RECEIVED
MAY 19 1998
STI

DOE/ER/40801--T1

Final Report on
The Nonlinear Beam Dynamics Studies at Indiana University
(DOE-FG02-93ER40801)

for the Funding Period Aug. 1, 1993 - July 31, 1997

S.Y. Lee

Department of Physics
Indiana University, Bloomington, IN 47405

March 31, 1998

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

Prepared for

The U.S. Department of Energy
Agreement No. DE-FG02-93ER40801

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

**Portions of this document may be illegible
electronic image products. Images are
produced from the best available original
document.**

Summary of Nonlinear Beam Dynamics Studies at Indiana University

During three-year funding period (Aug. 1, 1993- July 31, 1997), our achievements can be summarized as follows: (1) We have systematically studied the effect of phase modulation on bunch distribution. (2) We have found that a frequently observed proton beam instability when the velocity of cooling electrons differs from that of protons is due to Hopf bifurcation. The equilibrium proton beam distribution becomes a "limiting cycle attractor." This technique can be used to measure the temperature of cooling electrons. (3) We have studied mechanisms of controlled bunch dilution. Evolution of beam bunch profile is measured while the beam is perturbed by the modulation of a secondary rf system. Our experimental data reveal that mean square bunch length $\sigma^2(t)$ exhibits two characteristic time scales. We find that overlapping parametric resonances plays an important role in beam diffusion. (4) We have studied the effects of global chaos on the halo formation of space charge dominated beams. We have analysed the halo produced by a modulating mismatched space charge dominated beam in a uniform focusing channel. We found that the halo formation is determined by a single scaling parameter, the space charge perveance parameter divided by the phase advance per unit length. The condition of global chaos can thus be predicted.

In June 1994, Indiana University has served as the host of the USPAS (United States Particle Accelerator School). In July 1994, IUCF has also served as the host site of the workshop on the *future direction of hadron facilities*. In October 1994, we have organized a successful workshop on *space charge dominated beams and application of high brightness beams*. During this period of this funding, we have many visitors from SSC, SLAC, Fermilab, and Brookhaven National Laboratory. We have carried out many nonlinear beam dynamics experiments at the IUCF Cooler Ring. We have published about 30 refereed journal papers related to this project. These papers are listed in Sec. 3.

1 Progress and Results in Physics Research

During the funding period of this project, our research efforts centered on studies of nonlinear beam dynamics for particles in synchrotrons. In this summary report, I will identify some of our achievements.

1.1 Nonlinear beam dynamics experiments

We have performed a series of longitudinal beam dynamics experiments, with (1) rf phase modulation [1, 7], (2) rf voltage modulation [6, 22], and (3) a dipole field modulation [5, 8]. In the presence of rf phase and voltage modulations, the measured Poincaré maps revealed many interesting invariant tori of the Hamiltonian. The longitudinal particle motion is confined to resonance islands and, with the electron cooling, is damped to basins of these islands within a few damping periods. Bunch splitting into separate beamlets in different islands due to this effect was also clearly observed. When a dipole field at a high dispersion location is modulated, the synchro-betatron coupling can generate rf phase modulation to synchrotron motion. We have systematically studied the effect and set a limit to the tolerable dipole field modulation.

These experiments can shed new light on rf noise problems, ground vibration, beam diffusion due to rf sinusoidal modulation for very slow beam extraction, and the possibility of multi-bunch instability feedback through parametric resonances. Many of these works provide the framework of our theoretical investigation of space charge dominated beams.

