**Determination of the Uncertainties of Reflection Coefficient** Measurements of a Microwave Network Analyzer

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#### Abstract

A method that calculates the residual uncertainties of a microwave network analyzer for the frequency range of 300 kHz to 50 GHz is described. The method utilizes measurements on NISTcertified standards (such as an airline or load) plus additional measurements to estimate the combined standard uncertainties for measurements using the network analyzer. The uncertainties of the standards are incorporated by means of a Monte Carlo technique. The uncertainties assigned to a network analyzer then provide the basis for estimating the uncertainties assigned to devices measured using a network analyzer. The results of this method for characterizing network analyzer uncertainties are presented for several connector types.

# Introduction

Vector network analyzers (VNA) are widely used to measure the scattering parameters of a variety of microwave devices. The scattering parameters are related to reflection coefficients and transmission properties of the microwave devices. Typical devices that are measured include attenuators, terminations, mismatches, and directional couplers. Proper operation of the network analyzer requires a "calibration" or "accuracy enhancement" which is accomplished by measuring a set of known devices (typically, these devices are opens, shorts, and fixed and sliding loads contained in a calibration kit) and comparing the measurements to a model of the device in the instrument. Differences between the measurements and model result in a set of

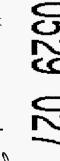
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Calibration or accuracy enhancement of the network analyzer does not eliminate all errors from the measurements. There still exist some residual errors that the calibration cannot entirely eliminate. This paper describes a scheme to estimate these residual uncertainties for the measurement of reflection coefficient of devices under test (DUT). The paper briefly describes requirements of this method both for characterizing the VNA and for the subsequent measurement of a DUT. The theory behind the two schemes (one is used for low frequencies, typically below 50 MHz, while the other is used from 50 MHz up to 50 GHz) is described and some typical uncertainty estimates for several connector types are presented.

# **Requirements for Calibration of VNA**

Proper use of a microwave network analyzer requires measuring the response of the analyzer to a set of known calibration standards. This process is called measurement calibration, accuracy enhancement or error correction. Measurement calibration reduces the systematic errors in the VNA measurements<sup>(1)</sup>. The same parameter settings used for the measurement calibration must also be used when obtaining measurements with the VNA. Changing any parameter settings invalidates the measurement; the measurements taken with the new settings on the VNA are no longer correct. The same equipment (adapters, cables, etc.) used for the calibration should be used for subsequent measurements. Finally, the same or greater averaging factor should be used for the measurements as was used for the calibration of the VNA.

# **VNA Measurements**

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This paper does not describe a new scheme for calibrating a VNA. Rather, it demonstrates a process where the measurement uncertainties of a VNA may be quantified. The measurement calibration only reduces (but does not eliminate) the systematic error. Errors due to drift and random errors increase the measurement error. These errors must be accounted for when measuring a DUT. The process described in this paper provides an estimate of the residual uncertainties in the VNA that are not corrected by calibration. Combining these uncertainty estimates with multiple DUT measurements provides a much better estimate of the combined standard uncertainty of the VNA measurements.

# Limitation of Uncertainty Analysis

The uncertainty analysis methods for reflection coefficient measurements presented here have the same general limitations as the measurement calibration. That is, the uncertainty measurements are valid only for the same parameters (power, averaging factor, etc.) of the VNA as were used for the calibration. Changes to these parameters requires repeating the uncertainty measurements and analysis.

# **Theory - TRL**

The Thru-Reflect-Line (TRL) technique offers an attractive way to determine the parameters of two error boxes at the test ports of a VNA. Because this technique uses an airline as one of the standards, it has a lower frequency limit of approximately 25 to 50 MHz. Conceptually, the VNA is viewed as shown in Figure 1. The conceptual model, illustrated in Figure 1, shows that when a DUT with a true reflection coefficient of  $\Gamma_1$  is connected to Test Port 1, the VNA will produce a reading of  $\Gamma_1^m$ . This reading is the result of the corrupting influence of Error Box X. In terms of the measured scattering parameters,  $S_{ij}^m$  (which are complex quantities), the reflection coefficient,  $\Gamma_1^m$ , is given by

$$\Gamma_1^m = S_{11}^m + \frac{S_{12}^m S_{21}^m \Gamma_1}{1 - S_{22}^m \Gamma_1} = \frac{(S_{12}^m S_{21}^m - S_{11}^m S_{22}^m) \Gamma_1 + S_{11}^m}{-S_{22}^m \Gamma_1 + 1} = \frac{a_X \Gamma_1 + b_X}{c_X \Gamma_1 + 1}.$$
 (1)

