NN INTERACTIONS IN QCD: OLD AND NEW TECHNIQUES

Richard F. Lebed
Jefferson Lab, 12000 Jefferson Avenue
Newport News, VA 23606, USA
E-mail: lebed@jlab.org

Since QCD is believed to be the underlying theory of the strong interaction, it is appropriate to study techniques that take into account more features of its rich and complex structure. We begin by discussing aspects of physics that are ill-reproduced by the usual one- or two-meson exchange approaches and identify the source of the deficiencies of these models. We then reveal promising methods for curing some of these ills, such as new quark potential models, baryon chiral perturbation theory, soluble strongly-interacting field theories, and large Nc QCD.

1 Introduction

As a particle theorist, I was invited to speak at this conference about what can be said regarding the nucleon-nucleon interaction in the context of quantum chromodynamics. Hadronic physics is probably the most difficult problem in the entire panoply of particle theory, and the primary quest of its practitioners (myself included) is to uncover some particularly simple physical picture that not only respects the physics of observed hadrons, but arises as a natural consequence of the substructure of quarks and gluons. To this day we remain hindered in our ability to provide an elegant and definitive solution to the problem.

However, even on that day of QCD's eventual solution, the meson exchange picture of nuclear physics will remain the natural picture for the NN interaction in almost all situations. In precisely the same way that NASA quite successfully employs Newtonian gravitation for calculating trajectories of spacecraft, the existence of general relativity as a more "fundamental" theory of gravitation notwithstanding. Quarks and gluons are certainly present in all NN interactions, but it is not always necessary to take them explicitly into account.

Nevertheless, we have learned quite a bit about QCD in its first quarter century. We know the Lagrangian and its symmetries, some properties of the quarks themselves, and a bit about the nature of color confinement. Moreover, one can study versions of QCD-like theories that have been simplified so that

some of the difficult physics becomes tractable. These sorts of advances can be applied to the NN problem, to constrain types of possible dynamics or to reduce allowed parameter spaces. The significance of such physics becomes most apparent when one considers circumstances in which the standard meson exchange picture begins to fail, so we begin with a discussion of the successes and problems of this picture. We then consider exactly what it means to obtain QCD improvements, and exhibit a number of techniques designed to accomplish this goal.

2 The Age of the Meson Exchange Picture

The NN interaction is perhaps the most studied of all problems in nuclear physics, and the decades of careful scrutiny and hard work are now coming to fruition in the convincing numerical success of various famous potential models. With only a score or two of parameters, the models of Paris,\(^1\) Bonn,\(^2\) Nijmegen,\(^3\) and Argonne\(^4\) have begun to achieve a global fit to the numerous experimentally observed partial waves and hundreds of extracted data points approaching the all-important statistical criterion of \(\chi^2/\text{d.o.f.} = 1\). The fundamental physics incorporated in these models is obtained from the current understanding of the dynamics of meson exchange: The unknown fit parameters appear in the context of Yukawa potentials, form factors, and so forth.

Are we then to conclude that the NN interaction is essentially a solved problem, with the few remaining discrepancies requiring only minor adjustments of functional forms or numerical values of the parameters? From a strictly reductionist point of view, as the fits of models to data become ever more precise, questions about the origin of these numerical values become more pressing. Moreover, one has in QCD the apparent underlying fundamental theory of strong interactions. Even if one looks askance at future prospects of describing a nuclear problem such as the NN interaction with its complicated phenomenology in the language of quarks and gluons, one cannot believe that all of the masses and couplings of the common mesons are independent quantities. The proper question is, how do nuclear phenomena arise as a limit of QCD?

In the traditional picture of the NN potential, at large interparticle separations (2–3 fm) one observes exponential saturation of nuclear forces with a residual attraction, which is explained by one-pion exchange. At intermediate ranges, between roughly 0.6 and 3 fm, there is an attractive region typically ascribed to exchange of a scalar \(\sigma\) meson or scalar combination of two \(\kappa\)'s, while at smaller distances one finds an effective short-range repulsion, which is identified with \(\omega\) exchange. Including the \(\rho\), whose primary role is to cancel most of the tensor interaction of the \(\pi\) at short distance, completes this picture. One is immediately faced with questions about the identity of these mesons, \(\pi, \sigma, \rho, \omega\). Are they the fundamental mesons appearing in the Particle Data Book, or do they merely serve as placeholders to parameterize more complicated physics? For example, the troublesome \(\sigma\) might actually indicate the exchange of scalar current between nucleons arising from many different sources at different momentum scales, including but not limited to \(\pi\) \& \(\pi\) pairs.