1.2 Bifurcation of parametric resonances in double rf system

We have performed beam dynamics experiments and measured the bifurcation of parametric resonance islands for double rf system with a harmonic ratio of 2, i.e. with harmonic numbers 3 and 6, where the circulating current was found to increase by a factor of 4 or about 1.7 mA at about 100 V of rf voltage. This experiment paves the way for the implementation of the barrier bucket concept in cooling storage rings. Our results are published in Ref. [13, 16, 20]

The beam dynamics associated with the double rf system is of interest to many synchrotrons. For low energy synchrotrons, a charged particle in a bunched beam may encounter enormous electromagnetic forces. The effects of the space charge force are manifested as transverse incoherent and coherent tune shifts, longitudinal impedance and potential well distortions etc. The beam intensities in booster synchrotrons at Fermilab, Brookhaven National Laboratory and CERN are known to be limited by the incoherent space charge tune shift. To improve beam intensity for low energy boosters, two rf cavities operating at harmonics 5 and 10 have been used to flatten out the longitudinal bunch shape. This improves the beam intensity by about 25-30%. However, the system exhibited coherent sextupole and decapole synchrotron mode instabilities. Microwave instabilities

can result from insufficient Landau damping due to a small local synchrotron tune spread. But, since the observed instabilities were found to be independent of the beam intensity, they might arise from single particle dynamics associated with intrinsic time dependent noise in the accelerator.

Let h_1, h_2 be harmonic numbers, V_1, V_2 be voltages of the primary and the secondary rf cavities respectively. We consider the stationary state case so that the synchronous particle does not gain or lose energy in either cavity. Let ν_s be the small amplitude synchrotron tune of the primary rf system alone, i.e. $\nu_s = \left(\frac{h_1 e V_1 |\eta|}{2\pi \beta^2 E} \right)^{1/2}$. Let (ϕ, δ) are the normalized conjugate phase space variables referenced to the primary rf system with $\delta = -\frac{h_1 |\eta|}{\nu_s} \frac{\Delta p}{p}$ as the normalized off momentum variable, $h = \frac{h_2}{h_1}$ is the ratio of the harmonic numbers, and $r = \frac{V_2}{V_1}$ is the ratio of rf voltages. The corresponding Hamiltonian, for $\frac{\eta}{|\eta|} = -1$, is $H_0 = \frac{1}{2} \nu_s \delta^2 + V(\phi)$ with $V(\phi) = \nu_s \left[(1 - \cos \phi) - \frac{r}{h} (1 - \cos h\phi) \right]$. Although the method we use can be applied for arbitrary r and h , we consider only the case $r = \frac{1}{h}$. Usually, a lower h value is preferred for attaining a larger bucket area. However, a lower h value also needs a higher second harmonic rf voltage.

The action is given by

$$J = \frac{1}{2\pi} \oint \delta d\phi, \quad (1)$$

and the phase space area of a given torus is $2\pi J$. Since the Hamiltonian is autonomous, a torus corresponds to a constant "energy", i.e. $H_0 = E$. The synchrotron tune is given by $Q_s = \frac{dE}{dJ}$. One notes that $Q_s(J) = 0$ at $J = 0$ and $J = J_{sx}$, where J_{sx} is the action of the separatrix orbit. Thus there exists an action value such that $dQ_s(J)/dJ = 0$. At this action, Landau damping, an essential mechanism for beam stability, also unfortunately disappears.

To study the particle beam stability, we apply a small perturbation to the system and measure the response of the particle motion. We consider here a small perturbation produced by external phase modulation, where the equation for the phase ϕ is replaced by, $\dot{\phi} = \nu_s \delta + \nu_m a \cos \nu_m \theta$. Here ν_m and a are respectively the modulation tune and the modulation amplitude. Such an external modulation may arise from synchro-betatron coupling, rf noise, or a wake field resulting from longitudinal impedances, etc. The corresponding Hamiltonian becomes,

$$H = H_0 + \nu_m a \delta \cos \nu_m \theta. \quad (2)$$

In the limit of small perturbation, i.e. $a \ll 1$, the solution can be expanded in terms of the action-angle of the unperturbed Hamiltonian H_0 . Using the generating function $F_2(\phi, J) = \int_{\hat{\phi}}^{\phi} \delta(\phi') d\phi'$, where $\hat{\phi}$ is an extremum of the phase angle for a given torus, the angle variable is then given by $\psi = \frac{\partial F_2}{\partial J} = \frac{\partial E}{\partial J} \int_{\hat{\phi}}^{\phi} \frac{\partial \delta}{\partial E} d\phi'$.

