EXPERIMENTAL-BASED MODELING OF A SUPPORT STRUCTURE AS PART OF A FULL SYSTEM MODEL

Thomas G. Carne and Clark R. Dohrmann

Sandia National Laboratories
P.O. Box 5800, Mail Stop 0557
Albuquerque, NM, 87185-0557

Abstract
Structural dynamic systems are often attached to a support structure to simulate proper boundary conditions during testing. In some cases the support structure is fairly simple and can be modeled by discrete springs and dampers. In other cases the desired test conditions necessitate the use of a support structure that introduces dynamics of its own. For such cases a more complex structural dynamic model is required to simulate the response of the full combined system. In this paper experimental frequency response functions, admittance function modeling concepts, and least squares reductions are used to develop a support structure model including both translational and rotational degrees of freedom at an attachment location. Subsequently the modes of the support structure are estimated, and a NASTRAN model is created for attachment to the tested system.

Nomenclature
- $a$: vector of platform accelerations at point $O$ (see Eq. 2)
- $\alpha_i$: angular acceleration of platform about axis $i$
- $d_{ij}$: direction cosine $i$ for input force $j$
- $e_{ik}$: direction cosine $i$ for measurement $k$
- $frf$: frequency response function (acceleration/force)
- $f$: vector of platform forces and moments (see Eq. 3)
- $G$: matrix of frfs for platform alone
- $H_m$: matrix of frfs for all measurements on platform
- $H_p$: matrix of frfs for six platform degrees of freedom
- $H_5$: matrix of frfs for support structure alone
- $H_6$: matrix of frfs for support structure and platform
- $I$: identity matrix
- $N$: number of accelerometers
- $M$: number of forces
- $O$: attachment point of platform to support structure
- $P_{ij}$: input force $j$ location relative to $O$ in dir. $i$
- $r_{ik}$: measurement $k$ location relative to $O$ in dir. $i$
- $\ddot{u}_i$: acceleration of $O$ in direction $i$

1. Introduction
Testing a structural dynamic system may require attachment to a support structure in order to simulate boundary conditions representative of its in-use environment. Ideally these boundary conditions can be modeled easily and included in an analytical model of the full system. For example, simulating free boundary conditions only requires inclusion of a model of the "soft" system which approximates the free conditions. In a more complex situation, one may need to preload the system in order to simulate the operating conditions, such as a rocket during launch. In such situations the support or preloading system may complicate the test and modeling considerably. However, the preload must be included in the test if it is important to the system dynamics. The support system needed for the preload could change the dynamics of the combined system significantly. Thus, it must be accounted for in the model of the combined system.

The issue of accounting for the effects of a support system on the dynamics of a primary system is addressed in this paper. As motivation of this approach, a dynamic system was required to be flexibly supported and preloaded in order to simulate its operating conditions. We wanted to model this primary system; and then test the system in order to validate the adequacy of the model. The flexible support system was fairly complex and included some nonlinearities that made finite element analysis of the support system very difficult. Consequently, we decided to model the support system using a best-fit linear model developed from measured frequency response function (frf) data. This model was then included with the model of the primary system.

Using frf data or admittance modeling to describe a dynamic system, particularly a substructure of a combined system, is not a new idea. It was perhaps electrical engineers who first used admittance modeling to describe how a component would be added to an electrical circuit using scalar admittance models. O'Hara [1] and Sykes [2] explored vector ad-
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
mittance modeling concepts for mechanical dynamic systems as early as the 1960's. Admittance modeling became more practical with the development of digital data acquisition systems, and activity in this area grew. In 1984 Crowly et al. [3] discussed the use of frfs in a structural modification procedure for adding constraints and assembling elements. In 1988, Kienholz and Smith [4] developed a capability for predicting the response of a combined model where part is modeled with finite elements and part with a test-based admittance model. They also developed a software product called GAMMA. Brassard and Massoud [5] used admittance modeling for system synthesis and examined responses at both junction and points within the substructures. Recently Tsuei et al. [6] developed the Modal Force Method using admittance modeling concepts.

