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SENSITIVITY STUDY ON HYDRAULIC WELL TESTING INVERSION USING SIMULATED ANNEALING

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Sensitivity Study on Hydraulic Well Testing Inversion using Simulated Annealing

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Abstract

Cluster variable aperture (CVA) simulated annealing has been used as an inversion technique to construct fluid flow models of fractured formations based on transient pressure data from hydraulic tests. A two-dimensional fracture network system is represented as a filled regular lattice of fracture elements. The algorithm iteratively changes an aperture of cluster of fracture elements, which are chosen randomly from a list of discrete apertures, to improve the match to observed pressure transients. The size of the clusters is held constant throughout the iterations. Sensitivity studies using simple fracture models with eight wells show that, in general, it is necessary to conduct interference tests using at least three different wells as pumping well in order to reconstruct the fracture network with a transmissivity contrast of one order of magnitude, particularly when the cluster size is not known a priori. Because hydraulic inversion is inherently non-unique, it is important to utilize additional information. We investigated the relationship between the scale of heterogeneity and the optimum cluster size (and its shape) to enhance the reliability and convergence of the inversion. It appears that the cluster size corresponding to about 20 ~ 40 % of the practical range of the spatial correlation is optimal. Inversion results of the Raymond test site data are also presented and the practical range of spatial correlation is evaluated to be 5 ~ 10 m from the optimal cluster size in the inversion.

1 Introduction

For environmental remediation, management of nuclear waste disposal or geothermal reservoir engineering, it is very important to evaluate the permeabilities, spacing and sizes of the subsurface fractures which control groundwater flow. Pressure interference testing using multiple wells is thought to be a powerful method to collect the information of the permeabilities of the fracture system. Once obtaining the pressure transient data, an inverse method to match the observed data is recently applied to construct the hydrological model [e.g., *Doughty, et al.*, 1994]. Conventional inversion methods are mainly based on the conjugate gradient method or maximum likelihood method [*Carrera and Neuman*, 1986]. Such methods, however, will often find a minimum in a close neighborhood of the starting (initial) model so that one needs to start very close to the global minimum or to try several starting models with the hope that one of them will lead to the best model.

Simulated annealing (SA) has been recently attracting attention as a powerful stochastic inverse method for finding the minimum near optimal values of a function to solve such a problem. SA has been found useful in a variety of optimization problems that involve finding optimum (minimum or maximum) values of a function of a very large number of independent variables. One example is the traveling salesman problem. A salesman travels through a number of cities but must visit each city once before returning to a starting point. The problem is optimized if the shorter route is found [e.g., *Kirkpatrick et al.*, 1983; *Geman and Geman*, 1984; *van Laarhoven and Aarts*, 1987]. In many of these applications, the function to be minimized is called the cost function (which is also referred to as the energy function). SA is advantageous in that annealing is not forced to converge toward the local minimum nearest to the starting point as occurs with conventional minimization methods such as conjugate gradient methods.

In hydrological application, Lawrence Berkeley National Laboratory has been developing a SA inverse method for well tests data analyses. *Maulden et al.* [1994] developed a SA inverse method in which a partially filled lattice represents a fracture network and the an individual element are changed randomly from conductive to nonconductive or vice versa. The effect of the change is examined by numerically simulating well tests and comparing them with field tests. In this technique, the finite element code TRINET [*Karasaki*, 1987]

was used as a subroutine to solve for the head distribution at each iteration. *Maulden et al.*'s method changed individual fracture element to be conductive or not, on the other hand, *Najita and Karasaki* [1995] developed a simulated annealing model that uses a variable aperture network, namely, the algorithm will change a cluster of elements instead of single element. They also applied the method to pressure transient data from synthetic hydraulic well tests.

One of the purposes of this paper is to conduct sensitivity studies in order to develop general guideline for a field test design. One subject is the relative number of pumping wells and observation wells. A second, equally important factor is the location of the wells. These two factors are clearly related to the ability to detect heterogeneity. The other purpose of this paper is to conduct sensitivity studies to examine relationship between the scale of heterogeneity and the optimal cluster size and shape to be specified in the inversion. These factors are expected to enhance reliability and convergence because it reduces the size of the space where the inversion searches for solutions. We will discuss these results using synthetic data. We will also discuss the inversion results of the Raymond test site data using the present technique.

2 Concept of Simulated Annealing

Metropolis et al. [1953] first used an annealing algorithm to simulate changes in a system of interacting molecules at a fixed temperature. This algorithm became popular after it was applied to combinatorial optimization problems by *Kirkpatrick et al.* [1983]. Simulated annealing (SA) is a stochastic search method which can be applied to a wide variety of problems such as allocation and scheduling problems in operations research, image restoration [*Geman and Geman*, 1984] and the well-known traveling salesman problem [*Kirkpatrick et al.*, 1983; *Press et al.*, 1986]. In addition to the applications cited above, SA has also been applied to geophysical exploration [*Basu and Frazer*, 1990; *Sen and Stoffa*, 1991; *Vasudevan et al.*, 1991; *Sen et al.*, 1993; *Carrion and Bohm*, 1994] and hydrology to develop a groundwater management strategy [*Dougherty and Marryott*, 1991].

