JLAB-PHY-00-03

Comment on the Generalized Gerasimov-Drell-Hearn Sum Rule in Chiral Perturbation Theory

Volker D. Burkert[a]
Thomas Jefferson National Accelerator Facility
Newport News, Virginia, 23606

Abstract

I comment on the application of the chiral expansion to generalize the Gerasimov-Drell-Hearn sum rule for finite Q^2. The observation, made by several authors, that the corrections to the leading order contributions are large and limit the applicability to a very small range in Q^2 are valid when considering the generalization of the sum rule for protons and neutrons, separately. However, when using the proton-neutron difference, the application of chiral perturbation theory may be expanded to substantially higher Q^2 .

The spin structure of the nucleon has been of central interest for nearly 15 years. Most studies have focused on the deep inelastic regime to measure the spin structure functions $g_1(x)$ and $g_2(x)$, and their respective first moments. In recent years the interest has shifted towards the lower Q^2 regime and the resonance region [1,2], and measurements have been untertaken to study the transition from the scaling regime to the regime of strong QCD [3,4,5]. These advances in experiments made it urgent to study theoretically the connection between these different regimes. While perturbative techniques and higher twist expansion approaches seem appropriate at $Q^2 > 0.5\mathrm{GeV}^2$ and invariant masses above the resonance region (W > 2.5GeV/c^2), new approaches are needed to study the low Q^2 and low W regimes. Numerous phenomenological models have been constructed to describe the resonance regime and the transition to the deep-inelastic regime[1,2,6,7,8,9,10]. All models show that the resonance region, especially the $\Delta(1232)$, plays an important role in the helicity dependence of the inclusive cross section at small Q^2. At $Q^2 = 0$ the sum rule by Gerasimov, Drell, and Hearn [11] (GDH SR) relates the energy-weighted integral of the helicity-dependent cross section to the anomalous magnetic moment of the target nucleon:

$$I_{GDH} = \int \frac{\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)}{\nu} d\nu = -\frac{1}{4}\kappa^2 \tag{1}$$

[a]e-mail: burkert@jlab.org

Recently, attempts have been undertaken to evolve this sum rule into the regime of finite Q^2 using chiral perturbation theory (χPT) [12,13]. Ji and Osborne[13] constructed a generalized sum rule using a dispersion relation for the invariant photon-nucleon Compton amplitude $S_1(\nu, Q^2 = 0)$ at non-zero Q^2,

$$\int G_1(\nu, Q^2)\frac{d\nu}{\nu} = \frac{1}{4}\bar{S}_1(0, Q^2) \qquad (2)$$

where $G_1(Q^2, \nu)$ is the spin-dependent structure function, and $\bar{S}_1(0, Q^2)$ the Compton amplitude, where the overline means that the elastic contribution has been subtracted. The authors calculate the right-hand side in chiral perturbation theory. In leading order $\bar{S}_1(0, Q^2)$ is independent of Q^2 [13]. The authors expected from power counting that the GDH SR could be evolved to a $Q^2 = 0.2\text{GeV}^2$. However, the next-to-leading order resulted in a strong Q^2 dependence[14] with a slope at $Q^2 = 0$

$$\frac{d\bar{S}_1(Q^2)}{dQ^2} = \frac{g_A^2 \pi}{12(4\pi f_\pi)^2 M m_\pi}[(1 + 3\kappa_V + 2(1 + 3\kappa_S)\tau^3] \qquad (3)$$

where $\kappa_V = 3.706$ and $\kappa_S = -0.120$ are the experimental values of the isovector and isoscalar anomalous magnetic moments of the nucleon, and τ^3 is +1 for the proton and -1 for the neutron, respectively. When converted ~~to the often used dimensionless quantity~~

$$\bar{I}_{GHD}(Q^2) = M^2 \int G_1(\nu, Q^2)\frac{d\nu}{\nu} \qquad (4)$$

the low Q^2 evolutions for the proton and neutron are given by

$$\bar{I}^p_{GHD}(Q^2) = -\frac{\kappa_p^2}{4} + 6.85Q^2(GeV^2) + .. \qquad (5)$$

$$\bar{I}^n_{GHD}(Q^2) = -\frac{\kappa_n^2}{4} + 5.54Q^2(GeV^2) + .. \qquad (6)$$

