*Measuring the Fill Height of
Sealed Cans with a Compound Pendulum*

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MEASURING THE FILL HEIGHT OF SEALED CANS WITH A COMPOUND PENDULUM

by

Phillip M. Rinard

ABSTRACT

A compound pendulum has been designed, fabricated, tested, and used to determine the fill height of material in sealed cans. The specific cans that stimulated this work are partially filled with uranium and plutonium oxide. Fill height affects nondestructive assays using fission neutrons, but corrections for various fill heights can be made once the height is known. Heights vary with use as the powder compacts or loosens, so it is necessary to determine the height at the time of the neutron measurement. The pendulum is small and readily portable so it can be taken to the location of the neutron measurement. Tests with open cans filled with sand to various known heights had accuracies generally within 3%. Factors that can affect the accuracy are examined and discussed. Experience in using the pendulum on sealed cans is related.

INTRODUCTION

There are occasions when it is desirable to know the fill height of the material in a sealed can. The technique reported here for doing so was stimulated by a need to improve the nondestructive assays for the mass of plutonium oxide powder in sealed cans. Assays are typically based on counting neutrons emitted by the oxide using special coincidence circuitry and applying a mature data analysis.¹

However, three parameters affect the neutron production rate and only two neutron count rates are measured in coincidence counting. The three parameters are the plutonium mass, the neutron multiplication within the plutonium, and the (α, n) reaction production rate relative to the spontaneous fission production rate. The two measured count rates are of the total and coincident neutrons (or singles and doubles, in the vernacular). If there are no impurities in the oxide, the ratio of neutrons from (α, n) reactions and spontaneous fissions is well known and the assay analysis for this "known-alpha" case is reduced to two equations for the two remaining unknowns, including the desired mass of plutonium.

But if impurities that have poorly known effects on the neutron production rate are mixed with the oxide, it becomes necessary to eliminate the multiplication parameter and find the (α, n) production rate as one of the two remaining unknowns. Such common impurities are water and fluorides, which even in small quantities can greatly affect the neutron production rate by enhancing the number of induced fissions and (α, n) reactions. The multiplication can be calculated with a Monte Carlo code if the geometry of the material is known. The can's shape will set the shape of its contents except for the fill height, but handling the can may change the fill height through compacting or loosening the oxide particles.

So an accurate assay of a can with low-Z impurities, based on neutron coincidence counting, requires knowing the fill height of the material at nearly the same time as the neutron coincidence count and at nearly the same location.

(A relatively new neutron counting technique, multiplicity counting,² measures the triple coincidences in addition to the single neutrons and double coincidences. This raises the number of measured quantities to the number of unknowns, giving relief to this problem just outlined. However, coincidence counting is still widely used and there are limitations to the application of multiplicity counting that will probably keep coincidence counting in wide use along with multiplicity counting.²)

PHYSICAL BASIS

The technique described here to determine the fill height is based on the properties of a compound pendulum³ (also known as a physical pendulum). Other techniques can be imagined, but they do not meet the requirements for the problem discussed in the Introduction. For example, radiography with x-rays or neutrons could be (and probably has been) used. But the free surface of the material will not be flat, complicating interpretations of the radiographs, and the fill height is likely to change as the can is moved between the neutron counter and the radiography facility.

The compound pendulum described later in this report is small, light, and portable, so it can be positioned very close to the neutron counter and determine the fill height immediately before or after a neutron count.

A simple pendulum³ is a point mass swinging from a support on the end of a massless arm. This is clearly an idealization of any real pendulum but it is often a useful approximation of an actual swinging object. A compound pendulum is a fully realistic pendulum in that the mass is distributed well beyond a small volume far from the pivot point. Figure 1 is a sketch showing the key components of a compound pendulum.

The period of oscillation of a pendulum depends on the mass and its distribution throughout the swinging object. The expression for the period P (for small amplitudes where $\sin \phi \approx \phi$) is deceptively simple:

$$P = 2\pi [I/(m \cdot g \cdot s)]^{1/2}, \quad (1)$$

where m is the mass of the can, its contents, and the supporting pendulum arm. I is the moment of inertia of this distributed mass about the pivot point, and s is the distance from the pivot point to the center of mass. The acceleration due to gravity, g , is known accurately at relevant locations

Fig. 1. Some key components of a compound pendulum are indicated in this sketch. A three-dimensional object of arbitrary shape is supported by a pivot axis. Its period of oscillation depends on its mass m , its moment of inertia I about the pivot axis, the distance s from the pivot axis to the center of mass, and the acceleration due to gravity g .

around the world. It is easy to measure P and m . The challenge of using a compound pendulum is to accurately calculate I and s in terms of the fill height of the powder within the can.

