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A New Method for Calculation of Low-Frequency Coupling Impedance*

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A NEW METHOD FOR CALCULATION OF LOW-FREQUENCY COUPLING IMPEDANCE

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In high-energy proton accelerators and storage rings the bunch length is typically at least a few times larger than the radius of the vacuum chamber. For example, the SSC will have an rms bunch length above 6 cm and a beam-pipe radius below 2 cm. The main concern for beam stability in such a machine is the low-frequency impedance, i.e., the coupling impedance at frequencies well below the cut-off frequency of the vacuum chamber. In the present paper we develop a new analytical approach for calculation of the low-frequency impedance of axisymmetric structures that allows us to give quick and reliable estimates of contributions to the impedance from various chamber discontinuities. Simple formulae for the longitudinal impedance of some typical discontinuities are obtained.

1 INTRODUCTION

The low-frequency impedance of axisymmetric structures has been studied by different approaches. In Reference 1 a matching method was applied to study the case of a beam pipe with a cylindrical cavity and a formula for the inductance of a small pill-box was derived. The numerical solution of the Maxwell equations in time-domain with the TBCI code² allows one to obtain the wake potential of a bunch in such structures. The dependence of the inductance of small pill-boxes and irises on their geometrical parameters was derived from fitting numerical results obtained by means of TBCI in Reference 3. In a similar way, formulae for the inductance of triangular-shaped insertions were obtained in Reference 4 using the semianalytical integral-equation approach. While practically important, these results do not represent a regular method for obtaining impedances of different structures. The present paper aims to fill this apparent gap and to develop a theory that allows effective derivation of the low-frequency impedance for small discontinuities on the beam pipe.

We will consider axisymmetric structures that consist of an infinitely long, perfectly conducting smooth pipe that has a localized discontinuity (e.g., a pill-box or an iris) on it. The frequency under consideration is much below the cutoff frequency of the pipe, $\omega \ll c/b$. In this limit, we show that the calculation of low-frequency impedance is reduced to solution of two quasistatic problems—electrostatic and

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magnetostatic ones. Assuming additionally that dimensions of the discontinuity (its width g and height h) are much smaller than the pipe radius b , we are able to use conformal mapping for solution of the electrostatic problem and to derive simple formulae for the impedance of several shapes of discontinuities. Comparison with numerical calculations using TBCI shows very good accuracy of our theory in the range of its validity.

2 THE IDEA OF THE METHOD

To evaluate the coupling impedance, we have to calculate the fields induced in the vacuum chamber by a given current perturbation. Let the current harmonic

$$\vec{I}(z, t) = I_0 \vec{e}_z \exp(ikz - i\omega t), \quad (1)$$

where $k = \omega/c$, propagate along the chamber axis. If one finds the scattered-field harmonic $E_z = E(r, z)$ (hereafter we omit the time factor $\exp(-i\omega t)$), then the longitudinal impedance is given by

$$Z(\omega) = -\frac{\mathcal{E}}{I_0} = -\frac{1}{I_0} \int_{-\infty}^{\infty} dz E(r, z) \exp(-ikz), \quad (2)$$

where $\mathcal{E}$ means the Fourier-harmonic of the induced voltage. For axisymmetric structures the monopole longitudinal impedance does not depend on the transverse displacement of the integration path, i.e., the integral in the RHS is independent of r . Since the current wavelength is large compared to the discontinuity size (low frequencies), the beam fields near the pill-box vary slowly and can be treated as almost static ones.

One can easily find the magnetic contribution to $\mathcal{E}$. For cavities, shown in Figs. 1(a) and 1(b), the beam azimuthal magnetic field $B_\varphi(r) = \mu_0 I_0 / (2\pi r)$ fills the volume of the cavity. So the presence of the pill-box produces an additional magnetic flux

$$\Phi = \iint_S B_\varphi(r) dS, \quad (3)$$

where the integration goes over the cross section S of the enlargement. Using the smallness of the dimensions of the cavity, we can simply put $\Phi \simeq B_\varphi(b)S$. From Faraday's law one can easily find the contribution of the solenoidal electric field, produced by oscillations of this flux with time, to the induced voltage (electromotive force) as

$$\mathcal{E}_{(m)} = i\omega\Phi. \quad (4)$$

From the physical point of view the additional magnetic flux due to the enlargement can be treated as if produced by a toroidal solenoid having the cross section of the cavity and attached to the smooth pipe. This solenoid is characterized by a dipole magnetic moment $M = 2\pi b\Phi/\mu_0$. The magnetic moment is uniformly distributed along the circumference of the solenoid and has only an azimuthal component. Let us introduce the magnetic moment per unit length, $m = M/(2\pi b)$, and