Now the task is to express the perturbation in terms of the action-angle of the unperturbed Hamiltonian, i.e. $\delta = \sum_n f_n(J) e^{in\psi}$. Here the expansion amplitude $f_n(J)$ can be

obtained from the inverse Fourier transform as

$$f_n(J) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \delta e^{-in\psi} d\psi, \quad (3)$$

which can be evaluated and parameterized in terms of J . Since $V(\phi)$ is an even function, ψ varies from $-\pi$ to π for a given torus, and δ is an odd function of ψ with reflective symmetry about the δ axis in the (ϕ, δ) phase space, the integral of Eq. (3) is zero except for odd integral n . Thus *phase modulation of the double rf system gives rise to only odd order excitations*, similar to that of the single rf system. For small amplitude oscillations in the single rf system, the dominant resonance driving term is the first harmonic.

Once the f_n coefficients are obtained, the Hamiltonian of Eq. (2) becomes,

$$H = E(J) + \nu_m a \sum_{n=\text{odd}} |f_n(J)| [\cos(n\psi - \nu_m \theta + \gamma_n) + \cos(n\psi + \nu_m \theta + \gamma_n)], \quad (4)$$

where γ_n is the phase of the Fourier amplitude f_n . For $a \ll 1$, we have $\dot{\psi} \approx Q_s$ and the resonance (stationary phase) condition occurs when the modulation tune equals an odd integral multiple of synchrotron tune. In a single resonance dominated regime, we transform coordinates into resonance rotating frame by using the generating function $F_2(\psi, I) = (\psi - \frac{\nu_m}{n}\theta + \frac{\gamma_n}{n})I$. The new conjugate action-angle variables (I, χ) are given by $I = J$, $\chi = \psi - \frac{\nu_m}{n}\theta + \frac{\gamma_n}{n}$, and the new Hamiltonian becomes

$$H = E(I) - \frac{\nu_m}{n}I + \nu_m a |f_n(I)| \cos(n\chi) + \Delta H(I, \chi, \theta). \quad (5)$$

Neglecting the time dependent term with $\langle \Delta H(I, \chi, \theta) \rangle = 0$, the stable and unstable fixed points (SFPs and UFPs) are given by

$$\sin n\chi_{\text{fp}} = 0, \quad nQ_s(I_{\text{fp}}) - \nu_m \pm n\nu_m a |g'_n(I_{\text{fp}})| = 0. \quad (6)$$

Here, the Poincaré surfaces of section around SFPs has n islands.

Using our data acquisition system, the synchrotron tune vs the amplitude of resonance islands can be measured. The bifurcation of resonance islands can be measured as a function of the modulation frequency. Our experimental data are reported in Ref. [16]

1.3 2-D nonlinear resonances

This DOE grant provides us the possibility of performing many 2D nonlinear beam dynamics experiments (see Ref. [9, 15, 23]). Here, we report results of 2-D nonlinear resonances at $\nu_x - 2\nu_z = -6$ and $\nu_x + 2\nu_z = 13$. The Hamiltonian for the accelerator operating near these nonlinear resonances can be expressed as

$$H(J_x, J_z, \phi_x, \phi_z) = H_0(J_x, J_z) + g_{\pm} J_x^{1/2} J_z \cos(\phi_x \pm 2\phi_z - \ell_{\pm}\theta + \chi_{\pm}), \quad (7)$$

