Finite-Element Analysis of Earing Using Non-Quadratic Yield Surfaces

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This paper was prepared for submittal to NUMIFORM'95
The 5th International Conference on Numerical Methods in Industrial Forming Processes
Cornell University, Ithaca, NY

June 18, 1995

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ABSTRACT: During deep draw cupping, the phenomenon known as earing may occur as the cup wall is formed, resulting in a periodic variation of cup wall height around the perimeter of the finished cup. This is generally due to planar anisotropy of flow in rolled sheet product. It is generally observed that the anisotropy parameter $R$ will vary in the plane of the sheet when ears are observed in cupping, with a parameter $\Delta R$ describing the variation of $R$ in the plane of the sheet. For many common textures in face-centered and body-centered materials, the ears form relative to the sheet rolling direction at $0^\circ$ and $90^\circ$ around the perimeter if $\Delta R > 0$, and at $-45^\circ$ and $+45^\circ$ if $\Delta R < 0$. There is extensive experimental evidence that ear height shows a linear correlation with $\Delta R/R$, but attempts to duplicate this using the finite-element method are highly dependent on both the methodology and yield surface used. It was shown previously that using a coarse mesh and the quadratic Hill yield surface tends to greatly under predict earing. In this study, we have used two different finite-element codes developed at LLNL to examine the predicted earing using both quadratic Hill and alternative non-quadratic yield surfaces. These results are compared to experimental data and conclusions drawn about the most desirable closed-form yield surfaces to duplicate the observed earing phenomena.

1. INTRODUCTION AND MOTIVATION

The forming of thin sheets is an important manufacturing process which is used in the production of a wide range of products from beverage cans to auto body parts to lightweight airframe and military aerospace components. There are two primary goals for the engineering analysis of a sheet metal forming process. First, analysis aims to reduce the trial and error in tooling and process design, and thereby reduce material waste and lead times to produce a new part. Second, analysis aims to influence the design of the desired part for ease of manufacture. Both of these goals ultimately lead toward the objective of faster production of better parts at minimum cost.

One of the causes of material waste in stamping is the often unanticipated formation of ears on the periphery of the blank from which the part is formed. The formation of ears in simplest form results from the stamping of a circular blank into a cylindrical cup. The essence of this process is illustrated by and studied numerically with the tooling and blank finite-element meshes in Fig. 1. This tooling is used with codes such as the explicit finite-element code DYNA3D to follow the formation of cylindrical cups, with ear formation due to planar anisotropy as shown in Fig. 2. The final cup in Fig. 2 clearly illustrates the potential material waste due to ear formation on the cups. The magnitude of the observed earing is often similar to the level as shown in Fig. 2. However, it is usually below this level for two reasons. The first is that the example shown was intentionally given contrived material properties of $R = 4$, $Q_{ab} = 1$, and $P = 4$, where $R$, $Q_{ab}$, and $P$ are the width-to-thickness strain ratios in tension tests on the sheet at $0, 45, 90$ degrees to the rolling direction. This level of planar anisotropy is higher than commonly observed in most body-centered cubic (bcc) or face-centered cubic (fcc) metals, but may be sometimes achieved in other crystal structures. A second reason for the large ears shown in Fig. 2 is the use of the 1948 Hill criterion (Hill, 1948) for anisotropic flow. It has been observed (Yang and Kim, 1986) that use of the 1948 Hill criterion can lead to overprediction of earing. Thus, an important motivation for this work was to examine, with fully 3-D simulations, the predicted vs. observed earing using both the 1948 Hill quadratic yield criterion and an alternative (Hosford, 1979) criterion in an effort to allow more accurate predictions for this simple case of cylindrical cups, as well as the stamping of complex parts where the proper blank shape to counteract earing is not at all obvious.
In this study we have chosen to begin with the Lawrence Livermore National Laboratory (LLNL) collaborator version of DYNA3D, our explicit, transient dynamics Lagrangian finite-element code (Whirley and Hallquist, 1991). Even though stamping processes fall on a time scale perhaps more suited in principle to quasi-static, implicit codes such as NIKE3D (Maker, Ferencz, and Hallquist, 1991).

Fig. 1. Tooling (blank and ‘rigid’ punch, die, and blankholder) used to model cup drawing.