magnetic polarizability of the discontinuity α_m per unit length of circumference as $m = \alpha_m H_\varphi(b)$. Then $\alpha_m = \Phi/(\mu_0 H_\varphi(b))$, and in the case of a small pill-box $\alpha_m = S$. Respectively, the magnetic contribution to the impedance takes the form

$$Z_{(m)}(\omega) = -\frac{\mathcal{E}_{(m)}}{I_0} = -i\omega \frac{Z_0 \alpha_m}{2\pi b c}, \quad (5)$$

where μ_0 is replaced by Z_0/c , $Z_0 = 120\pi \Omega$. Substituting $\alpha_m = S$ we get the well-known inductive impedance of a small pill-box.¹

In addition to the magnetic term given by Eq. (5) due to generating of the solenoidal electric field, there is also another contribution to the impedance that comes from distortion of the beam-potential electric field by the chamber discontinuity. Because the distorted field is the potential one, $\text{rot } \vec{E} = 0$, its longitudinal component can be written as $E_z = -\partial\phi(z)/\partial z$. It gives the following contribution to $\mathcal{E}$:

$$\begin{aligned} \mathcal{E}_{(e)} &= \int_{-\infty}^{\infty} dz E(z) \exp(-ikz) \\ &= -\int_{-\infty}^{\infty} dz e^{-i\omega z/c} \partial_z (\phi(z) - \phi_\infty). \end{aligned} \quad (6)$$

In Eq. (6) we have taken into account that the perturbed field is localized near the discontinuity, and $\phi(z) \rightarrow \phi_\infty$ when $z \rightarrow \pm\infty$. Integrating by parts one finds

$$\begin{aligned} \mathcal{E}_{(e)} &= -i\frac{\omega}{c} \int_{-\infty}^{\infty} dz e^{-i\omega z/c} (\phi(z) - \phi_\infty) \\ &\simeq -i\frac{\omega}{c} \int_{-\infty}^{\infty} dz (\phi(z) - \phi_\infty). \end{aligned} \quad (7)$$

In the last step we have used the fact that the wavelength c/ω is large compared to the pipe radius and, therefore, to the size of the region where the perturbed field is localized.

To perform integration in Eq. (7) it is convenient to shift the integration path from the axis closer to the wall. If the distance from the pill-box to the integration path is such that $g, h \ll |\vec{R}| \ll b$, where $\vec{R} = (z, x)$ and $x = r - b$, the main component of the perturbed electrostatic potential is a dipole one,

$$\phi(z) = \phi_\infty + \phi_{\text{dip}}(z) = \phi_\infty + \frac{1}{2\pi\epsilon_0} \frac{px}{x^2 + z^2}, \quad (8)$$

where the last term corresponds to the electric dipole in radial direction, distributed along the circumference of the chamber cross section, and p is the dipole moment per unit length. From Eq. (7) we find immediately ($x < 0$):

$$\mathcal{E}_{(e)} = i\frac{\omega}{c} \frac{p}{2\epsilon_0}. \quad (9)$$

It is natural to introduce the electric polarizability per unit length α_e of the discontinuity as $p = \epsilon_0 \alpha_e E_r$, where $E_r = I_0/(\epsilon_0 c 2\pi b)$ is the beam electric field at the

wall. Then the electrostatic contribution to the impedance is

$$Z_{(e)}(\omega) = -\frac{\mathcal{E}_{(e)}}{I_0} = -i\omega \frac{Z_0 \alpha_e}{4\pi b c} . \quad (10)$$

It turns out (see below) that the induced dipole moment for the pill-box is opposite to the radial beam field, i.e., $\alpha_e < 0$, and the electric contribution to the inductance is negative in this case.

Summing up Eqs. (5) and (10) we find the low-frequency impedance of a discontinuity as a sum of the magnetic and electric terms that are proportional to the magnetic and electric polarizability respectively. This result is analogous to that for the impedance of a pumping hole at low frequencies,⁵ which has been derived in a rigorous but more complicated way using the Bethe theory of diffraction by small holes and an expansion over waveguide modes.

3 RECTANGULAR PILL-BOX AND A STEP

While the calculation of the magnetic polarizability is straightforward, in order to find the electric one we have to solve an electrostatic problem, namely, to calculate the dipole moment induced on the pill-box by a homogeneous electric field. For small discontinuities ($g, h \ll b$), the dipole moment does not depend on b , and one can solve the problem considering the limit $b \rightarrow \infty$. In this limit, the cylindrical geometry reduces to the plane one and we have to find the distribution of the potential created by an infinitely long rectangular groove in a perfectly conducting plane wall that is generated by a uniform electric field perpendicular to the wall. This field is in fact the beam electric field at the wall, E_r .