The concept of SA is analogous to thermodynamics, in the manner in which liquid metals cool and anneal. In physical annealing, a metal is heated and allowed to cool very

slowly in order to obtain a regular molecular configuration having the lowest possible energy state. In order to encourage the formation of this crystalline structure, a schedule of temperatures is used to govern the rate at which the metal cools. In SA, energy state and molecular configuration have exact analogies. The cost or energy function is analogous to energy state and the set of free parameters (configuration) is analogous to the arrangement of molecules. As in physical annealing, a transition to a new configuration has an associated energy change. The following criterion known as the Metropolis algorithm is applied in determining whether a transition to another configuration occurs at the current temperature. For the $n-1^{\text{st}}$ configuration X_{n-1} and the n^{th} configuration X_n with energies (energy function) E_{n-1} and E_n respectively, the transition probability at some system temperature T is given by

$$\Pr(X_{n-1} \rightarrow X_n) = \begin{cases} 1 & \text{if } E_n - E_{n-1} < 0 \\ \exp[-(E_n - E_{n-1})/T] & \text{if } E_n - E_{n-1} > 0. \end{cases} \quad (1)$$

This criterion always allows a transition to a configuration if system energy is decreased and sometimes allows a transition to a configuration with higher energy. The probability of a transition to a higher energy state decreases exponentially with the size of the energy increase. This is a stochastic relaxation step that allows SA to search the space of possible configurations without always converging to the nearest local minimum. If the temperature T is held constant, the system approaches thermal equilibrium and the probability distribution for the configuration with energy E approaches the Boltzman probability

$$\Pr(E) = \exp(-E/kT) \quad (2)$$

where k is the Boltzman constant. As the system temperature is lowered, the probability of accepting configurations with higher energy decreases so that only configurations with small energy perturbations are allowed at low temperatures.

In general, the SA algorithm consists of the following tasks; 1) the element to generate or change system configuration randomly, 2) calculations of an objective function (energy or error function), 3) stochastic test to determine whether a randomly changed configuration

is accepted or not (Metropolis algorithm), 4) adjustment of current temperature according to the cooling schedule.

Several choices of temperature schedule are possible, and are currently a topic of research (e.g., Dougherty and Marryott, 1991). A computationally practical schedule is the widely used decrement rule (Press et al., 1986). Given an initial temperature T_0 , assign

$$T_k = T_0 \alpha^k; \quad k=0,1,2,3 \dots \quad (3),$$

where α is between 0 and 1. This general form has been implemented with $\alpha = 0.55$ to 0.99. In this schedule, the current temperature is kept fixed until a finite number of transitions, L_k , have been accepted, then the temperature is lowered.

3 Application to Well Test Inversion

To use simulated annealing for the inverse of well tests, we consider the two dimensional fracture network model as shown in Figure 1. Each element represents a simplified fracture. Transmissivity(m^2/sec), storativity(m^{-1}), aperture(m), unit thickness(1 m) and length are imposed on each element. In Figure 1, thick elements represent fractures with higher transmissivity. Well locations are specified at node points and pressure transients due to pumping or injection are calculated at each node using the finite element code TRINET[Karasaki, 1987]. Fluid flow along fractures is assumed to be laminar. It is also assumed that apertures are large enough so that the transmissivity of any fracture element follows the cubic law

$$T = \rho g b^3 / 12\mu, \quad (4)$$

where ρ is the density of water, g is acceleration due to gravity, μ is dynamic viscosity of water and b is aperture of the element.

In cluster variable aperture (CVA) annealing, we would like to find the optimum fracture network geometry by modifying clusters of element apertures and calculating transient curves to simultaneously match all observed transient curves. At each step of the

algorithm, the apertures for one cluster of elements are selected randomly using the Wolff algorithm [Wolff, 1989]. This step is analogous to the perturbation step in the general SA algorithm. The number of elements in the cluster is limited by the maximum cluster size which is specified by the user. Following the perturbation step, well tests are simulated on the fracture network and the calculated pressure transient curves are compared with observed data. The system energy is the sum of the squared differences between the calculated and observed pressure transients.

The Metropolis algorithm is applied to determine whether the current fracture configuration is accepted or not, based on the energy change and the temperature (Equation 1). When L_k acceptances at T_k have been achieved, the temperature is reduced to T_{k+1} according Equation (2) with $\alpha = 0.9$, until L_{k+1} transitions have been accepted at the temperature T_{k+1} . This process is continued until the temperature schedule is exhausted or the number of iterations has reached a user-specified maximum. In the following, we refer to "minimum energy", however, this is not used to describe the global minimum. Rather it is the smallest energy found after completing the maximum number of iterations. We expect this to be close to the global minimum (i.e., near optimal).

4 Sensitivity Study

4.1 Relation between Number of Pumping Wells and Resolution

Sensitivity studies are conducted to examine the relation between the number of drawdown wells and the resolution of transmissivities, where we define the resolution as the degree of heterogeneity of the fracture network (contrast of the transmissivities and geometry of the distribution) that can be extracted by the analysis. As shown in Figure 1, consider the 20 m x 20 m two-dimensional region filled with fractures 1 m apart and consisting of 441 node points and 840 fracture elements. It is a simple fracture network model where the transmissivity of the upper region (Zone 1) is T_1 (aperture b_1) and that of the lower region (Zone 2) is T_2 (aperture b_2). All of the following results shown in this section are performed using the same fracture model. In all cases, total number of the observation wells is eight including pumping wells. By changing the number of pumping

Table 1 Summary of numerical experiment on resolution (NE-1 ~ NE-6).

