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Solution Adaptive Methods for Low-Speed and All-Speed Flows

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Abstract

The goal of this work was to design new fast algorithms that could be used to solve fluid flows at all speeds by building upon the best approaches now available for solving very low speed flows and high speed flows. Furthermore the algorithms developed must be appropriate for use on complex moving geometries and for use with adaptive mesh refinement. The algorithms must also be extendible to chemically reacting (combustion) flows. To this end we have developed new methods for efficiently computing fluid problems that involve low-speed flows and problems that are a mixture of low-speed and high-speed flows. The algorithms have been implemented in 2D and 3D on moving overlapping grids and will be a fundamental component of the chemically reacting flow solvers that we are now developing for industrial applications.

Background and Research Objectives

The computation of low-speed (i.e., slightly compressible) flow is a significant computational challenge. It is of great importance since almost all industrial flow problems and many flows found in nature are low speed. The difficulty is directly related to the fact that a slightly compressible flow is, to first approximation, a combination of an incompressible flow together with rapidly moving sound waves. To compute the fine spatial structure of the vortices in an incompressible flow requires high spatial resolution but a relatively large time step, while computing the rapidly moving sound waves requires a very small time step but not as fine spatial resolution. Applying standard methods to the combined problem requires both a fine grid and a very small time step—thus making it infeasible to solve many problems of interest.

There are a number of approaches that have been tried to treat all-speed flows. The simplest is to use a standard explicit method for compressible flows. This approach quickly becomes too expensive when the “Mach number,” M , (the ratio of the fluid velocity, u to the speed of sound of the fluid c , $M=u/c$) becomes small since the time step required to keep the scheme stable will have to be proportional to the Mach number. To overcome this time step restriction one can try to use implicit schemes. These are well known to overcome the stability requirements of explicit schemes. An implicit scheme can

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help for moderately low Mach numbers although it will be quite expensive to apply to the full three-dimensional Navier-Stokes equations. Straight forward implicit schemes further suffer from the fact that the implicit matrix that needs to be inverted becomes highly skewed (far from symmetric), which can lead to loss of accuracy. To overcome this problem to some extent it is possible to “precondition” the matrix [1]. This has the effect of artificially slowing down the fast sound waves. However even the best preconditioners still have problems for very small Mach numbers.

Another approach to solving low Mach number flows is the method of artificial compressibility, first introduced by Chorin. In this approach an artificial time is introduced (and an artificial time scale) in order to slow down the fast sound waves. In this approach it is often difficult to choose the optimal new time scale and the approach is difficult to apply to time dependent flows. There is another approach for slowing down the sound waves by scaling the pressure gradient term in the momentum equations. This pressure gradient scaling method (PGS) is used in the KIVA combustion code [2], to some success although it is somewhat problematic to choose the appropriate parameters.

Starting from the works of [6],[7] and extended by others [4],[12],[13] it became apparent that it was not necessary to treat the entire system of equations implicitly (in 3D the compressible Navier-Stokes equations are a system of five coupled partial differential equations). Since only two of the five wave speeds associated with this system are large (corresponding to sound waves) it was possible to derive a nonlinear equation for the pressure (a sort of wave equation) that could be solved implicitly. This method worked fairly well although there could still be difficulties for very low Mach number and Mach numbers of order one (since the methods were not optimal when shocks were present).

The previous approaches have started from the fully compressible equations. Another class of methods use asymptotic expansions to derive equations valid for low Mach number. The basic idea here is that a slightly compressible flow consists of an incompressible piece ($M=0$) with a correction that is proportional to the Mach number. Thus one can compute corrections to incompressible flow by solving extra equations that are derived by formal asymptotic expansions, treating the Mach number as a small parameter [11],[5],[10].

In recent years, some very good numerical procedures have been developed for computing the solution to problems that are either truly “high-speed” (i.e., compressible) or truly incompressible (in which case the sound waves have been removed entirely). There are many problems, however, that are a combination of low-speed and high-speed flow. The high-speed flow may be restricted to a part of the domain or it may only appear

periodically in time. In an internal combustion engine, or pulsed combustor, for example, the flow is slightly compressible, except for part of a cycle in part of the domain.