The fundamental problem with one-meson exchanges as a universal explanation for NN phenomena can be described in terms of the following simple "parable" (Fig. 1): Think of hadrons as classical hard spheres. The charge radius of the proton is \(r_\text{p} \approx 0.8 \text{ fm}\), while the typical size of light mesons is set by the QCD scale \(\Lambda_{\text{QCD}} \approx 1/(300 \text{ MeV}) \approx 0.5 \text{ fm}\). Then a meson cannot mediate an NN interaction at distances below \((0.6 \text{ fm} + 0.5 \text{ fm} + 0.6 \text{ fm}) = 2.2 \text{ fm}\): It simply won't fit between the nuclei! Of course, real hadrons are quantum mechanical, but the point of the parable remains: One-meson exchange only makes sense inasmuch as the meson (and, to a smaller extent, the nucleon) can be treated as a point particle. At distances smaller than a couple of fermi, one probes the inner structure of the hadrons, which is to say their short distance or high momentum components. In meson models this is typically taken into account with form factors. But which type of form factor is correct? Obviously, this is a situation in which more input from QCD would provide valuable clarification.

Perhaps you are unconvinced by this parable, and would prefer a more direct demonstration of some situation where one-meson exchange fails. To be suitably rigorous, one needs a theory in which quark and hadron degrees of freedom can be handled equally well. There in fact exists a completely soluble, strongly interacting theory, which is called the 't Hooft model.\(^5\) By definition, it is QCD in one space and one time dimension with a large number of color charges \(N_c\). "Completely soluble" here means that one can obtain meson masses, wavefunctions, and transition amplitudes precisely in terms of
work masses. Within this framework, consider a typical hadronic quantity, the meson electromagnetic form factor:

\[ F(q^2) = \sum_{\mu=0}^{\infty} \frac{A_{\mu} \mu^2}{q^2 - \mu^2 + i\epsilon}. \]  

(1)

Here \( \mu_n \) and \( A_n \) are the mass and pole residue of the \( n \)th meson. That the factor may be written as a sum of pole terms is a consequence of large \( q^2 \). Single-meson exchange models tell us to expect \( A_0 \gg A_1, A_2, \ldots \), i.e., the contribution of the lightest meson is the most important. Carrying through the calculation in the 't Hooft model, one finds that this is true only for the quarks (Fig. 2). As the quark mass increases, one sees not only that the number of poles becomes significant, but that they alternate in the sense of their residues, meaning that a one-meson exchange picture becomes progressively more inadequate. Indeed, as the quark mass becomes very large, we can show that the residues arrange themselves to give the same predictions as the nonrelativistic quark model.

Including More QCD.

We have argued that there are circumstances in which input from QCD is essential for physical understanding, one must define exactly what is meant by QCD improvement. Let us adopt the broadest definition possible, in order to encompass as many features of the strong interaction as QCD improvements fall into two basic categories. First, since QCD is a quantum field theory, it must satisfy the properties of causality, relativistic invariance, crossing symmetry, unitarity, and unrestricted production of virtual particles. Second, a number of field-theoretic properties are special to QCD, namely, the presence of quarks and gluons (of course), as well as gauge invariance and color conservation, color confinement, and the discrete symmetries of parity, charge conjugation, and time reversal invariance. In addition, QCD provides for many kinds of hadrons with numerous types of nonlinear interactions, and — very importantly — approximate chiral symmetry.

