LOCKHEED MARTIN

ORNL/TM-13280

**Ultrasonic Thickness Sampling Plan for the
Depleted Uranium Hexafluoride Program**

B.F. Lyon
M.L. Lykins

RECEIVED
JUL 10 1997
OSTI

MASTER *zoo*

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MANAGED BY
LOCKHEED MARTIN ENERGY SYSTEMS, INC.
FOR THE UNITED STATES
DEPARTMENT OF ENERGY

UCN-13675 (6 8-95)

This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from the Office of Scientific and Technical Information, P.O. Box 62, Oak Ridge, TN 37831; prices available from 615-576-8401, FTS 626-8401.

Available to the public from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Rd., Springfield, VA 22161.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

**Portions of this document may be illegible
electronic image products. Images are
produced from the best available original
document.**

Ultrasonic Thickness Sampling Plan for the Depleted Uranium Hexafluoride Program

B. F. Lyon
M.L. Lykins

Date Issued--July 1996

Prepared by
Risk Analysis Section
Health Sciences Research Division
Oak Ridge National Laboratory

Prepared for
U.S. Department of Energy
Office of Environmental Restoration and Waste Management
under budget and reporting code EW 20

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831-6285
managed by
LOCKHEED MARTIN ENERGY RESEARCH CORP.
under contract no.DE-AC05-96OR22464
with the U.S. DEPARTMENT OF ENERGY

CONTENTS

1. BACKGROUND AND PURPOSE	1
2. METHODS	1
3. RESULTS AND RECOMMENDATIONS	4
APPENDIX A: DETERMINING THE NUMBER OF SAMPLES TO DETERMINE CURRENT CONDITION OF CYLINDERS	
A-1. Methodology	A-1
A-2. Results	A-3
A-3. References	A-3
APPENDIX B: DETERMINATION OF SAMPLE SIZE NECESSARY FOR MODELING ...	
B-1. Calculation of Confidence Limits	B-1
Confidence Limits for the Case $P(t) \sim N(at+b, \sigma)$	B-2
Confidence Limits for the Case $P(t) \sim \text{Log}(\ln A + n \ln t, \sigma_t)$	B-5
B-2. The General Term	B-7
B-3. Results	B-7
B-4. References	B-8

TABLES

Table 1. Basic cylinder subpopulations considered	3
Table 2. Suggested ultrasonic thickness sampling plan	5
Table A-1. Number of samples required to determine current condition of cylinder populations with specified accuracy.	A-4
Table B-1. Intermediate results used in determining sample sizes for modeling.	B-9

1. BACKGROUND AND PURPOSE

The United States Department of Energy (DOE) currently manages depleted uranium hexafluoride that is stored in approximately 50,000 carbon steel cylinders located at three DOE sites. The disposition of any particular cylinder for storage, handling, and transfer is based on the condition of the cylinder, where "condition" is ultimately reflected by the minimum wall thickness of a cylinder.

Currently, the wall thickness of a cylinder may be measured using either a hand-held ultrasonic transducers or an automated scanner. At the Portsmouth site, the cylinder program is currently committed to a sampling plan that requires sampling 10% of the cylinders moved during the cylinder relocation efforts. This plan was agreed to with the Ohio Environmental Protection Agency in FY 1995 in the Consent Decree.

The purpose of this report is to present a statistically-based sampling plan to be considered for use within the three site cylinder management program. This plan is designed to meet the following objectives:

- 1) allow determination of the current condition of the cylinder populations within the accuracy and confidence specified by cylinder program management, and
- 2) be sufficient for the models to be used for modeling purposes.

The first objective does not require "modeling" in the sense of making assumptions about the corrosion process for the populations involved. By avoiding such additional assumptions, this may result in stronger statements to be made about the populations in question. Assumptions must be made regarding corrosion of the cylinders through time.

The second objective depends on the particular model used. In this report, two basic methods are used in determining sample sizes. The sample sizes are intended to be conservative because it may be that other models are developed for use within the Program.

This plan has been developed to guide the cylinder program towards the point where the current corrosion rates can be characterized for the important cylinder subpopulations. By so doing, the corrosion modeling can be simpler and more defensible than is currently possible. Indeed, a major gap in the data at present is an estimate of the current corrosion rates for the various cylinder populations. This necessitates the use of cylinder age in order to characterize the dependence of corrosion on time. The history of many of the cylinders is such that the age may not be relevant to the corrosion that has occurred, thereby increasing the uncertainty in the reliability of the results based on these assumptions. Use of information from inspections of the visual conditions of the cylinders allows combining populations into groups for inventory modeling.

2. METHODS

2.1. Basic Approach

For details about the methods and models used to determine sample size, please refer to

Appendices A and B. A brief summary is provided below.

To develop a sampling plan, it is necessary to specify the accuracy desired. Based on input from cylinder program management, a general criterion of 10% of the population was determined to be sufficient to address the objectives of the program. In essence, this is equivalent to determining the proportion of the population in a given thickness class with an error of at most 10%.

The first objective for determining the current condition of the cylinders is addressed by using a nonparametric method analogous to determining how many red marbles are present in a collection of red and black marbles. In this context, the collection of marbles is a population of cylinders, and a red marble would be a cylinder with a particular attribute (e.g., shippable).

Upper bounds on how many cylinders have an attribute in specific populations were made using estimates based on the available data. It is not intended that these estimates be precise, but it is intended that they be conservative, because the sample sizes required generally increase as the number with the attribute increase.

The number of samples for predictive modeling is determined based on likely values for the fitted model parameters. These values were determined using the currently available data. The number of samples is chosen so that the difference between the upper bound on the upper 95% confidence limit on the number of cylinders with a minimum wall thickness below 250 mils and the maximum likelihood values is less than 10% of the total population in 2001. If the actual model parameters are found to be substantially different from those observed so far (e.g., the variability of pit depth), this could affect the accuracy of the results.