The authors of[14] point out the very large Q^2 variation in (3), (5) and (6), much larger than expected from simple power counting. This fact will limit the usefulness of the chiral expansion to very small Q^2 values. However, it has gone unnoticed that the situation is considerably more favorable for the proton-neutron difference rather than for proton and neutron separately. This is obvious when taking the proton-neutron difference in (3). While the values in the bracket are 13.4 and 10.84 for proton and neutron, repectively, the p-n difference is 2.56, yielding a much smaller slope at $Q^2 = 0$. For the proton-neutron difference of the generalized GDH SR one obtains

$$\bar{I}^{p-n}_{GDH} = \frac{\kappa_n^2 - \kappa_p^2}{4} + 1.31Q^2 + ... \qquad (7)\ .$$

In comparison with (5) and (6), a much reduced Q^2 dependence is predicted for this quantity compared to the proton and neutron, respectively. A possible explanation for this is the absence of the $\Delta(1232)$ contribution in the proton-neutron difference, while this is an important contribution to the proton and neutron sum [1] at small Q^2. Taking the proton-neutron difference is quite natural in analogy with the deep inelastic regime. While the Bjorken sum sule [16] for the proton-neutron difference has been tested experimentally [18], the corresponding Ellis-Jaffe sum rule for proton and neutron separately [17] failed the experimental tests [18]. In order to compare with existing data, (7) is converted to the usual first moment

$$\Gamma_1^{p-n}(Q^2) = \frac{Q^2}{2M^2}\bar{I}_{GDH}^{p-n} \tag{8}$$

In figure (1), $\Gamma_1^{p-n}(Q^2)$ is shown with the data from SLAC and the pQCD evolution of the Bjorken sum rule to order α_s^3. [15]. The next-to-leading order term in the χPT expansion for the proton - neutron difference has the correct sign and reproduces better the trend of the data compared to the leading term, which just represents the GDH sum rule.

A similar conclusion can be drawn for the next-to-leading-order expansion by Ji et al. [14] applied to the generalized GDH sum rule proposed by Bernard et al. [12]. A much reduced Q^2 dependence is obtained for the proton-neutron difference in this case as well.

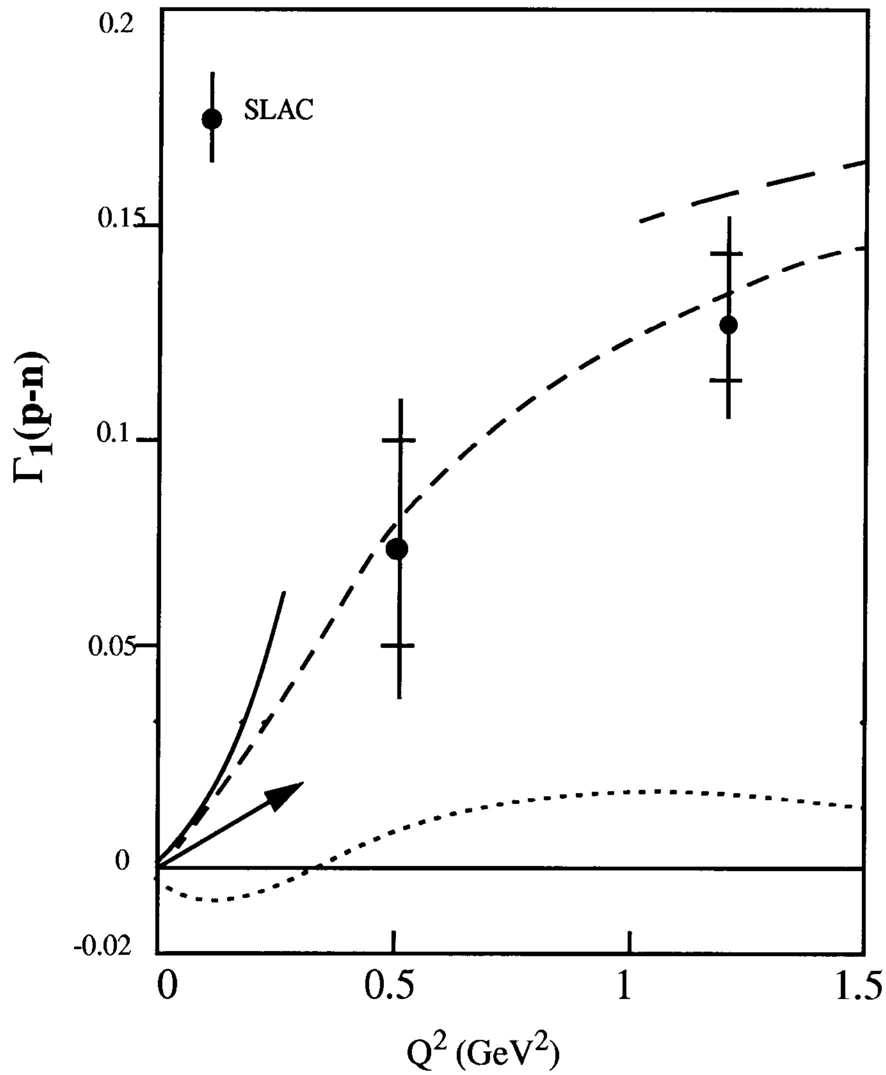
To understand the convergence of the chiral expansion at finite Q^2, it is essential to evaluate the next-to-next-to-leading order corrections for the proton-neutron difference. With the accurate data expected at small and medium Q^2 from experiments at Jefferson Lab within this year, stringent tests of these predictions will be possible. From the higher Q^2 end, QCD sum rule expansions may be used to extend the range down to $Q^2 = 0.5\text{GeV}^2$ [14]. If the remaining gap can be bridged using QCD lattice calculations it would mark the first time that nucleon structure is described within fundamental theory from small to large distances, a worthwhile goal.

