Quench Propagation Velocity for Highly Stabilized Conductors

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Abstract

Quench propagation velocity in conductors having a large amount of stabilizer outside the multifilamentary area is considered. It is shown that the current redistribution process between the multifilamentary area and the stabilizer can strongly effect the quench propagation. A criterion is derived determining the conditions under which the current redistribution process becomes significant, and a model of effective stabilizer area is suggested to describe its influence on the quench propagation velocity. As an illustration, the model is applied to calculate the adiabatic quench propagation velocity for a conductor geometry with a multifilamentary area embedded inside the stabilizer.

Introduction

The development of conductors with aluminium superstabilizer for applications, such as detector magnets for high energy physics,¹ energy storage devices,²,³ and others, has led to some new problems. One of them is the effect of the current redistribution process between the superconductor and the stabilizer on the quench propagation.⁴ This effect can also be important for the conductors considered for the next generation of high energy particle accelerator magnets, which contain a large amount of stabilizer outside the multifilamentary area.

The quench propagation velocity is determined by the Joule heating in the vicinity of the transition front. During the transition from the superconducting to the resistive state, the current is redistributed from the superconductor to the stabilizer. This redistribution occurs in two phases. First, the current is expelled from the superconducting filaments to the copper in the multifilamentary area. Second, the current diffuses into the stabilizer outside the multifilamentary area. If the interfilament spacing is small, the first phase is very fast. On the other hand, if most of the stabilizer is located outside of the multifilamentary area, the second phase can be
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relatively long. In the vicinity of the transition front, where the quench-driving heat release occurs, the current may thus remain confined in a small fraction of stabilizer around the multifilamentary area. This results in a relatively high local value of Joule heating, leading to high quench propagation velocity.

In this paper, we shall consider the case where the quench propagation is effected by the current redistribution process. We shall introduce the characteristic velocity at which this process becomes significant. We shall introduce a model of effective stabilizer area for fast quench propagation, and we shall find a transcendental equation for the quench propagation velocity in highly stabilized conductors.

Concept of Highly Stabilized Conductor

Most of the papers on quench propagation velocity consider the current redistribution process as instantaneous (see the review in reference 5). To discuss the applicability of this assumption, let us estimate the characteristic times of the phenomena involved. The current redistribution time, $t_d$, may be estimated as

$$t_d = \frac{\mu_0 d^2}{\rho_n}$$  \hspace{1cm} (1)

where

$$d = \frac{A_n}{P_n}$$  \hspace{1cm} (2)

Here $d$, $A_n$, and $P_n$ are the effective thickness, the cross-sectional area, and the contact perimeter of the stabilizer (see figure 1), and $\rho_n$ is the resistivity of the stabilizer.

The characteristic time associated with the quench propagation, $t_p$, is given by

$$t_p = \frac{L}{v}$$  \hspace{1cm} (3)
where \( v \) is the quench propagation velocity, and \( L \) is the thickness of the zone where the quench-driving heat release occurs. In other words, \( L \) is the thickness of the region, in the vicinity of the transition front from the resistive to the superconducting state, where the Joule heating determining the propagation velocity takes place.

In this paper, we shall represent the power of the Joule heating in the superconductor as a step function of temperature. In other words, we shall assume that the Joule heating is equal to zero for temperatures below a certain temperature, \( T_t \), and is non-zero above \( T_t \). In the following, we shall represent \( T_t \) by

\[
T_t = \frac{i}{2} T_0 + \left(1 - \frac{i}{2}\right) T_c
\]  

(4)

Here \( T_0 \) is the coolant temperature, \( T_c \) is the critical temperature at the given field and zero current, and \( i \) is the dimensionless current defined as

\[
i = \frac{I}{I_c}
\]  

(5)

where \( I \) is the transport current, and \( I_c \) is the critical current at the given field and \( T_0 \). It was shown\(^6\) that Eq. (4) leads to satisfactory results while computing the quench propagation velocity.

In most cases of practical interest, the cooling conditions are weak. Then, \( L \) is determined by the thermal diffusion along the conductor, and can be estimated as\(^5\)

\[
L = \frac{k}{C v}
\]  

(6)

where \( k \) and \( C \) are the thermal conductivity and the heat capacity per unit volume averaged over the conductor cross-section, and taken at the given field and \( T_t \).