Numerical Experiment #	transmissivity for T1 (m ² /sec)	transmissivity for T2 (m ² /sec)	T1/T2	number of drawdown wells	initial energy E ₀	energy for minimum E(min.)	iteration number for minimum
NE-1	10 ⁻⁶	10 ⁻⁸	100	1(in T ₂)	51113	0.26	9988
NE-2	10 ⁻⁶	10 ⁻⁸	100	1(in T ₁)	2331	0.23	9791
NE-3	10 ⁻⁶	10 ⁻⁷	10	1(in T ₂)	619	0.09	6869
NE-4	10 ⁻⁶	10 ⁻⁷	10	1(in T ₁)	2067	0.67	9660
NE-5	10 ⁻⁶	10 ⁻⁷	10	2(in T ₁ & T ₂)	2662	0.71	8092
NE-6	10 ⁻⁶	10 ⁻⁷	10	3(in T ₁ & T ₂)	4705	0.30	9900

wells from one, to three, we evaluate inversion results for the cases where the contrast T1/T2 is 10 and 100. The procedure of the pumping test is that the well is pumped at the constant flow rate until drawdown reaches near steady state and then the well is shut in until steady state is reached. For two and three pumping tests, the same procedure is simulated in series.

Table 1 summarize the numerical experiments with an ID number, values of T1 and T2, T1/T2, the number of pumping wells, energy of the initial model (configuration), the minimum energy and iteration number at which the minimum energy was reached. The initial model (configuration) used here is a homogeneous fracture network in which each fracture element has a transmissivity of 10⁻⁵ m²/sec (aperture 2.3 x 10⁻⁴ m). The pumping rates are constant, 0.06 l/min. for wells in zone 1 and 0.6 l/min. for wells in zone 2. The initial temperature is set to be 150. Since this algorithm changes configurations by clusters of fracture elements, the number of acceptances at each temperature is set to be the average number of non-overlapping clusters: the value obtained by dividing the number of elements (840) by a cluster size. In this case it is 80, because cluster size is 10 for all cases. The temperature is reduced by 10% and totally 10,000 iterations are scheduled. For simplicity two replacement apertures are specified. These are the apertures associated with T1 and T2 after applying the cubic law.

Numerical experiment 1 (NE-1) is the case where the transmissivity contrast $T1/T2$ is 100, and one pumping well is located in the low permeability zone 2. The minimum energy, 0.26, was reached at iteration 9,988 (Figure 2). Comparison of pressure transients between synthetic observed data and those of the minimizing configuration is illustrated in Figure 3. A good fit of pressure transients is achieved by SA, and the minimizing configuration is close to the true configuration in that the boundary between $T1$ and $T2$ is clearly defined. Figure 4 gives the history of energy transitions at each iteration. We can observe that the algorithm allows many transitions to higher energy with an overall decreasing trend in energy. The transmissivity contrast for NE-2 is the as same as that of NE-1, $T1/T2$ is 100, however, one pumping well is located in the high permeability zone 1. The minimum energy of the system is reached at the iteration 9,791 with energy 0.23. The boundary between $T1$ and $T2$ is clearly reconstructed (Figure 5). From these results, we see that it is possible to reconstruct such a simple fracture network model by using single pumping well if the transmissivity contrast is 100.

The next four numerical experiments (NE-3 through NE-6) are cases with a transmissivity contrast $T1/T2 = 10$. NE-3 has a single pumping well in the zone 2, NE-4 has a single pumping well in the zone 1, NE-5 has two pumping wells in series and NE-6 has a series of three pumping wells. The results are shown in Figure 6(a) ~ (d), respectively. For all cases, minimum energies are less than one. As the number of pumping wells increases, the degree to which network configurations match also increases. For NE-3 and NE-4, it is noteworthy that the minimum energies are low enough (0.09 and 0.67), however, the resulting configurations differ greatly from the "solution". This is caused not by a problem in SA itself, but by the inherent nature of pressure transient testing, where a large number of equivalent models exists that can match one set of observed data (non-uniqueness). Although the value of minimum energy can be one of the guidelines to check the degree of convergence and reliability of the inversion, we have to be careful when comparing the minimum energy of the result. As the total number of the well is fixed as eight in these calculations, increasing the number of pumping wells has a similar effect as increasing the number of ray paths in seismic tomography. From these results, it is shown that setting three pumping wells out of eight wells is necessary to reconstruct the fracture network with a transmissivity contrast of one order of magnitude. Moreover, in the case of a single

drawdown well, locating the pumping well in the higher transmissivity zone makes the resulting configuration appear more like the synthetic network than of the pumping well is located in the lower transmissivity zone. This finding also holds true for the case of $T_1/T_2 = 100$.

On the basis of these limited number of cases considered, it appears that setting three pumping wells out of eight wells is necessary to reconstruct the fracture network with a transmissivity contrast of one order of magnitude. As the actual flow geometry will be more complicated than the cases considered, one must conduct pumping tests on more than three wells to resolve any kind of heterogeneity.