Contributions to the moment of inertia come from the arm, the can, and the powder inside the can:

$$I = I_a + I_c + I_p . \quad (2)$$

I_a can be calculated from the mass distribution in the arm. I_c can be calculated from the geometry of the can, so it is necessary that the dimensions of the can be well known. I_p depends on the mass distribution of the material in the can; it is assumed that all the dimensions of the material are known (from the geometry of the can's interior) except the fill height.

The center-of-mass distance s also depends on the mass distribution:

$$s = (m_a s_a + m_c s_c + m_p s_p) / (m_a + m_c + m_p) . \quad (3)$$

The masses and center-of-mass distances of the arm, the can, and the powder are indicated by the subscripts a , c , and p .

The fill height enters into I_p , m_p , and s_p . For example, assume that the powder forms a right cylinder of radius r and height h with a uniform density ρ . The powder's mass is

$$m_p = \rho \pi r^2 h . \quad (4)$$

In practice, this mass will be known and need not be considered a function of fill height. Even if the net weight of a can's contents is not given, it can be determined from the gross weight and the weight of an empty can. (If the mass and dimensions of an empty can are not known, the compound pendulum cannot be applied.)

The moment of inertia and the center of mass are relative to the pivot point, so assume that the bottom of the powder is a distance L from the pivot point. The center of gravity of the powder is then at

$$s_p = h/2 - L , \quad (5)$$

when the positive direction is taken as upwards. The moment of inertia of this cylinder of powder about the same pivot point is

$$I_p = m_p [(r/2)^2 + (h/12)^2 + (L - h/2)^2] . \quad (6)$$

For this common-place geometry of the powder, the fill height is a solution of the following quadratic equation:

$$A h^2 + B h + C = 0 , \quad (7)$$

where

$$A = m_p/3 , \quad (8)$$

$$B = (P/2\pi)^2 g m_p/2 - m_p L , \quad (9)$$

and

$$C = I_a + I_c + m_p(r^2/4 + L^2) - (P/2\pi)^2 g (m_p L + m_a s_a + m_c s_c) . \quad (10)$$

It is possible that both roots are real and positive, but only one can be correct. A decision can be made by using a second value for L and adopting the value that appears in both sets of roots.

The pendulum should be designed so that the period of oscillation is as greatly affected by small changes in the fill height as possible. In other words, $|dP/dh|$ should be as large as practical. Study of this derivative leads to these conclusions. (a) If the arm of the pendulum is much larger than the height of the can, a small change in fill height will have little effect on the period of oscillation because I and s will hardly change. (b) If the arm's length places the powder's surface near the elevation of the pivot point, a change in fill height will again have little effect because mass near the pivot point contributes very little to I and s . (c) The best length of the arm is short enough to place the powder's surface *above* the pivot point as far as possible while still keeping the total center of mass below the pivot point. Much of the powder's mass is near the pivot point and has a small impact on the period, giving a small change in the fill height the largest impact on the period. A short arm also lengthens the period of oscillation, making the period's measurement more precise.

IMPLEMENTATION

A compound pendulum built from these principles is shown in Fig. 2. It was designed for a variety of can sizes and weights, so the platform on which cans rest can be placed at six different distances from the pivot. The values of L range from 2 to 17 cm, with 3-cm steps. The pivot is a pair of knife edges, allowing cans to rest between and above the edges and to restrain the oscillations to a plane. A small pin hangs from the bottom of the arm and passes through an infrared sensor as the pendulum oscillates; this is the signal to the electronic timer.