For almost any model used, the uncertainty in the predictions will generally increase with time. This is primarily because one is forced to predict corrosion at ages that have not been observed. For the models used here, the number of samples and the ages at which the cylinders are measured can be controlled. Essentially, the larger the "spread" in ages used in the sampling, the smaller the uncertainty in the predictions, although this uncertainty is unbounded with increasing time.

2.2. Cylinder Subpopulations

The entire inventory of 10- and 14-ton cylinders is subdivided into different populations, with the populations being based primarily on the storage history of the cylinders. There are three broad categories: 1) cylinders that may have been in ground contact and/or standing water, 2) top row cylinders, and 3) bottom row cylinders that have not been in ground contact or standing water. The populations are further divided into subpopulations based on the ages of the cylinders: 1) those manufactured before 1968, 2) those manufactured between 1968 and 1983, and 3) those cylinders manufactured after 1983. The age groups are based on the age distributions for the populations. The age distributions are usually bimodal or trimodal, a reflection of distinct periods of elevated purchasing activity. There is some leeway in the definition of the particular boundaries of the age groups. The age groups defined here are chosen so that they are consistent across populations. While this consistency is not necessary, it may assist the modeling efforts because there will be some time delays in the sampling. The data for one group may be used as a surrogate for other populations until more data are acquired.

Table 1. Basic Cylinder Subpopulations Considered

Cylinder Population	Description
C-745-F yard top row (previously in bottom row) C-745-G yard bottom row (prior to relocation effort conducted during FY 1995 and FY 1996)	Cylinders that may have been in ground contact or standing water at Paducah
C-745-F yard bottom row (previously in top row) C-745-B/C/D/G/K/L/M/N/P top row K-1066-B/E/J/L yard top row All Portsmouth top row	Cylinders that have primarily been in the top row for most of their past storage history
C-745-B/C/D/K/L/M/N/P bottom row K-1066-B/E/J/L bottom row All Portsmouth bottom row	Cylinders that have primarily been in the bottom row not in ground contact or standing water
K-1066-K	Narrow age range of cylinders, many of which were previously in ground contact for at least 15 years while in K-1066-G yard

There are two subpopulations that consist of cylinders that may have been in ground contact and/or standing water at some point in their storage history. One of the subpopulations is at Paducah, and one is located at K-25.

The K-1066-K yard represents a unique situation. This population consists of cylinders manufactured in the period 1958-1964. These cylinders were previously located in an adjacent yard (the now non-existent K-1066-G yard), with many in ground contact. They were relocated to their current location during 1982. Many of the cylinders were put in the opposite row (i.e., top and bottom row switched) that they were in while they were in K-1066-G yard, but this was not performed consistently. These cylinders are treated separately from all other cylinder populations, with the goal being to determine the distribution of *current* corrosion rates. This may be possible because approximately 100 of these cylinders were evaluated in 1994 with the same ultrasonic equipment used currently. By sampling a number of these cylinders again in FY 1997 or FY 1998, it may be possible to determine the current corrosion rates for these cylinders. Combined with estimates of the current wall thicknesses for these cylinders (also collected during FY 1997 or FY 1998), this can be used to model future conditions for these yards requiring fewer assumptions to be made about the dependence of the corrosion rates on the age of the cylinder (which may be obscured by other factors in the storage history of the cylinder).

Another interesting factor is how to address the C-745-F yard cylinders at Paducah. The top and bottom rows for these cylinders were switched in 1992 in order to reduce the corrosion rates for the cylinders that had been in the bottom row for a significant period of time. This means that the corrosion rates for these cylinders probably changed (both top and bottom row) when the movements occurred. It is suggested that the cylinders currently in the top row of C-745-F yard be combined with the C-745-G yard bottom row cylinders for modeling at this time. The age distribution for these yards is similar, and both have had some proportion of their population in ground contact and/or standing water at some point in their storage history. This is in contrast to the rest of the bottom row population at Paducah,

Portsmouth, and most of the K-25 yards. Further, both of these populations can be evaluated during FY 1997 with minimal additional cylinder movement. For the C-745-F yard top row cylinders, the automated scanner can be used for cylinders that are in the stack. For the C-745-G yard cylinders, measurements can be obtained when these cylinders are relocated back to the refurbished C-745-G yard during FY 1997, and the cylinders selected for sampling can be evaluated when they are moved.

The two other populations considered are: 1) cylinders that have been in the bottom row in good storage conditions (i.e., not in ground contact or standing water); and 2) cylinders that have been in the top row for most of their storage history. The latter includes those cylinders that are currently in the bottom row of C-745-F yard at Paducah. These cylinders are included because they were in the top row until 1992. For the purpose of determining the current condition (without any modeling), these cylinders are most similar to the other cylinders that are located in the top row currently.

3. RESULTS AND RECOMMENDATIONS

Table 2 provides a sampling plan for the different subpopulations of cylinders. The number of cylinders suggested for sampling is generally larger for those populations that are in the poorest condition. This plan is designed to meet the objectives discussed above. In particular, this plan is intended to be sufficient for both modeling purposes and determining the current condition of the cylinders within the accuracy specified by cylinder management. It is acknowledged that there may be time delays in getting the data for some of the populations, and it may not be possible to conduct all of the sampling during FY 1997- FY 1998. It is noted that the sampling is to be based on the age group of the cylinders as well, as discussed in Appendices A and B. In particular, the sampling should be proportional to the number of cylinders in each of the three age groups, as defined in Appendix A. Only the total number of samples is provided in the summary table below.

Sampling beyond FY 1998 is not provided here because it is not known if these plans can correspond with the cylinder movement plans by FY 1998. Beginning in FY 2000, it will be possible to reevaluate cylinders sampled as part of this program. This will provide a characterization of the "current" corrosion rates for the relevant populations. There have also been cylinders sampled during FY 1996 as part of the previous sampling plan which can be evaluated in FY 2000. This population consists of those cylinders evaluated at Portsmouth as part of the "1 in 10" plan agreed to with the Ohio EPA in 1995, as well as cylinders set aside at Paducah during the relocation of the C-745-G yard in FY 1995 and FY 1996, and evaluated prior to the implementation of the plan provided here.