The right side of Eqn. 1 displays the equation in terms of the bi-linear or linear fractional transformation<sup>(2)</sup>. In order to determine the corrupting effects of the Error Box X, the values of the complex parameters  $a_x$ ,  $b_x$ , and  $c_x$  must be determined from measurements of the scattering parameters. The equivalent expression for Error Box Y is

$$\Gamma_2^m = S_{22}^m + \frac{S_{12}^m S_{21}^m \Gamma_2}{1 - S_{11}^m \Gamma_2} = \frac{(S_{12}^m S_{21}^m - S_{11}^m S_{22}^m) \Gamma_2 + S_{22}^m}{-S_{11}^m \Gamma_2 + 1} = \frac{a_Y \Gamma_2 + b_Y}{c_Y \Gamma_2 + 1}.$$
 (2)

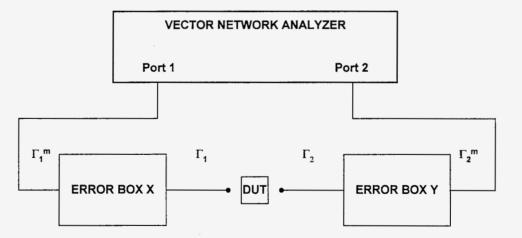


Figure 1. Conceptual diagram of a VNA and the error boxes at each port.

The TRL technique provides an efficient method to obtain the required measurements while requiring only one standard - the complex characteristic impedance,  $Z_i$ , of a transmission line. The advantage of the TRL method is that only one certified complex value is required and that

the uncertainty on this quantity is very small for an air dielectric coaxial line (airline). It therefore follows that the residual uncertainties of the VNA may be very well characterized.

In order to obtain the error box parameters  $a_X$ ,  $b_X$ ,  $c_X$ ,  $a_Y$ ,  $b_Y$ , and  $c_Y$ , measurements of a thru line, an airline, an uncalibrated open, and an uncalibrated short are made at the test ports of the VNA. Using the measurements, Eqns. 1 and 2 are solved for the error box terms. Next, it is necessary to account for the fact that the true reflection coefficient is relative to impedance  $Z_l$ . This is accomplished by relating the true reflection coefficient computed from the measurements,  $\Gamma_1$ , to the true reflection coefficient in a 50 ohm system,  ${}^{50}\Gamma_1$ . Eqn. 1 may be inverted to yield

$$\Gamma_1 = \frac{\Gamma_1^m - b_X}{a_X - c_X \Gamma_1^m}.$$
(3)

The true reflection coefficient relative to  $Z_l$  may be related to  ${}^{50}\Gamma_1$  by

$$\Gamma_{1} = \frac{{}^{50}\Gamma_{1}(50+Z_{l}) + (50-Z_{l})}{{}^{50}\Gamma_{1}(50-Z_{l}) + (50+Z_{l})}.$$
(4)

Substituting Eqn. 4 into Eqn. 1 and rearranging gives

$$^{50}\Gamma_{1} = \frac{B - \Gamma_{1}^{m}}{C\Gamma_{1}^{m} - A},$$
(5)

where

$$A = \frac{a(50 + Z_1) + b(50 - Z_1)}{c(50 - Z_1) + (50 + Z_1)} \quad B = \frac{a(50 - Z_1) + b(50 + Z_1)}{c(50 - Z_1) + (50 + Z_1)} \quad C = \frac{c(50 + Z_1) + (50 - Z_1)}{c(50 - Z_1) + (50 + Z_1)}.$$
 (6)

Therefore, Eqn. 5 is used to estimate the value of  ${}^{50}\Gamma_1$  and then to compute the errors in the measurement. The incorporation of the error in the standards is discussed in a subsequent section.

#### **Theory - TMS**

The following approach develops a new method to construct the VNA S<sub>ii</sub> uncertainties at low frequencies (typically, below 50 MHz, but it could be used at higher frequencies as well)<sup>(3)</sup>. It uses the basic method of Silvonen<sup>(4,5)</sup> to construct an eight-term error model using scattering parameters measured by the VNA. The method requires the measurement of all S-parameters of a thru, a load (or match), and a short. The short only requires a measurement at one test set port<sup>(6)</sup>. Figure 2 shows the conceptual diagram used for the derivation.