where $H_0 = \nu_x J_x + \nu_z J_z + \frac{1}{2} \alpha_{xx} J_x^2 + \alpha_{xz} J_x J_z + \frac{1}{2} \alpha_{zz} J_z^2$ is the unperturbed non-resonant Hamiltonian, J_x, J_z are actions of the horizontal and the vertical betatron oscillations and ϕ_x, ϕ_z are conjugate variables. In the single resonance approximation, the Hamiltonian of Eq. (7) is integrable. The Poincaré map in the resonance rotating frame will trace out a torus, which is characterized by a Courant-Snyder invariant circle and a resonance circle. Particle motion near resonance is permanently perturbed by the resonance circles. For a difference resonance, $\nu_x - 2\nu_z = \ell_-$, the motion is bounded, i.e. $2J_x + J_z = \text{constant}$. Experimental results have been used to model nonlinear fields in the Cooler [15].

On the other hand, the sum resonance, $\nu_x + 2\nu_z = \ell_+$, is intrinsically unstable because the invariant of the Hamiltonian is $2J_x - J_z = \text{constant}$. Therefore, both J_x and J_z can both increase and become "unbounded." Particles at the sum resonance are lost to the vacuum chamber walls in less than 100 turns. Thus the study of a sum resonance is very difficult. At IUCF, we have built a ferrite Panofsky quadrupole in order to change the betatron tune by about 0.03 within 1 μ s. In this way, the accelerator can be placed at the resonance condition suddenly. The centroid of the beam motion can be tracked to observe the effect of the unstable nonlinear sum resonance in action. The beam loss in this resonance occurred within about 100 turns. Although our preliminary experimental data on the $\nu_x + 2\nu_z$ resonance can not be used yet to derive the nonlinear Hamiltonian, we are confident that the Panofsky quadrupole works and future experiments will provide the necessary data [23].

1.4 Mechanism of emittance dilution

RF phase modulation is an effective technique for the controlled bunch beam dilution. This technique is routinely applied to blow up the phase space area of high intensity proton beam in CERN PS. The physics of the beam dilution mechanism has been attributed to parametric resonances. The Hamiltonian for particle motion in the presence of a modulating secondary rf cavity is given by

$$H = \frac{1}{2} \nu_s \delta^2 + \nu_s (1 - \cos \phi) + \frac{\nu_s r}{h} [1 - \cos(h\phi + \hat{\phi}_m \sin \nu_m \theta)],$$

where ϕ and $\delta = -\frac{h|\eta|}{\nu_s} \frac{\Delta p}{p}$ are conjugate phase space coordinates, ν_s is the synchrotron tune of the primary rf system, $r = \frac{V_2}{V_1}$ is the ratio of the rf cavity voltages, $h = \frac{h_2}{h_1}$ is the ratio of the harmonic numbers, ν_m is the modulation tune, $\hat{\phi}_m$ is the amplitude of the phase modulation, and θ is the orbiting angle used as the time coordinate. Our data show that the rms bunch length σ obeys a scaling law:

$$\sigma^2 \propto t^\alpha$$

with $\alpha \neq 1$. This means that the diffusion process does not obey the Einstein relation. Such a diffusion process is usually called anomalous diffusion. Most experimental results

show two characteristic diffusion region. Particle diffusion mechanism in such a dynamical system is an interesting topic of future research.

1.5 Physics of space charge dominated beams

Studies of nonlinear dynamics relevant to the Kapchinskij-Vladimirskij (KV) envelope equation for an intense charged particle beam are important in order to achieve high current and high brightness beams. For a mis-matched beam, the envelope function presumably obeys the KV equation with a periodic longitudinal modulation. Therefore, solutions of such a KV equation, in the presence of self-electromagnetic fields, can offer important insight toward understanding high brightness beam transport problems.

We have developed a Hamiltonian formalism for solving KV equation, where we reformulate the KV envelope equation in terms of *envelope Hamiltonian* [17,19]. Parametric resonances appear naturally when the envelope Hamiltonian is expressed in action-angle variables.