Table 2. Suggested Ultrasonic Thickness Sampling Plan

Cylinder Population	Population Size	Number of Cylinders to be Evaluated in FY 1997 and FY 1998
C-745-F yard top row (previously in bottom row) C-745-G yard bottom row (prior to relocation effort conducted during FY 1995 and FY 1996)	5636	242
C-745-F yard bottom row (previously in top row) C-745-B/C/D/G/K/L/M/N/P top row K-1066-B/E/J/L yard top row All Portsmouth top row	23064	225
C-745-F yard top row C-745-B/C/D/K/L/M/N/P bottom row K-1066-B/E/J/L bottom row All Portsmouth bottom row	19446	225
K-1066-K	2942	200

The sampling of the cylinders must be selected at random from the individual populations, and this includes subpopulations determined by cylinder age. This may be logistically difficult for some yards. If it is not possible, then separate subpopulations that can be accessed need to be determined, and then probability statements can be made for these subpopulations. If a group of cylinders cannot be considered when the random sampling is performed, then statistical inferences cannot be made for this group.

Depending on the nature of the data collected from the populations considered the best storage conditions, it may be necessary to perform additional evaluations of cylinders from these populations. Data collected at a later date can be combined with those data collected using the plan suggested here.

Data for fixed cylinders at two different dates can become available if sampling is performed in the K-1066-K yard. Due to the uncertainty in the measurement process, it is probably not useful to measure the same cylinder more often than every three years. The sampling conducted during FY 1997 can be used to characterize the corrosion rates for cylinders evaluated in FY 1994, as well as to characterize the current distribution of minimum wall thicknesses. This may serve as the distribution of "initial thickness" for these cylinders which, in conjunction with the distribution of corrosion rates determined by resampling the cylinders measured in 1994, will allow predictions to be made that utilize the most relevant information. Further, this will avoid the complications of modeling the cylinders in this yard with a narrow age range and when it is not known how similar this yard is to other yards. Subsequent measurements in this yard would then not be required until at least FY 2000, when the corrosion rates during the period from 1997 to the next year of measurement may be evaluated.

It is hoped that by FY 2001 a database will exist that will require fewer assumptions to be made about the corrosion rates for the cylinders. In particular, by FY 2001 data should be available for fixed cylinders at two different times for all cylinder populations. This will allow determination of the distribution of current corrosion rates for the cylinders. In conjunction with a database for the distribution of minimum wall thicknesses, this will allow all corrosion modeling to be based on a better characterization of the current corrosion of the cylinders in the yards, eliminating (sometimes tenuous) assumptions that must be made with the presently available data.

APPENDIX A: DETERMINING THE NUMBER OF SAMPLES TO DETERMINE CURRENT CONDITION OF CYLINDERS

One of the objectives of the sampling plan is to determine the current conditions of the cylinders. In this appendix, the method used in order to address this problem is discussed.

A-1. Methodology

For a given population of size L of objects, assume that M have a particular attribute (e.g., minimum thickness less than 250 mils), and assume that n objects are to be selected at random. Let m denote the number of objects in the sample that have the attribute. The basic problem is to determine how large n should be so that the difference between M/L and m/n (this is the fraction of the population that is estimated to have the attribute based on the sample) is sufficiently small, for a specified confidence.

It is known that m follows the hypergeometric distribution. In particular,

$$\text{Probability of obtaining } m \text{ objects with the attribute} = \frac{\binom{M}{m} \binom{L-M}{n-m}}{\binom{L}{n}}$$

where $\binom{i}{j} = \frac{i!}{j!(i-j)!}$. The probability of obtaining m or fewer objects with the attribute is calculated as¹

$$\text{Probability of obtaining } k \text{ or fewer objects with the attribute} = P(k;n;M,L) = \sum_{m=0}^k \frac{\binom{L}{m} \binom{L-M}{n-m}}{\binom{L}{n}}$$

As the sample size n increases, the distribution of possible values for the estimated number of objects with the attribute, given by $(m/n)N$, becomes more tightly distributed about the expected value M . In particular, the standard deviation of (m/n) goes to zero as n increases (Evans et al. 1993). This means that, for a given tolerance T , if n is large enough then most of the distribution will be within T of M . Conversely, for a given confidence level α and number of samples n , one can determine how accurate the sample estimate of the total number of objects with the attribute is.

Let k_α denote the smallest nonnegative number such that $P(k;n;M,L) > \alpha$, i.e., an approximate

¹ Simple programs were written in Fortran to calculate the probability density function, the cumulative distribution, and the inverse distribution for the hypergeometric distribution. The factorials are calculated using the logarithm of the gamma function, the code for which was obtained from the *NETLIB* library of software routines and was written by W.J. Cody and L. Stolz.

upper $100\alpha\%$ percentile on the number of objects k with the attribute in a given sample². For a given sample size n , the interval $(k_{1-\alpha}L, k_{\alpha}L)$ is at least³ a $100(2\alpha-1)\%$ confidence interval for k . This means that if one is going to use the quantity $(k/n)L$ as an estimate of the total number of objects with the attribute, then one can conclude with at least $100(2\alpha-1)\%$ confidence that

$$\frac{k_{1-\alpha}-1}{n}L \leq \frac{k}{n}L \leq \frac{k_{\alpha}}{n}L$$

This means that one can conclude with at least $100(2\alpha-1)\%$ confidence that the maximum error between $(k/n)L$ and the actual number M is bounded by

$$Tol(n, M, \alpha) = \max \left\{ \left| \frac{k_{1-\alpha}-1}{n}L - M \right|, \left| \frac{k_{\alpha}}{n}L - M \right| \right\}$$

This quantity does not strictly decrease with decreasing M . Therefore, if one has only an upper bound U on the number of objects with the attribute, then one must use the maximum over all M up to and including U , and so

$$\text{Maximum error using } n \text{ samples if there are at most } U \text{ objects with attribute} = E(n, U, \alpha) = \max_{0 \leq M \leq U} Tol(n, M, \alpha)$$

This method can also be used to determine the sample size necessary simply by setting the sample size n so that the maximum error $E(n, U, \alpha)$ is sufficiently small.