Inverse Scattering

Next consider an improvement that takes into account only field theoretic QCD properties. The premise of inverse scattering is that one uses data directly from \( S \)-matrix and phase shifts as functions of momentum transfer \( k \) via

\[ S(k) = \exp(2i\delta(k)), \]

(2)

which, by construction, automatically satisfies field theoretic aspects of QCD, since the \( S \)-matrix obeys unitarity, crossing symmetry, and so forth. One then inverts (2) using standard mathematical techniques such as Gell-Mann-Levy and Marchenko inversion, to obtain an equivalent local potential \( V(r) \) that, by construction, agrees with the data. If one then compares to usual potential models, the agreement is quite good for most partial waves, since these potentials were designed expressly to fit the phase shifts. In particular, \( V(r) \) obtained in this method exhibits a repulsive core. Moreover, the inversion may be continued off-shell to produce interesting results relevant to processes like nuclear bremsstrahlung. However, a local potential \( V(r) \) depends on
only one coordinate \( r \), which is the separation of the two nucleon centers. This is a natural picture when the nucleons may be considered (nonrelativistic) point particles, but may be inadequate when nucleonic substructure is taken into account, in which case there is more than just one relevant separation coordinate.

### 3.2 Local and Nonlocal Potentials

The nucleonic substructure of quarks and gluons can create nonlocality in the NN potential, which may be expressed as an energy dependence \( V(E, r) \). It is true that relativistic effects also produce an energy-dependent potential, but one can study the effects of substructure separately by considering a nonrelativistic toy model. An interesting example of this approach appears in Ref. 16, where the \( p \Sigma^+ \) potential, known to be have a highly nonlocal potential in the quark model, is considered. The nonlocality is introduced through a potential term

\[
\Delta V(r_N, r_{\Sigma}) = \exp \left( -\frac{(r_N + r_{\Sigma})^2}{4\alpha^2} \right) \exp \left( -\frac{(r_N - r_{\Sigma})^2}{4\alpha^2} \right),
\]

which depends on not only the separation \( r_N - r_{\Sigma} \) but also the average position \( (r_N + r_{\Sigma})/2 \). The phase shifts obtained from this nonlocal potential can then be used to generate an equivalent local potential. The result of this calculation shows that the height of the repulsive core in the nonlocal potential is greatly reduced compared to that from the local potential (see exp. their Fig. 3). Such a conclusion suggests that the repulsive core of potential models is actually due to nucleon substructure; it would certainly agree with our earlier comments that single-meson exchange at short distance should not be a good description of the NN interaction.

### 3.3 Quark Models

One form of quark model phenomenology uses nothing more than valence quarks either interacting in some phenomenological potential, or with some chosen wavefunctions within the nucleon. Such quark model studies of features of the NN interaction have a very old history, dating back to the dawn of the quark model itself in the mid 1960s. The unique feature of quarks, however, is that they possess the color degree of freedom. Once one determines that color exists, situations become inevitable in which the color degree of freedom must be considered explicitly. As an example, consider a second parable in the form of Fig. 3. Starting with one-meson exchanges in the NN interaction (Fig. 3a), one can decide that \( N \) is a 3-quark state while mesons are \( \phi \) states, the interpretation of the meson exchange in terms of colored quark lines becomes clear (Fig. 3b). However, just as likely are diagrams in which the quark lines are tangled (Fig. 3c). In such a case, the intermediate state is clearly not a single meson, nor is it even a color singlet. It is still possible to describe it in the meson language, but to do so requires a large number of carefully correlated meson exchanges; this is the same phenomenon that we saw in the 't Hooft model (Fig. 2) for large quark masses. The promotion of this argument from parable to rigor involves the inclusion as well of all possible gluon exchanges, but it seems reasonable that such a modification cannot completely screen all color from our notice.

To date, however, the best quark model studies still only include the gluon degrees of freedom through field-theoretic reductions of one-gluon exchanges. Nevertheless, this is enough to capture quite a bit of physics, such as spin-orbit couplings and hyperfine terms. Consider, for example, the results of one particular study, in which six-quark states are placed in an interaction derived from single-gluon exchange plus an explicit confining potential. It is then found that the repulsive core originates as anti-binding from the spin-spin coupling of the hyperfine interaction, while the intermediate attraction is a result of the excitation of color nonsinglet P-wave clusters of quarks. Clearly, these are phenomena that have no simple interpretation in terms of one-meson exchange.

