Predicting Time-Dependent Interfacial Behavior with a Viscoelastic Cohesive Zone Model

J.W. Foulk
Sandia National Laboratories
Livermore, CA, 94551, USA

D.H. Allen
Department of Aerospace Engineering
Texas A&M University
College Station, TX 77843-3141, USA

Summary

A time-dependent cohesive zone model is employed to model adhesive failure and grain-boundary cracking. Through the incorporation of viscoelasticity and evolving damage, the viscoelastic cohesive zone model (VCZM) yields a time-dependent critical energy release rate. A double-cantilever beam configuration is investigated. Crack growth curves are presented for the Xu-Needleman and VCZM model. In addition, a fifty-five grain polycrystal is simulated in compression. Time-dependent grain boundary prying and sliding result in inter-granular separation parallel to the major axis of loading.

Introduction

Many material systems exhibit time-dependent fracture. Although much research has focused on predicting the bulk behavior of organics and metals, fewer works have addressed rate-dependent crack growth. The situation is further complicated by the introduction of material interfaces on different length scales.

The complications associated with material interfaces can be simplified through the introduction of cohesive zone models. First proposed by Dugdale and Barenblatt, cohesive zone models provide an elegant transition from crack initiation to traction-free boundaries. Since the works of Needleman [1], numerous investigators have employed cohesive zones to model de-adhesion in composite systems. These studies, however, have traditionally been performed with rate-independent cohesive zone laws. Knauss and Losi [2] proposed a rate-dependent cohesive zone constitutive law incorporating viscoelasticity and damage. In addition, Knauss [3] addressed crazing in thermoplastic polymers within the context of the model.

Utilizing the Helmholtz free energy, Yoon and Allen [4] proposed a cohesive zone constitutive equation in the form of a single hereditary integral for a linear viscoelastic material. The model relies on an internal state variable which reflects the damage state of the cohesive zone. Allen and Searcy [5] have also arrived at a similar formulation through micromechanics. In addition, Zocher [6] and Allen [7] have proven that the incrementalized equations can be integrated analytically for a linear viscoelastic material. Recently, Rahulkumar [8] has also proposed a rate-dependent cohesive zone constitutive law.

Given a time-dependent material system with multiple interfaces and the current state of the literature, the engineer naturally gravitates towards a rate-dependent bulk material model and a rate-independent interface model. While this methodology may be sufficient for some boundary-value problems, research emphasizing rate-dependent interface models is needed to understand the source of dissipation.
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Theoretical Foundation

The cohesive zone constitutive law employed in this work was derived by Yoon and Allen [4]. Equation (1) represents a local form of the traction relationship where \( p \) is indexed over the normal and tangential coordinate directions. \( \Delta_p \) is the gap function and \( \lambda \), expressed in Equation (2), represents the norm of the gap function where \( \delta_n, \delta_t, \) and \( \delta_s \) establish the length scale and mode-mixity associated with \( \lambda \). The damage state is reflected by \( \alpha \). One can observe that the crack faces become traction free as \( \alpha \) approaches 1. A simple damage evolution law has been proposed by Allen and Searcy [5]. Equation (3) reflects a power-law based on \( \lambda \) where \( \alpha_1 \) and \( m \) are fitting parameters. Note that damage does not accumulate on unloading. Finally, \( \sigma^f \) represents an initiation criterion and \( E_{cc} \) constitutes the relation modulus of the cohesive zone. In the context of polymeric materials, one may view \( \lambda \) as a measure of fibril stretching in a crazed zone. Void growth sheds load to the extended fibrils. As the load bearing area decreases, \( \alpha \) increases. The critical fiber diameter coincides with \( \alpha=1 \).

Numerical Implementation

In order to implement the cohesive zone constitutive law, one must develop an incremental form of the traction-displacement relation. A general procedure for a viscoelastic solid is presented by Zocher [6], while Allen and Searcy [7] address Equation (1) specifically. First, one must assume that the relaxation modulus is well described by a prony series, Equation (4). Second, the rate of stretch, \( \lambda_0 \), must be constant over the chosen time step. Equation (5) is an acceptable condition provided small time steps are taken during the simulation. Given these conditions, the convolution integrals can be integrated analytically and a recursive relationship can be established to capture the load history. The incremental traction-displacement equations were incorporated into the Sandia National Labs (SNL) code, JAS3D.
Example Simulations

The following examples emphasize time-dependent model capability with minimal complexity. The first study examines a weak adhesive joining two elastic, aluminum beams (aspect ratio = 20). Figure 1 illustrates the double-cantilever beam (DCB) geometry and VCZM material parameters. The viscoelastic behavior is represented by a standard-linear-solid for simplicity. The end displacement, \( h \), was applied at a constant rate of 0.01. Because the interface is loaded through bending, the normal opening rate for each cohesive zone element along the interface will decrease with increasing beam length. The authors also performed simulations employing the Xu-Needleman [9] constitutive law for comparison. A prior study confirmed that the mesh composed of 3280 hex elements (one-element thick) was converged. Please note that the DCB geometry was not pre-cracked.

