Title: Deriving the Structure of Pre-Supernovae and Delta Scuti Star Using Nonradial Oscillations

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Abstract

This is the final report of a three-year Laboratory Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). Our objective is to learn more about the internal structure of two classes of variable stars, the delta Scuti variables and the massive Luminous Blue Variable (LBV) stars, by using the observational data afforded by their pulsation properties. We updated our one-dimensional computer codes to calculate the evolution and pulsation frequencies of representative delta Scuti and LBV models. We compared the observed pulsation properties with model predictions in an iterative process to find a model (or models) with interior structures that matched the observational constraints for several delta Scuti stars. We carried out nonlinear hydrodynamic modeling of LBV envelopes and proposed a mechanism for their periodic “outbursts”. Finally, we began validation of a two-dimensional stellar evolution code that will be used to investigate the effects of rotation and hydrodynamic instabilities on the interior structure of these stars.

Background and Research Objectives

We performed computer modeling of the evolution and pulsation properties of two important classes of variable stars, the Luminous Blue Variables and the delta Scuti stars. Both types of stars pulsate in multiple radial and nonradial modes. These modes have significant amplitudes throughout the stellar interior, and are therefore effective probes of stellar structure. We used the observed properties of these pulsation modes, in particular their frequencies and regions of instability in luminosity versus effective surface temperature (also known as Hertzsprung-Russell) diagrams, as clues to inferring the internal structure of these stars. Our

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objectives are: 1) to compute the evolution and pulsation frequencies for stellar models with masses representative of the delta Scuti stars, and determine the interior conditions necessary to reproduce the dozen or more frequencies reported for several delta Scuti stars; 2) to use evolution and pulsation modeling to explain the small irregular variations in radial velocity and brightness of Luminous Blue Variables (LBVs), determine their evolutionary state, and understand the mechanism for “outbursts”, accompanied by large mass outflows, that are observed every few decades for typical LBVs; 3) to begin validation of a two-dimensional stellar evolution code written by Robert Deupree to investigate the effects of rotation and hydrodynamic instabilities on the interior structure and evolution of delta Scuti and LBV stars.

The Luminous Blue Variables

The Luminous Blue Variables (LBVs) are massive stars with initial masses of 30 to perhaps over 100 times the mass of the Sun, and they are on their way to becoming Type II supernovae. These stars exhibit three distinct types of variability with different magnitudes and timescales. Some LBVs, such as the famous southern hemisphere object Eta Carinae, have giant eruptions, in which their brightness suddenly increases by a factor of 100 or more, then fades over decades. During these eruptions, LBV stars eject shells of material of up to several solar masses. Most LBVs also exhibit moderate variability of 1-2 visual magnitudes on decade-long timescales, during which the mass loss rate increases by several orders of magnitude above those of normal supergiants. LBVs also exhibit low-amplitude variability (5-30%) with periods of weeks to months. One proposed cause of this variability is nonradial gravity-mode (g-mode) pulsations. We focused first on searching for the g-mode pulsations, but we also confirmed the work of Kiriakidis et al. that radial “strange modes” with periods somewhat longer than the g-modes are also predicted to be unstable under certain conditions. These modes grow much more rapidly than the g-modes, and are likely to dominate the pulsations. The conditions needed for pulsation instability provide clues to the star’s structure and evolution history.

Surprisingly, the evolutionary state of LBV stars is uncertain. We do not know if LBV stars are making their first crossing of the Hertzsprung-Russell (HR) diagram between the main sequence and red giant branch, or are undergoing one or more “blue loops” bringing them from the red giant region back to the LBV instability strip. Theoretical models do not yield blue loops consistently; different investigators derive completely different evolutionary tracks. Many physical parameters and processes have been proposed as the key to determining whether or not a star blue loops. We proposed to explore the effects of element abundances, mass loss rates, and new opacities recently available from the OPAL project at Lawrence Livermore National Laboratory, on LBV star evolution.
We are becoming convinced that high-mass LBVs develop rapidly growing pulsational instabilities (the aforementioned "strange modes") and undergo outbursts the first time they evolve to cooler surface temperatures, and that they never become red supergiants or blue loop. This is consistent with the observation that no red supergiants are observed at very high luminosities\(^1\). We are conducting nonlinear hydrodynamics calculations including time-dependent convection\(^5\) to investigate the growth of these strange mode pulsations, and potential for pulsation-driven mass outflows/ejections.