In order to use this method, it is necessary to determine an upper bound U on the number of cylinders there are with a particular attribute. This is done using the available data to estimate upper bounds on the fraction of cylinders in each subpopulation that have the attribute of interest.

In general, the number of samples necessary increases as the number of the objects with the attributes increases. If it is possible to divide population up into subgroups, each of which having a different estimate of the number of objects with the attribute, then it may be possible to reduce the samples necessary. This is because fewer samples will be required from subpopulations for which there are considered to be few objects with the attribute.

Numerous methods exist for determining sampling fractions from each subpopulation that are optimal with respect to some measure (e.g., variance of sampling distribution for proportion is minimized), but none found seem to be suitable for the problem at hand. Further, there are at present few subpopulations considered, and the accuracy desired is such that a simple method should be sufficient⁴.

²Since the hypergeometric distribution is discrete, it is not always possible to find an integer k such that $P(k; n; M; L) = \alpha$.

³It may happen that $k_{1-\alpha} = 0$, in which case 0 is used for $k_{1-\alpha}L$.

⁴This may not be the case if one considers the entire cylinder population as a whole. However, the sample sizes determined in the manner suggested here should be conservative.

Basically, for each population for which distinct subpopulations have been defined, the number of samples is determined so that the sum of the errors for each subpopulation is sufficiently small. The particular size of the error tolerated is specified by cylinder program management at 10% of the total population considered; this is equivalent to estimating the fraction of cylinders in a given class with an error of at most 10%. In many cases, no samples are necessary based on the upper bound estimates for the number of cylinders with the attribute of interest. However, sampling may still be necessary from these populations for modeling purposes (see Appendix B), as adequate sampling is needed from the different age groups.

A-2. Results

Currently, the accuracy desired by cylinder program management does not depend on the particular criterion of interest. This means that the number of samples determined to ensure the desired accuracy in estimating the number of cylinders with minimum thickness below 250 mils will be sufficient for estimating the number of cylinders in thinner wall thickness categories, because the upper bounds will be necessarily be lower for the thinner categories.

Table A-1 shows the results for the cylinder subpopulations considered. Based on the upper bound estimates used, only a few of the cylinder subpopulations require sampling at all in order to estimate the number of cylinders with the specified accuracy. These subpopulations are the K-1066-K yard top and bottom rows (at least 100 from each population if treated separately, or only 110 samples if these populations are combined), and the C-745-F and C-745-G bottom row cylinders manufactured before 1968 (at least 150 samples required).

It is stressed that these results are based on the currently specified accuracy desired (i.e., estimate proportion of a given population within 10%), and the upper bound estimates on the number of cylinders with minimum thickness below 250 mils. If it is deemed that more (or less) accuracy is necessary, then this sampling plan will need to be updated.

A-3. References

Evans, M., N. Hastings and B. Peacock (1993). *Statistical Distributions*, 2nd edition, John Wiley & Sons, Inc. New York.

Table A-1. Number of samples required to determine current condition of cylinder populations with specified accuracy.⁵

Table A-1. Number of samples required to determine current condition of cylinder populations with specified accuracy.					
Population	Size	Sampling for Current Condition Characterization			Upper Bound on Error
		Upper Bound for Cylinders with Minimum Thickness Below 250 mils	Samples from each subpopulation		
C-745-F top row and C-745-G bottom row					10%
	< 1968	3483	2400	150	325
	1968-1983	2150	500	92	231
	> 1983	3	0	0	0
C-745-F bottom row, C-745-B/C/D/G/K/L/M/N/P top row, K-1066-B/E/J/L top row, All Portsmouth top row					3%
	< 1968	6384	192	0	192
	1968 - 1983	9936	298	0	298
	> 1983	6744	202	0	303
C-745-B/C/D/K/L/M/N/P bottom row, K-1066-B/E/J/L bottom row, All Portsmouth bottom row					3%
	< 1968	3990	120	0	120
	1968 - 1983	8715	261	0	261
	> 1983	6741	202	0	202
K-1066-K Top and Bottom Rows					10%
	1956-1964	2492	1300	110	249

⁵Because the accuracy desired by cylinder management does not depend on the thickness category of interest, and because the number of cylinders in thinner wall thickness categories is smaller than those that are less than 250 mils, the sampling necessary to determine the number of cylinders with minimum thickness less than 250 mils will also be sufficient for determining the number of cylinders in the thinner categories.

APPENDIX B: DETERMINATION OF SAMPLE SIZE NECESSARY FOR MODELING

In this appendix, the number of samples necessary for modeling purposes is estimated. How many samples are required depends on the method used. For the purpose of determining sample sizes, two basic methods are used to model the penetration depth $P(t)$. The first consists of a model of the form $P(t) = N(at+b, \sigma)$, where $N(\mu, \sigma)$ is the normal distribution with mean μ and standard deviation σ . The second model is of the form $P(t) = \text{Log}(\ln A + n \ln t, \sigma_L)$, where $\text{Log}(\mu_L, \sigma_L)$ is the lognormal distribution with μ_L and σ_L being the mean and standard deviation of the logarithm of the distribution. Such relatively simple models are used to determine sample size because it is relatively straightforward to quantify the effect of sample size on the results.

Before determining sample size, it is first necessary to detail how the sample size affects the accuracy of the fitted models.

B-1. Calculation of Confidence Limits

For a given z , the basic problem is to obtain a confidence limit on the probability that the penetration depth at time t exceeds z . The sample size affects the accuracy of the estimated model parameters, and this then affects the accuracy of the estimated confidence limits.