Another interesting idea for the suppression of the repulsive core arises in the context of Moscow potentials. One begins with the physical observation that it is rare to observe baryons with very small separation. The standard explanation, of course, is that one is seeing a potential with a repulsive core. However, many researchers suggest that the same physics may be obtained
through an NN wavefunction with a node at small separation, effectively sup-
pressing such observations. In the case of the Moscow potential, the same sort
of wavefunction suppression is achieved through a “two-phase” model: Starting
at six quarks, at large separation there is a high probability for segregation
into three-quark nucleon clusters, with combined wavefunction $\Psi$. At small
separation there is a high probability to form a “bag-like” six-quark state $\Phi$.
These wavefunctions $\phi$ and $\Psi$ are then taken to be orthogonal. Thus, it is not
particularly difficult to push six quarks into a very small volume, but then the
missing part of the wavefunction no longer resembles two distinct nucleons.
Such models allow for good fits to many of the phase shifts, and the residual
interaction potentials (to account for long-distance physics not incorpo-
rated into the quark interactions) may then be taken as local. Moreover, the
$\pi N$ coupling fails to values consistent with that predicted from SU(3) sym-
metry, since the $\pi$ is no longer required to serve the special role of providing
a repulsive core interaction.

Quark models typically explain the short- and intermediate-distance fea-
tures of the NN interaction; both nuclear and particle physicists can agree that
a long-distance tail is due to single-pion exchange. Nevertheless, it is fruit-
ful to compare the tortuous discovery of this fact with its current explanation
in many textbooks. Historically, Yukawa proposed in 1935 the exchange of
mesons to explain nuclear binding; the $\pi$ was the first true meson discovered
in 1947, and was subsequently found to explain the long-distance behavior
of NN interactions well. Since the $\pi$ is still the lightest observed meson, by
the Heisenberg principle it has the longest range. This bottom-up process of
building up from our current top-down understanding of the $\pi$—one has
the QCD Lagrangian possession chiral symmetry, which spontaneously breaks
an approximate flavor symmetry. The breaking induces a multiplet of pseudoscalar
Nambu-Goldstone bosons, of which the $\pi$ is the lightest, since it contains no heavy
strange quarks. Therefore, again the Heisenberg principle, it should have the largest range of any strongly
interacting particle.

4 Effective Theories

The discovery of the approximate chiral symmetry of strong interactions has
been one of the primary achievements of the extensive efforts placed in under-
standing the NN interaction over the years. It is exploited to great effect in
efficient Lagrangians and chiral perturbation theory ($\chi$PT), and yet such theories
are only a specific type of what are now collectively called effective theories.
Let us explain how such theories are constructed in general, with reference to
the familiar $\chi$PT case.

1. Choose a set of fields as dynamical degrees of freedom. In $\chi$PT these are
pions, as well as $K$'s and $\eta$'s in the 3-flavor case, and nucleons can also
be incorporated into this scheme.

2. Identify the symmetries obeyed by interactions of these fields. In $\chi$PT
these are Lorentz covariance, (approximate) chiral symmetry, and the

3. Express fields in forms that transform appropriately under the given sym-
metries. For example, one convenient representation containing the pion
field $\pi$ in $\chi$PT is $\Sigma \equiv \exp(2i\pi \cdot r/\Lambda)$, for then under $\text{SU}(2)_L \times \text{SU}(2)_R$
chiral rotations $L$ and $R$ one has $\Sigma \to \Sigma^n$. Here $r$ are the isospin
generators and $\Lambda$ is the pion decay constant.

4. Construct the Lagrangien that explicitly obeys all symmetries. In general,
this procedure produces an infinite list of terms, which gives the
initial naive impression that the theory has no predictive power at all.
In the case of $\chi$PT, however, more complicated terms with more fields or
derivatives enter, by virtue of simple dimensional analysis, with more in-
verse powers of some characteristic mass scale $\Lambda$. In practice, $\Lambda$ for $\chi$PT
is typically taken as the scale of $m_\pi$ or 1 GeV, where describing physics
solely in terms of pion interactions is no longer adequate. Therefore, all
but a small finite number of the possible infinite set are insignificant for a
given physical process. In the general case, an effective theory is useful if
the more complicated terms are suppressed numerically in physical quan-
tities, which means that the characteristic moments of the process must
be below some scale $\Lambda$. In this sense, $\Lambda$ acts as a radius of convergence
for the perturbative organization of the series.