The purpose of the work done here was to devise, analyze and implement techniques for the efficient computation of problems involving low-speed (slightly compressible) fluid flows and for problems that contain regions of both low-speed and high-speed (compressible) flow. We have built upon the efficient and accurate methods that have been developed for incompressible flows, such as the projection methods of Bell, Collela and Glaz [3] and the fourth-order accurate method of Henshaw, Kreiss and Reyna [9]. Incompressible flow methods such as these usually incorporate an efficient Poisson solver for the pressure. Typically when compressible solvers are adapted to the slightly compressible case they implement an implicit time stepping method that effectively solves a Poisson equation for the pressure but usually with a lot more work (requiring a nonlinear iteration, for example) than in the incompressible case and often the methods suffer from a high frequency numerical instability in the pressure. The instabilities can be treated by designing special discretizations (cf. the ICE method of Harlow and Amsden [6]) or by adding filters (cf. the method used in the KIVA [2]) but the work required to solve the implicit system or nonlinear pressure equation is significant. By adapting incompressible methods to the slightly compressible case we have devised a superior method for low speed flows.

To tackle the problem that involves both low- and high-speed flow we have chosen the approach of using a “solution-adaptive” hybrid method. We will adaptively choose the most appropriate method for the current state of the solution. In regions in space and time when the flow is low speed we will use the slow-speed method proposed here and described in more detail below; in regions of high-speed flow we will smoothly change over to a state-of-the-art method for compressible flows. In the past it was probably too difficult to take this solution adaptive approach since the programming details became overwhelming. We have taken advantage of our experience with new object-oriented programming techniques using new computer languages (C++) to allow us to implement much more complicated programs that not only solve different equations on complicated three-dimensional geometries but also allow for moving and adaptive grids. Moreover since we use the Overture (see publication 2) framework the codes will also be able to run on both serial and parallel machines.

Importance to LANL's Science and Technology Base and National R&D Needs

The algorithms developed in this work will be a fundamental component of the all-speed chemically reacting flow solver that we are now developing for a follow-on DOE funded project in collaboration with Caterpillar and Ford. This solver will handle complicated moving geometries using overlapping grids and the Overture object-oriented framework. Adaptive mesh refinement will allow the accurate and efficient computation of complicated flows. A major impact will be to provide state-of-the-art algorithms and numerical methods to industry.

These solvers will have many industrial and weapons technology applications beside these combustion computations. They will likely be used by one or more of the ASCI funded university initiatives. Researchers at the University of Utah are planning to use the work in their modeling of accidental fires and explosions while the University of Illinois has plans for utilizing the work in their rocket modeling.

Scientific Approach and Accomplishments

We have developed two new approaches for solving all-speed flows. The first approach is a novel extension of the Godunov-projection algorithm for incompressible flows. The velocity and pressure fields are decomposed into two new fields, one representing the incompressible part of the flow with corrections for the effects of compressibility that are of primary interest while the second part contains the information of the fast sound waves that are normally not of interest. To avoid having a small time step based on the sound speed the part of the solution containing the fast sound waves is advanced implicitly. The second approach we have developed couples a partially implicit method for low Mach number with a high-order Godunov method for intermediate and high Mach number. The partially implicit method requires the solution of only one scalar elliptic problem. In the limit of small Mach number it reduces to a nice discrete approximation to the incompressible equations. Thus in the low Mach limit the method for the full compressible equations is essentially just as efficient as if we were solving the incompressible equations (there is a little extra work required as there are some extra variables present in the compressible case). The solution adaptive all-speed solver has been implemented in 2 and 3 space dimensions for moving geometries using the Overture object-oriented framework. Extensions have been added for simulating all-speed chemically reacting flows.

A Projection Method for Low-Speed Flows

To some degree of approximation a low-speed flow has three basic components—an incompressible part, a compressible part containing effects on time and length scales that are often of practical interest, and a third part holding the very fast sound waves. In many applications this last part is not important to compute. To develop a numerical method useful in this regime of interest, we have made a decomposition of the equations of inviscid, compressible Navier-Stokes equations. These equations are rewritten in terms of a Hodge decomposition of the velocity field and in terms of auxiliary pressures. The Hodge decomposition allows one to separate the flow velocity into a part which is divergence free (basically the incompressible part of the flow) and a part that contains effects of compressibility. With the new equations, we will separate the flow into the divergence-free part, one that varies on a time scale determined by the flow speed, and a part that may contain fast sound waves. The former part may be advanced with time step determined solely by the flow speed. Since the fast sound waves are only present in the latter part, we can advance much of the flow using an explicit method, and apply an implicit method only to the compressible part.