However, this traditional implicit method produces a system of nonlinear algebraic equations which are then solved at each load step using a linearization and iteration technique. The implicit formulation yields a large linear matrix equation which must be solved at each iteration of each load step. Since many sheet forming problems have large bandwidths arising from large workpiece/tooling contact areas, these matrices may tax even the storage capacity of supercomputers.

Fig. 2. DYNA3D simulation of earing development during deep drawing, $R, Q_{ab}, P = 4, 1, 4$, and $a = 2$. 
This situation is being overcome with recent improvements in solution methods and storage capability, yet there remains a constant rivalry between the use of explicit and implicit codes for sheet stamping. We find it is often efficient to implement a methodology in our explicit DYNA family first, bearing in mind that the formulation must be appropriate for subsequent implementation into the implicit NIKE family as well. Such is the case in this work, which examines the usefulness of a new yield surface implemented into an internal W-DYNA version of our DYNA code family.

It was shown in a previous related work (Whirley, Engelmann, and Logan, 1992) that the cupping problems shown below can be solved effectively by applying loads slowly to minimize dynamic effects, so a nearly quasistatic solutions may be obtained.

This work uses implementations of anisotropic yield surfaces in our DYNA family of codes to show that alternate, non-quadratic yield criteria can provide more accurate predictions of the earing observed in stamping processes.

2. YIELD SURFACE IMPLEMENTATION

This section gives a brief overview of the explicit finite element approach for solving the equations of motion, and describes the form and implementation of the yield surfaces implemented into the LLNL public DYNA3D [1948 Hill as Model 33] and the internal W-DYNA [1979 Hosford as Model 33x]. First, we begin with the continuum equations, written as:

\[ \nabla \cdot \sigma + \rho b = \rho \ddot{u} \]  

(1)

where \( \sigma \) is Cauchy stress, \( b \) is the body force density field, \( \rho \) denotes the current material mass density, \( u \) is the displacement field, and a superimposed dot denotes differentiation with respect to time. Applying the finite element method to spatially discretize Eq. (1) yields a coupled set of nonlinear ordinary differential equations in time,

\[ M \ddot{u} = f^{ext}(t) - f^{int}(u, t), \]  

(2)

where \( M \) is a mass matrix, \( u \) is now a vector of nodal displacements, \( f^{ext} \) is a vector of externally applied time-dependent nodal forces, \( f^{int} \) is a vector of internal nodal forces arising from stresses existing in the elements, and \( t \) is time. Even if higher-order differential operators are included, such as those arising in beam, plate, and shell formulations, the resulting set of ODEs still retains the form of Eq. (2).

At this point the assumptions of explicit analysis are introduced to numerically integrate these ODEs in time. Almost all nonlinear explicit finite element programs integrate Eq. (2) using the central difference method because of its accuracy, simplicity, and computational efficiency. Assume that all quantities are known at time \( t = t_n \) and it is desired to advance the solution to time \( t = t_{n+1} \). The first step is to find the acceleration \( \ddot{a}_n = \ddot{u}(t_n) \) from the discrete version of Eq. (2) at time \( t = t_n \):

\[ M \ddot{a}_n = f^{ext}_n - f^{int}_n, \]  

(3)

where \( f_n = f(t_n) \). Introducing a nodal lumped mass approximation, \( M \) becomes a diagonal matrix, and the evaluation of \( a_n \) from Eq. (3) is very inexpensive since the equations are now uncoupled and all quantities on the right-hand side are known. The central difference method gives update equations for the nodal velocities \( v \) and displacements \( u \) as

\[ v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + a_n \Delta t \]  

(4)

\[ u_{n+1} = u_n + v_{n+\frac{1}{2}} \Delta t \]  

(5)

Now that the updated kinematic variables are known, the next step is to evaluate the forces on the right-hand-side of Eq. (3) at time \( t = t_{n+1} \). Since external loads are usually prescribed functions of time, the evaluation of \( f^{ext}_{n+1} \) is straightforward. The bulk of the computations within a time step are expended to evaluate the internal force \( f^{int}_{n+1} \).