Thus, the current redistribution process can be considered as instantaneous, only if the dimensionless parameter \( \tau = t_d/t_p \) is less than one. Using Eqs. (1) to (3), and (6), it is convenient to rewrite \( \tau \) as
\[ \tau = \frac{\mu_0 C d^2}{k \rho_n} v^2 = \left( \frac{v}{v_c} \right)^2 , \]  

(7)

where we have introduced the characteristic velocity \( v_c \) defined as

\[ v_c = \frac{1}{d} \sqrt{\frac{k \rho_n}{\mu_0 C}} = \frac{p_n}{A_n} \sqrt{\frac{k \rho_n}{\mu_0 C}} . \]  

(8)

As an illustration, let us estimate \( v_c \) for the case of an aluminium superstabilized conductor.\(^7\) Using the data from Table I and Eq. (8), one gets:

\[ v_c = 0.7 \text{ m/s} . \]  

These values appear to be of the same order magnitude or even less than the quench propagation velocities measured experimentally.

It follows from the above discussion that actual conductors can exhibit quench propagation velocities larger than \( v_c \). In these cases, the dimensionless parameter \( \tau \) is larger than one, and the current redistribution has to be taken into account while calculating \( v \). The conductors where \( v \) occur to be of the order of, or higher than \( v_c \) will be defined as highly stabilized.

**Criterion of Highly Stabilized Conductor**

Let us first consider the case of instantaneous current redistribution. The quench propagation velocity is determined by the Joule heating in the vicinity of the transition front. The thickness of this region is \( L \), and, the power of the quench-driving heat release, \( q \), can be written as

\[ q = \rho \frac{I^2}{A} L . \]  

(9)

where, \( \rho \) is the longitudinal electrical resistivity of the conductor, defined as

\[ \rho = \frac{A \rho_n \rho_s}{A_n \rho_s + A_s \rho_n} . \]  

(10)
In Eq. (10), $A$ is the conductor cross-sectional area, and $A_s$ and $\rho_s$ are the cross-sectional area and the resistivity of the multifilamentary area. While deriving Eq. (9), we considered that the currents $I_n$ and $I_s$ flowing in the stabilizer and the multifilamentary area was uniform and the ratio $r = I_n/I_s$ was equal to

$$r = \frac{A_n \rho_s}{A_s \rho_n} \quad (11)$$

In all cases of practical interest, the resistance per unit length of the stabilizer, $\rho_n/A_n$, is much smaller than that of the multifilamentary area, $\rho_s/A_s$, and thus, $r >> 1$.

On the other hand, $q$ is equal to the heat flux which heats up the superconducting zone to the transition temperature. It follows

$$q = v A \Delta H \quad (12)$$

where

$$\Delta H = \int_{T_0}^{T_t} C \, dT \quad (13)$$

is the difference in enthalpy per unit volume of conductor between $T_0$ and $T_t$. If we assume adiabatic cooling conditions, the value of $L$ is given by Eq. (6). Equating the two expressions of $q$, and substituting the expression of $L$, lead to

$$v = \frac{I}{A} \sqrt{\frac{k \rho}{C \Delta H}} \quad (14)$$

(The same formula can be derived by considering a non-zero heat transfer coefficient to the coolant and letting the Stekly parameter tends towards infinity.)

In this model, the maximum of the quench propagation velocity, $v_m$, is obtained for $I = I_c$, and thus, $T_t = (T_0 + T_c)/2$. Let us estimate $v_m$ for the superstabilized conductor considered above. Using the data from Table I and
Eq. (14), one gets: $v_m = 13 \text{ m/s}$. As we can see, for actual conductors, the value of $v_m$ can be much higher than the value of $v_c$. In these cases, the current redistribution process has to be taken into account while calculating the quench propagation velocity.

For adiabatic cooling conditions, a criterion defining highly stabilized conductors may be derived by comparing $v_m$ and $v_c$. Let us define the dimensionless parameter $\beta$

$$\beta = \left(\frac{v_m}{v_c}\right)^2 .$$

(15)

Then, a highly stabilized conductor is a conductor with $\beta$ larger than one. Combining Eqs. (8) and (14) leads to the following criterion

$$\beta = \frac{\rho}{\rho_n} \left(\frac{A_n}{A}\right)^2 \frac{\mu_0 I_c^2}{\Delta H_c P n^2} \geq 1 ,$$

(16)

where $\Delta H_c$ is calculated by mean of Eq. (13) at $T_t = (T_0 + T_c)/2$, i.e., for $I = I_c$. For the superstabilized conductor considered above: $\beta = 300$, which is much larger than 1.

Model of Effective Stabilizer Area

Let us now consider the case where the current redistribution has to be taken into account while calculating the quench propagation velocity, i.e., $v > v_c$. Then, in the vicinity of the transition front, the current remains confined to a certain fraction of stabilizer around the multifilamentary area, leading to non-uniform quench-driving heat release. The cross-sectional area occupied by the current is determined by the ratio, $\tau$, of the characteristic times associated with the current redistribution and the quench propagation. The larger $\tau$, i.e., the larger the ratio of $v$ to $v_c$, the smaller the fraction of stabilizer where the current has diffused.