4.2 Effect of Cluster Size

In the previous section, we used very simple models to examine the resolution of the annealing solution. In this section, we will discuss the results of sensitivity studies using spatially correlated fracture models to investigate the relationship between the cluster size to be chosen in the inversion and the correlation length. Heterogeneity is introduced to the same scale network as seen in Figure 1 (20 m x 20 m region with 1 m fracture elements). Transmissivity values are assigned to each fracture element from a lognormal transmissivity distribution, with a log mean of 10^{-6} (m^2/sec) and a standard deviation in $\log_{10}$ of 0.4343. The range of transmissivity is from 10^{-7} to 10^{-5} with a mean of 10^{-6} (m^2/sec). Two cases are considered in which the spatial correlation of the transmissivity is isotropic, with a correlation length of 1.0 m and 1.67 m, respectively. The transmissivity distribution is generated using an exponential autocovariance model. Therefore, the practical range of the spatial correlation is 3.0 m and 5.0 m, respectively. The stochastic transmissivity field is generated by the numerical code COVAR which uses the covariance matrix decomposition method [Williams and El-Kadi, 1986].

Two different realizations of transmissivity fields with correlation length of 1.0 m (cases 1A and 1B) are shown in Figure 7 and 8. Because transmissivity distributions of these models are continuous (thicker lines indicate larger transmissivity), it is difficult to decipher the differences in figures. An equivalent discrete transmissivity representation is also shown in Figure 7 and 8 for reference, with two threshold values of 3×10^{-6} and 3×10^{-7} (m^2/sec). A semi-variogram of transmissivity for each case is calculated. It reaches its sill at about 3.0

m which coincides with the practical correlation range chosen for the random fields (Figure 9). Eight wells are set up in the model and three drawdown tests are simulated in series. Pumping wells are indicated by filled circles in Figure 7 and 8. Synthetic observed data for the wells was generated based on continuous transmissivity distribution models. Five inversions were conducted with cluster sizes of 10, 20, 30, 40 and 60. To examine the effect of cluster size alone, other parameters such as the temperature schedule, time steps and flow rate for the pumping wells are kept the same for all inversions. Replacement apertures are set to 2.3044×10^{-4} m, 1.0696×10^{-4} m and 4.9646×10^{-5} m which correspond to transmissivities of 10^{-5} , 10^{-6} and 10^{-7} (m^2/sec) respectively. A homogeneous transmissivity distribution ($10^{-7} \text{ m}^2/\text{sec}$) is assigned for the initial configuration.

First, consider the inversion result for the case of the correlation length of 1.0 m (cases 1A and 1B). Figure 10 and Figure 11 show the annealing solutions found using the 5 different cluster sizes for case 1A and case 1B. In these figures, three types of line thickness (thick, thin and none) represent element transmissivities of 10^{-5} , 10^{-6} and $10^{-7} \text{ m}^2/\text{sec}$, respectively. Table 2 gives the cluster size along with the initial energy, the minimum energy and number of iterations at which the minimum energy was reached. The lowest minimum energy was reached by a cluster size of 10 for both cases. When the cluster sizes are smaller than 60, the minimum energy reaches less than 0.1. As the cluster size decreases, the minimum energy also decreases except for the cluster size of 20 for case 1A and cluster size of 40 for case 1B. This trend is clearly observed in drawdown matches shown in Figure 12. Plots of annealing energy versus iteration number are given in Figure 13 (case 1A) for cluster sizes of 10, 20, 30 and 40. Inversions with larger cluster size show a faster energy decrease at first, however, convergence tends to be slow when energies are less than 1.0.

By visual examination of Figures 10 and 11, it is possible to check the reliability of the results. It shows that the results of the cluster size = 10 for both cases appear to give a good match to the flow geometry. For case 1B a cluster size of 20 also gives a good match, although the size of high transmissivity regions are overestimated. Semi-variograms of each annealing solution are given in Figure 14 (case 1A) and Figure 15 (case 1B). Practical ranges of the spatial correlation for case 1A are 2.9 m, 4.0 m, 4.8 m, 5.8 m and 7.0 m for cluster sizes of 10, 20, 30, 40 and 60, respectively. On the other hand, they are 2.5 m, 4.5 m, 4.3 m, 5.1 m and 7.0 m for cluster sizes of 10, 20, 30, 40 and 60, respectively for case 1B. It

Table 2 Annealing energies for several cluster sizes for cases 1A and 1B (correlation length of 1.0 m)

Case 1A					
Cluster Size	10	20	30	40	60
Initial Energy	1.41×10^3	1.41×10^3	1.41×10^3	1.41×10^3	1.41×10^3
Minimum Energy	0.014	0.038	0.032	0.067	0.112
Number of Iterations for Convergence	9923	9918	9999	9844	9581
Case 1B					
Cluster Size	10	20	30	40	60
Initial Energy	1.39×10^3	1.39×10^3	1.39×10^3	1.39×10^3	1.39×10^3
Minimum Energy	0.013	0.024	0.061	0.047	0.122
Number of Iterations for Convergence	9867	9901	9904	7773	9914

is noteworthy that the same cluster size reaches the annealing solution with approximately the same practical range of the correlation for both cases. From these results, we observe that cluster sizes of 10 and possibly 20 are the most suitable size to obtain reliable results. Since a practical range of 3.0 m includes about 45 fracture elements in this case, the optimal cluster size seems to be about 20 ~ 40 % of the number of fractures within the practical range of correlation (equivalently, the optimal cluster dimension is about equal to the prescribed correlation length, which is 20 ~ 40 % of the practical range for an exponential model).

