Bandwidth requirements for fine resolution squinted SAR

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ABSTRACT

The conventional rule-of-thumb for Synthetic Aperture Radar is that an RF bandwidth of $c/(2\rho_r)$ is required to image a scene at the desired slant-range resolution ρ_r , and perhaps a little more to account for window functions and sidelobe control. This formulation is based on the notion that the total bandwidth required is the same bandwidth that is required for a single pulse. What is neglected is that efficient processing of an entire synthetic aperture of pulses will often require different frequency content for each of the different pulses that make up a synthetic aperture.

Consequently, the total RF bandwidth required of a Synthetic Aperture Radar may then be substantially wider than the bandwidth of any single pulse. The actual RF bandwidth required depends strongly on flight geometry, owing to the desire for a radar to maintain a constant projection of the Fourier space collection surface onto the ω_y axis. Long apertures required for fine azimuth resolution, and severe squint angles with steep depression angles may require total RF bandwidths well beyond the minimum bandwidth required of any single transmitted pulse, perhaps even by a factor of two or more. Accounting for this is crucial to designing efficient versatile high-performance imaging radars.

This paper addresses how a data set conducive to efficient processing might increase the total RF bandwidth, and presents examples of how a fixed RF bandwidth might then limit SAR geometries.

Keywords: SAR, Radar, Bandwidth, Squint, Motion Compensation

1. INTRODUCTION

The evolution of Synthetic Aperture Radar (SAR) is towards high-performance real-time light-weight extremely-fine resolution imaging, with a user interface that might best be described as for a digital radar camera.¹ This would include the ability to be pointed and operated over a wide range of 'pan' angles and 'tilt' angles. In radar language this would of course be described as squint and depression angles.

The requirement to efficiently form images over such a large range of geometries has a profound impact on the design of a SAR system. Ideally, image formation algorithms would not suffer any performance degradation even for substantial depression angles and non-broadside squint angles. Historically, forming high-quality SAR images at substantial non-broadside squint angles has been a rather difficult task, and more so at finer resolutions. Much of this difficulty can be mitigated with proper real-time motion compensation, that properly considers the radar-to-target geometry changes during a synthetic aperture, and collects data on the proper grid in Fourier space. Image formation is then considerably simplified.

The penalty for easy image formation is the requirement for highly accurate and precise waveform control, both for the transmitted signal, and the local oscillator de-chirping signal (presuming stretch processing is employed). In general we need to control phase, frequency, and chirp rates (or equivalent) to compensate for antenna phase-center motion. The net result is that the pulse-to-pulse frequency and bandwidth will change over the course of an aperture, further resulting in an expansion of the total RF bandwidth required of the hardware, increasing perhaps substantially beyond that for any single pulse.

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2. BACKGROUND - A SINGLE PULSE - STRETCH PROCESSING

Consider the geometry of figure 1. Also consider a radar using a Linear-FM chirp, stretch processing, and quadrature sampling. The video signal from an echo of an ideal point target located at S can be modeled approximately as

$$X_V(t) \approx E \left(\frac{t - \frac{2}{c}|r_c| - \frac{2}{c} \frac{r_c \cdot S}{|r_c|}}{T} \right) \exp \left\{ j \frac{4\pi}{c} \left(f_T + \gamma_T \left(t - \frac{2}{c}|r_c| \right) \right) \frac{r_c \cdot S}{|r_c|} \right\} \quad (1)$$

where

$$\begin{aligned} E() &= \text{pulse envelope function of unit width, centered at 0,} \\ T &= \text{total pulse width,} \\ f_T &= \text{nominal center frequency of transmitted pulse, and} \\ \gamma_T &= \text{chirp-rate of transmitted pulse.} \end{aligned} \quad (2)$$

For a target located with x-y-z coordinates (s_x, s_y, s_z) , we can expand

$$\frac{r_c \cdot S}{|r_c|} = s_x \cos \psi_c \sin \alpha - s_y \cos \psi_c \cos \alpha + s_z \sin \psi_c. \quad (3)$$

Note that both the video frequency, and the envelope of the video signal, depends on the relative target range $(r_c \cdot S)/|r_c|$. This leads to the familiar parallelogram of figure 2. For the phase histories to be complete for all target locations passed by the video filter, the transmitted pulse width needs to be extended, leading to the ultimate result that the actual transmitted bandwidth needs to exceed the effective bandwidth by an amount equal to at least twice the video bandwidth. That is

$$B_T \geq B_{\text{eff}} + 2B_V, \quad (4)$$

where the effective bandwidth B_{eff} determines the slant-range resolution at the scene center (in the slanted direction away from the radar). Doing this allows the sampled video signal to be described by