An electronics box has a microprocessor-based circuit that displays the average time after a number of consecutive oscillations set by thumb switches. A lamp is normally on, but goes off when the pin interrupts the infrared beam; this is useful in preparing the pendulum for a measurement, as described in the next paragraph. A reset switch starts a new set of timing

Fig. 2. The compound pendulum used in this study is shown in this photograph. The platform is shown at the $L = 5$ cm position. The cans shown are typical of the smaller varieties encountered; much taller cans are accommodated by using lower positions of the platform.

intervals for a new average period of oscillation. The precision of the display is 0.1 ms, which is slightly better than the accuracy of an average period.

The lamp displaying the status of the infrared beam has two uses. The original design of the pendulum had the knife edges resting in V-shaped troughs that made positioning the arm easily reproducible. It was found that the knife edges were not moving entirely freely in the troughs, so they now rest on flat metal surfaces. But positioning the arm relative to the infrared beam is not as easily done as with the troughs. It takes a few seconds to shift the knife edges so that the infrared beam is blocked by the pin when the pendulum is not swinging. Satisfactory knife edges and troughs could be designed to simplify the pendulum's setup.

This lamp also helps in flattening the surface of the powder, which cannot be seen in a sealed can. The surface shape can be changed by gently shaking or tapping the tilted can. If the can is as symmetrically placed on the platform as possible but the pendulum hangs at an angle from the vertical, some taps can correct this problem (with the lamp indicating the exact vertical). After success in the first orientation, the process is repeated after rotating the can by 90° . This rotation and tapping regimen can be repeated until the entire pendulum hangs nearly vertically regardless of orientation. This only guarantees that the surface now has a shape symmetric about the vertical center line, but this will probably be flatter than the original surface. Some practice with sand in a topless can will make the process entirely clear.

The highest position of the platform is chosen while still keeping the composite center of gravity below the pivot point. Periods of oscillation are often between 0.6 and 1.5 s. Periods

longer than 2 s may not be reliable because the center of mass of the arm and can is so near the pivot point that air currents could affect the oscillations. If a second value of L is needed, as mentioned just below Eq. (10), the next lowest height should be used.

A circular bubble level was built into the pendulum's base, but leveling has not been critical. As long as the bubble is not pressed against the outside of the level, the measured periods are unaffected.

The amplitudes of oscillation must be kept small for Eq. (1) to apply. Pendulum periods lengthen with amplitude, but if the amplitude stays under 5° of arc the approximations behind Eq. (1) are well met. Amplitudes used in this study were under 2° .

APPLICATION RESULTS

The pendulum was tested using simple, known objects before moving on to experimental cans of powder and then sealed cans with oxides of fissile materials.

Pendulum Arm

The simplest case is the pendulum's arm alone; the mass distribution is complex, but it must be described accurately for all other cases. It was treated as a collection of simple shapes (cylinders, rectangular parallelepipeds, triangular prisms) so that I_a and s_a could be calculated readily. The periods calculated with Eq. (1) and measured for the six platform heights are compared in Table I.

The calculated periods are all slightly larger than the measured periods, so there is a small but persistent error in the understanding of the arm's mass distribution. However, when a can or other massive object is placed on the arm, the effect of this problem on the period is greatly reduced.

TABLE I. Measured and Calculated Periods of the Pendulum Arm Alone			
L (cm)	Period(s)		Percent Difference
	Calculated	Measured	
2	0.822	0.814	1.0
5	0.759	0.749	1.4
8	0.755	0.747	1.0
11	0.781	0.774	0.93
14	0.820	0.816	0.50
17	0.867	0.862	0.58

Add-A-Mass Technique

Instead of using the detailed mass distribution of the pendulum arm to calculate its values of I_a and $m_a s_a$, they can be deduced from periods measured with and without a well-defined additional mass on the platform. Equation (1) is used twice to solve for the two unknowns of the arm.

One and two small lead bricks (the bricklets described in the next section) were placed on the platform. The deduced parameters of the platform are compared in Table II with the values calculated from the mass distribution.

The comparison is generally satisfying, but the values at $L = 11$ and 14 cm are not as uniform as at other levels. But the add-a-mass technique can be used to either check or replace calculated values.