In order to confirm the result mentioned above, we repeated this procedure for a correlation length of 1.67 m (case 2). Figure 16 shows the synthetic fracture network model with correlation length of 1.67 m, discrete network model and well location. The practical range of the spatial correlation is approximately 5.7 m, a little larger than the value specified (Figure 18). This is probably an effect of the grid spacing. Ideally, the grid spacing should be considerably smaller than the correlation length. If not, assigning values on a coarse mesh can artificially inflate (or deflate) the correlation length.

Table 3 Annealing energies for several cluster sizes for case 2 (correlation length of 1.67 m)

Cluster Size	10	20	30	40	60
Initial Energy	1.28×10^3	1.28×10^3	1.28×10^3	1.28×10^3	1.28×10^3
Minimum Energy	0.021	0.037	0.034	0.064	0.068
Number of Iterations for Convergence	9818	9233	9660	9884	9971

Figure 17 shows the annealing solutions found using the 5 different cluster sizes (10, 20, 30, 40 and 60). Table 3 gives the cluster size along with the initial energy, the minimum energy and the number of iterations at which the minimum energy was reached. Although the minimum energy is quite small (less than 0.1) for all cluster sizes, a 50 % energy difference is evident between cluster size 30 and 40. As in cases 1A and 1B, inversion runs with larger cluster size achieve faster energy decrease at first, however, the convergence tends to slow down below energies less than 1.0. A visual examination of Figure 17 shows that the results of the cluster size = 10, 20, 30 and 40 appear to give a good match to the flow geometry. Semi-variograms of each solution are given in Figure 18. The practical ranges of the spatial correlation are 3.1 m, 4.5 m, 5.3 m, 5.9 m and > 10.0 m for cluster sizes of 10, 20, 30, 40 and 60, respectively. From reviewing these results, cluster sizes less than 40, especially 30 and 40 are suitable for obtaining good results. Since a practical range of 5.0 m covers an area equivalent to about 130 fracture elements, the optimum cluster size seems to be 20 ~40 % of the practical range, as was found above.

4.3 Effect of Cluster Shape

In the sensitivity studies described above, the annealing algorithm was designed to select a cluster of fracture elements at random locations (Figure 19). This seems to be a reasonable approach for perturbing the network model if information about the system geometry is not known a priori. On the other hand, if we have a priori information, namely, geological information about regional fractures such as strikes and scales, including such information in the inversion as shapes of a cluster might improve convergence and result in a more reliable solution. Hence, we added an option in the cluster selection so that the cluster

shape can be anisotropic (ellipse). Input parameters for an ellipse shape (one can define as many ellipses as one like) are the minor axis, major axis and counter-clockwise angle to the X direction. After the location of the cluster origin is determined, the shape of the ellipse is selected randomly (Figure 19).

To examine the effect of the cluster shape, we employed a spatially correlated transmissivity model. The heterogeneity is introduced to the network of the same scale as seen in Figure 1 (20 m x 20 m region with 1 m fracture elements). Transmissivity values are assigned to each fracture element from a lognormal transmissivity distribution, with a log mean of 10^{-6} (m^2/sec) and a standard deviation in $\log_{10}$ of 0.4343 (i.e. 2.0 in natural log). Practical ranges of the correlation are 2.0 m in the X direction and 4.0 m in the Y direction. Figure 20 shows the realization of the correlated transmissivity model and its discretized model. To isolate the effect of the cluster shape, other parameters such as the temperature schedule, time steps and flow rates for the pumping wells are unchanged between two inversions. One of the inversions uses isotropic clusters and the other uses anisotropic clusters. Replacement apertures chosen are set to be 2.3044×10^{-4} m, 1.0696×10^{-4} m and 4.9646×10^{-5} m which correspond to the element transmissivities of 10^{-5} , 10^{-6} and 10^{-7} (m^2/sec) respectively. A homogeneous transmissivity distribution (10^{-7} m^2/sec) is assigned for the initial configuration. For the case of anisotropic cluster shape, the ellipse with a minor axis of 1.2 m and major axis of 2.5 m are specified. These parameters are chosen so that the ratio of cluster length in the Y and X direction (i.e., 2.5 m/1.2 m) is close to the ratio of correlation lengths in the Y and X direction.

The network geometries obtained by inversion using isotropic clusters and anisotropic clusters are given in Figure 21, where the maximum cluster size of 10 is specified. After 10,000 iterations, the minimum energy of 0.068 is reached at 9,255 iterations for isotropic cluster shapes and 0.057 at 9,836 iterations for anisotropic cluster shapes. Although the difference in minimum energies between the two cases is not significant, a comparison of the configurations shows that the case with anisotropic cluster shapes seems to reconstruct the original fracture geometry better than that of isotropic cluster shapes. Figure 22 and Figure 23 show the semi-variograms of transmissivity for isotropic and anisotropic cluster shapes, respectively. We can observe that each case preserves its anisotropy of practical range of the spatial correlation, which is about 1.0 for the isotropic case and about 2.0 for

the anisotropic case. Thus, if geological information of fracture geometry is available, it may be possible to incorporate such pieces of information into the inversion.