Because LBV stars eventually become supernovae, an improved knowledge of their structure is crucial to understanding the evolution of supernovae progenitors. Until recently, the progenitor stars of Type II supernovae were believed to be massive red supergiants. However, the first two identified supernovae progenitors, SN 1987A\(^6\) in the Large Magellanic Cloud and SN 1993J\(^7\) in the nearby galaxy M81, are blue (LMC) and yellow-orange supergiants (M81). This underlines the need to reassess massive star evolution, in particular the physical mechanisms for a star's expansion into a red giant and possible blue-looping. The Hubble Space Telescope is resolving stars in more distant galaxies, expanding the possibility of matching progenitor stars to supernovae as they occur. Coupled with these observations, we hoped to shed light on the conditions under which massive stars evolve into the LBV instability region, and how much mass a star loses before reaching the LBV phase. We also proposed to understand the details of the pulsation driving, and investigate whether pulsations are responsible for initiating LBV outbursts, causing them to eject shells of material with mass of \(10^{-3}-10^{-4}\) solar masses. A possible, but remote prospect that we hoped to investigate along the way was whether deep-seated nonradial pulsations might trigger the actual supernova explosions.

In summary, our goals are: to understand the evolution of pre-supernovae stars of 30 to over 100 times the mass of the Sun; understand the mechanism for the small amplitude variations in LBV stars, and infer their evolution state, amount of mass lost in stellar winds, envelope composition, and conditions in the pulsation driving region; and develop an explanation for LBV outbursts.

\textit{δ Scuti Variables}

The delta Scuti variables are 1.4-2.5 solar mass stars of spectral types A–F that are burning hydrogen either in their core, or in a shell outside a hydrogen-exhausted core. These stars pulsate in dozens of radial and nonradial modes, with periods of a few hours, and a wide range of amplitude variations from less than 1% to over 50% from their mean luminosity. The
driving mechanism for these modes is cyclic partial ionization of hydrogen and helium. The low spherical harmonic index nonradial modes (degree $l=1-3$) can be observed via photometric variations, and they propagate throughout the entire star. The frequencies of many of these are split by rotation. This gives us the possibility of using them to determine the interior rotation profile as well as the interior structure. Higher-degree modes ($l=8-16$) probe the stellar envelope nearer the surface and are seen as traveling bumps in spectral lines.

The delta Scuti stars appear to be ideal laboratories to study the physics of hot dense plasmas. Their interior conditions are similar to the Sun's, so that the evolutionary modeling and pulsation analysis tools we have developed to derive detailed, and widely accepted, models of solar structure are applicable to delta Scuti stars. However, delta Scuti stars differ in several ways from the Sun. For example, they have convective cores, shallower surface convection zones, and more rapid rotation (up to 200 km/sec at the surface), and they burn hydrogen into helium in their cores via a different nuclear reaction network. We therefore expect to considerably confirm and extend our understanding of stellar structure and evolution. Our seismological analysis of delta Scuti stars will provide important information on the size of their convective cores, the extent of possible convective overshooting, and the effect of relatively rapid rotation on stellar evolution.

One deficiency in the astrophysical literature that we proposed to correct is the lack of an up-to-date grid of one-dimensional evolution models, and complementary pulsation analysis including consistent physics for delta Scuti stars. Most published tables give only radial mode periods, whereas observations show that nonradial modes are ubiquitous. In addition the sensitivity of ground-based observations using a network of sites spanning the globe, as well as proposed space-based observations, will continue to improve our ability to detect much smaller amplitude variations and add to the 2-10 modes seen in a typical delta Scuti star. It is even possible that nonradial g-modes, which have large amplitudes only outside the convective core and smaller amplitudes elsewhere, have observable amplitudes. During the course of this investigation, we participated in multisite campaigns on several delta Scuti stars (FG Vir, CD-24° 7599, and 4 CVn) that have considerably increased the number of detected modes for these stars (up to 13, 32, and over 30 respectively). Comparisons between predicted and observed periods can be used to constrain the entire structure of the star, just as the nonradial p-modes are used to derive the structure and rotation profile of the Sun. The only available nonradial pulsation survey has been that of Fitch (1981), but his periods are not widely accepted due to his choice of boundary conditions. Very few of the models in the literature incorporate the latest opacities, which strongly affect the pulsation driving and periods of delta Scuti stars.
Despite the advent of fast computers, most stellar evolution and pulsation codes to date have been one-dimensional, and therefore only able to address spherical, non-rotating, non-magnetic stellar models. Some progress has been made with one-dimensional evolution codes that include the effects of slow rotation; these studies suggest that rotation-induced flows can play a role in mixing elements up to the surface or down to nuclear burning regions, but these codes contain too many free parameters to be of predictive value. We are using a fully implicit two dimensional stellar evolution program that has been developed and already successfully applied to high-mass stellar models by Robert Deupree at Los Alamos. In addition to including rotation and accompanying angular momentum transfer, this code has the ability to handle the hydrodynamics of rotationally induced instabilities. Applying this code to the variable star modeling will be especially useful because the pulsation data, particularly the rotational frequency splitting discussed above, provides additional constraints on the models. We will also learn how rotation velocities of up to 100 km/sec, differential internal rotation, and possible rotationally-induced mixing affect the structure and pulsation periods of delta Scuti stars.