1. V.D. Burkert and Zhujun Li ; Phys.Rev.D47,46(1993)
2. V.D. Burkert and B.L. Ioffe, Phys.Lett.B296,223(1992)
3. V. Burkert, D. Crabb, R. Minehart, et al., JLAB experiment E92-023
4. S. Kuhn, M. Taiuti, et al., JLAB experiment E93-009
5. Z.E. Mezziani, J.P. Chen, et al., JLAB experiment E94-010
6. V. D. Burkert and B.L. Ioffe, J. Exp.Theo.Phys.78,619(1994)
7. J. Soffer and O. Teryaev, Phys. Rev. Lett.70, 3373(1993)
8. W.X. Ma, D.H. Lu, A.W. Thomas, Z.P. Li, Nucl.Phys.A635, 497(1998)

9. O. Scholten and A. Yu Korchin, Eur.Phys.J.A6,211 (1999)
10. D. Drechsel, S.S. Kamalov, G. Krein, L. Tiator, Phys. Rev.D59:094021 (1999)
11. S.B. Gerasimov; Sov. J. Nucl. Phys. 2, 430 (1966); S.D. Drell and A.C. Hearn, Phys. Rev. Lett.16 (1966) 908
12. V. Bernard, N. Kaiser, U. Meissner, Phys.Rev.D48,3062(1993)
13. X. Ji and J. Osborne, hep-ph/9905410
14. X. Ji, C.W. Kao, and J. Osborne, Phys.Lett.B472,1(2000)
15. K. Abe et al., Phys.Rev.D58:112003 (1998)
16. J.D. Bjorken, Phys. Rev. 179, 1547 (1969)
17. J. Ellis and R.L. Jaffe, Phys.Rev.D9(1974)1444
18. S.A. Larin and J.A.M. Vermaseren, Phys.Lett.B172, 109(1986); Particle Data Group, R.M. Barnett et al., Phys.Rev.D53,6100(1996)

Figure caption.

First moment difference $\Gamma_1^p - \Gamma_1^n$. Data are from SLAC [15]. Long-dashed line - Bjorken sum rule, corrected to $O(\alpha_s^3)$ [18], solid - equ.(8) for the proton-neutron difference of the χPT prediction [14], short dashed - model by Burkert and Ioffe [6], dotted - resonance contributions [1], solid arrow - slope defined by the GDH SR



0.2
0.15
0.1
0.05
0
-0.02
SLAC
Γ_1(p-n)
0
0.5
1
1.5
Q^2 (GeV^2)