5 Application to Field Data

5.1 Site Description

Simulated annealing was applied to hydraulic data from the Raymond field site, which is located in the foothills of Sierra Nevada, approximately 3.2 km east of Raymond, California. In order to develop field testing techniques and analysis methods for characterizing flow and transport properties of fractured rocks, the Raymond test site was established in collaboration between U. S. Geological Survey, Denver and Lawrence Berkeley National Laboratory as part of a task under the bilateral agreement between US/DOE and Atomic Energy of Canada(AECL). The site lies within the Knowles granodiorite which is light-gray, equigranular and non-foliated, and is widely used as a building material in California [Bateman and Sawka, 1981; Bateman, 1992].

Nine boreholes were drilled in a reverse V pattern with increasing spacing between boreholes (Figure 24). Driller's logs indicate that relatively unweathered granite is located beneath less than 8 meters of soil and regolith. The wells are cased to approximately 10 meters and vary in depth between 75 and 100 meters. The water level is normally between 2 and 3 meters below the casing head. Various geophysical logs, geophysical imaging techniques and hydraulic tests have been conducted to image the hydrologic connection of the fractured rock mass [Cohen, 1993; Karasaki *et al.*, 1995; Cook, 1995; Cohen, 1995; Vasco *et al.*, 1996]. Regional characterization and site specific fracture measurements show that there are two sets of subvertical tectonic fractures: one of the set strikes at N30W and the other strikes at N60E [Cohen, 1995]. Two major conductive zones have been detected: one occurring near a depth of 30 m and the other between 54 to 60 m, which are also reconfirmed by the ground penetrating radar reflection and seismic tomography survey (Figure 25).

5.2 Data of Hydraulic Well Testing and Model

Hydraulic well testing has conducted at the Raymond test site in several occasions [e.g.,

Cohen, 1993; Karasaki et al., 1995; Cook, 1995; Cohen, 1995]. The data used in this study was taken in the summer of 1995. Each of the nine wells was injected systematically with fresh water using a straddle packer system. The distance between the straddle packers was roughly 6 meters. A typical injection test was, on the average, ten minutes. The pressure in the water tank was controlled and maintained at a constant pressure using compressed air. Neither the flow rate nor the downhole pressure was actively controlled; they spontaneously adjusted themselves accordingly to the transmissivity of the injection interval. The advantage of this method are the simplicity of the set-up and the ease of test execution. After each test, the packer string was lowered by approximately 6 meters. Depth intervals sealed by packers during a particular injection were kept unobstructed during the next, so that the entire length of the well was tested. There were approximately 15 injection tests per well in all nine wells. While these injections were conducted, the pressures in the remaining 31 intervals were simultaneously monitored. As a result, a total of 4,200 interference pressure transients were recorded. A schematic of the packer set up in the site is shown in Figure .26. To construct a hydraulic model using such a large number of interference data, a binary inversion method was developed [*Karasaki et al., 1995; Cook, 1995*]. In their method each set of the pressure transient data was reduced down to a binary set: 1(yes) if an observation zone responds to an injection, and 0(no) otherwise, and they successfully visualized connections between wells.

From such a large number of interference data, we selected three sets of injection data; injection into wells 0-0, SE-1 and SE-3. These injection intervals are located approximately between depths of 20 to 30 meters. Intervals of the interference response are also located in these depths. We believe these intervals are confined in the upper conductive zone shown in Figure 25. Injection flow rates were, on the average, 6.4 l/min. for well 0-0, 5.9 l/min. for well SE-1 and 6.5 l/min. for well SE-3. It should be noted that for a pair of wells, injecting at the first and observing at the second does not always give the same pressure response when their roles are reversed (i.e., injecting at the second and observing at the first). For example, Injection into well 0-0 gives about 1.5 meters change of head in well SE-3, on the other hand, injection into well SE-3 gives 0.2 meters change of head in well 0-0. This is probably because the injection interval is much shorter than the observation intervals.

The template (model) of the Raymond test site consists of a 32 x 32 regular grid with 2.5 meters fracture elements which covers an 80 m x 80 m area (Figure 27). The total numbers of nodes and elements are 1089 and 2112, respectively. The observation data of wells SW-4 and SE-4 is not included in the model, since the interference response is not observed for a 600 second injection into well 0-0, SE-1 nor SE-3. In all the calculations discussed below, we impose an initial head value of 0 meters for all nodes and elements, so that the relative head changes of wells are examined. We also assign constant head conditions (0 meter) on the outer boundary of the mesh. As an initial (starting) configuration, all fracture elements have transmissivity 10^{-5} (m^2/sec), aperture 2.3044×10^{-4} m and length 2.5 m. The equivalent specific storage of the fracture elements are set to be 0.1 m^{-1} and held constant throughout the inversion. If we set the specific storage of the fracture element to be 10^{-5} m^{-1} (a typical value for rigid fractures), a steady state is reached at a very early time. The relatively large specific storage value we use is more consistent with the time scales observed in the field data. The imposed injection flow rates are based on the averaged measured data. Replacement apertures specified in the inversion are 1.0696×10^{-3} , 4.9646×10^{-4} , 2.3044×10^{-4} and 1.0696×10^{-4} m, which correspond to element transmissivities of 10^{-3} , 10^{-4} , 10^{-5} and 10^{-6} (m^2/sec), respectively. Other parameters such as the temperature schedule and time steps are held fixed for all the calculations in the following section.