For the delta Scuti variables, our objectives were to develop theoretical models that match the radial and nonradial mode frequencies for several stars with a relatively large number of detected pulsation frequencies (FG Vir, CD-24° 7599, 4 CVn and delta Scuti). This analysis will improve our knowledge of the internal structure and evolution of stars slightly more massive than the Sun, including the extent of convection, the importance of mass loss and diffusive element settling, and the effects of differential rotation on evolution, structure, and pulsation properties. We also expected to test the validity of the new opacity and equation of state data by comparing pulsation and evolution results to observations.

**Importance to LANL's Science and Technology Base and National R&D Needs**

The study of the stellar interiors and pulsation contributes to the understanding of fundamental physics such as theoretical equations of state (EOS) and opacities for hot dense plasmas, radiative and convective energy transport, turbulent mixing, and shear instabilities. For this project, we implemented the newest Livermore OPAL opacities made available in 1996, as well the new Livermore equation of state that complements these opacities. We and others find that the sensitivity of pulsation driving and periods to the stellar structure requires high accuracy in the input physics. In many cases, comparisons with pulsation observations have helped to guide improvements to opacity and equation of state calculations. The same opacity and equation of state treatments used to produce astrophysical opacities are also applied
to producing EOS and opacity tables for modeling of thermonuclear weapons, which have
temperatures and densities similar to those found in stellar interiors.

Regarding the impact of this project on national astrophysics efforts, the Hubble Space
Telescope is allowing astronomers to resolve stars in distant galaxies, and identify many more
supernova progenitors as supernovae occur. We hope to contribute to the understanding of the
evolution of supernova progenitors. During the past three years, data analysis of ground-based
network observations has progressed to the point where several delta Scuti stars have several
dozen detected pulsation frequencies that require theoretical interpretation. A new consortium
(Stellar Oscillation Network Group) has formed to assess the prospects for using recent
 technological advances to detect oscillations in sun-like stars. Until this project was initiated,
there were no delta Scuti evolution/pulsation models with modern physics available to interpret
the existing data, or predict possible frequencies of any additional modes that might be found.
In this project, we have completed the preliminary work necessary to obtain direct information
on the internal rotation profile, the convective core, and the degree of convective overshooting
in delta Scuti stars. This information can settle decades-long disputes over the importance of
these processes.

The two-dimensional evolution/hydrodynamics code of R. Deupree has now been
validated on a solar model, and represents a major advance in stellar evolution. We are in a
position to take advantage of this unique Los Alamos capability to produce 2-D evolution
models that can be constrained by pulsations. We intend to work toward a complementary 2-D
nonradial stellar pulsation code, which will be a major advance in stellar pulsation modeling.

This research has also proven an effective avenue for recruiting prospective staff
members into the Applied Theoretical and Computational Physics (X) Division at Los Alamos.
Two persons who were postdoctoral research associates and contributed heavily to this LDRD
project recently became X-Division staff members. We have also been able to fund two
summer students who have made excellent contributions to the project, and who will now
contribute to the Laboratory’s programmatic efforts. Finally, due to the international visibility
of this project and reputation of Los Alamos, we have been invited to give review talks at
several conferences, and also hosted at Los Alamos an international stellar pulsation meeting in
honor of retired Laboratory Fellow Arthur Cox in June 1997. Contacts made at this conference
resulted in the recruiting of two postdoctoral candidates for X-Division, who will begin work
in the summer and fall of 1998.