Assuming that one can calculate confidence limits on a given percentile p of a distribution $P(t)$, calculation of an upper 95% confidence limit on the probability that $P(t) > z$ is performed as follows: find that percentile $p = p(z)$ of $P(t)$ such that the 95% confidence limit on $p(z)$ is equal to z . One can then conclude with 95% confidence that

$$P_{p(z)}(t) < z$$

where $P_a(t)$ is the upper a th percentile on $P(t)$; i.e.,

$$\text{Prob}\{P(t) < P_a\} = a$$

Therefore, we can conclude with 95% confidence that

$$p(z) < \text{Prob}\{P(t) < z\}$$

which is equivalent to

$$1 - p(z) \geq \text{Prob}\{P(t) > z\}$$

Therefore, the number $1 - p(z)$ is an upper 95% confidence bound on the probability that $P(t)$ is larger than z . This shows that calculation of confidence limits on percentiles is sufficient to allow calculation of confidence limits on the probability of exceeding a given value.

Confidence Limits for the Case $P(t) \sim N(at+b, \sigma)$

The case where $P(t) \sim N(at+b, \sigma)$, where $N(\mu, \sigma)$ is a normal distribution with mean μ and standard deviation σ , is similar to that for $P(t) = Rt$ in that the non-central t -distribution plays a fundamental role, although there are additional factors that depend on the ages at which the measurements are made. We note that this method was suggested for use to model pit depth in Rosen and Glaser (1995).

Assume that we have pairs of samples of the penetration depth at various ages t : (t_i, p_i) , $i=1, \dots, N$. Estimates of the regression coefficients a and b are given by

$$\hat{a} = \frac{\sum_{i=1}^N p_i t_i - \frac{1}{N} \left(\sum_{i=1}^N p_i \right) \left(\sum_{i=1}^N t_i \right)}{\sum_{i=1}^N t_i^2 - \frac{1}{N} \left(\sum_{i=1}^N t_i \right)^2}$$

$$\hat{b} = \bar{p} - \hat{a} \bar{t}$$

where $\bar{p} = \frac{1}{N} \sum_{i=1}^N p_i$ and $\bar{t} = \frac{1}{N} \sum_{i=1}^N t_i$. An unbiased estimate of the standard deviation σ is

$$S^2 = \frac{1}{N-2} \sum_{i=1}^N (p_i - \hat{a} t_i - \hat{b})^2$$

Assuming that $P(t) \sim N(at+b, \sigma)$, the following is known about the sampling distributions for the estimates of the parameters (Casella and Berger, 1990; pp. 569-575):

$$\hat{a} + t \hat{b} \sim N \left(a + bt, \sigma \sqrt{\frac{1}{n} + \frac{(t - \bar{t})^2}{S_{tt}}} \right)$$

$$S^2 \sim \sigma^2 \frac{\chi_{N-2}^2}{N-2}$$

where

$$S_u = \sum_{i=1}^N (t_i - \bar{t})^2$$

and χ^2_v is the χ -squared distribution with v degrees of freedom. Based on the above, it is possible to obtain confidence limits on the percentiles of $N(at+b, \sigma)$ in terms of the non-central t -distribution, following the discussion in Lawless (1982, pp. 226-228) as described in Appendix B of Lyon (1995).

The p th percentile of a normal distribution with mean μ and standard deviation σ is given by $\mu + u_p \sigma$, where u_p is the p th percentile of the standard normal distribution $N(0, 1)$. Thus, the p th percentile for $N(at+b, \sigma)$ is given by $at+b + u_p \sigma$. We will only be able to approximate a, b and σ using the formulas above, and the accuracy of these approximations depends on the sample size and the times t_i .

Set

$$Z_p = \frac{(\hat{a}t + \hat{b} + u_p S) - (at + b + u_p \sigma)}{S}$$

Because for any z

$$\begin{aligned} \text{Prob}\{Z_p \leq z\} &= \text{Prob}\{at + b + u_p \sigma \geq at + b + (u_p - z)S\} \\ &= \text{Probability that } p\text{th percentile} \geq \hat{a}t + \hat{b} + (u_p - z)S \end{aligned}$$

probability statements for Z_p produce confidence limits for the p th percentile $at + b + u_p \sigma$ of the distribution $N(at+b, \sigma)$.

$$\begin{aligned}
\text{Prob}\{Z_p \leq z\} &= \text{Prob}\left\{\frac{(\hat{a}t + \hat{b} + u_p S) - (at + b + u_p \sigma)}{S} \leq z\right\} \\
&= \text{Prob}\left\{\frac{\hat{a}t + \hat{b} - (at + b) - u_p \sigma}{S} \leq z - u_p\right\} \\
&= \text{Prob}\left\{\frac{\sigma}{S} \left(\frac{\hat{a}t + \hat{b} - (at + b)}{\sigma} - u_p\right) \leq z - u_p\right\} \\
&= \text{Prob}\left\{\frac{\sigma}{S} \left(\frac{\hat{a}t + \hat{b} - (at + b)}{\sigma e_N(t)} - \frac{u_p}{e_N(t)}\right) \leq \frac{z - u_p}{e_N(t)}\right\} \\
&= \text{Prob}\left\{\frac{\left(\frac{\hat{a}t + \hat{b} - (at + b)}{\sigma e_N(t)} - \frac{u_p}{e_N(t)}\right)}{S/\sigma} \leq \frac{z - u_p}{e_N(t)}\right\}
\end{aligned}$$

where we define $e_N(t)$ by

$$e_N(t) = \sqrt{\frac{1}{N} + \frac{(t - \bar{t})^2}{S_{tt}}} \quad (1)$$

From the results above for the sampling distributions,

$$\frac{\hat{a}t + \hat{b} - (at + b)}{\sigma e_N(t)} \sim N(0, 1)$$

$$S^2/\sigma^2 \sim \chi_{N-2}^2/(N-2)$$

and so we have that the random variable

$$\frac{\left(\frac{\hat{a}t + \hat{b} - (at + b)}{\sigma e_N(t)} - \frac{u_p}{e_N(t)}\right)}{S/\sigma}$$

is distributed with a noncentral t -distribution with $N-2$ degrees of freedom and noncentrality parameter $\lambda = -u_p/e_N(t)$ ⁶, which we denote by $t_{N-2}(u_p/e_N(t))$. Therefore,

$$\text{Prob}\{Z_p \leq z\} = \text{Prob}\left\{t_{N-2}\left(-u_p/e_n(t)\right) \leq \frac{(z-u_p)}{e_n(t)}\right\}$$