A standard approach for solving this kind of electrostatic problem is to apply the conformal mapping technique, e.g., Reference 6. For the small pill-box the conformal mapping that relates the coordinates (x, y) with the complex variable $\psi + i\phi$ is given by the following equation:

$$z + ix = CE \left(-\arcsin \frac{\psi + i\phi}{CuE_r}, u \right) + ih , \quad (11)$$

where $C = g/(2E(u))$ and parameter u is defined by the equation

$$\frac{g}{2h} = \frac{E(u)}{K(\sqrt{1-u^2}) - E(\sqrt{1-u^2})} . \quad (12)$$

In the equations above K and E are the elliptic integrals of the first and second kind, ϕ is the potential, and ψ is the electric flux. Inverting the dependence given by Eq. (11) and taking the imaginary part, one can find ϕ as a function of x and y . From asymptotic expansion of the elliptic integral, we find that at large distances, $\sqrt{x^2 + z^2} \gg g, h$, the potential is

$$\phi = -E_r \left[x + \frac{C^2}{2}(1-u^2) \frac{x}{x^2 + z^2} \right] + const , \quad (13)$$

and hence the electric polarizability is

$$\alpha_e = -\pi C^2(1 - u^2) \equiv -ghf(g/h) . \quad (14)$$

The function $f(g/h)$ gives the dependence of α_e on the aspect ratio g/h and is plotted in Fig. 2. The function α_e has the following asymptotic behavior

$$\alpha_e = \begin{cases} -g^2/\pi & \text{when } g \leq h \\ -2gh + 2h^2/\pi(\ln(2\pi g/h) + 1) & \text{when } g \gg h \end{cases} \quad (15)$$

The conformal mapping and polarizability for other shapes of the discontinuity (Figs. 1(b)–1(e)) are discussed in Section 6.

Collecting together Eqs. (5) and (10) and using the asymptotic expansion from Eq. (15) we obtain the low-frequency impedance of the small short pill-box ($g \leq h$):

$$Z(\omega) = -i\omega \frac{Z_0}{2\pi bc} \left(gh - \frac{g^2}{2\pi} \right) , \quad (16)$$

which is inductive. The electric contribution to the inductance is negative and small in this case. The comparison of Eq. (16) with numerical computations by means of the ABCI code⁷ is shown in Fig. 3. As follows from this figure, the electric contribution to the impedance is small but noticeable.

For deeper pill-boxes one must correct the above formula, taking into account that the magnetic field in the pill-box varies with radius as $1/r$. When the ratio h/b is not very small, one has to use the exact expression (3) for the magnetic flux. Performing simple integration, one finds that the approximate result $\alpha_m = gh$, which is valid in the limit $h \ll b$, has to be replaced by $\alpha_m = gb \ln(1 + h/b)$ (the solid curve in Fig. 3). The impedance Z_m with this log-term coincides exactly with the expression obtained in Reference 1 by the field-matching method. As for the electric term, we do not need such corrections in this case, $g \ll h$, since the electric field is distorted in the region with a typical size of the order of g .

In the opposite extreme, $g \gg h$, the situation is different: the electric and magnetic contributions almost cancel each other, and the low-frequency impedance of the shallow enlargement takes the form

$$Z(\omega) = -i\omega \frac{Z_0 h^2}{2\pi^2 bc} (2 \ln(2\pi g/h) + 1) , \quad (17)$$

where g should still be below b . The h^2 -dependence of the shallow-cavity inductance was found earlier by numerical methods, e.g., References 3 and 4, but Eq. (17) also gives an additional logarithm-type dependence on the aspect ratio that was hard to extract from fitting in the previous works. Fig. 4 shows the comparison of ABCI computations and our results for shallow cavities. Minor differences between the analytical and numerical results for $g \geq b$, especially for the deeper cavity, are explained by the fact that our approach is valid only when $g < b$.

Increasing the length g of the pill-box beyond the value of b , we arrive at the situation corresponding to two separate steps. Apparently, in this limit the impedance

will not depend on g because a very long pill-box is actually composed of two independent steps—one that increases and one that decreases the beam-pipe radius. Since for the shallow pill-box the impedance depends on the length g only logarithmically (see Eq. (17)), we can obtain the impedance of a step simply by putting $g = b$ in Eq. (17) and dividing it by a factor of 2. This gives the following result ($h \ll b$):

$$Z(\omega) = -i\omega \frac{Z_0 h^2}{4\pi^2 b c} (2 \ln(2\pi b/h) + 1). \quad (18)$$

It should be noted that in addition to the imaginary part given by Eq. (18) there is also a real part of the impedance of a single step associated with the potential jump through the transition from the broad to the narrow part of pipe. The real part of the impedance is proportional to $\ln(1 + h/b)$ being positive for the transition "narrow-to-broad" and negative in the opposite case. For shallow cavities the contributions to the real part from the two steps exactly cancel each other.