**Scientific Approach and Accomplishments**

We divide our approach and accomplishments into four subcategories: 1) validating
new opacities and equation of state input using solar models; 2) asteroseismology of delta Scuti
stars; 3) two-dimensional evolution models with rotation and hydrodynamics; and 4) microvariations, outbursts, and evolution scenario for Luminous Blue Variables.

A. Validating New Opacities and Equation of State Using Solar Models

The frequencies of thousands of individual nonradial frequencies of the Sun have been measured\textsuperscript{21} to nearly 1 part in 10\textsuperscript{5}. Matching these frequencies is an extremely stringent test of solar modeling tools. New astrophysical opacities\textsuperscript{18} and equation of state (EOS)\textsuperscript{19} tables from Lawrence Livermore National Laboratory became available in 1995. The improvements in these tables included extending the number of elements for the opacity calculation from 14 to 21, adopting the 1993 solar element abundances of Grevesse and Noels\textsuperscript{22}, computing tables with finer grids, and incorporating more physical effects than were included in earlier tables.

For this project, we first updated our stellar evolution and pulsation codes to use these new tables, as we had been modifying analytical fits developed in the early 1970s to match the then-available opacity data.\textsuperscript{23} We also investigated the use of a new analytical equation of state developed by F. Swenson and collaborators that incorporated most of the physics of the new OPAL tables, but allowed for variable element mixtures, and for various atomic physics refinements to be turned on and off to investigate their relative importance. Finally, because the OPAL opacity tables do not extend to temperatures less than 6000 K, we considered several available options for low-temperature opacities and adopted the Alexander and Ferguson\textsuperscript{24} tables.

The solar models proved very useful for debugging our implementation of these new opacity and EOS treatments. The low-degree frequency predictions for our solar models agree with observed frequencies to within a few microhertz out of several thousand, as well as or better than the models of other research groups (Figure 1). We summarized results of detailed comparisons between models with different opacity and equation of state treatments in Publications 28-30. With this evidence that our codes are producing good evolution and pulsation frequency predictions for the Sun, we could progress with more confidence to applying these codes to the delta Scuti stars, which are somewhat more massive and luminous than the Sun.

B. Asteroseismology of delta Scuti Stars

The goal of stellar seismology, or asteroseismology, is to use observed properties of acoustic (p-mode) or gravity (g-mode) waves to infer the internal structure of stars. In general, we calculate the evolution of stellar models with a range of initial mass, composition, or input physical parameters (for example the mixing length parameter used in the convection treatment), and identify models with the observed surface temperature, surface gravity, and
effective temperature. We then calculate the pulsation frequencies of these models and compare with observed frequencies to find a model with frequencies that match the observations. Of course a significant consideration is the uniqueness of the model fit—a range of models with different interior structures may match all of the available observational constraints. It is then necessary to seek additional observational information to rule out some of these models.

Another potentially more useful outcome occurs when no standard models match the observations. Then we can use the discrepancies as clues to eliminate deficiencies in the model physics.

After updating the opacities in our evolution and pulsation codes and validating these codes for solar models, we next compared the evolution of delta Scuti models with old (1970) and new (1996) opacities. We showed that the evolution tracks, and therefore the initial mass and interior structure that would be derived from the position of these stars in the Hertzsprung-Russell diagram, is significantly different (Figure 2). This result highlighted the need for updated models using the new opacities (Publication 23).

We presented results of our model matches to the 21 observed frequencies of the delta Scuti star FG Vir, and the 13 observed frequencies of CD-24° 7599 at several workshops and conferences (Publications 4,5,22,23,25,32,36). These two delta Scuti stars have not yet exhausted the hydrogen in their convective cores and are in a similar point in their evolution as the Sun. Their predicted frequency spectrum is not as dense as for the more evolved delta Scuti stars, which makes developing models that match the observed frequencies more straightforward. We also investigated the effect of metallicity (the abundance of elements heavier than hydrogen and helium, also known as Z) on the inference of internal structure for FG Vir (Publication 36). We illustrated our point by comparing the predicted frequency spectrum of a lower-mass, less-evolved model for FG Vir with Z=0.02 that has the same overall observable properties (luminosity, surface temperature and gravity, and radial mode frequencies) as a model slightly more massive and less evolved, but with Z=0.03. Figure 3 compares the predicted radial and nonradial mode frequencies for these two models. If all of the frequencies were identical, the points would lie on a straight line. However, differences in core structure between the models can shift a few nonradial mode frequencies considerably or introduce additional frequencies, as indicated by the arrows. If enough frequencies in the right range can be observed for FG Vir, we hope that comparisons such as these can be used to discriminate between models with different convective core sizes, different metallicities, or different prescriptions for convective overshooting in the core.