Figure 1. Target scene Geometry.

$$X_V(i) \approx E\left(\frac{i}{I}\right) \exp\left\{j \frac{4\pi}{c} \left(f_T + B_{\text{eff}} \frac{i}{I}\right) \frac{\mathbf{r}_c \cdot \mathbf{s}}{|\mathbf{r}_c|}\right\} \quad (5)$$

where

$$i = \text{ADC sampling index } (-I/2 \leq i \leq I/2 - 1). \quad (6)$$

Within the Fourier space of the target scene, the video samples from a single pulse describe a linear sequence of samples over a radial segment $(4\pi/c)(f_T + B_{\text{eff}} i/I)$ at polar angles α and ψ_c .² This is shown in figure 3.

Figure 2. Video signal time-frequency parallelogram for stretch processing.

Figure 3. Fourier space of target scene.

3. A SYNTHETIC APERTURE - BROADSIDE COLLECTION GEOMETRY

Now consider a collection of pulses that define a synthetic aperture. That is, let each pulse be controlled and sampled such that

$$X_V(i, n) \approx E\left(\frac{i}{J}\right) \exp\left\{j\frac{4\pi}{c}\left(f_{T,n} + B_{\text{eff},n}\frac{i}{J}\right)\frac{\mathbf{r}_{c,n} \cdot \mathbf{S}}{|\mathbf{r}_{c,n}|}\right\} \quad (7)$$

where

$$\frac{\mathbf{r}_{c,n} \cdot \mathbf{S}}{|\mathbf{r}_{c,n}|} = s_x \cos \psi_{c,n} \sin \alpha_n - s_y \cos \psi_{c,n} \cos \alpha_n + s_z \sin \psi_{c,n} \quad (8)$$

with

$$n = \text{pulse index } (-N/2 \leq n \leq N/2 - 1). \quad (9)$$

The radar flight path is embodied in the pulse-to-pulse variations in $\mathbf{r}_{c,n}$, and ultimately $\psi_{c,n}$ and α_n .

Now consider a straight and level flight path, broadside to the scene center at the center of the synthetic aperture, and at a 45° depression angle to the scene center at the center of the synthetic aperture. If the pulse center frequency $f_{T,n}$ and effective bandwidth $B_{\text{eff},n}$ are held constant, the collection surface in Fourier space would be as shown in figure 4(a). This is NOT a segment of an annular ring, but rather a 3-dimensional warped version thereof, due to the changing angles α_n and $\psi_{c,n}$. For this geometry, depression angle $\psi_{c,n}$ is maximum at the aperture center, and minimum at either end of the synthetic aperture.

The projection of this collection surface onto the plane $\omega_z = 0$ is shown in figure 4(b). It is this projection that is typically resampled to a convenient grid and then inverse-transformed to the spatial image domain. The question that must be dealt with is "Although the data projects to the region indicated, what region is actually processed into the image?" Typically, the processed region is some rectangular area (to facilitate convenient 2-dimensional DFT processing) that is fit to the data region in some manner.³

If the processed region is a rectangle fit within the data region, then some data will not be processed, resulting in a cropped Fourier space, resulting ultimately in lost resolution and Signal-to-Noise Ratio (SNR) reduction. If the processed region includes area outside the data region, then resolution is enhanced to the data limits but at the expense of easy sidelobe

Figure 4. Fourier space for broadside imaging with constant waveform parameters.

control and perhaps otherwise unnecessary processing. A typical practice is to fit a rectangle that both crops some data, and includes some empty Fourier space.

A more interesting approach is to collect data in a manner that requires a minimum of resampling, and projects to a more favorable region in Fourier space. This is easily done with modern high-performance Digital Waveform Synthesizers by allowing the pulse center frequency and effective bandwidth to change on a pulse-to-pulse basis.