4 OTHER SHAPES OF DISCONTINUITIES

The conformal mappings for a symmetric trapezoid, Fig. 1(b), and triangular insertions, Figs. 1(d) and 1(e), are given in Section 6. Considering a shallow trapezoid ($h \ll g$) one can find the inductance of a single sloping step (transition) in the same way as that of the abrupt step, Eq. (18). The inductance produced by the transition with the slope angle $\theta = \pi\nu$ has the form:

$$Z(\omega) = -i\omega \frac{Z_0 h^2}{2\pi^2 b c} \left[\ln \pi\nu \left(\frac{b}{h} - 2 \cot \pi\nu \right) + \frac{3}{2} - \gamma - \psi(\nu) - \frac{\pi}{2} \cot \pi\nu - \frac{1}{2\nu} \right],$$

where $\gamma = 0.5772 \dots$ is Euler's constant, $\psi(\nu)$ is the psi function and the transition is assumed to be short compared to the chamber radius, i.e. its length $l = h \cot \pi\nu \ll b$. The ratio of this inductance to that of the abrupt step ($\nu = 1/2$, Eq. (18)) with the same height is plotted in Fig. 5 as a function of the slope angle.

In the same way, one can consider discontinuities that protrude inside the chamber like an iris or a collimator, Fig. 1(c). It is clear that in this case both the magnetic and electric dipole moments must change sign. However, it turns out that in this case the electric contribution is larger than the magnetic one, and the low-frequency impedance is again inductive with the positive value of the inductance. Making use of conformal mapping in Eq. (23) for $\nu = -1/2$ one can find the inductance for an iris of the length g and depth h . When $h \ll g$, the impedance of the iris coincides with that of a shallow cavity, Eq. (17). This result seems quite natural and, indeed, was found earlier^{3,4} from numerical results. For the case of a thin (or deeper) iris, $g \ll h$, the asymptotic expression for $Z(\omega)$ takes the form

$$Z(\omega) = -i\omega \frac{Z_0}{4bc} \left[h^2 + \frac{gh}{\pi} (2 \ln(8\pi g/h) - 3) \right]. \quad (19)$$

Fig. 6 shows the comparison of this formula with numerical computations. It is

seen that Eq. (19), derived in the limit $h \ll b$, works amazingly well even for very deep irises.

As one more example we consider discontinuities of a triangle-shaped cross section with height (depth) h and base g , shown in Figs. 1(d) and 1(e) (cf. Section 6). When $g \ll h$, the low-frequency impedance of such a chamber enlargement is

$$Z(\omega) = -i\omega \frac{Z_0}{4\pi bc} \left(gh - \frac{g^2}{\pi} \right), \quad (20)$$

and that of a triangular iris is

$$Z(\omega) = -i\omega \frac{Z_0}{4bc} \left[h^2 + \frac{2gh}{\pi}(1 - \ln 2) \right]. \quad (21)$$

For the case of shallow triangular perturbations, $h \ll g < b$, both the enlargement and contraction of the chamber have the same inductance,

$$Z(\omega) = -i\omega \frac{Z_0 h^2 2 \ln 2}{\pi^2 bc}, \quad (22)$$

which is independent of g .

5 LARGE CAVITIES

In the above considerations, we assumed that the dimensions of the discontinuities on the beam pipe are much smaller than the beam-pipe radius. We can relax this restriction and apply our method for large axisymmetric discontinuities. However, as before, the wavelength will be assumed to be large compared to a typical discontinuity size, which now can be of the order of or larger than the pipe radius.

The magnetic contribution for a large pill-box can be easily found for an arbitrary shape of the boundaries if one uses the exact expression (3) of the magnetic flux while calculating the magnetic polarizability.

As for the electric contribution, one has to solve the electrostatic problem exactly without invoking the plane approximation. This part of the problem has been solved using a standard electrostatic code, POISSON.⁸ After having calculated the potential distribution produced in an axisymmetric structure by a linear charge located on the chamber axis, we integrate it numerically at $r = b$ according to Eq. (7). Effectively, this integration involves only a part of the integration path that crosses the cavity opening, since on the wall $\phi(z) = \phi_\infty$. For the illustration of the method, we have calculated the inductance of the cavity having the shape shown in Fig. 1(a) with the length $g = 5$ cm and depth $h = 3$ cm on the pipe of radius $b = 2$ cm. The magnetic part of the impedance is $L_{(m)} = Z_0 g \ln(1 + h/b)/(2\pi bc) = 9.2$ nH, the electric one (from POISSON computations) is $L_{(e)} = -3.5$ nH, which amounts to the total impedance $L = 5.7$ nH. The ABCI code gives us a very close value, $L = 5.78$ nH.