Our models match all but one or two of the observed frequencies of FG Vir and CD-24 7599 fairly well. However, to date we have only incorporated estimates of rotational splitting of the frequencies, assuming that first and second order perturbation theory is adequate (i.e. the
rotation is relatively slow) and that these stars rotate as solid bodies. To first order, the Coriolis force splits each nonradial mode frequency of degree \( l \) into \( 2l+1 \) equally spaced frequencies. In second order theory the centrifugal force, which distorts the star from a sphere, is taken into account. This shifts both radial and nonradial frequencies, and destroys the symmetrical splitting. For CD-24° 7599, which rotates fairly rapidly (75 km/sec), second order corrections are significant. Figure 4 illustrates the changes in frequencies due to first and second order rotational splitting for a model of CD-24° 7599, assuming uniform solid body rotation (Publication 4).

As can be seen, the final predicted pattern of frequencies is very complex, with modes of different multiplets overlapping in frequency. It will be very difficult to identify the modes of such a complex observed spectrum, based on knowing the pulsation frequencies alone.

We have also analyzed data from a 5-year photometric campaign on the prototype star delta Scuti (Publication 40). This analysis revealed six pulsation frequencies. The luminosity and temperature of delta Scuti indicate that it is a highly evolved star that has exhausted hydrogen in its core and is now burning hydrogen in a shell outside this inert helium core. We have considered models that match the observations and find that a dense spectrum of nonradial g-mode (gravity-modes) is predicted that is not observed. Understanding the absence of these g-modes, or why only a few modes are excited to observable amplitudes, remains an outstanding problem for delta Scuti stars and pulsating stars in general.

We also participated in multisite observational campaigns for FG Vir, CD-24° 7599, 4 CVn, and the slightly lower mass gamma Doradus variable star HD10800 as part of the Whole Earth Telescope and Delta Scuti Network observing collaborations, which promise to yield more frequencies (Publications 8,9,10,11,37,38). The 4 CVn data analysis is still in progress, but revealed over 30 pulsation modes—a new record for delta Scuti variables. However, the pattern is more consistent with a widely spaced spectrum of nonradial p-mode (pressure, acoustic mode) pulsations, than with the expected dense spectrum of g-mode (gravity mode) pulsations, posing another challenge for theorists.

One of our stated goals was to publish a grid of updated delta Scuti models for preliminary comparisons to observations. While we have calculated the evolution and pulsation frequencies of a large number of models, and attempted to match the observed frequencies of a few delta Scuti stars, we determined that such a grid may be impractical. Only a few stars have enough observed frequencies at present to provide meaningful discrimination between models, and we have had limited success in matching the frequencies of even these few stars. And as explained above, moderately rapid and probably differential rotation alters the frequency spectrum enough to destroy symmetric regular patterns, making it difficult to identify modes by comparison to frequency predictions for non-rotating models. Also, the role
of overshooting from the convective core, which alters the evolution and structure of delta Scuti stars, is still uncertain. Finally, the enigma of the dense spectrum of predicted (but unobserved) gravity modes for the more evolved delta Scuti stars remains. Therefore, we decided for now to focus attention on detailed matches to the observed frequencies of a few well-studied stars. Nevertheless, we believe that a coarse grid showing trends in frequency changes with mass and composition (ignoring the effects of rotation and convective overshoot) for the less-evolved-core, hydrogen-burning delta Scuti stars is in order, and we are working to publish these results in the near future.

C. Two-Dimensional Stellar Evolution and Hydrodynamics Code

Differential rotation, rotation-induced hydrodynamic instabilities, and core convective overshoot may significantly affect the internal composition, structure and pulsation frequencies of delta Scuti stars. The two-dimensional stellar evolution and hydrodynamics code ROTORC being developed by Robert Deupree offered the ability of possibly modeling convective overshoot and hydrodynamic instabilities during the evolution of delta Scuti stars, and also to derive self-consistent internal rotation profiles.