An upper 95th confidence limit for a given percentile Z_p , denoted here by $y_{0.95}(p)$, is then given by

$$\begin{aligned} \text{Upper 95\% Confidence Limit on the } p\text{th percentile of } P(t) = N(at+b, \sigma) \text{ is} \\ = \hat{a}t + \hat{b} - T_{0.05}(p, N) S e_N(t) \end{aligned} \quad (2)$$

where $T_{0.05}(p, N)$ is the number satisfying

$$\text{Prob}\{t_{N-2}(-u_p/e_n(t)) \leq T_{0.05}(p, N)\} = 0.05$$

Confidence Limits for the Case $P(t) \sim \text{Log}(\ln A + n \ln t, \sigma_t)$

In this case we assume that the penetration depths are lognormally distributed at each time. This can also be expressed as $\ln P(t) \sim N(\ln A + n \ln t, \sigma_t)$, which shows how it is related to the previously discussed approach. For this model, the median is equal to At^n , the arithmetic mean μ is $At^n \exp[0.5\sigma_t^2]$, and the arithmetic standard deviation is $\mu [\exp(\sigma_t^2) - 1]^{1/2}$. While the arithmetic standard deviation is not constant with time, as is the case for the previous model, the coefficient of variation (ratio of the standard deviation to the mean) is constant with time, and is equal to $[\exp(\sigma_t^2) - 1]^{1/2}$.

Calculation of confidence limits is based on that of the previous model because after taking logarithms the models are mathematically equivalent.

Assume that we have pairs of samples of the penetration depth at various ages t : (t_i, p_i) , $i=1, \dots, N$. Let $A_L = \ln A$. Estimates of the regression coefficients A_L and n are given by

⁶ By definition, if $Z \sim N(0,1)$ and W is a χ^2 distribution with v degrees of freedom, then the ratio $(Z+\lambda)/(W/v)^{1/2}$ is distributed with a noncentral t -distribution with noncentrality parameter λ (Lawless 1982).

$$\hat{n} = \frac{\sum_{i=1}^N \ln p_i \ln t_i - \frac{1}{N} \left(\sum_{i=1}^N \ln p_i \right) \left(\sum_{i=1}^N \ln t_i \right)}{\sum_{i=1}^N (\ln t_i)^2 - \frac{1}{N} \left(\sum_{i=1}^N \ln t_i \right)^2}$$

$$\hat{A}_L = \overline{\ln p} - \hat{n} \overline{\ln t}$$

where $\overline{\ln p} = \frac{1}{N} \sum_{i=1}^N \ln p_i$ and $\overline{\ln t} = \frac{1}{N} \sum_{i=1}^N \ln t_i$. An unbiased estimate of the standard deviation σ_L is

$$S_L^2 = \frac{1}{N-2} \sum_{i=1}^N (\ln p_i - \hat{n} \ln t_i - \hat{A}_L)^2$$

Following the identical discussion as above, an upper 95th confidence limit on the p th percentile of $\ln P(t) \sim N(\ln A + n \ln t, \sigma_L)$,

$$\hat{n} \ln t + \hat{A}_L - T_{L,0.05}(p, N) S_L e_{LN}(t)$$

where

$$e_{LN}(t) = \sqrt{\frac{1}{N} + \frac{(\ln t - \overline{\ln t})^2}{\sum_{i=1}^N (\ln t_i - \overline{\ln t})^2}} \quad (3)$$

and $T_{L,0.05}(p, N)$ is the number satisfying

$$\text{Prob}\{t_{N-2}(u_p/e_{LN}(t)) \leq T_{L,0.05}(p, N)\} = 0.05$$

An upper 95th confidence limit on the p th percentile of $P(t) \sim \text{Log}(\ln A + n \ln t, \sigma_L)$ is then given by the exponential of the confidence limit for the p th percentile of $\ln P(t)$:

Upper 95% Confidence Limit on the p th percentile of $P(t) = \text{Log}(\ln A + n \ln t, \sigma_L)$ is

$$\begin{aligned}
 &= \exp\left[\hat{n} \ln t + \hat{A}_L - T_{L,0.05}(p, N) S_L e_{L,N}(t)\right] \\
 &= \hat{A} t^{\hat{n}} \exp\left[-T_{L,0.05}(p, N) S_L e_{L,N}(t)\right]
 \end{aligned} \tag{4}$$

where $\hat{A} = \exp[\hat{A}_L]$.

B-2. The General Term

The actual quantity of interest is the total number of cylinders with a minimum thickness below a given level at some time t . This is a sum of the form $\sum_j M_j(t) N_j$, where the sum is over all age classes j , $M_j(t)$ is the fraction of the population in age class j that has a minimum thickness below the level of interest at time t , and N_j is the number of cylinders in age class j . Even if one can calculate upper confidence limits on each individual term, determination of an upper confidence limit on this sum is more involved than one might suppose. For example, by using upper 95% confidence limits for each term, one cannot in general conclude with 95% confidence that the resulting sum is an upper 95% confidence limit. Thus, the confidence limits reported in Lyon (1995, 1996) as upper 95% confidence limits are more accurately described as simply upper bounds using upper 95% confidence limits for each term.