Computation of \( f^{int}_{n+1} \) begins with the calculation of the rate of deformation \( d_{n+\frac{1}{2}} \) from

\[ d_{n+\frac{1}{2}} = \frac{1}{2} \left[ \nabla v_{n+\frac{1}{2}} + (\nabla v_{n+\frac{1}{2}})^T \right] = B v_{n+\frac{1}{2}} \]  

(6)

where \( \nabla v \) denotes the gradient of the velocity with respect to the geometry at time \( t = t_{n+1} \), and \( B \) is the "strain-velocity operator." Next, the updated Cauchy stress \( \sigma_{n+1} \) is found from

\[ \sigma_{n+1} = \sigma_n + \int_{t_n}^{t_{n+1}} \ddot{\sigma} dt. \]  

(7)
where $\sigma$ is computed from an objective stress response function using the rate of deformation $d_{n+1}$ and material history variables. This incremental formulation easily accommodates material non-linearities such as elastoplasticity and viscoplasticity. Finally the new internal force vector for an element $e$ is found from the updated stresses using

$$f_{n+1}^{int} = \int_{\Omega_e} B^T \sigma_{n+1} d\Omega_e, \quad (8)$$

and the global force vector $f_{n+1}^{int}$ is found by assembling contributions from all elements. This completes the update of all quantities from time $t=t_n$ to time $t=t_{n+1}$. The stress tensor term in eqn. (8) depends in principle on the material constitutive equation chosen. One of the most straightforward and commonly used is the 1948 Hill equation for anisotropic plastic flow:

$$\sigma^2 = \frac{F(\sigma - \sigma_o)^2 + G(\sigma - \sigma_o)^2 + H(\sigma - \sigma_o)^2 + D}{R + 1} \quad (9)$$

Eqn. (9) relates the effective stress to the three normal components of Cauchy stress, with the term $D$ containing the shear stress terms:

$$D = 2L\sigma_{bc}^2 + 2M\sigma_{oa}^2 + 2N\sigma_{ab}^2 \quad (10)$$

The values for the constants in Eqns. (9) and (10) can be expressed in terms of the strain ratios $R$, $Q$, and $P$ as described above, with the following additional relations needed:

$$F = R/P \quad (11)$$
$$G = 1 \quad (12)$$
$$H = R \quad (13)$$
$$L = (Q_{bc} + \frac{1}{2})(R + 1) \quad (14)$$
$$M = (Q_{oa} + \frac{1}{2})(R + Z) \quad (15)$$
$$N = (Q_{ab} + \frac{1}{2})(1 + Z) \quad (16)$$

In addition to being comparatively straightforward to implement, the quadratic 1948 Hill criterion permits the relatively simple calculation of the ratio, $X(\theta)/X$, of the yield stress in a direction at an angle in the plane of the sheet to the rolling direction, as well as the calculated $R$-value in that direction:

$$X(\theta) = \left[ \frac{R + 1}{2Ns^2c^2 + R(c^4 - s^4) + c^4 + Zs^4} \right]^{\frac{1}{2}} \quad (17)$$

$$R(\theta) = \frac{R + ((2Q_{ab} + 1)(1 + Z) - Z - 1 - 4R)s^2 c^2}{Zs^2 + c^2} \quad (18)$$

where:

$$c = \cos(\theta)$$
$$s = \sin(\theta) \quad (19)$$

Eqns. (17)-(18) are relatively easy to comprehend, but show several trends that are not usually borne out by experimental data. For example, at 45 degrees to the rolling direction, 1948 Hill predicts (if $R=P$):

$$X(45^\circ) = \left[ \frac{R + 1}{Q_{ab} + 1} \right]^{\frac{1}{2}} \quad (20)$$

Eqn. (20) shows a high dependence of yield stress on orientation in the plane of the sheet. This dependence is a likely factor in overprediction of earing as in Fig. 2, as the strong material in the 45 degree direction tends to pull in to form the wall of the punch (forming a deep trough), while compressing the 0 degree and 90 degree walls (forming high ears). One yield criterion which in principle should show better agreement with experiment in the 1979 Hosford equation, extending 1948 Hill to a non-quadratic form with values of the exponent $a$ in the range of $a=8$ for fcc (Hosford, 1979), and $a=6$ for bcc (Logan and Hosford, 1980) metals:

$$\sigma^a = \frac{F(\sigma - \sigma_o)^a + G(\sigma - \sigma_o)^a + H(\sigma - \sigma_o)^a}{R + 1} \quad (21)$$

