The Effects of General Relativity on Core Collapse Supernovae

K. R. De Nisco,† S. W. Bruenn,† and A. Mezzacappa†

† Physics Department, Florida Atlantic University, Boca Raton, FL 33431
‡ Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831

Abstract. The effects of general relativity (GR) on the hydrodynamics and neutrino transport are examined during the critical shock reheating phase of core collapse supernovae. We find that core collapse computed with GR hydrodynamics results in a substantially more compact core structure out to the shock, the shock radius at stagnation being reduced by a factor of 2. The inflow speed of material behind the shock is also increased by a factor of 2 throughout most of the evolution. We have developed a code for general relativistic multigroup flux-limited diffusion (MGFLD) in static spacetimes and compared the steady-state neutrino distributions for selected time slices of post-bounce models with those computed with Newtonian MGFLD. The GR transport calculations show the expected reductions in neutrino luminosities and RMS energies from redshift and curvature effects. Although the effects of GR on the hydrodynamics and neutrino transport seem to work against shock revival, the core configurations are sufficiently different that no firm conclusions can be drawn, except that simulations of core collapse supernovae using Newtonian hydrodynamics and transport are not realistic.

1. Introduction

General relativity (GR) is an essential component in the realistic modeling of core collapse supernovae because of the very strong gravitational fields in the vicinity of the collapsed core of a star. Hydrodynamics and neutrino transport are closely connected in this problem, and as we will show, GR can have a profound effect on each of these, especially in the critical phase of shock reheating. The detection of neutrinos from supernova 1987A (Bionta et al. 1987, Hirata et al. 1987) and the hope of detecting neutrino signatures from future supernovae, with next-generation detectors, is additional motivation for an accurate general relativistic treatment of the neutrino transport in numerical simulations.

1 A.M. was supported at the Oak Ridge National Laboratory, which is managed by Lockheed Martin Energy Research Corporation under DOE contract DE-AC05-96OR22464. S.W.B. and A.M. gratefully acknowledge the hospitality and support of the Institute for Theoretical Physics in Santa Barbara, which is supported by the National Science Foundation under contract PHY94-07194.
We have developed a code for general relativistic multigroup flux-limited diffusion (MGFLD) that computes the neutrino transport in a static background metric. This is the first step in the development of a fully GR MGFLD code. The GR metric used is \( ds^2 = a^2c^2dt^2 - b^2dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \). This metric is allowed to evolve in the hydrodynamics calculations, but is “frozen” in the transport calculations, which are then performed treating the metric as static. Two precollapse models, a 15\( M_\odot \) model (S15s7b) and a 25\( M_\odot \) model (S25s7b) (Woosley & Weaver 1995; Weaver & Woosley 1997), were evolved through core collapse, bounce, and to approximately 800 ms after bounce in three sets of simulations. The first was with Newtonian hydrodynamics and Newtonian radiation transport, the second was with GR hydrodynamics and Newtonian transport, and the third was with both GR hydrodynamics and transport. In addition to these three sets of simulations, stationary-state neutrino distributions were computed for various post-bounce time slices in these models in order to isolate the effects associated with each of the metric components.

**Figure 1.** Shock and gain radii vs. post-bounce time for model S15s7b. Both cases are calculated with Newtonian radiation transport.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
2. The Role of General Relativity

The effects of GR are seen quite clearly in the hydrodynamic evolution of the initial models. GR hydrodynamics produces a much more compact post-bounce structure than Newtonian hydrodynamics. After $t_{pb} = 0.4$ s in both models ($t_{pb}$ being the post-bounce time), the radius of the shock and the gain radii are reduced by a factor of 2 in the GR calculations, as shown in Figure 1.

Also strongly affected by GR is the flow velocity between the shock and the proto-neutron star. Because matter falls through a greater potential well to reach the shock in the GR calculation, GR preshock and therefore postshock velocities are larger than their Newtonian counterparts, again by a factor of approximately 2.

The main effect of GR on the neutrino rms energies is the redshift of the neutrinos after they decouple from the matter, which is governed by the metric parameter $a$. The rms energies are reduced by a factor of $a$ evaluated at the $\nu$-sphere. A smaller effect is a slight outward shift of the $\nu$-sphere resulting from the (non-unity) value of the metric parameter $b^{-1}$, which causes the neutrinos to decouple outside the $\nu$-sphere at a lower temperature. These effects are shown in Figure 2. Also shown are the independent

---

**Figure 2.** RMS energy vs. radius for the $\nu_e$'s in model S15s7b. This figure contrasts stationary state GR and Newtonian transport at $t_{pb} = 114$ ms.
effects of $a$ and $b$ on the neutrino transport.

GR reduces the neutrino luminosity by three effects: redshift, governed by the metric parameter $a$; the difference in the local clock rates at the emission surface and the observer radius, also governed by the metric parameter $a$; and the reduction of the neutrino flux, governed by the metric parameter $b^{-1}$. All three of these effects are of roughly equal magnitude and reduce the luminosities by a total factor of $a^2b^{-1}$ evaluated at the $\nu$-sphere, as shown in Figure 3. This figure also shows the independent effects of $a$ and $b$ on the stationary-state neutrino transport.

The shock heating rate is proportional to the product of the luminosity and the square of the $\nu_\alpha$ rms energy. This means that any percentage change in these quantities add together. Looking at Figures 2 and 3, we see a decrease of 8\% in the $\nu_\alpha$ luminosity and a 3\% reduction in the $\nu_\alpha$ rms energy. These combine to give a 14\% reduction in the heating rate. Similar results are obtained for the $\nu_\beta$’s.

These are significant differences [e.g., see Burrows & Goshy (1993), Janka & Müller (1996), Mezzacappa et al. (1998), and Messer et al. (1998)] and serve to illustrate the point that modeling core collapse supernovae without GR hydrodynamics and transport leads to results that cannot be interpreted as realistic. For more information on this
subject, the reader is referred to Bruenn, De Nisco, & Mezzacappa (1998).

3. Nucleosynthesis

r-process nucleosynthesis is believed to occur in a neutrino-driven wind emanating from the proto-neutron star after the successful launch of the shock. The r-process yields are a function of the $\nu_e$ and $\bar{\nu}_e$ luminosities and rms energies. The luminosities affect the entropy, mass loss rate, and expansion time scale associated with the wind, and the rms energies determine the neutronization of the wind. As an example of the impact of GR on the r-process, consider its effect on the rms energies. Because the $\bar{\nu}_e$-sphere lies below the $\nu_e$-sphere, the $\bar{\nu}_e$'s suffer a greater emergent redshift. As can be seen in Figure 4, general relativistic transport and hydrodynamics affects the ratio of the $\nu_e$ and $\bar{\nu}_e$ rms energies. This differential redshifting affects, in turn, the ratio of the number of $\nu_e$'s to $\bar{\nu}_e$'s, at a given neutrino energy, and therefore, the neutronization of the wind. This suggests that general relativistic hydrodynamics and transport will be required to obtain accurate r-process yields.
References

Messer, O. E. B., Mezzacappa, A., Bruenn, S. W., & Guidry, M. W., this volume
Weaver, T. A. & Woosley, S. E. 1998, in preparation