The normalized paraxial KV envelope equation is given by

$$\frac{d^2 R}{ds^2} + k(s)R - \frac{K}{R} - \frac{1}{R^3} = 0,$$

where R is the normalized beam radius, K is the normalized space charge perveance factor, $k(s)$ is the normalized linear focusing field, and s is the normalized longitudinal distance along the accelerator. For a periodic focusing field, one has $k(s) = k(s + 1)$.

To cast KV equation into a general framework of a periodic dynamical system, *à la* circular synchrotrons, we use $\theta = 2\pi s$ as the time variable. Let (R, P) be the *envelope phase space* coordinates, where P is conjugate to R . The Hamiltonian becomes $H = \frac{1}{4\pi}P^2 + V(R)$ with the potential $V(R) = \frac{1}{4\pi}k(\theta)R^2 - \frac{1}{2\pi}K \ln R + \frac{1}{4\pi R^2}$, with $k(\theta) = k(\theta + 2\pi)$. Now, we decompose the Hamiltonian into $H = H_0 + \Delta H$ with

$$H_0 = \frac{1}{4\pi}P^2 + V_0(R) = \frac{1}{4\pi}P^2 + \frac{1}{4\pi}\mu^2 R^2 - \frac{1}{2\pi}K \ln R + \frac{1}{4\pi R^2},$$

$$\Delta H = \frac{1}{4\pi}[k(\theta) - \mu^2]R^2.$$

The unperturbed autonomous Hamiltonian H_0 describes amplitude oscillations of a mis-matched beam, and the perturbation arises from the remnant of the periodic focusing field. This dynamical system is equivalent to time dependent perturbation on the Hamiltonian flow. We will show later that the parameter μ should be chosen as the phase advance of the particle motion in the focusing field.

The minimum of the potential well $V_0(R)$ is located at $R_0 = \left(\frac{2}{\sqrt{K^2 + 4\mu^2} - K}\right)^{1/2}$, which corresponds to the average radius of a matched beam. Since the autonomous unperturbed Hamiltonian is integrable, a mis-matched beam radius will follow a torus of the

Hamiltonian flow. The action of the torus is given by

$$J = \frac{1}{2\pi} \oint P dR = \frac{1}{\sqrt{\pi}} \oint (E - V_0(R))^{1/2} dR,$$

where E is the “energy” of the unperturbed Hamiltonian given by $H_0 = E(J) = \nu J + \frac{1}{2}\alpha J^2 + \dots$. The tune of the unperturbed Hamiltonian, defined as the number of oscillations in one period, is given by

$$Q(J) = \frac{dE}{dJ} = \nu + \alpha J + \dots,$$

where the nonlinear detuning parameter, $\alpha = \frac{3}{32\pi^3 R_0^4 \nu^2} (K + \frac{10}{R_0^2}) - \frac{5}{192\pi^5 R_0^6 \nu^4} (K + \frac{6}{R_0^2})^2 + \dots$, can easily be obtained by the canonical perturbation method, and

$$\nu = \frac{1}{2\pi} [4\mu^2 - K(\sqrt{K^2 + 4\mu^2} - K)]^{1/2}$$

is the tune at a zero oscillation amplitude of a matched beam. At a zero space charge limit, the phase advance of the unperturbed envelope function in one period is 2μ . Since the envelope of a linear system oscillates at twice of the particle oscillation amplitude, the parameter μ should therefore be chosen as the phase advance obtained from a proper Floquet transformation. At the infinite space charge limit, the envelope tune becomes $\sqrt{2}\mu$. The nonlinear detuning α arises solely from the space charge force.