As one more example, we have considered a cavity of the same shape and size as above, but shielded from the beam pipe by a screen in such a way that only

an axisymmetric short gap of length 0.4 cm couples the cavity and beam pipe. In such a layout the induced electric dipole is very small and the inductance has to be due only to the magnetic term, $L \simeq L_{(m)} = 9.2$ nH, while the ABCI result is $L = 9.4$ nH. These two examples show that our approach works well for large discontinuities also.

6 CONFORMAL MAPPING

The conformal mapping from the upper half-plane $w = \psi + i\phi$ on (z, x) -plane in the case when the discontinuity has the symmetric trapezoid shape with height h , base g , and slope angle $\pi\nu$, Fig. 1(b), is given by the integral

$$z + ix = C \int_0^w d\xi \left(\frac{\xi^2 - 1}{\xi^2 - u^2} \right)^\nu + ih, \quad (23)$$

where $g \geq 2h \cot \pi\nu$, $C = 2hu / (\pi\nu(1 - u^2) {}_2F_1(\frac{1}{2}, 1 - \nu, 2, 1 - u^{-2}))$, and u is defined as a root ($0 \leq u \leq 1$) of the equation

$$\begin{aligned} & \frac{g}{2h} \sin \pi\nu - \cos \pi\nu \\ &= \frac{\sqrt{\pi}}{\Gamma(\frac{3}{2} - \nu) \Gamma(1 + \nu)} \frac{u^{2-2\nu}}{1 - u^2} \frac{{}_2F_1(\frac{1}{2}, -\nu, \frac{3}{2} - \nu, u^2)}{{}_2F_1(\frac{1}{2}, 1 - \nu, 2, 1 - u^{-2})}. \end{aligned} \quad (24)$$

In the equations above ${}_2F_1$ is the Gauss hypergeometric function and Γ is the gamma-function. The electric polarizability per unit length of circumference is given by

$$\alpha_e = -2\pi\nu C^2(1 - u^2). \quad (25)$$

In the case of a pill-box $\nu = 1/2$ and Eq. (23) can be reduced to Eq. (11).

Equations (23) and (24) work not only for chamber enlargement but also for chamber contraction if we replace $h \rightarrow -h$ ($h > 0$) for $\nu < 0$. In this case, $\nu = -1/2$ corresponds to an iris.

The particular case $u = 0$ ($g = 2h \cot \pi\nu$) gives us triangular perturbations, Figs. 1(d) and 1(e). Then the electric polarizability (25) can be reduced to

$$\alpha_e = -2\nu \left[\frac{\pi h}{\sin \pi\nu \Gamma(1/2 - \nu) \Gamma(1 + \nu)} \right]^2. \quad (26)$$

7 CONCLUSIONS

A method is developed that allows one to calculate the low-frequency impedance for a small discontinuity of the beam vacuum chamber. There are two contributions to the impedance—the electric and the magnetic, which have the opposite sign—but the resulting low-frequency impedance is always inductive. This approach gives

a clear physical picture and allows us to reproduce and explain results obtained earlier from numerical simulations. In many cases, simple analytical formulae for the low-frequency impedance are derived.

We have to emphasize here that being perturbative in small parameters g/b and h/b , our method includes all the important terms in the lowest order, so that neglected terms are much smaller than those retained. In that sense, our result is accurate in the range of its validity.

We have also demonstrated that our method allows to compute easily the low-frequency impedance for large cavities. As a future generalization we intend to apply the approach to study the impedance of small discontinuities at frequencies above the cut-off frequency. We believe that in this frequency range the idea of effective dipole moments will be fruitful as well.

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FIGURE CAPTIONS

FIGURE 1: Discontinuities of the vacuum chamber.

FIGURE 2: The electric polarizability of a pill-box versus the aspect ratio.

FIGURE 3: The inductance of a deep pill-box versus its depth: without (dash-dotted) and with (dashed) the electric term; with the electric term and smeared magnetic moment (solid).

FIGURE 4: The inductance of shallow pill-boxes versus the length.

FIGURE 5: The inductance of a shallow step versus the slope angle.

FIGURE 6: The inductance of an iris versus its depth.

a

b

c

d

e