31-48, 1981.

Muhammad Iqbal, An Introduction to Solar Radiation, Academic Press, NY, 1983.

In Young Lee, G. Hanel, H.R. Pruppacher, A Numerical Determination of the Evolution of Cloud Drop Spectra due to Condensation on Natural Aerosol Particles, *J. Atmos. Sci.*, pp 1839-1853, 1980.

Kuo-Nan Liou, An Introduction to Atmospheric Radiation, International Geophysics Series, Volume 26, Academic Press, Inc., Orlando, FL, 1980.

K.N. Liou, Radiation and Cloud Processes in the Atmosphere, Theory, Observation, and Modeling, Oxford University Press, Inc., New York, 1992.

Milton Van Dyke, An Album of Fluid Motion, The Parabolic Press, Stanford, CA, 1982.

H.C. Van de Hulst, Light Scattering by Small Particles, Dover Pub, Inc., New York, 1957.

T.M.L. Wigley and P.R. Slawson, On the Condensation of Buoyant, Moist, Bent-Over Plumes, J. Appl. Met., vol 10, pp 253-259, 1971.

TABLE 1

VARIATION OF EMISSIVITY WITH OPTICAL PROPERTIES

Band	τ	ω_0	g	$\bar{\epsilon}_p$
3.5-4.0	0.50	0.900	0.700	0.09243
	1.00			0.17327
	2.00			0.30700
	3.00			0.41017
	4.00			0.48907
	5.00			0.54889
	10.00			0.68490
	20.00			0.72144
	30.00	0.90		0.72310
	30.00	0.50	0.70	0.94790
	30.00	0.60		0.92511
	30.00	0.70		0.89138
	30.00	0.80		0.83569
	30.00	0.92	0.70	0.68374
	30.00	0.95	0.70	0.59863
	30.00	0.90	0.50	0.63983
	30.00	0.90	0.55	0.65748
	30.00	0.90	0.60	0.67697
	30.00	0.90	0.65	0.69867
8.0-8.8	0.50	0.65	0.55	0.27352
	0.75			0.37299
	1.00			0.45519
	2.00			0.66747
	3.00			0.77148
	4.00			0.82301
	5.00			0.84875
	10.00			0.87380
	30.00			0.87463
10.0-11.5	0.50	0.50	0.45	0.35774
	1.00			0.56434
	2.00			0.77019
	3.00			0.85237
	4.00			0.88610
	5.00			0.90021
	10.00			0.91032
	30.00			0.91047
	30.00	0.20	0.45	0.97453
	30.00	0.30	0.45	0.95771
	30.00	0.40	0.45	0.93696
	30.00	0.60	0.45	0.87561
	30.00	0.50	0.20	0.87749
	30.00	0.50	0.30	0.89025
	30.00	0.50	0.40	0.90353
	30.00	0.50	0.50	0.91756

TABLE 2

VARIATION OF PLUME OPTICAL PROPERTIES
WITH DROPLET SIZE DISTRIBUTION

Band	$r_m(\mu m)$	α	τ	ω_0	g	$\bar{\epsilon}_p$	$\bar{\epsilon}'_p$
3.5-4.0	1	5	24.797	0.96938	0.73129	0.518	0.526
		10	21.035	0.96546	0.67420	0.506	0.514
		20	17.908	0.96069	0.62246	0.502	0.511
		30	16.633	0.95820	0.60026	0.502	0.512
	5	5	6.854	0.89078	0.69498	0.646	0.696
		10	8.512	0.90643	0.68203	0.641	0.675
		20	9.572	0.91380	0.66864	0.633	0.660
		30	10.007	0.91604	0.66333	0.631	0.655
	10	5	3.393	0.82383	0.69927	0.615	0.729
		10	3.986	0.83973	0.70528	0.631	0.729
		20	4.538	0.85118	0.71020	0.643	0.728
		30	4.767	0.85515	0.71206	0.648	0.728
10-11.5	1	5	3.527	0.15692	0.05178	0.955	0.962
		10	3.326	0.10153	0.01861	0.965	0.972
		20	3.218	0.07145	0.00846	0.970	0.978
		30	3.182	0.06153	0.00606	0.972	0.980
	5	5	4.540	0.51061	0.45937	0.892	0.901
		10	6.670	0.47750	0.41583	0.911	0.907
		20	4.709	0.45177	0.38223	0.906	0.911
		30	4.698	0.44124	0.36844	0.908	0.913
	10	5	3.421	0.55607	0.52512	0.851	0.881
		10	3.872	0.55778	0.52370	0.867	0.887
		20	4.228	0.55574	0.51884	0.876	0.891
		30	4.352	0.55389	0.51581	0.879	0.893
	20	5	1.707	0.51481	0.49616	0.721	0.827
		10	2.018	0.52422	0.50337	0.760	0.844
		20	2.312	0.53369	0.51120	0.789	0.855
		30	2.437	0.53811	0.51503	0.798	0.859

$\rho_l = 10^{-6} \text{ gms/cm}^3$ Plume Thickness = 30m

Effect of Inlet Relative Humidity on Enthalpy Change

Figure 1
 2 graphs

Figure 2

Figure 3

Scattering Optical Depth per Km

Figure 4

$$0.75 \cdot \rho \cdot Q_s / (\rho_{\text{H}_2\text{O}} \cdot d)$$

Figure 5

of output

Wavelength
microns

3.5 microns for 0.5°K Change

$$\frac{\delta e}{e} = \frac{\frac{c}{\lambda T} + 1}{\frac{c}{\lambda(T + \delta T)} + 1} - 1$$

3.5 microns

$\delta T = 0.5^\circ K$

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Asymmetry Parameter

Figure 12

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Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

DROPLET GROWTH RATE

Figure 19

Figure 20

Figure 21

Figure 22

PLUME CENTERLINE LOCATION

Figure 23

Figure 24

25
Figure 26

Figure 26
sup 7

Figure 27