ROTORC had been developed to model the evolution and hydrodynamics of massive (8-30 solar mass), rapidly rotating, main-sequence stars without convective envelopes. At the beginning of this project, this code had never been tested for sun-like stars. We first incorporated the new Livermore OPAL opacities and EOS tables into ROTORC. We also developed a methodology for handling envelope convection zones necessary for the lower-mass delta Scuti stars and the Sun. We evolved two-dimensional solar models with and without rotation, and compared the structure to the results of our one-dimensional code without rotation. The solar models evolved with ROTORC required very nearly the same initial helium abundance and mixing length/pressure scale height ratio to produce the observed solar luminosity and radius at the current solar age of 4.6 billion years; the models also had a convective envelope depth in very good agreement with the convection zone depth derived from solar oscillation frequency data (Publication 27). We are now in a position to evolve delta Scuti star models with significantly more rapid rotation (20-200 km/sec) and investigate the amount of core convective overshooting, test for rotationally induced instabilities, and study the evolution of the internal rotation profile given different initial rotation assumptions.

D. Microvariations, Outbursts, and Evolution Scenarios for Luminous Blue Variables

We began our study of the Luminous Blue Variables (LBVs) by evolving representative high-mass models of initial mass 50 and 80 solar masses from the onset of core hydrogen burning, through core hydrogen exhaustion (including mass loss via stellar winds), into the effective temperature range of the observed LBV instability region. The high-mass models are
intended to be representative of LBVs found just to the left (hot side) of the Humphreys-Davidson (H-D) limit, a line in the H-R diagram to the right of which no supergiant stars are found. The low-mass star models are intended to be representative of LBVs with luminosities well below the H-D limit where red supergiants are observed.

We tested these models for pulsational instability, and found that unstable nonradial g-mode pulsations do occur, and that they have periods similar to the observed LBV microvariation periods of 5-30 days. However, we found that the models were also unstable to radial mode pulsations, driven by the ionization of iron for the high-mass stars, or helium for the lower-mass stars. The periods of these modes were also in the range of the LBV microvariation periods. Furthermore, the linear growth rates of these modes were extremely large, as much as several hundred percent per period. Based on this finding, we determined that the linear pulsation analysis was inadequate, and we turned to nonlinear hydrodynamic modeling of the envelopes of LBVs to follow the growth of the pulsations to large amplitude.

We began with the high-mass models, which have final masses 47-55 solar masses after wind mass loss and that are on their first crossing of the H-R diagram. We considered models with heavy element abundances $Z = 0.02$ and 0.01, representative of the abundances in our galaxy and in the Large Magellanic Cloud, where high-mass LBVs are found. We also considered a series of helium mass fractions, from 0.28-0.29 (no He enrichment) to 0.49-0.58, representative of an LBV that has lost about half of its mass and has begun to expose material that has been processed by earlier nuclear burning in the core. We also note that the opacity of the envelope material decreases by about 5% for an increase in helium mass fraction of 0.1.

We found that for models with a high helium mass fraction, the pulsation amplitude grows rapidly, but after a few cycles reaches a limiting amplitude of 50-200 km/sec. The pulsations appear to be multiperiodic, with several pulsation modes excited simultaneously, but the quasi-periods and amplitudes agree well with the observed LBV microvariation periods of 5-30 days, and bolometric amplitude of 0.1-0.2 magnitudes (Figure 5). However, if the helium mass fraction is low enough, we find that the pulsations begin to grow, until the radial velocity of the outer layers of the star suddenly increases (Fig. 6), and the outer layers begin to move monotonically outward over several would-be pulsation cycles. We cannot follow these layers in our code as they leave the star, but it is plausible that they will eventually escape.

The physical mechanism for this “outburst” appears to be the exceeding of the Eddington luminosity limit in relatively deep layers of the envelope (the force of radiation pushing outward exceeds the force of gravity pulling inward). The key to obtaining this behavior in these models is the inclusion of time-dependent convection. Normally, when the amount of radiation that needs to traverse a layer of the envelope gets too large and the
temperature gradient becomes too high, convection turns on to transport the excess luminosity and lower the temperature gradient. However, since these stars are pulsating and the conditions in the outer layers are changing periodically, convection takes some time to turn on and off during each pulsation cycle. Until the convection is fully turned on, the radiation is blocked, and the radiation pressure pushes outward on these layers of the envelope.