TABLE II. Calculated and Measured Parameters of the Pendulum Platform			
L (cm)	Number of Bricks	I_a (g cm ²)	$m_a s_a$ (g cm)
2	1	30269	1806
	2	30253	1805
	Calc.	30189	1801
5	1	33202	2404
	2	33135	2399
	Calc.	35350	2476
8	1	44573	3153
	2	44557	3152
	Calc.	44566	3152
11	1	51173	3462
	2	52684	3564
	Calc.	57836	3827
14	1	63240	3886
	2	60803	3737
	Calc.	75159	4503
17	1	96883	5198
	2	96720	5189
	Calc.	96536	5179

Lead Bricklet

Another test was to find the height of a small 1.1139-kg rectangular parallelepiped of lead: a "bricklet" with dimensions $2.54 \times 3.81 \times 10.16$ cm. The moment of inertia and center of mass of this object are very simple to calculate, assuming a uniform density. Any one of these dimensions can be the height by placing the bricklet on the platform suitably.

With the bricklet "standing up," its height is 10.16 cm. With the 3.81-cm dimension parallel to the line between the knife edges, the pendulum was unstable at the top platform position and Eq. (7) had no real, positive solution with $L = 11$ and 14 cm. But with $L = 5, 8,$ and 17 cm, the

heights were measured to be 10.23, 10.08, and 10.64 cm, respectively. (A second solution of 18.95 cm at $L = 17$ cm is ignored as extraneous.) It was argued earlier that the most accurate result is obtained with the highest possible platform position, so the best bricklet height in this case is 10.23 cm. This is 0.7% larger than the known value.

Rotating the bricklet 90° so that the 2.54-cm dimension is parallel to the knife edges gave heights of 10.17, 9.98, and 10.66 cm (after ignoring the extraneous 18.87 cm) at the platform positions of the previous paragraph. The best value of 10.17 cm is now less than 0.1% in error.

The bricklet was also laid "flat" on the platform, making its height 2.54 cm. The 3.81-cm edge was parallel to the knife edges. All six platform positions could be used, but at $L = 5$ cm a solution to Eq. (7) did not exist. The other heights, in order of increasing L , were 2.552, 3.937, 2.880, 2.817, and 2.683 cm. The height when L was 2 cm is taken as the most accurate, and indeed it is less than 0.5% too large.

Two identical bricklets were placed "upright" and their height from the $L = 2$ cm platform position was 10.20 cm, which is less than 0.4% in error.

Box of Sand

The complexity was increased by using a rectangular parallelepiped box of sand. The inside horizontal dimensions of the box were 5.08 x 10 cm. The lightweight cardboard box had a negligible effect. Its top was kept open to allow complete control of the sand's surface.

With 373.4 g of fine sand in the box, the fill height was measured with a ruler to be 4.90 cm (to within a fraction of a millimeter). With the 10-cm side parallel to the knife edges, the height was found to be 5.01 cm, which is 2% larger than expected. With the 5.08-cm side parallel to the knife edges, the measured height was 4.92 cm, in excellent agreement with the ruler's value.

Another test with 301.2 g of the sand and a measured fill height of 3.80 cm gave results independent of the box's orientation: 3.93 and 3.91 cm. These are about 3% high.

The surface of the 373.4 g of sand was deliberately slanted to determine the impact on accuracy. The long side of the box was placed parallel to the knife edges and the gradient of the sand's surface was perpendicular to the knife edges. A measure of the slant is the largest deviation from the flat height. Deviations from 0.4 to 1.2 cm were generated; the latter is at the angle of repose of the sand and thus is the largest possible deviation. Under these conditions the various slants had minor effects on the measured fill heights. Even with the 1.2-cm deviation, the measured fill height was only about 3% larger than the value for 4.90 cm with a flat surface. But this is not proof that the surface shape is unimportant in general, so it would be good practice to attempt to flatten the surface as much as possible.

Cylindrical Can with Sand

An open-topped metal can (identical to some used to store oxides of uranium and plutonium) was used with sand as another step toward measurements on sealed cans. The can's I_c and s_c were calculated from simple cylindrical shapes of the appropriate dimensions. The inner radius was 5.24 cm with a wall and bottom thickness of 0.00354 cm.

The calculated periods for the combined can and pendulum arm were within 1% of the measured periods at the four lower positions and were 5% and 3% high at the two highest platform positions. This was taken as sufficiently accurate for the present time but further development of the pendulum should include improving the description of a can.

After adding 373.4 g of sand to a can, the fill height measured with a ruler was 3.0 cm, to within a millimeter. The pendulum with the platform at the highest position gave 3.3 cm, which is 10% high; lower positions were less accurate, as expected.