5.3 Results and Discussion

The inversion was used four times to find configurations that matched the pressure transient data. Each inversion was run with the same temperature schedule from the initial temperature of 30, for 10,000 iterations. Four inversion results are shown in Figure 28 to Figure 31. The first result was found using a cluster size of 10 with isotropic cluster shapes, the second and third results were found using cluster sizes of 20 and 40 with isotropic cluster shapes. The fourth result was found using a cluster size of 20 with anisotropic cluster shapes, with strikes at N30W and N60E. Note that the transmissivity of the outer region of the grid (beyond a radius of 37.5 meter from the origin) was held at 10^{-5} (m^2/sec) throughout the inversion. We can observe that annealing results consistently show a strong connection with a trend of about N30W to N45W, between wells SE-2 and SW-1. We can also see that SW-3 is relatively isolated in all results. This is because SW-3 had a very low interference response from all of three well injections (see Figure 32). Aside from these two features, the results of isotropic cluster shapes all seem to show relatively

Table 4 Annealing energies for several cluster sizes (Raymond test site data)

Cluster Size	10	20	40	20 (anisotropic cluster)
Initial Energy	3.33×10^3	3.33×10^3	3.33×10^3	3.33×10^3
Minimum Energy	3.205	4.109	5.795	4.187
Number of Iterations for Convergence	9469	9225	9937	9964

random transmissivity distributions, while the result of anisotropic cluster shapes show the N30E to N45E trend of low transmissivity zones.

Table 4 gives the cluster size along with the initial energy, the minimum energy and number of iterations at which the minimum energy was reached. The lowest minimum energy, 3.205, was reached with a cluster size of 10. As the cluster size decreases, the minimum energy also decreases. Comparison of pressure transients between observed data and calculated data for the minimum energy configuration for the case of cluster size = 10 is illustrated in Figure 32. A relatively good match between the observed and calculated data was obtained. In this figure, symbols and lines represent the observed and calculated data, respectively.

In order to evaluate the inversion results, we conducted a cross validation test. We modeled two other injection tests, with injection into wells SW-2 and SW-3, which were not included in the inversion as injection data. Table 5 shows the case number (CV1 ~CV4) along with the cluster size and its shape, and the minimum energy for well SW-2 and SW-3 injection. For injection into well SW-2, the energy becomes approximately 20 for cases CV1 and CV2, while the minimum energy becomes approximately one hundred for cases CV3 and CV4. For injection into well SW-3, all of the energies obtained are approximately 280, which indicates that the annealing solutions do not predict the behavior around well SW-3 accurately. This might be caused by the fact that the SW-3 had a very low interference response from all of three injection wells used in the inversion. If we include the well SW-3 injection data in the inversion, reliable transmissivity distribution around SW-3 will be obtained.

Table 5 Predicted energies of injections into well SW-2 and SW-3 for four annealing solutions

Case Number	CV1	CV2	CV3	CV4
Cluster Size	10	20	40	20
Cluster Shape	Isotropic	Isotropic	Isotropic	Anisotropic
Energy for SW-2 Injection	22.39	26.75	99.82	91.25
Energy for SW-3 Injection	285.53	286.51	285.37	286.33

The cross validation result of the injection into well SW-2 indicates that the configuration obtained using a cluster size of 10 or 20 with isotropic cluster shapes (cases 5A and 5B) are the most suitable among the inversion results. This suggests that the transmissivity distribution of the upper conductive zone of the Raymond test site is rather isotropic. Moreover, the practical range of the spatial correlation of the site can be estimated from the optimal cluster size, and it appears to be approximately 5 ~ 10 meters. Semi-variograms of each solution are given in Figure 33. Practical ranges of the spatial correlation that correspond to the cluster sizes of 10 and 20 are approximately 6.0 m and 10.0 m, respectively.

As mentioned before, the system energy, reflecting the difference between the observed and calculated pressure transients, is the only function being evaluated and minimized in this algorithm. We showed that the information of spatial correlation provides additional available data for determining the transmissivity distribution model of the site, in that the optimal cluster size for an inversion approximately corresponds to the spatial correlation length. This means that we are able to include information about spatial correlation into the inversion as a cluster size (and/or cluster shape) if that parameter is known a priori. Even if parameters describing spatial correlation are unknown a priori, the process of selecting reasonable cluster size using this algorithm may lead us to estimate a possible range of spatial correlation.

6 Summary and Conclusion

The inversion of hydraulic pressure transient well testing using Cluster Variable Aperture (CVA) simulated annealing has been applied as an inverse technique to construct fluid flow models in fractured formations. Sensitivity studies using a simple fracture model with eight wells show that, in general, it is necessary to conduct drawdown (or injection) tests with at least three wells in order to reconstruct the fracture network with a transmissivity contrast of one order of magnitude, particularly when the cluster size is not known a priori. As the real flow geometry will be more complicated than the cases considered, it is recommended to conduct pumping (or injection) testing on more than three wells.