The support system was used to preload our primary dynamic system. Because the support system was somewhat nonlinear, it was decided that the admittance testing should both preload and mass load the support system so that it simulated as close as possible the operating conditions. With the mass loading of the support system, its dynamic response should be close to that observed in actual operations. The effect of the added mass is accounted for using admittance modeling.

A platform was added to the support system that created a mass loading and allowed for a preload to be applied to the support system. The added mass platform was designed so that it would be rigid for the frequency band of interest and included a bungee cord attachment to ground through which the preload was applied. With a platform elastically attached to ground, its admittance model could be derived analytically without the requirement of further testing. We created two platforms; one of aluminum and the other steel, that were identical in shape. This was done so that our procedures could be evaluated and compared for platforms with two different masses. The platforms were modeled analytically based on their inertial properties and elastic connections to ground.

The support system, of which an admittance model is desired, is preloaded and mass loaded by a platform connected to it at a single point. It is possible to transfer both forces and moments through the connection point. Thus, measurements of rotational, as well as the translational degrees of freedom, are required for an adequate admittance model if moments are important. A number of investigators have examined the importance of including moments and rotations in the synthesis of system models. References [7] and [8] show their importance, along with the residual compliances, when synthesizing analytical models; in [9] the importance of rotations is emphasized for nonredundant interfaces. Including rotational degrees of freedom from a finite element model is simple, but measuring rotations during a test is not common practice without the use of rotational transducers. Many investigators over a number of years have devised various techniques to estimate the rotational degrees of freedom. These include: attached rigid masses [10], finite element expansion techniques [11], curve-fitting techniques [12-13], and finite difference approaches [14]. In this work a variation of the ideas presented in [10] were used, since our platform is rigid over the desired frequency range. An overdetermined measurement and force input system is created; then a least squares analysis is performed to derive the driving point frf of the connection point for all six degrees of freedom.

The next section of this paper develops the theory used in the least squares reduction process to obtain the driving point frf admittance model of the combined support structure/platform system. The equations required for removing the effects of the platform are then presented. As part of the development, equations are presented for the comparison of reconstructed frfs based on the least squares reduction and original frf measurements. Such comparisons help to reveal the accuracy of the reduction process and of the original measurements. In the third section a number of results are shown and discussed for the application that motivated this work. These include least squares reductions and a comparison of the admittance models derived using the steel and aluminum platforms.

2. Theory

Consider a platform that is attached at a single point to the support structure at point $O$ of the platform. Accelerations of various points on the platform are measured in specified directions. Known forces are also applied to various points on the platform in specified directions. These measurements are processed to obtain frfs for all measurement location and force input pairs. It is assumed that the platform is rigid for the frequency range of interest. Consequently, its motion can be described by three rigid-body translations and three rigid-body rotations. The goal is to determine the driving point frf matrix $H_6$ for the combined platform/support structure system at point $O$ using the frfs described above. That is,

$$a = H_6 f,$$  \hspace{1cm} (1)

where

$$a = \begin{bmatrix} \ddot{u}_1 & \ddot{u}_2 & \ddot{u}_3 & \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix}^T ;$$ \hspace{1cm} (2)

and

$$f = \begin{bmatrix} f_1 & f_2 & f_3 & m_1 & m_2 & m_3 \end{bmatrix}^T .$$ \hspace{1cm} (3)

The terms $\ddot{u}_i$ are components of the acceleration of $O$, and $\alpha_i$ are components of the platform angular acceleration. The terms $f_i$ and $m_i$ are components of the resultant force and moment about $O$.