Consequently, we force the pulse-to-pulse dependence of frequency and bandwidth as

$$\begin{aligned} f_{T,n} &= \kappa_n f_{T,0}, \text{ and} \\ B_{\text{eff},n} &= \kappa_n B_{\text{eff},0} \end{aligned} \quad (10)$$

where $f_{T,0}$ and $B_{\text{eff},0}$ are nominal constant values, and

$$\kappa_n = \frac{\cos \psi_{c,0}}{\cos \psi_{c,n} \cos \alpha_n} \quad (11)$$

with $\psi_{c,0}$ representing a nominal constant depression angle. The actual transmitted waveform has parameters that are compensated for the motion of the radar. Doing so alters the Fourier-domain collection surface by radially shifting and stretching the data in Fourier space such that the projection of the collection surface onto the plane $\omega_z = 0$ is trapezoidal. This is illustrated in figure 5 for the same flight path as was used to generate figure 4. In fact, the projection of the collection surface onto the ω_y axis is held constant.

Furthermore, constant values for index i define a constant spatial frequency ω_y , indicating that the only resampling required to achieve a rectangular grid is in the ω_x direction. That is, a substantial part of polar reformatting is accomplished by motion compensation of the waveform parameters.

Furthermore yet, a rectangular processing region inscribed within the projection area of the data need not crop any ω_y frequencies, thereby offering excellent control of resolution and sidelobes in the y-direction. An inscribed rectangle would, however, require cropping some data in the ω_x direction. The extreme alternative would be to circumscribe a rectangle, and process regions outside the data space. Full resolution would be achieved, but perhaps at the expense of extra processing.

Some algorithm implementations do naturally process exclusively the trapezoidal data region. These include Polar Format processing using a chirped-z-transform in azimuth prior to range processing, and Overlapped Subaperture image formation algorithms.

Figure 5. Fourier space for broadside imaging with motion compensated waveform parameters.

In any case, since κ_n has generally increased the transmitted frequency content at the edges of the aperture with respect to the center of the aperture, the total RF bandwidth as measured by the difference between the highest transmitted frequency and the lowest transmitted frequency, has increased somewhat. The relative increase depends on the length of the synthetic aperture, and its attendant effects on α_n and $\psi_{c,n}$.

The required total RF bandwidth due to both the motion compensation by κ_n , and the video bandwidth, increases to

$$B_T \geq f_0(\max(\kappa_n) - \min(\kappa_n)) + \left(\frac{B_{\text{eff}}}{2}\right)(\max(\kappa_n) + \min(\kappa_n)) + 2B_V \quad (12)$$

Figure 6 shows how κ_n changes versus α for the geometry of figure 5. The required RF bandwidth will increase even more. Nevertheless, we note that this is often not a big deal for broadside imaging. For example, 4-inch resolution at Ku-band (16.7 GHz) would require only about a 1.4% total bandwidth increase (neglecting the video filter). However, flight-path deviations from the straight and level may cause further excursions in κ_n , and ultimately in B_T .

The big problem is with non-broadside squint angles.

4. A SYNTHETIC APERTURE - SQUINT COLLECTION GEOMETRY

For a straight-line flight path, we define the squint angle θ_s as the difference between the ground-track of the radar, and the ground bearing to the scene center, from the angular center of the aperture ($\alpha_0 = 0$). This makes broadside at $\pm 90^\circ$. A forward squint ($|\theta_s| < 90^\circ$) means the radar is approaching the target scene. Straight and level flight then means that the depression angle $\psi_{c,n}$ is increasing throughout the synthetic aperture, and perhaps substantially. A rearward squint has the opposite effect. Nevertheless, squinting has the effect of twisting the collection surface in Fourier space.

For a radar operating with constant frequency and bandwidth ($\kappa_n = 1$), and in a geometry with depression angle of 45° and a squint angle of 45° , the collection surface in Fourier space is illustrated in figure 7, along with its projection onto the plane $\omega_z = 0$. Clearly, the data region is more cumbersome. Inscribing a rectangular processing region is more problematic, and circumscribing a rectangular region is a substantially larger area, with lots of data voids. In addition, defining resolutions is also more problematic.

However, employing the motion compensation scheme presented earlier (pulse dependent κ_n), frequencies and bandwidths are again adjusted to yield a desired trapezoidal projection of the collection surface, as is illustrated in figure 8. The resulting projection onto the plane $\omega_z = 0$ is identical to that collected at a broadside squint angle. Consequently, image formation is identical. However, the required RF bandwidth has increased substantially. For the same example as before, 4-inch resolution at Ku-band, the bandwidth increase becomes 71%, neglecting the video filter and flight path deviations.