We investigated the postulated use of the correlation length (or observable information regarding the scale of the heterogeneity) as an input parameter to enhance the reliability or convergence of the inversion. It reveals that the optimal cluster size seems to be about 20 ~40 % of the practical range of spatial correlation for the transmissivity. This suggests that analyzing the optimal cluster size which gives the minimum energy makes it possible to estimate the spatial correlation parameters for the transmissivity distribution in the region.

Since hydraulic inversion results are inherently non-unique, it is important to utilize any relevant information. We showed that a priori information such as fracture strike and scale can also be taken into account to conduct inversions with the annealing by specifying this information in the cluster shape. This can be effective to enhance the reliability or convergence of the inversion.

We applied the inversion algorithm to the hydraulic well test data at the Raymond test site. We estimated that the practical range of the spatial correlation of transmissivity to be 5 ~ 10 meters, since a cluster size of 10 to 20 gives the minimum energy. This result was confirmed by the cross validation of the injection test data which was not used in the inversion.

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Figure 1 Example of synthetic fracture network. Each element represents a fracture. Circles on the upper left figure show the well locations.

Figure 2 Inversion result (minimizing element configuration) for NE-1. Open circles show location of observation wells and black circle show well location of the pumping well on the upper left figure.

Figure 3 Drawdown match for the minimizing configuration for NE-1.

Figure 4 Energy (error) versus iteration for NE-1.

Figure 5 Inversion result (minimizing element configuration) for NE-2.

Figure 6 Inversion result (minimizing element configuration) for transmissivity contrast of 10; (a) NE-3, (b) NE-4, (c) NE-5, (d) NE-6.

Figure 7 The synthetic fracture network model with 1.0 m correlation length (case 1A): (a) continuous transmissivity distribution and (b) its equivalent discrete distribution. Filled circles indicate pumping wells.

Figure 8 The synthetic fracture network model with 1.67 m correlation length (case 1B): (a) continuous transmissivity distribution and (b) its equivalent discrete distribution. Filled circles indicate pumping wells.

Synthetic fracture network (case 1A)

Synthetic fracture network (case 1B)

Figure 9 Omnidirectional semi-variogram of transmissivity distribution for the synthetic fracture network (case 1A and 1B).

Figure 10 Inversion results (minimizing element configuration) for case 1A shown for different cluster sizes. The discretized synthetic fracture network is also shown for reference.

Figure 11 Inversion results (minimizing element configuration) for case 1B for different cluster sizes. The discretized synthetic fracture network is also shown for reference.

Figure 12 Drawdown match of minimizing configuration for case 1A for cluster size 10, 20, 30 and 40.

Figure 13 Energy versus iteration for case 1A for cluster size of 10, 20, 30 and 40.

Figure 14 Omnidirectional semi-variogram of transmissivity distribution of the inversion results for case 1A for different cluster sizes.

Figure 15 Omnidirectional semi-variogram of transmissivity distribution of the inversion results for case 1B for different cluster sizes.

Figure 16 The synthetic fracture network model with 1.67 m correlation length (case 2). (a) continuous transmissivity distribution and (b) its equivalent discrete distribution. Filled circles indicate pumping wells.

Synthetic fracture network

Cluster size = 10

Cluster size = 20

Cluster size = 30

Cluster size = 40

Cluster size = 60

Figure 17 Inversion results (minimizing element configuration) for case 2 for different cluster sizes. The discretized synthetic fracture network is also shown for

Figure 18 Omnidirectional semi-variogram of the inverted transmissivity distribution for case 2 for different cluster sizes.

Cluster of Random Shape

Anisotropic Cluster

Figure 19 Example of an isotropic fracture cluster (left) and an anisotropic fracture cluster (right).

Figure 20 The synthetic fracture network model with 2.0 m correlation length in the X direction and 4.0 m in the Y direction: (a) continuous transmissivity distribution and (b) its equivalent discrete distribution. Filled circles indicate pumping wells.

Synthetic fracture network

Isotropic cluster

Anisotropic cluster

Figure 21 Inversion results (minimizing element configuration) for the anisotropic transmissivity field. The discretized synthetic fracture network is also shown for reference.

Figure 22 Semi-variogram of the inverted transmissivity distribution (isotropic cluster shape).

Figure 23 Semi-variogram of the inverted transmissivity distribution (anisotropic cluster shape).

Figure 24 Well configuration at the Raymond test site. Large dots and small dots indicate 25 cm and 15 cm diameter wells, respectively.

Figure 25 Conceptual model of hydrogeologic structure at the Raymond test site (after Cohen, 1995).

Figure 26 Unfolded view of the wells and typical packer locations during the systematic injection tests. Note the length of the injection interval is much shorter than the packed-off zones in non-injection wells.

Figure 27 Initial model for the Raymond test site and well locations.

Figure 28 Inversion results (minimizing element configuration) with cluster size 10 for the Raymond test site.

Figure 29 Inversion results(minimizing element configuration) with cluster size 20 for the Raymond test site.

Figure 30 Inversion results (minimizing element configuration) with cluster size 40 for the Raymond test site.

Figure 31 Inversion results (minimizing element configuration) with cluster size 20 and anisotropic shapes for the Raymond test site.

Drawdown match at annealing minimum

Figure 32 Pressure match for minimizing configuration for cluster size of 10.

Cluster size = 10

Cluster size = 20

Figure 33 Omnidirectional semi-variogram of transmissivity distribution of the inversion results for the Raymond test site for cluster size = 10 and 20.

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