5. Each term in the Lagrangian has an unknown coefficient, expected to be
of order unity, which must be fit to data. Once the effective Lagrangian
has been truncated by the process described above, a (hopefully small)
number of such coefficients remain. Of course, the usefulness of the theory
depends on how many coefficients remaining that the Lagrangian
may then be used to predict other observables. The expectation that
the coefficients are of order unity once the known physics is taken into
account is called the naturality assumption; if a coefficient turns out to
be too small, one suspects a hidden symmetry, while if it is too large,
one suspects that important physics has not been taken properly into
account.
The relevance of this construction in the current context is that a great deal of effort has recently been invested in developing effective chiral theories to compute nucleon properties. A nice talk on the importance of chiral symmetry in nuclear interactions appears in [18], while [14] provides a very thorough review through 1995. The subtlety in the nucleon case is that the development of the effective theory runs into complications because of the presence of several mass scales. For suppose, in the construction described above, one finds not one but two scales of physics, \( A_1 < A_2 \), relevant to a given process. Then it is not enough to merely choose processes with characteristic momenta \( p \) satisfying \( p \ll A_1 \) and \( p \ll A_2 \), for the combination \( A_2/A_1 > 1 \) might appear in the dimensionless unknown coefficients, making them unnaturally large and thus defeating the predictivity of the theory.

In the single nucleon case, in addition to the scale of the onset of non-pionic interactions \( A_1 \), one must also deal with the appearance of the nucleon mass, as discussed first in [18]. One particularly successful treatment [18] is to use a Foldy-Wouthuysen transformation to remove nucleon mass terms from the Lagrangian, a method that effectively replaces nuclear momenta with velocities.

However, in the case of two or more nucleons, one typically has a three-scale problem: momentum \( p \), nucleon mass \( M \), and nuclear binding energy \( p^2/2M \). In this case, a typical approach is to remove the scale \( M \) as described above, and then to sum up diagrams with the small scale \( p^2/2M \) in nucleon propagators — a chain of loop diagrams — using nonperturbative quantum mechanics in the form of the Lippmann-Schwinger or Schrödinger equation, into an effective potential. [18] A very new approach [18] eliminates the small binding scale by regularizing loop integrals using minimal subtraction near \( D = 3 \) dimensions rather than \( D = 4 \), as is usually done in field-theoretic calculations. Then the loop diagrams are summed by means of renormalization group equations, thus avoiding the necessity of picking a kernel for a particular wave equation. That such an approach might work is perhaps not so surprising: Binding energy scales are very small compared to the nucleon masses, so the fundamental dynamics of the problem involves perturbations about an essentially static nucleon, and therefore is three-dimensional.

Before leaving this topic, it should be pointed out that many theories and models can be promoted to an effective theory. All that is needed is a set of symmetry principles for deciding what interactions are allowed, and an organizing principle (e.g., a perturbation series) for deciding which of these interactions are important. For example, meson potential models have neglected corrections in the form of nontrivial form factors or meson-meson couplings, while quark potential models are typically organized in a series in \( 1/m_q \).

All in all, the concept of the effective theory is not unlike the famous Wigner-Eckart theorem. Both divide physics into a symmetry part and a dynamical part. In the case of the W-E theorem, the symmetry part is represented by spin SU(2) Clebsch-Gordan coefficients, while the dynamical part is the so-called reduced matrix element. In effective theories, the symmetry part consists of Lorentz, chiral, and parity invariances, and other conditions we impose upon the interactions, while the dynamical part is represented by the unknown coefficients that must be fit to data. In this sense, effective theories are very minimal in their dynamical content, but provide a very useful starting point for deeper inquiries into the dynamics.

\section{Large \( N_c \) QCD}

It is a remarkable fact that considering the limit in which the number \( N_c \) of QCD color charges, which is 3 in our universe, becomes infinite, [2] actually simplifies strong interaction physics. How can increasing the number of degrees of freedom actually lead to a simplification? Think of statistical mechanics as an analogy, where Avogadro's number of particles can be described by just a few collective quantities, such as pressure, temperature, etc. In large \( N_c \) QCD, baryons are treated similarly, in a Hartree-Fock picture. [11] To first approximation, each of the \( N_c \) quarks feels only the collective effect of the other \( N_c - 1 \) mesons.