Traction-displacement relations for both models are shown in Figure 2. VCZM parameters were chosen such that fracture energy, \( \Phi_n \), associated with the intermediate rate, VCZM(2), would roughly correspond to the Xu-Needleman input fracture energy. In the VCZM, the viscoelastic constitutive response and the damage evolution law combine to yield a rate-dependent fracture energy. The variance in the initial slope is indicative of the viscoelastic response while the general curvature of the traction-displacement relation is a function of damage evolution.

\[
\begin{array}{cccc}
\delta_n & 0.001 \\
E_{nt} & 100.0 \\
E_1 & 100.0 \\
\eta_1 & 1.0 \\
\alpha_1 & 0.50 \\
m & 2.0 \\
\sigma' & 0.0 \\
\end{array}
\]

Figure 1. Schematic of DCB geometry with VCZM material properties.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \Delta \xi/\xi )</th>
<th>( \Phi_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCZM(1)</td>
<td>0.100</td>
<td>1.64</td>
</tr>
<tr>
<td>VCZM(2)</td>
<td>0.050</td>
<td>1.01</td>
</tr>
<tr>
<td>VCZM(3)</td>
<td>0.025</td>
<td>0.63</td>
</tr>
<tr>
<td>Xu-Needleman</td>
<td>0.100</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 2. Traction-displacement relations and fracture energies as a function of loading rate.
One can track the crack tip as a function of opening displacement, \(0.1\delta_n\). Growth curves for both models are presented in Figure 3. The Xu-Needleman model yields a curve consistent with linear elastic fracture mechanics. VCZM differs in that \(G_{1c}\), the critical energy release rate, is not a constant. Initially, the VCZM lags the Xu-Needleman prediction. Because the normal opening rate is a function of beam length, \(G_{1c}\) decreases with crack growth. Soon, the VCZM prediction surpasses the Xu-Needleman result. To further illustrate this point, we plot the peak traction in the cohesive zone. Notice that the peak tractions illustrated in Figure 2 span the behavior shown in Figure 4. In fact, the DCB end displacement rate and VCZM(1) are identical, 0.01. The Xu-Needleman model remains unchanged.

![Figure 3. Rate-dependent crack growth for a DCB with a constant end-displacement rate.](image)

![Figure 4. VCZM global peak traction decreases with increasing crack length.](image)
The second study investigates the evolution of microstructural interfaces through the incorporation of cohesive zone elements along grain boundaries. Some polycrystalline material systems such as rocksalt exhibit rate dependence at room temperatures. Unable to measure bulk and grain-boundary behavior separately, the analyst must predict time-dependent grain-boundary fracture through the incorporation of rate-dependent bulk and interface models. If we assume that the grains behave elastically, we maximize the geometric effects of prying and sliding along grain boundaries. To further simplify matters, we assume that all rate dependence stems from the damage evolution law. Figure 5 illustrates the trial RVE, corresponding microstructure, and fictitious material parameters. The applied displacement and simulated reaction load are plotted in Figure 6. Although the ends are held fixed after 50 s, $G_{1c}$ continues to decrement with time. The strain energy available in the bulk drives grain-boundary separation and eventual failure. While macroscopic unloading is illustrated in Figure 6, Figure 7 depicts load redistribution at the granular level. In addition, the deformed geometry mirrors a frequent observation made in polycrystalline materials under compressive loads - cracks form parallel to the major axis of loading.

Figure 5. Schematic of 55 grain RVE, corresponding micrograph, and chosen material parameters.

Figure 6. Grain-boundary fracture causes trial RVE to unload under fixed-grip conditions.
Figure 7. Stored energy drives crack growth, load redistribution, and eventual failure.

Conclusion

In order to predict time-dependent fracture in material systems with interfaces, one must not only rely on rate-dependent bulk models, but also rate-dependent interface models. The calculations presented herein support a framework for incorporating time-dependence into a cohesive zone traction-displacement relation. Through material inelasticity and damage, one can account for interfacial evolution and thus predict a time-dependent critical energy release rate.

References