Using the generating function $F_2(R, J) = \int_{\hat{R}}^R P dR$, where $\hat{R}$ is the maximum amplitude of a Hamiltonian torus, the conjugate phase variable is given by $\psi = \frac{\partial F_2}{\partial J} = 2\pi \frac{\partial E}{\partial J} \int_{\hat{R}}^R \frac{dR}{P}$. Although the Hamiltonian H with a periodic time dependent modulation is not integrable, the envelope dynamics can be studied by expanding the perturbing potential in action-angle variables with $R^2 = \sum_{n=-\infty}^{\infty} G_n e^{in\psi}$. Here the strength function G_n is obtained from the inverse Fourier transformation as $G_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} R^2 e^{-in\psi} d\psi$ with $G_{-n} = G_n^*$. Without loss of generality, we assume a symmetric periodic focusing field so that

$$k(\theta) = \sum_{\ell=0}^{\infty} k_{\ell} \cos \ell\theta.$$

The Hamiltonian becomes

$$H = E(J) + \sum_{n=0, \ell=-\infty}^{\infty} h_{n,\ell} \cos(n\psi - \ell\theta + \gamma_n).$$

where γ_n is the phase of G_n , and $h_{n,\ell} = k_{\ell}|G_n|/2\pi$. We neglect first the $n = 0$ terms, which is related with the closed orbit solution. The parametric resonances generated by the time dependent perturbation are given by the stationary phase condition $n\dot{\psi} = \dot{\ell}$, which can cause coherent perturbation to Hamiltonian tori. Numerical simulations confirm our theoretical analysis [17, 19]. Experimental measurements would be highly desirable.

2 Participating Scientists

M. Ball^a, J. Budnick^a, D.D. Caussyn^{a,g}, J. Collins^a, V. Derenchuck^a, G. East^a
M. Ellison^a, D. Friesel^a, B. Hamilton^a, H. Huang^a, W.P. Jones^a, S.Y. Lee^a
D. Li^a, J.Y. Liu^a, A. Riabko, T. Sloan^a, Y. Wang^a, A. Chao^b, W. Chou^b, S. Dutts^b
M. Syphers^b, Y. Yan^b, P.L. Zhang^b, P. Colestock^c, A. Gerasimov^c, G.P. Jackson^c
K.Y. Ng^c, D. Reistad^d, Tor Lofnes^d, K. Hedblom^d, M.G. Minty^e, X. Pei^f

^a IUCF, Indiana University, Bloomington, IN 47405

^b SSC laboratory, 2550 Beckleymeade Avenue, Dallas, TX 75237-3946

^c Fermilab, Box 500, Batavia, IL 60510

^d Uppsala University, The Svedberg Laboratory, Box 533, S-75121, Uppsala, Sweden

^e SLAC, MS 26, Box 4349, Stanford, CA 94309

^f Brookhaven National Laboratory, Upton, N.Y. 11973

^g present address: University of Michigan.

3 Journal Publications of AP Group in 1993-1996

1. M. Ellison *et al.*, *Driven Response of a Beam*, Phys. Rev. Lett. **70**, 591 (1993).
2. S.Y. Lee, K.Y. Ng, and D. Trbojevic, *Minimizing dispersion in flexible momentum compaction lattices*, Phys. Rev. **E48**, 3040 (1993).
3. S.Y. Lee, *Spin Depolarization Mechanisms due to Overlapping Spin Resonances in Synchrotrons*, Phys. Rev. **E47**, 3631 (1993).
4. M.G. Minty, S.Y. Lee, *Effect of a Type-3 snake and the $G\gamma = 2$ Imperfection resonance on the Spin Motion in a Proton Synchrotron*, Particle Accelerators, **41**, 71 (1993).
5. M. Syphers, *et al.*, *Experimental studies of Synchro-betatron coupling induced by dipole modulation*, Phys. Rev. Lett. **71**, 719 (1993).
6. D. Li *et al.*, *Experimental measurement of resonance islands induced by the rf voltage modulation*, Phys. Rev. **E48**, R1638 (1993).
7. H. Huang *et al.*, *Experimental determination of Hamiltonian for synchrotron motion with rf phase modulation*, Phys. Rev. **E48**, 4678 (1993).
8. Y. Wang, *et al.*, *Effects of the synchro-betatron coupling induced by Dipole Modulations*, Phys. Rev. **E49**, 1610 (1994).
9. J.Y. Liu *et al.*, *Determination of the Linear Coupling Resonance Strength using 2D invariant Tori*, Phys. Rev. **E49**, 2347 (1994).
10. A. Gerasimov and S.Y. Lee, *Random driving of nonlinear oscillators*, Phys. Rev. **E49**, 3881 (1994).
11. Y. Wang, *et al.*, *Effects of tune modulation on particles trapped in 1D resonance islands*, Phys. Rev. **E49**, 5697 (1994).
12. S.Y. Lee, *Single particle dynamics at synchro-betatron coupling resonances*, Phys. Rev. **E49**, 5706 (1994).
13. S.Y. Lee *et al.*, *Parametric resonances of double rf system*, Phys. Rev. **E49**, 5717 (1994).
14. D.D. Caussyn *et al.*, *Negative Resistance Instability due to nonlinear damping in synchrotrons*, Phys. Rev. Lett. **73**, 2696 (1994).
15. M. Ellison *et al.*, *Experimental measurements of Invariant surface near a 2D nonlinear resonance*, submitted to the Phys. Rev. **E50**, 4051 (1994).