More sand was added to increase the mass to 674.6 g and the fill height to 5.5 cm. The pendulum's two topmost positions gave fill heights of 5.67 and 5.47 cm, respectively. These are 3% high and 0.5% low.

The surface of the 674.6 g of sand was slanted as before with deviations from 1 to 2.5 cm. (The can's diameter is larger than the box's width, so at the angle of response the height at the edge of the can was farther from the flat surface position than possible with the box.) All measurements were made at the highest platform position where the most accuracy is expected with a flat surface. The slant was oriented both parallel and perpendicular to the knife edges; no difference was seen between the two orientations. Measured fill heights ranged from 5.8 to 6.2 cm, which are 5% to 16% larger than the flat-surface height of 5.5 cm. Attempting to flatten the surface of the powder in such a can seems clearly worthwhile.

Double Cans

The problem that prompted the development of the pendulum was the case of powder inside a can, which in turn was inside a plastic bag, all of which was inside a second can. The plastic bag is thick-walled and typically has a bulky tie-off. Two cans without the intervening bag were studied first. Some double cans are in fact used without a bag and the position of the inner can is fixed by plastic spacers.

Each of the two empty cans was first studied individually. The calculated periods were from 1% to 5% of the measured periods at the different platform levels. With the two empty cans nested together the calculated periods ranged from 12% low to 17% high, compared to the measured periods. Errors are obviously being compounded as the cans are combined, so descriptions of individual cans should be refined before working with their combination. The add-a-mass technique could be of value here, but it has not yet been applied.

A bag was used around the inner can with a lump of gathered plastic at the top; it defied easy description and predictions of periods were uselessly poor. The inner can's orientation was impossible to control. A rigid, well-understood distribution of the mass of the pendulum arm and the whole container is clearly imperative.

Sealed Cans

Some sealed single cans of uranium oxide have been studied with the pendulum. Such a can's actual fill height cannot be measured with a ruler but it can be estimated from the can's radius, the mass of oxide, and an assumed density of the oxide powder.

A can with 1000 g of uranium oxide was studied to determine the powder densities (inversely proportional to the fill heights) after attempts to compact and loosen the powder. Density affects the transport of neutrons and gamma rays through the powder, so it is an important parameter in the nondestructive assays of such materials. This was a fairly squat can with a radius of 6.375 cm and a height of 9.90 cm. An empty can without a lid had a period of 0.9164 ± 0.0003 s when the platform was at its highest position; the period calculated from the can's geometry was an identical 0.9168 s. After tamping the can on a table top several times to maximize the density, we found the average period to be 0.6758 ± 0.0002 s. After shaking the can vigorously to reduce the density, we found the average period rose to 0.6865 ± 0.0002 s (the uncertainties are standard deviations of

repeated measurements). There is clearly a difference in the two periods far beyond the uncertainties, so a density change has unquestionably occurred. Corresponding fill heights were 3.90 cm and 4.42 cm, respectively; the 0.5-cm difference is probably close to the limit of the pendulum's accuracy under these conditions because the surface shape could not be observed. (Flattening of the surface was attempted and checked with the pendulum.) The densities of the oxide with these two fill heights are 2.02 g/cm³ after tamping and 1.78 g/cm³ after shaking, both of which are quite plausible. Tamping and shaking have long been qualitatively known to change densities from the effects on gamma-ray and neutron measurements, but deducing quantitative density information from such measurements has not been done. The densities from the pendulum application seem worthy of trust.

Another can had a gross mass of 430.2 g and an oxide mass of 196.0 g. Its inner radius was 11.1 cm. The bottom and top bowed inward slightly, but this was ignored and they were taken to be flat. If the density is 2 g/cm³, the fill height would be very close to 1 cm. The pendulum with different platform levels gave heights from 1.7 to 5 cm, corresponding to oxide densities of 1.2 and 0.4 g/cm³. The larger of these two densities is not impossible, but densities below 1 g/cm³ are unlikely. It appears that the pendulum gave fill heights that are too large, probably because the powder's surface was well below or too near the pivot point.

Cans of "ash" were measured and fill heights near 10 cm were obtained. These are plausible values, but actual fill heights were too poorly known for a useful comparison.

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