It is straightforward to calculate an upper bound on the upper 95% confidence limit for the sum using the so-called Bonferroni inequality. This requires using higher confidence limits for each individual term so that one can conclude that the sum is bounded at 95% confidence. Because this is conservative, estimation of sample sizes based on the Bonferroni inequality guarantees that the sample size is sufficient. The Bonferroni inequality is a rather general result, and as such much of the structure of the current model is not incorporated. If necessary, more sophisticated methods (e.g., Taylor series approximations, simultaneous tolerance intervals as described in Lieberman and Miller 1963) will be utilized.

B-3. Results

The number of samples required depends on the model used. There are two main factors that can be controlled: the total number of samples made, and the ages at which the samples are made. The ages at which the samples are made affect the magnitude of the terms $e_N(t)$ (Eq. 1) or $e_{L,N}(t)$ (Eq. 3). Basically, the larger the "spread" in the ages at which the cylinders are measured, the closer the confidence limits are to the sample confidence limits.

For the purpose of determining sample sizes necessary, the following method was used:

- * The available data were analyzed in order to determine the likely range for the parameters in the two models
- * It was assumed that the sampling from the different age groups was proportional to the

size of the age group

- * The sample sizes were determined so that the difference between the confidence bounds and the results using the parameter values in the year 2001 was less than 10% of the population considered (this particular accuracy is identical to that specified for determining the current condition of the cylinders as described in Appendix A)

The errors for a variety of sample sizes (with proportional sampling) are shown in Table B-1. In general, if the fitted model parameters are within the ranges used here, and the ages at which the cylinders are measured is consistent with the age distribution for that population, then approximately 225 samples from each population is sufficient. In all cases, this is less than 5% of the population.

In approximately three years (FY 1999 or FY 2000), it is suggested that the cylinders evaluated in FY 1996 and FY 1997 be measured again to estimate the corrosion rates for the period 1997-2000. Sampling should also be performed for the population as a whole to determine the current conditions as discussed in Appendix A, and the distribution of minimum wall thicknesses be used as the "initial thickness" for years subsequent to FY 2000.

B-4. References

Casella, G. and R.L. Berger (1990). *Statistical Inference*, Wadsworth & Brooks/Cole Advanced Books, Pacific Grove, CA.

Lawless, J.F. (1982). *Statistical Models and Methods for Lifetime Data*, John Wiley & Sons, New York.

Lieberman, G.J. and R.G. Miller (1963). Simultaneous tolerance intervals in regression, *Biometrika*, **50**:155-168.

Lyon, B.F. (1995). *Prediction of External Corrosion for UF₆ Cylinders: Results of an Empirical Method*, ORNL/TM-13012, Oak Ridge National Laboratory, Oak Ridge, TN.

Lyon, B.F. (1996). *Prediction of External Corrosion for Steel Cylinders at the Paducah Gaseous Diffusion Plant: Application of an Empirical Method*, ORNL/TM-13192, Oak Ridge National Laboratory, Oak Ridge, TN.

Rosen, R. And R. Glaser (1995). Letter to Charles E. Bradley dated November 8, 1995.

Table B-1. Intermediate results used in determining sample sizes for modeling.

		Parameters for $N(at+b, \sigma)$						Results								
		Parameters for $N(at+b, \sigma)$			Parameters for $\text{Log}(\ln A + n \ln t, \sigma_L)$			$N(at+b, \sigma)$			$\text{Log}(\ln A + n \ln t, \sigma_L)$					
Date	Population	$\hat{a}$	$\hat{b}$	s	$\hat{A}_L$	$\hat{n}$	S_L	Using parameters	95%	% Diff	Using parameters	95%	% Diff	Pop. Size	Sample Size N	Min. Thickness Predicted (mils)
2001	C-745-F/G yards, bottom row	0.5	0	8	1	0.4	0.2	0	0		0	0		5636	225	250
2001	C-745-F/G yards, bottom row	0.5	0	35	1	0.4	0.5	570	975		2	11		5636	225	250
2001	C-745-F/G yards, bottom row	0.5	18	8	1	1	0.2	5	24	0%	4894	5051	3%	5636	225	250
2001	C-745-F/G yards, bottom row	0.5	18	35	1	1	0.5	1254	1816	10%	4289	4650	6%	5636	225	250
2001	C-745-F/G yards, bottom row	2.5	0	8	2.1	0.4	0.2	4629	4783	3%	11	42	1%	5636	225	250
2001	C-745-F/G yards, bottom row	2.5	0	35	2.1	0.4	0.5	4150	4537	7%	625	1016	7%	5636	225	250
2001	C-745-F/G yards, bottom row	2.5	18	8	2.1	1	0.2	5555	5589	1%	5635	5635	0%	5636	225	250
2001	C-745-F/G yards, bottom row	2.5	18	35	2.1	1	0.5	4828	5085	5%	5612	5627	0%	5636	225	250
2001	C-745-F/G yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	5636	200	250
2001	C-745-F/G yards, bottom row	0.5	0	35	1	0.4	0.5	570	1004	8%	2	12	0%	5636	200	250
2001	C-745-F/G yards, bottom row	0.5	18	8	1	1	0.2	5	26	0%	4894	5060	3%	5636	200	250
2001	C-745-F/G yards, bottom row	0.5	18	35	1	1	0.5	1254	1853	11%	4289	4669	7%	5636	200	250
2001	C-745-F/G yards, bottom row	2.5	0	8	2.1	0.4	0.2	4629	4792	3%	11	45	1%	5636	200	250
2001	C-745-F/G yards, bottom row	2.5	0	35	2.1	0.4	0.5	4150	4557	7%	625	1044	7%	5636	200	250
2001	C-745-F/G yards, bottom row	2.5	18	8	2.1	1	0.2	5555	5590	1%	5635	5635	0%	5636	200	250
2001	C-745-F/G yards, bottom row	2.5	18	35	2.1	1	0.5	4828	5098	5%	5612	5628	0%	5636	200	250
2001	C-745-F/G yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	5636	225	187
2001	C-745-F/G yards, bottom row	0.5	0	35	1	0.4	0.5	6	28	0%	0	0	0%	5636	225	187
2001	C-745-F/G yards, bottom row	0.5	18	8	1	1	0.2	0	0	0%	1015	1406	7%	5636	225	187
2001	C-745-F/G yards, bottom row	0.5	18	35	1	1	0.5	30	98	1%	1687	2212	9%	5636	225	187
2001	C-745-F/G yards, bottom row	2.5	0	8	2.1	0.4	0.2	17	55	1%	0	0	0%	5636	225	187
2001	C-745-F/G yards, bottom row	2.5	0	35	2.1	0.4	0.5	1003	1486	9%	27	85	1%	5636	225	187
2001	C-745-F/G yards, bottom row	2.5	18	8	2.1	1	0.2	1078	1478	7%	5599	5618	0%	5636	225	187
2001	C-745-F/G yards, bottom row	2.5	18	35	2.1	1	0.5	1804	2367	10%	5190	5360	3%	5636	225	187
2001	C-745-F/G yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	5636	200	187
2001	C-745-F/G yards, bottom row	0.5	0	35	1	0.4	0.5	6	31	0%	0	0	0%	5636	200	187
2001	C-745-F/G yards, bottom row	0.5	18	8	1	1	0.2	0	0	0%	1015	1432	7%	5636	200	187
2001	C-745-F/G yards, bottom row	0.5	18	35	1	1	0.5	30	104	1%	1687	2245	10%	5636	200	187
2001	C-745-F/G yards, bottom row	2.5	0	8	2.1	0.4	0.2	17	58	1%	0	0	0%	5636	200	187
2001	C-745-F/G yards, bottom row	2.5	0	35	2.1	0.4	0.5	1003	1519	9%	27	91	1%	5636	200	187
2001	C-745-F/G yards, bottom row	2.5	18	8	2.1	1	0.2	1078	1503	8%	5599	5619	0%	5636	200	187
2001	C-745-F/G yards, bottom row	2.5	18	35	2.1	1	0.5	1804	2402	11%	5190	5369	3%	5636	200	187
2001	K-1066-B/E/I yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	17428	225	250
2001	K-1066-B/E/I yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	35	1	0.4	0.5	1314	2448	7%	3	20	0%	17428	225	250
2001	K-1066-B/E/I yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	8	1	1	0.2	10	58	0%	7355	8053	4%	17428	225	250