The position vector from $O$ to measurement point $k$ is given by
where $\hat{b}_i$ are unit vectors fixed in the platform. Assuming that the squared magnitude of the platform angular velocity is much smaller than its angular acceleration, the acceleration of the measurement point is given by

$$\ddot{\mathbf{k}} = (\ddot{u}_1 + \alpha_2 r_{3k} - \alpha_3 r_{2k}) \hat{b}_1 +$$

$$+ (\ddot{u}_2 + \alpha_3 r_{1k} - \alpha_1 r_{3k}) \hat{b}_2 +$$

$$+ (\ddot{u}_3 + \alpha_1 r_{2k} - \alpha_2 r_{1k}) \hat{b}_3 ,$$

(5)

$\alpha$, are direction cosines for the accelerometer axis. Consideration of all $N$ accelerometers leads to the system of equations

$$A\mathbf{a} = \mathbf{s} ,$$

(7)

where

$$A = \begin{bmatrix} e_{11} & e_{21} & e_{31} & c_{11} & c_{21} & c_{31} \\ e_{12} & e_{22} & e_{32} & c_{12} & c_{22} & c_{32} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{1N} & e_{2N} & e_{3N} & c_{1N} & c_{2N} & c_{3N} \end{bmatrix} ,$$

and

$$\mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{bmatrix} .$$

(8)

The acceleration terms in the vectors $\mathbf{a}$ and $\mathbf{s}$ in Eq. (7) can be interchanged with frfs. Thus,

$$A\mathbf{a} = \mathbf{s} .$$

(10)

where the six rows of $\mathbf{H}_p$ correspond to the six platform degrees of freedom; and its columns correspond to the $M$ inputs. Likewise, each row of $\mathbf{H}_m$ corresponds to a specific measurement and each column to a specific input. The least squares solution to Eq. (10) is given by

$$\mathbf{H}_p = \hat{A} \mathbf{H}_m ,$$

(11)

where $\hat{A}$ denotes the pseudo-inverse of $A$. The least squares solution is calculated for each frequency line.

We now have $\mathbf{H}_p$, which is the frf of the platform degrees of freedom owing to each input force. To calculate $\mathbf{H}_6$ (see Eq. 1) which relates to platform forces and moments at $O$, consider the following. The position vector from $O$ to the location of input force $j$ is given by

$$\mathbf{P}_j = p_{1j} \hat{b}_1 + p_{2j} \hat{b}_2 + p_{3j} \hat{b}_3 .$$

(12)

The direction cosines for the direction of force $j$ are denoted by $d_{ij}$. Based on the net force and moment about $O$ of each input force, one obtains

$$\mathbf{H}_p^T = B \mathbf{H}_6^T ,$$

(13)

where

$$B = \begin{bmatrix} d_{11} & d_{21} & d_{31} & g_{11} & g_{21} & g_{31} \\ d_{12} & d_{22} & d_{32} & g_{12} & g_{22} & g_{32} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{1M} & d_{2M} & d_{3M} & g_{1M} & g_{2M} & g_{3M} \end{bmatrix} .$$

(14)

(15)

The least squares solution to Eq. (13) is given by

$$\mathbf{H}_6^T = \hat{B} \mathbf{H}_p^T ,$$

(16)

where $\hat{B}$ denotes the pseudo-inverse of $B$. Thus, with Eqs. (16) and (11), we have the desired result $\mathbf{H}_6$. The frf matrix, $\mathbf{H}_m$, can be reconstructed from the least squares solutions in Eqs. (11) and (16) and is given by

$$\mathbf{H}_m = \hat{A} \mathbf{H}_6 \hat{B}^T .$$

(17)

The reconstructed $\mathbf{H}_m$ can be compared with the original $\mathbf{H}_m$ to obtain a measure of the accuracy of the least squares reduction.