The system of flow equations with time evolution equations for the density, velocity and pressure is thus extended to a system for the density, the nearly-incompressible velocity, a potential velocity, a potential pressure, and an acoustic pressure. The key to developing an efficient scheme for this new system is based on the knowledge of the characteristics of the different components. The potential velocity and acoustic pressure contain the very fast time scales associated with the speed of sound and since we are not interested in resolving this part of the solution we can use a time discretization (backward-Euler) that will damp these fast sound waves. This allows us to compute with a much bigger time step. The equations are coupled, but they can be advanced in a particular order so that the appropriate information is known when it is required. Care must be taken when defining discrete approximation to the projection operators and their boundary conditions since the solution to the extended system must be solutions to the original equations.

This new algorithm successfully extends the incompressible projection algorithm to the low Mach number case. Numerical convergence tests have verified the accuracy and convergence properties of the schemes (see publication 1). The scheme gives accurate results for low Mach number and a time step can be taken that is solely based on the flow speed; the fast sound waves are damped by the backward Euler time stepping.

A Solution Adaptive Approach for All-Speed Flows

The second approach we have developed couples a partially implicit method for low Mach number with a high-order Godunov method for intermediate and high Mach number. The implicit method solves the full Navier Stokes equations with the energy equation replaced by an equation for the pressure. A scaling of the equations for low Mach number in which the leading order term for the pressure is constant in space but of order $1/M^2$ shows that the terms in the equations that must be treated implicitly are the pressure gradient term in the momentum equations and the term involving the pressure times the divergence of the velocity in the pressure equation. All other terms can be treated explicitly if desired. To solve these equations in an efficient way we first discretize the equations in time using a backward differentiation formula. This time discrete implicit system (for the components of the velocity and the pressure) can be algebraically rearranged to eliminate the velocity. The result is a single scalar equation for the pressure. The big advantage of this approach is thus that the implicit method requires the solution of only one scalar elliptic problem for the pressure, even though the velocity is treated implicitly as well. Once the pressure is known the velocity can be computed. The second advantage of this technique is that the discrete approximation to the pressure can be chosen to be an accurate scheme with a compact stencil. This prevents the appearance of the checkerboard instability that plagues many similar approaches since the pressure equation they derive has a wider stencil that allows every other point to decouple. In the limit of small Mach number the scheme reduces to the approach we prefer for solving the incompressible equations. An important issue here is that of choosing the boundary conditions. This is a difficult issue since the limiting incompressible equations have different boundary conditions from the compressible ones, but as known from previous analysis it is of primary importance to get the correct boundary conditions. However we have shown how to choose the boundary conditions and thus in the low Mach limit the method for the full compressible equations reduces to the incompressible case and is essentially just as efficient as if we were solving the incompressible equations (there is a little extra work required as there are some extra variables present in the compressible case).

This solution adaptive approach has been incorporated into a general purpose solver for overlapping grids with moving geometries in two- and three-space dimensions. The solver uses the Overture class libraries that we have developed, which provide a high-level interface to solving PDEs on moving overlapping grids (see publication 3). This means that there is significant reuse of code between the solvers and that future solvers will be easier to write. The requirements of the all-speed solvers have also influenced the design and capabilities of the Overture library. Those capabilities that are more generic and that

can be used in other applications are added to the framework, rather than being only usable by the all-speed flow solver. Thus, for example, Overture contains implementations of the many different “elementary” boundary conditions from which the all-speed solver boundary conditions are built (see publication 4). Significant progress has also been made toward treating chemically reacting flows in a quite general fashion by utilizing the Chemkin package. This work has demonstrated that the algorithm can be generalized to the more difficult case when chemical reactions are present.

Publications

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Figure 1: Results from the all-speed projection algorithm. Top: the grid used in the converging channel computation. Bottom: the Mach number Ma at $t=3.0$. $u_{\infty} = 4.0$.

Figure 2: A comparison of the all-speed flow solver (top) versus an incompressible flow solver (bottom) for flow past a cylinder. The horizontal component of the velocity is shown.