								Results								
Date	Population	Parameters for $N(at+b,\sigma)$			Parameters for $\text{Log}(\ln A + n \ln t, \sigma_L)$			$N(at+b,\sigma)$			$\text{Log}(\ln A + n \ln t, \sigma_L)$			Pop. Size	Sample Size N	Min. Thickness Predicted (mils)
		$\hat{a}$	$\hat{b}$	S	$\hat{A}_L$	$\hat{n}$	S_L	Using parameters	95%	% Diff	Using parameters	95%	% Diff			
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	35	1	1	0.5	3072	4686	9%	7571	8854	7%	17428	225	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	8	2.1	0.4	0.2	6168	6727	3%	16	69	0%	17428	225	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	35	2.1	0.4	0.5	7636	8991	8%	1077	2001	5%	17428	225	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	8	2.1	1	0.2	10649	11121	3%	17255	17333	0%	17428	225	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	35	2.1	1	0.5	10435	11688	7%	16148	16580	2%	17428	225	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	35	1	0.4	0.5	1314	2531	7%	3	22	0%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	8	1	1	0.2	10	63	0%	7355	8095	4%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	35	1	1	0.5	3072	4795	10%	7571	8930	8%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	8	2.1	0.4	0.2	6168	6762	3%	16	74	0%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	35	2.1	0.4	0.5	7636	9072	8%	1077	2071	6%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	8	2.1	1	0.2	10649	11152	3%	17255	17337	0%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	35	2.1	1	0.5	10435	11763	8%	16148	16602	3%	17428	200	250
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	35	1	0.4	0.5	12	70	0%	0	0	0%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	8	1	1	0.2	0	0	0%	1278	1758	3%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	35	1	1	0.5	60	238	1%	2197	3221	6%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	8	2.1	0.4	0.2	329	605	2%	0	0	0%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	35	2.1	0.4	0.5	1309	2208	5%	38	153	1%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	8	2.1	1	0.2	1861	2217	2%	11824	12331	3%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	35	2.1	1	0.5	2420	3546	6%	11442	12510	6%	17428	225	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	8	1	0.4	0.2	0	0	0%	0	0	0%	17428	200	187

Date	Population	Parameters for $N(at+b, \sigma)$						Results								
		Parameters for $N(at+b, \sigma)$			Parameters for $\text{Log}(\ln A + n \ln t, \sigma_L)$			$N(at+b, \sigma)$			$\text{Log}(\ln A + n \ln t, \sigma_L)$			Pop. Size	Sample Size N	Min. Thickness Predicted (mils)
		$\hat{a}$	$\hat{b}$	s	$\hat{A}_L$	$\hat{n}$	s_L	Using parameters	95%	% Diff	Using parameters	95%	% Diff			
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	0	35	1	0.4	0.5	12	77	0%	0	0	0%	17428	200	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	8	1	1	0.2	0	0	0%	1278	1789	3%	17428	200	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	0.5	18	35	1	1	0.5	60	257	1%	2197	3290	6%	17428	200	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	8	2.1	0.4	0.2	329	625	2%	0	0	0%	17428	200	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	0	35	2.1	0.4	0.5	1309	2269	6%	38	165	1%	17428	200	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	8	2.1	1	0.2	1861	2236	2%	11824	12362	3%	17428	200	187
2001	K-1066-B/E/J yards, C-745-B/C/D/K/L/M/N/P yards and all PORTS yards, bottom row	2.5	18	35	2.1	1	0.5	2420	3619	7%	11442	12370	6%	17428	200	187