On the other hand, as we mentioned before, in most cases of practical interest, the cooling is weak, i.e., the Biot parameter, $Bi$
where \( P \) and is the cooling perimeter, \( k_t \) is the transverse thermal conductivity of the stabilizer, and \( h \) is the heat transfer coefficient to the coolant. In particular, for the superstabilized conductor considered above, and for \( h = 10^3 \ \text{W/m}^3 \), we get: \( Bi = 0.006 \), which appears to be much smaller than 1.

At the same time, the dimensionless ratio, \( \delta \), of the transverse thermal diffusivity, \( D_t = k_t/C \), to the magnetic flux diffusivity, \( D_m = \rho_n/\mu_0 \), is much larger than one, \( i.e., \)

\[
\delta = \frac{D_t}{D_m} = \frac{\mu_0 k_t}{\rho_n C} \gg 1
\]

For example, for the superstabilized conductor considered above, \( \delta \approx 10^5 \), which is much larger than 1. It results from Eqs. (17) and (18) that the temperature is uniform over the conductor cross-sectional area, even if the heat release is non-uniform.

As the temperature is uniform over the conductor cross-sectional area, we shall treat the temperature distribution as one-dimensional, depending only on the coordinate along the conductor. The main difference between highly stabilized and conventional conductors is thus the non-uniformity in the quench-driving heat release. To find the exact expression of the Joule heating, we should solve the system of Maxwell's and heat diffusion equations. For most cases of practical interest, it cannot be done analytically, and is a complicated problem for numerical analysis.

In this paper, we shall calculate the Joule heating considering that the current is uniformly redistributed between the multifilamentary area and a certain area of the stabilizer, which we shall introduce as an effective area, \( A_{\text{eff}} \). As the fraction of the stabilizer where the current has diffused depends on the quench propagation velocity, the effective area of the stabilizer is determined by the ratio \( v/v_c \), \( i.e., \)

\[
A_{\text{eff}} = A_n f \left( \frac{v}{v_c} \right)
\]  

(19)
To find an expression for $f$, let us first discuss its asymptotic behavior for small and large values of $v/v_c$.

When the ratio $v/v_c$ is small, the current redistribution process is almost instantaneous, and the current occupies the whole stabilizer cross-sectional area. It means that $A_{\text{eff}}$ tends towards $A_n$, and

$$ f \left( \frac{v}{v_c} \right) = 1 , \quad \text{for} \quad \frac{v}{v_c} \to 0 . \quad (20) $$

On the other hand, when the ratio $v/v_c$ is large, the current only diffuses into a thin layer of stabilizer, $l$, and the current redistribution process can be treated as in the case of a semi-infinite slab of stabilizer. Then, $l$ is determined by the magnetic flux diffusion length for a characteristic time of the order of $t_p$

$$ l = \sqrt{D_m t_p} = \frac{A_n v_c}{P_n v} . \quad (21) $$

Thus, the effective area, i.e., the cross-sectional area of stabilizer occupied by the current is

$$ A_{\text{eff}} = l P_n = A_n \frac{v_c}{v} , \quad (22) $$

and, it comes

$$ f \left( \frac{v}{v_c} \right) = \frac{v_c}{v} , \quad \text{for} \quad \frac{v}{v_c} \to \infty . \quad (23) $$

Having determined the asymptotic dependencies for small and large values of $v/v_c$, we shall now define $f$ for the full range of velocities. To match smoothly Eqs. (20) and (23), we suggest the following function

$$ f \left( \frac{v}{v_c} \right) = \tanh \left( \frac{v_c}{v} \right) . \quad (24) $$
Thus, we shall calculate the quench-driving heat release considering that the stabilizer cross-sectional area is equal to $A_{\text{eff}}$ as given by Eqs. (19) and (24).