16. J.Y. Liu *et al.*, *Bifurcation of Resonance islands and Landau damping in the double rf system*, Phys. Rev. E **50**, R3349 (1994).
17. S.Y. Lee, A. Riabkov, *On the hamiltonian of an intensed charge particle beam in periodic solenoidal fields*, Phys. Rev. E **51**, 1609 (1995).
18. S.Y. Lee, *Nonlinear Beam Dynamics Experiments at IUCF Cooler Ring, Accelerator physics at the SSC*, AIP Proceedings No. 326, edited by Y. Yan and M. Syphers, p.13 (1995).
19. A. Riabko *et al.*, *Hamiltonian formalism for the halo formation in a space charge dominated beam*, Phys. Rev. E **51**, 3529, (1995).
20. J.Y. Liu *et al.*, *Analytic solution of particle motion in a double rf system*, Particle Accelerators. **49**, 221-251 (1995).
21. D.D. Caussyn *et al.*, *Effects of Nonlinear Damping force in synchrotrons with electron cooling*, Phys. Rev. E **51**, 4947 (1995).
22. D. Li *et al.*, *Effects of rf voltage modulation on particle motion*, Nucl Inst. Methods **A 364**, 205 (1995).
23. J. Budnick *et al.*, *Design, fabrication, and experimental test of a multi-purpose Panofsky magnet*, Nucl. Inst. and Methods, **A368**, 572 (1996).
24. S.Y. Lee *et al.*, *Effects of magnetized cooling on Hopf bifurcation*, Phys. Rev. E **53** 1287 (1996).
25. S.Y. Lee and M. Berglund, *Overlapping synchrotron sideband resonances*, Phys. Rev. E **54**, 806 (1996)
26. A. Riabko *et al.*, *Particle dynamics in quasi-isochronous storage rings*, Phys. Rev. E **54**, 815 (1996).
27. S.Y. Lee, *Emittance optimization for the three and multiple bend achromats*, Phys. Rev. E **54**, 1940 (1996).
28. D. Jeon *et al.*, *The role of parametric resonances in global chaos*, Phys. Rev. E **54**, 4192 (1996)
29. C.M. Chu *et al.*, *A method of detecting coherent synchrotron modes*, Nucl. Inst. and Methods **A 381**, 215 (1996)
30. S.Y. Lee, and K.Y. Ng, *Particle Dynamics in barrier rf systems*, Phys. Rev. E **55**, 5992 (1997).

31. S.Y. Lee and A. Riabko, *The envelope dynamics of intensified charge particle beams in solenoidal focusing channels*, the Chinese Journal of Physics, **35**, 387 (1997)
32. M. Bai *et al.*, Phys. Rev. E**56**, 6002 (1997)