The present goal is to use the calculated frf matrix, $\mathbf{H}_6$, for the combined platform/support structure system to determine the frf matrix, $\mathbf{H}_5$, for the support structure alone. It is assumed that the connection of the platform to ground can be modeled analytically. Thus, for the platform alone one has

$$\mathbf{a} = G \mathbf{f}_p ,$$

(18)

where $\mathbf{f}_p$ is the force applied to the platform at point $O$; and $G$ can be determined analytically from the mass properties and connection of the platform to ground. For the support structure alone, one has
\[ a = H_5 f_5 , \]  
where \( f_5 \) is the force applied to the support structure at point \( O \). Equilibrium of point \( O \) implies

\[ f = f_p + f_5 . \]  

It follows from Eqs. (1) and (18-20) that

\[ \begin{align*}
H_5 &= (I - H_6 G^{-1})^{-1} H_6 ,
\end{align*} \]

where \( I \) denotes the identity matrix. In addition to the driving point frf matrix \( H_6 \) for point \( O \), we are interested in the frf \( H_{6P} \) of another point \( P \) on the support structure owing to input at \( O \). For the combined platform/support structure system, one has

\[ a_P = H_{6P} f , \]

and for the support structure alone,

\[ a_P = H_{5P} f_5 . \]

It follows from Eqs. (1), (19), and (22-23) that

\[ H_{5P} = H_{6P} H_6^{-1} H_5 . \]

3. Application

Let us now illustrate the developed procedure with some results from a specific application. Figure 1 shows the steps which were taken to create the admittance model of the support system. Starting at the upper-left corner, the diagram shows the starting point with the measured frfs, \( H_m \). These are the original experimental data obtained from the responses and inputs on the rigid platform. For our application we measured four triaxial and four uniaxial accelerometers for a total of 16 responses. The 16 responses overdefine the six degrees of freedom of the rigid platform, allowing the least squares reduction. We used seven distinct inputs to the platform which were only slightly redundant. So the first step, as shown in Figure 1, is to transform the \( (16 \times 7) \) frfs to the \( (6 \times 7) \) platform frfs, \( H_p \), using the \( \hat{A} \) matrix. The \( H_p \) frfs are for the six degrees of freedom at point \( O \) owing to the seven distinct input forces.

A reconstructed frf obtained from the least squares solution in Eq. (11) was compared with a measured frf in Figure 2. Note that the comparison is between a specific row and column in \( AH_p \) and \( H_m \). The frfs are displayed to 250 Hz (their Nyquist frequency), but we are actually only interested in the data to 200 Hz, beyond which the frfs are very noisy owing to the antialiasing filters. The favorable comparison shown in the figure was typical of other input/output pairs. This comparison is an important check of the validity of the least squares procedure. If there were any errors in signs, locations, sensitivities, or formulation, these comparisons would show poor results for some of the frfs.

The next step is to use the second least squares reduction to reduce the \( (6 \times 7) H_p \) frf matrix to the \( (6 \times 6) H_6 \) frfs. The seven inputs for \( H_p \) are equivalent to combinations of inputs at point \( O \) on the platform. The \( H_6 \) are the frfs for the six responses at point \( O \) owing to the six inputs at point \( O \), both forces and moments. Note that \( H_6, H_p, \) and \( H_m \) are for the combined system, both the platform and the support system.

We can again evaluate the accuracy of this least squares transformation with a comparison of the reconstructed frfs to the original measured frfs. The reconstructed frfs are obtained from Eq. (17) and include both of the least squares reductions. Figure 3 shows one such comparison for a specific row and column of \( H_m \). The favorable comparison shown in the figure is typical of other input/output pairs and validates the accuracy of the measurement system and the assumption of a rigid platform for the frequency band of interest.

The next step indicated by \( P1 \) in Figure 1 used the admittance modeling technique shown in Eq. (21) to remove the effect of the platform, both its inertia and stiffness to ground. At this
point one also requires the (6x6) frf matrix, G, for the platform. As discussed earlier, G was developed analytically from the known inertia, geometry, and stiffness.

The step indicated by P2 in Figure 1 is where the additional point P is included in the model using Eq. (24). P is on the support structure but not on the interface between the platform and the support structure.