**Adiabatic Quench Propagation Velocity for Highly Stabilized Conductors**

In this section, we shall apply the above model of effective stabilizer area to the computation of the adiabatic quench propagation velocity. To do it, we have to calculate the quench-driving heat release. In the case of adiabatic cooling conditions, it is given by Eq. (9) where the expression of $\rho$ is given by Eq. (10). Then, substituting $A_n$ by $A_{\text{eff}}$ in Eq. (10), and combining Eqs. (9) and (10), it comes

$$q = \frac{\rho_n \rho_s I^2}{A_{\text{eff}} \rho_s + A_s \rho_n} \frac{k}{C v} \quad \text{(25)}$$

where we have replaced $L$ by Eq. (6). An equation determining $v$ can be derived by equating Eqs. (25) and (12), and replacing $A_{\text{eff}}$ by Eq. (19). It comes

$$v = \frac{I}{A} \sqrt{\frac{k \rho}{C \Delta H}} \sqrt{\frac{1 + r}{1 + r f \left(\frac{v}{v_c}\right)}} \quad \text{(26)}$$

Note that Eq. (26) is similar to the equation derived in reference 8.

Let us now qualitatively discuss the dependence of the quench propagation velocity on the transport current.

When the ratio $v/v_c$ is small, the current occupies the whole cross-sectional area of the stabilizer, i.e., $f$ is approximately equal to 1. In this case, and as expected, the dependence of $v$ on $I$ coincides with that given by Eq. (14). Let us note that Eq. (14) gives the lower limit of the quench propagation velocity.

When the ratio $v/v_c$ is large, $f$ is approximately equal to $v_c/v$. It follows from Eq. (26) that the dependence of $v$ on $I$ is given by the solution of the following second order equation

$$v^2 + r v v_c - \left(\frac{I^2}{A^2} \frac{k \rho r}{C \Delta H}\right) = 0 \quad , \quad v_c < v \quad , \quad \text{(27)}$$
where we assumed \( r >> 1 \).

To illustrate these results, figure 2 shows plots of the quench propagation velocity as a function of the dimensionless current \( i \) for the superstabilized conductor considered above. The solid line represents the velocity calculated by the combination of Eqs. (24) and (26), which takes into account the current redistribution process. The dashed line represents the velocity calculated by means of Eq. (14), which assumes an instantaneous current redistribution. As can be seen in figure 2, the difference in the results can be up to 8 times. Note that the solution of the approximate Eq. (27) practically coincides with the solution of the complete Eq. (26) for the whole range of quench propagation velocities larger than \( v_c \). Figure 3 shows a plot of the dimensionless ratios \( A_{\text{eff}}/A_n \) and \( I_n/I \) as a function of the dimensionless current \( i \). It can be seen that, even for relatively low values of effective stabilizer area, \( i.e., A_{\text{eff}}/A_n < 0.01 \), more than 40% of the transport current is still flowing in the stabilizer.

**Conclusion**

For conductors having a large amount of stabilizer outside the multifilamentary area, the current redistribution process has to be taken into account while calculating quench propagation velocity. For doing so, we developed a model of effective stabilizer area. We applied this model to the case of weak cooling conditions, and to a conductor geometry where the multifilamentary area is embedded inside the stabilizer. We derived a transcendental equation, Eq. (26), which determines the quench propagation velocity.

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Table I.

Multifilamentary Area

<table>
<thead>
<tr>
<th>Material</th>
<th>Cu:NbTi</th>
<th>Cu RRR</th>
<th>$A_s$ (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.35:1</td>
<td>200</td>
<td>8</td>
</tr>
</tbody>
</table>

Superstabilizer

<table>
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<tr>
<th>Material</th>
<th>Al RRR</th>
<th>$A_n$ (mm$^2$)</th>
<th>$P_n$ (mm)</th>
<th>$P$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2200</td>
<td>118</td>
<td>12</td>
<td>3.6</td>
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</table>

Critical Current at the Given Field $B$

\[
I_c = \frac{B_0}{B + B_0} I_0
\]

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<tr>
<th>Field ($B_0$)</th>
<th>Current ($I_0$)</th>
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<tbody>
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<td>1.04</td>
<td>25820</td>
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</table>

All estimations are done at

<table>
<thead>
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<th>Temperature ($T_0$)</th>
<th>Current ($I$)</th>
<th>Field ($B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>5000</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Figure Captions

Fig.1 Example of Highly Stabilized Conductors: a) cross-sectional view of the Aluminium stabilized conductor used for the ALEPH solenoid, b) cross-sectional schematic of a highly stabilized conductor.

Fig.2 Quench Propagation Velocity as a Function of Dimensionless Current $\tilde{i}$: a) taking into account the current redistribution, b) assuming instantaneous current redistribution.

Fig.3 Dimensionless Ratios Characterizing the Effectiveness of the Stabilizer as a Function of Dimensionless Current $\tilde{i}$: a) effective stabilizer area normalized to total stabilizer area, $A_{\text{eff}}/A_n$, b) current in stabilizer normalized to total transport current, $I_n/I$. 
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Dimensionless Ratios $A_{\text{eff}}/A_n$, $I_n/I$