However, taking the large \( N_c \) limit seriously means that one expands physical quantities in a series in \( 1/N_c \). If we apply this to our universe, the expansion parameter is \( 1/3 \), which certainly does not seem small! However, for many quantities, the first correction to the large \( N_c \) limiting value appears not at relative order \( 1/N_c \) but at \( 1/N_c^2 \), which is arguably a small parameter. Even if this does not occur, one may simply adopt the expansion anyway, fit to the data using the \( 1/N_c \) expansion and set \( N_c = 3 \) at the end of the calculation. Then one can see a posteriori whether the factors of \( 1/3 \) truly are supported by experiment. A simple example was first pointed out in [22] where it is observed that the relative mass splitting between nucleons and \( \Delta \) resonances is suppressed by \( 1/N_c^2 \).

Writing this relation in a scale-independent way,

\[ \frac{m_\Delta - m_N}{m_\Delta + m_N} = O \left( \frac{1}{N_c^2} \right). \]

(4)

Experimentally, the l.h.s. is 0.27, whereas the r.h.s. is \( 3/N_c^2 \), which is 3 if we dismiss the factors of \( N_c \) as irrelevant, but 0.33 if they are retained. In fact, one can study the entire spectrum of the ground state baryons this way. [13]
and indeed the explicit factors of $N_c$ are essential to account properly for all uses.

In fact, studies of the large $N_c$ expansion for nuclear (as opposed to nucleon) systems are in their infancy; only a handful of papers studying this problem have yet appeared, but the prospects look quite promising. The basic reason is that large $N_c$ provides a kind of effective theory for nuclear systems, that the old spin-flavor $SU(3)$ is known to hold in the large $N_c$ limit (the symmetry), while interaction operators suppressed in this limit are accompanied by powers of $1/N_c$ (the organizing principle).

One direction that such a theory may be used is to note that, if the leading interactions in $1/N_c$ obey some symmetry, then so do the corresponding physical observables. For example, nature obeys an approximate symmetry under interchanges of the states $(p \uparrow, p \downarrow, n \uparrow, n \downarrow)$, the famous Wigner supermultiplet. In fact, this phenomenon has a large $N_c$ explanation in that the operators that would lift the degeneracy are suppressed by powers of $1/N_c$.

Another direction is to find exactly which operators appear at leading order in $1/N_c$ for a given process, and study their symmetry properties. This what is done in the first large $N_c$ analysis of the NN interaction, where it shown that one of the leading operators acting on nucleons 1 and 2 is the mixed spin-isospin operator

$$\langle \sigma_1 \cdot \sigma_2 \rangle \langle r_1 \cdot r_2 \rangle.$$ (5)

is important to realize that this sort of analysis is independent of the particular dynamical origin of the given operator. If one assumes that the dynamics arise from one-meson exchanges, for example, then one concludes that mesons strongest couplings to the given operator will be important for the NN interaction. In the given example, we know that the tensor couplings split into pieces like (5), and so large $N_c$ explains why we might have expected at meson potential models require large tensor couplings to these mesons. In any model, meson exchange or not, that successfully describes the NN interaction must recognize the importance of nucleon operators such as (5).

Conclusions

At the final analysis, we return to a variant of our original question: Why is the meson exchange picture work so well for the NN interaction, when underlying theory of QCD is so much more complicated? The ultimate attribution of QCD to the understanding of the NN interaction will almost certainly not be in the form of a solution to some as yet unknown field equation, rather the realization of how a complicated collection of quarks and gluons possesses some limiting case in which the system achieves a collective degree of simplicity, which we observe as a pair of interacting nucleons. We have already begun to see such simplifications take place in effective theories, and especially in large $N_c$, QCD.

Even though we cannot yet solve the strong interaction problem, we have begun to nibble at the edges. For example, we have argued that the famous "repulsive core" of the NN potential appears to be due to quark effects. It would be exciting to find more evidence for a six-quark "bag" or colored particle exchanges QCD suggests, phenomena that are quite exotic from the one-meson exchange perspective. Obviously these are topics of interest to both the nuclear and particle communities.

This last observation lies at the crux of my optimism on the future of NN studies: After following divergent paths for some decades, nuclear and particle physics are again making great strides together. We will see much more of the fruits of this combined effort in the future.

Acknowledgments

I would like to extend a special thank you to the conference organizers for their kind invitation and hospitality, and to Franz Gross and Wally van Orden for valuable comments on the content of the talk. This work was supported by the United States Department of Energy under contract No. DE-AC05-84ER40150.

References

7. See, for example, R. G. Newton, Scattering Theory of Waves and Particles (McGraw-Hill, New York, 1982).