(Geometric distortions in the image of large scenes due to the squint is a separate matter entirely.)

Figure 6. Variation in κ_n for broadside squint angle.

5. NON-STRAIGHT-LINE FLIGHT PATH

Equations 11 and 12 are flight-path independent, and therefore apply to non-straight-line flight paths as well. The effect of flight path deviations are variations in depression angle $\psi_{c,n}$. These, of course, can increase κ_n , and ultimately the total RF bandwidth B_T required for real-time motion compensated data (for trapezoidal $\omega_z = 0$ plane projections).

Figure 9 shows how κ_n extremes vary with squint angle and depression angle, for straight-line flight and up to 100 m deviations from straight-line flight.

Figure 7. Fourier space for squint imaging with constant waveform parameters.

Figure 8. Fourier space for squint imaging with motion compensated waveform parameters.

Interestingly, B_T can also be reduced back to its minimum by selecting the proper flight path. One might consider an optimal flight path as any one that keeps $\cos \psi_{c,n} \cos \alpha_n = \cos \psi_{c,0}$.

6. BANDWIDTH CONSTRAINTS - EXAMPLES

What follows is a discussion of some examples to illustrate the bandwidths required to employ the real-time motion compensation scheme previously discussed.

Our first example is an X-band (nominal 9.8 GHz) SAR with range and azimuth resolution at 0.1 m. Allowing for a window impulse-response bandwidth of 1.2 pixels, aperture-center nominal range of 5 km, flight path deviations of up to 100 m, and 50 MHz video bandwidth, lets us plot the total RF bandwidth requirements as illustrated in figure 10.

(a) Straight-line flight path

(b) 100 m deviations

Figure 9. Extremes for κ_n (0.1 m resolution, 5 km range).

(a) Bandwidth surface plot

(b) Bandwidth contours

Figure 10. Bandwidth requirements at 9.8 GHz, 5 km range, 100 m deviation, 50 MHz video bandwidth, 0.1 m resolution.

Figure 11 illustrates the requirements at Ku-band (16.7 GHz) with other parameters the same. Figure 12 illustrates the requirements for Ka-band (35 GHz).

Clearly, for these examples, bandwidth requirements increase with frequency, owing to f_0 increasing faster than κ_n limits are decreasing. But this isn't always so. Consider 0.3 m resolution at UHF (0.75 GHz) and L-band (1.5 GHz), as illustrated in figures 13 and 14.

Figure 11. Bandwidth requirements at 16.7 GHz, 5 km range, 100 m deviation, 50 MHz video bandwidth, 0.1 m resolution.

Figure 12. Bandwidth requirements at 35.0 GHz, 5 km range, 100 m deviation, 50 MHz video bandwidth, 0.1 m resolution.

Note how a total RF bandwidth of 800 MHz allows a larger range of geometries at L-band than at UHF, although this advantage disappears for wider bandwidths.

Figure 13. Bandwidth requirements at 0.75 GHz, 5 km range, 100 m deviation, 50 MHz video bandwidth, 0.3 m resolution.

Figure 14. Bandwidth requirements at 1.5 GHz, 5 km range, 100 m deviation, 50 MHz video bandwidth, 0.3 m resolution.

7. CONCLUSIONS

From the point of view of Fourier space, long apertures of wide-bandwidth highly-squinted data define a twisted collection surface. If frequencies and bandwidths are held constant, then the projection of the collection surface onto the plane $\omega_z = 0$ defines a somewhat problematic area to be processed into an image. The shape of the collection surface projection in conjunction with the region actually processed defines the sidelobe characteristics and resolution within the resulting image, and will vary considerably with squint angle and depression angle.

Processing can be considerably simplified, with less variation in resolution and sidelobe characteristics by forcing the collection surface to project to a consistent trapezoidal region on the $\omega_z = 0$ plane. This can easily be accomplished with modern state-of-the-art digital waveform synthesizers by adjusting the radar center frequency and bandwidth on a pulse-to-pulse basis as a function of the radar motion. In this manner, the data from each pulse in the Fourier domain projects identically onto the ω_y axis, thereby also considerably simplifying any polar reformatting that might be employed. However, for this to occur, the total RF bandwidth required for an entire synthetic aperture might be considerably increased, and more so for severe squints at steep depression angles.

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