This equation predicts much milder dependencies of strength ratios in stress states and directions other than tension in the rolling direction. For example, in biaxial tension, where both $\sigma_e = \sigma_e = B$, both 1948 Hill and 1979 Hosford predict the relatively simple value:

$$\frac{B}{X} = \left( \frac{R + 1}{Z + 1} \right)^{\frac{1}{2}} \quad (22)$$

Clearly, values of $a > 2$ will give a much milder dependence often observed experimentally. Further, if we assume the $R$-value orientation dependence as in eqn. (18), the dependence of uniaxial flow stress takes on a much milder dependence. This ratio is important in the flange of the cup during draw-in, and thus
indicates promise for greater accuracy in earing calculations. However, difficulties do indeed arise in the implementation of the 1979 Hosford criterion for cases in other than principal stress/strain space. This stems from the lack of shear terms in the criterion. Eqn. (21) must remain in principal stress space to be used without spurious results and non-convexity problems. To do so, we must make several approximations in updating the Cauchy stress tensor (expressed as a vector of six) when using eqn. (21). First and foremost is a rotation to the principal stress coordinate space. This does not normally coincide with either the material (rolling and transverse direction) coordinate system, nor with the axes of principal strain. Our first assumption is that the axes of principal stress and strain coincide, although for planar isotropy we know that they normally will not. However, this assumption, which leads us to ignore cross-terms in the constitutive matrix, is believed to lead only to small errors for the degree of anisotropy observed in most sheet metals. This is the assumption we will use below in obtaining the updated stresses. The first step in doing so is a calculation of contact stresses and updated elastoplastic stresses as follows:

\[ \sigma_i^e = \sigma_i^e + C_i^e d\epsilon_i^e \]  
\[ \sigma_i^{ep} = \sigma_i^{ep} + C_i^{ep}(d\epsilon_j - d\epsilon_j^e) \]  

Here, \( d\epsilon_j^e \) are the elastic portions of the strain increment, and \( C_i^e \) is the elastic constitutive matrix. To obtain the updated stresses, we proceed further by applying the remainder of the strain increment \( (d\epsilon_j - d\epsilon_j^e) \) using the elastoplastic matrix \( C_i^{ep} \):

\[ C_i^{ep} = C_i^e - \frac{C_i^e q_i (C_i^e q_i)^T}{p_i q_i + q_i^T C_i^e q_i} \]  

The yield surface \( F \) directly affects the calculation of the matrix \( C_i^{ep} \), since

\[ q_i = \frac{dF}{d\sigma_i} \]  
\[ p_i = \frac{dF}{d\epsilon_i} \]

In the following section we will demonstrate the effect of the chosen yield surface (Eqn. (9) or (21)) on the extent of earing in cupping.

3. EARING DEVELOPMENT AND FINITE-ELEMENT PREDICTIONS

To demonstrate the correlation of DYNA earing simulations with experiments, we compare the extensive data obtained by (Wilson and Butler, 1962) with that obtained using simulations with either \( a = 2 \) or \( a = 8 \) in eqn. (21). Numerous runs with \( R = P \) were made as outlined in Table 1 below:

<table>
<thead>
<tr>
<th>( a )</th>
<th>( R )</th>
<th>( Q )</th>
<th>( P )</th>
<th>( h )</th>
<th>( \Delta h )</th>
<th>( \Delta R/R )</th>
<th>( \Delta h/h )</th>
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<tr>
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<td>4.0</td>
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<td>1.20</td>
<td>.407</td>
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<td>2.0</td>
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<td>0.67</td>
<td>.267</td>
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<td>0.40</td>
<td>.166</td>
</tr>
<tr>
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<td>7.08</td>
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<td>-0.67</td>
<td>-0.212</td>
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<tr>
<td>2</td>
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<td>1.0</td>
<td>7.18</td>
<td>-1.72</td>
<td>-0.67</td>
<td>-0.240</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>2.0</td>
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<td>7.26</td>
<td>-2.40</td>
<td>-1.20</td>
<td>-0.330</td>
</tr>
<tr>
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<td>1.0</td>
<td>4.0</td>
<td>7.62</td>
<td>2.34</td>
<td>1.20</td>
<td>.312</td>
</tr>
<tr>
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<td>1.0</td>
<td>4.0</td>
<td>7.84</td>
<td>1.54</td>
<td>1.20</td>
<td>.196</td>
</tr>
<tr>
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<td>1.0</td>
<td>2.0</td>
<td>7.64</td>
<td>0.73</td>
<td>0.67</td>
<td>.096</td>
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<tr>
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<td>1.0</td>
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These results are plotted as \( \Delta h/h \) vs. \( \Delta R/R \) in Fig. 3, along with the original data from Wilson and Butler. It is strikingly clear that the use of \( a = 2 \) (identically 1948 Hill) overpredicts earing by about a factor of two. In contrast, the use of \( a = 8 \) matches almost exactly the observed earing data; in fact \( a = 8 \) seems to underpredict earing slightly. However, recall that we were forced to ignore cross-terms in the matrix \( C_i^{ep} \) above when using \( a = 8 \). We do not need to do this when \( a = 2 \), but we can choose to do so to examine the effect. There is indeed some difference, as shown by the data labeled \( a = 2x \) in Table 1 and in Fig. 3. Thus, if the proper cross-terms existed for \( a = 8 \), we might expect to see slightly more calculated earing, and perhaps slightly overpredict the data instead of slightly underpredicting it. Either way, the value of \( a = 8 \) is much more accurate than \( a = 2 \).
Fig. 3. Plot of earing ($\Delta h/h$ vs. $\Delta R/R$) including Wilson and Butler data compared to DYNA results with $a=2$ and $a=8$.

Several variations of runs were made in the process of compiling the DYNA comparisons in Fig. 3. These included variations of the SVE (Selective Velocity Enhancement) factor from 20x to 50x, with very little difference noticed in earing. The runs presented here were run at SVE=20, and otherwise the parameters used previously (Whirley, Engelmann, and Logan, 1992). Meshing consisted of 24 nodes radially along the blank, and 9 nodes around the quarter-symmetry hoop direction from 0 to 90 degrees. Meshing of 12x9 or even 12x5 nodes gave reasonable ear development early on, but 5 nodes in the hoop direction showed difficulties in completing the cupping operation. A much more important fact is that in the DYNA3D codes there is obviously a degree of freedom in the hoop direction in the flange. Early simulations by the author (Logan, 1984) considerably under predicted earing, even though $a=2$ was used in the MARMOT code of that work. However, a key constraint of the MARMOT sheet forming code was the assumption of no displacement in the hoop direction. When sliding boundary planes were employed in DYNA to preclude motion in the hoop direction at $\theta=0$, 22.5, 45, 67.5, and 90 degrees, earing is cut drastically (see the data for '2m' in Table 1 and 'u2=0' in Fig. 3). This explains why the early MARMOT code under predicted earing even with 1948 Hill, and shows the importance of hoop displacement and shear in the earing process. This is borne out graphically in Fig. 4, which compares ear development at $R, Q, P = 4,1,4$ with $a=2$. In the upper view, hoop motion is allowed, and lines of constant $\theta$ become distorted as the ears form. In the lower view, hoop motion is precluded as described above, and both earing and overall $\theta$-direction motion are greatly reduced.

Fig. 4. Quarter-symmetry view of ear development in full 3-D (upper), and with hoop motion restricted (lower). Grid of original blank shown as background.
4. CONCLUSIONS

Using full 3D finite-element simulations of the cupping process, it is clearly demonstrated that use of the 1948 Hill criterion \( (a = 2) \) will greatly over predict earing compared to typical experimental data. This requires the use of a suitable mesh, and large motions in the hoop direction. Use of the 1979 Hosford criterion with \( a = 8 \) gives very good correlation with the data of Wilson and Butler. It is possible that other yield criteria such as the generalized Hill (Hill, 1979) or the tricomponent criterion (Barlat and Lian, 1989) would show similar improvements in agreement in earing calculations such as these.

ACKNOWLEDGMENTS

The author wishes to thank Dr. W.F. Hosford at U. Michigan for assistance in the \( a=2 \) implementation, and to T.P. Slavik of LLNL for help with the never ending tasks of tensor coordinate transformations. This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract W-7405-Eng-48.

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