In our application we actually created two platforms, one from steel and the other from aluminum, so that the platforms were identical in shape but differed significantly in mass. The two separate platforms were built so that the accuracy of the admittance modeling could be evaluated using platforms that would have significantly different frf matrices (G). At this next step we compared the $H_5$ created using the steel platform with $H_5$ created using the aluminum platform. One of these comparisons is shown in Figure 4. Examine first the bottom of the figure. Here we see a comparison of an $H_5$ frf for the steel and aluminum platform. This particular frf is for the vertical response owing to a moment. The bottom figure shows that the frfs are quite different for the aluminum and steel platforms, as one would expect, since there is vastly different mass in the platforms. However, after applying Eq. (21) with the appropriate G matrix, the top figure shows that both the aluminum and steel data are converted to nearly identical $H_5$ frfs for just the support structure. This comparison of frfs is again only for one specific row and column, but similar comparisons were shared by other input/output pairs.

At this point we now have the validated admittance model of the support system, which was our original goal. The support system had been preloaded and included mass loading as well. The mass of the platform has been removed. We now want to develop an analytical model of the support system for connection to our primary structure. Our approach here was to extract the modal parameters from the $H_5$ frf. We used a modal analysis procedure called SMAC [15] to estimate these modal parameters and create a modal model of the support system. The last step was to create a NASTRAN modal model of the support structure that could then be attached to a model of our primary structure.

Figure 5 shows a comparison of frfs. The dashed line is the NASTRAN modal model analysis of the support system with a platform attached to it, whereas the solid line is the original $H_6$ frf. This figure shows just one component of the frf matrix, and other components compared similarly. The comparison in Figure 5 is not nearly as good as obtained in previous figures. The nonlinearity of the support system is revealed in the measured data, which violates an assumption of the admittance modeling. However, the modal model does capture most of the features of the experimental data with a linear model. Also, it appears that the estimated damping in the modal model is too low as compared to the test data. The damping could have been manually adjusted to improve the comparison but was not. With the application of Eq. (21), the damping in the system did not transform accurately, again probably owing to the nonlinearity of the support structure because the damping is dependent on the amplitude of the response.

4. Uncertainty and Accuracy

In modeling the support structure using admittance functions, one is concerned about uncertainty or approximations that are part of the modeling process. In this development we have built in systematic checks to validate the modeling process as it proceeds. Figures 2 and 3 are representative comparisons that validate the least squares transformation and all the assumptions included in that process. Of course, Figure 3 shows just one of the comparisons; there are actually 112 (16 outputs by 7 inputs) different rows and columns that were systematically compared for both the aluminum and steel platforms.
To transform the $H_6$ frfs to the $H_5$ frfs (removal of the platform effects), we have used an admittance modeling procedure in which there are inherent assumptions including linearity. To validate or measure the uncertainty of this process was the entire motivation for including two different platforms with significantly different masses. The bottom part of Figure 4 shows how different the frf data were for the different platforms; and the top part of Figure 4 reveals the accuracy and uncertainty of the modeling process. It clearly is not perfect, but comparisons like that shown in Figure 4 reveal the level of uncertainty in this admittance modeling.

5. Conclusions

In this paper we have shown a procedure for using measured frfs to develop an admittance model of a support structure that is attached to the primary structure of interest. A model was required of the support structure because it significantly affected the response of the total system. Admittance modeling is an alternative to developing a finite element model of the support structure. Both rotational and translational degrees of freedom were required at the attachment location; and a least squares procedure was used to obtain the full 6x6 frf matrix for the attachment point, using only translational acceleration responses and force inputs (no isolated moments).

A platform was attached to the support structure for the testing so that a preload and mass loading could be applied. The effects of the platform were removed using admittance modeling concepts so that the model represented just the support structure with the appropriate preload. Subsequently, a modal analysis was performed on the admittance model, and then a NASTRAN modal model of the support structure was developed for attachment to the primary system.

Acknowledgments

This work was supported by the United States Department of Energy under contract No. DE-AC04-94AL85000. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy.

References