We also checked this scenario for lower-mass Luminous Blue Variables and models with lower initial $Z = 0.004$, representative of the Small Magellanic Cloud. We found that the models do not lose enough mass on their first crossing of the H-R diagram into the LBV instability region to exceed the Eddington limit in their envelope. The models pulsate with typical microvariation periods and amplitudes, but outbursts do not occur. It is only after the star evolves to the red supergiant phase after losing additional mass, that the luminosity to mass ratio becomes high enough for radiation pressure to overcome the force of gravity and cause outbursts.

We propose the following scenario for LBV star evolution and outbursts: If the initial mass of the star is large enough (greater than about 60 solar masses), the star loses nearly half its mass by the time it evolves to the LBV instability region. If the envelope helium abundance is low enough, the high opacity of the hydrogen will cause the Eddington luminosity limit to be exceeded during each pulsation cycle, and "outbursts" accompanied by significant mass loss will occur. When the star has lost enough mass to bring material that is more helium-rich to the surface, the opacity of the envelope decreases and the Eddington limit will no longer be exceeded. The star will continue to undergo pulsations, but will no longer experience outbursts.

For stars with lower initial mass (less than 60 solar masses), the star does not lose enough mass on its first crossing of the H-R diagram into the LBV instability region to exceed the Eddington limit. After losing additional mass as a red supergiant, however, the star "blue-loops" back to the LBV instability region. This time, with a higher luminosity to mass ratio, it does periodically exceed the Eddington limit and experiences outbursts. The results of our work on Luminous Blue Variables are summarized in Publications 13,14,15,16,20, and 31.

**E. Conclusions**

We have made considerable progress in the course of this LDRD project on solar models, delta Scuti star pulsations, and understanding the mechanism for LBV pulsations and outbursts. While funded for small fractions of our time, we have been able to contribute to the frontiers of research in these fields. The evidence that we are maintaining our standing shows in our invitations to give review talks on solar modeling at the SOHO/GONG workshop in June 1998; on LBV stars at an International Astronomical Union Colloquium in Heidelberg.
also in June 1998; and on the theory of delta Scuti pulsations at the IAU General Assembly in Kyoto in August 1997 and at a delta Scuti workshop in August 1999.

Publications


References


Figure 1. Observed minus calculated (O-C) frequencies versus calculated frequency for solar models using two different equations of state: the Livermore OPAL (1996) equation of state and an older (1988) equation of state by Mihalas, Hummer and Dappen (MHD). The calculated frequencies for either model agree with the observed frequencies to within a few microHertz.
Figure 2. Evolution tracks in the luminosity versus effective temperature plane for stellar models of 1.6 and 2.0 solar masses, using the Cox-Stewart LANL (1970) opacities and the latest (1996) Livermore OPAL opacities. The new opacities considerably alter the evolution tracks and hence inferences of stellar mass and evolutionary state from position on this diagram.
Figure 3. Calculated radial ($l=0$) and nonradial ($l=1$ and $l=2$) frequencies for two prospective models of the delta Scuti star FG Vir. Both models have the same luminosity, effective surface temperature, and surface gravity, but different initial masses and compositions. The radial mode frequencies of the two models are nearly identical, but differences in the core structure shift and add some nonradial mode frequencies (as indicated by the arrows).
Figure 4. Observed frequencies (horizontal lines) and calculated frequencies (at positions 1, 2, and 3) for prospective model of the delta Scuti star CD-24° 7599. The frequencies at position 1 assume no rotation; the frequencies at position 2 account for the first-order rotational effects of the Coriolis force, assuming solid body rotation of 75 km/sec; the frequencies at position 3 include both the first-order effects and the second-order rotational effects of distortion from a sphere.
Figure 5. Radial velocity versus time for a Luminous Blue Variable model with mass 54.9 solar masses, surface temperature 16,000 K, and initial mass fractions of helium and heavier elements 0.49 and 0.01, respectively. The pulsations were initiated with radial velocity $10^5$ cm/sec, and rapidly grew to amplitudes over $10^7$ cm/sec. This model continues to pulsate with period of approximately 8.3 days, typical of observed LBV microvariation periods.
Figure 6. Radial velocity versus time for a Luminous Blue Variable model identical to the model in Fig. 5, except for a lower envelope helium mass fraction 0.29. In this model, zones in the pulsation driving region periodically exceed the Eddington luminosity limit, the point at which radiation pressure pushing outward is greater than the gravitational force pulling inward. The pulsation amplitude begins to grow but at $2 \times 10^6$ seconds the outward velocity of the outer layers suddenly becomes very large. The outer layers of this model continue to move outward, and will eventually escape from the star.