Using the nomenclature of Silvonen, the VNA can be configured with a left, L, and a right, R, error box or network. In order to certify the VNA, it must first be calibrated using the appropriate

cal kit. (Hewlett Packard<sup>(7)</sup> network analyzers use a 12 term error model whose values are calculated from measurements of standards in the cal kit. For example, the HP 8753 VNA uses a SOLT (short-open-load-thru) set of devices to produce the error terms.) Three

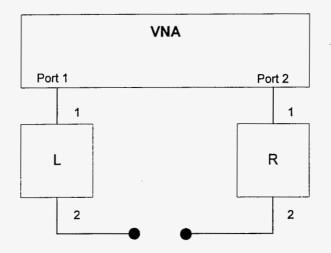


Figure 2. Conceptual diagram of the VNA used in the Thru-Match-Short (TMS) derivation.

calibration standards, A, B, and C, must be measured to provide the eight error terms which are used to estimate the VNA uncertainties. For the known standards A and B:

$$\begin{bmatrix} 1 & A_{11}M_{A11} & -A_{11} & 0 & A_{21}M_{A12} & 0 \\ 0 & A_{12}M_{A11} & -A_{12} & 0 & A_{22}M_{A12} & 0 \\ 0 & A_{11}M_{A21} & 0 & 0 & A_{21}M_{A22} & -A_{21} \\ 0 & A_{12}M_{A21} & 0 & 1 & A_{22}M_{A22} & -A_{22} \\ 1 & B_{11}M_{B11} & -B_{11} & 0 & B_{21}M_{B12} & 0 \\ 0 & B_{12}M_{B11} & -B_{12} & 0 & B_{22}M_{B12} & 0 \\ 0 & B_{11}M_{B21} & 0 & 0 & B_{21}M_{B22} & -B_{21} \\ 0 & B_{12}M_{B21} & 0 & 1 & B_{22}M_{B22} & -B_{21} \\ 0 & B_{12}M_{B21} & 0 & 1 & B_{22}M_{B22} & -B_{22} \end{bmatrix}$$

where

 $M_{A11}$  is the measured  $S_{11}$  for standard A, etc.,

 $A_{11}$  is the known  $S_{11}$  value for standard A, etc.,

 $L_{11}$ , and  $R_{11}$  are the error terms for  $S_{11}$  for the Left and the Right error box, respectively,

$$\Delta L = L_{11}L_{22} - L_{12}L_{21}, \ \Delta R = R_{11}R_{22} - R_{12}R_{21}, \text{ and } k = \frac{L_{21}}{R_{21}}.$$

All the terms in these and subsequent equations are complex quantities. Equations 7-14 may be written in matrix form as

where

$$\mathbf{G} = \begin{bmatrix} \mathbf{M}_{A11} \\ \mathbf{0} \\ \mathbf{M}_{A21} \\ \mathbf{0} \\ \mathbf{M}_{B11} \\ \mathbf{0} \\ \mathbf{M}_{B21} \\ \mathbf{0} \end{bmatrix} \quad \text{and} \quad \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{A12} \\ \mathbf{0} \\ \mathbf{M}_{A22} \\ \mathbf{0} \\ \mathbf{M}_{B12} \\ \mathbf{0} \\ \mathbf{M}_{B12} \\ \mathbf{0} \\ \mathbf{M}_{B22} \end{bmatrix}$$
(16)

The aim is to solve for the error vector, **E**. This may be done by first setting k=0 in Eqn. 15 obtaining

$$\mathbf{X} = \mathbf{N}^{-1} \mathbf{G},\tag{17}$$

where  $N^{-1}$  is the inverse of the N matrix. Next, letting k=1, gives

$$\mathbf{Z} = \mathbf{N}^{-1} \left( \mathbf{G} + \mathbf{H} \right) \tag{18}$$

The six-term difference vector is constructed from

$$\mathbf{Y} = \mathbf{Z} - \mathbf{X} = \mathbf{N}^{-1}\mathbf{H} \tag{19}$$

The desired error vector is found from

$$\mathbf{E} = \mathbf{X} + \mathbf{k}\mathbf{Y}.$$
 (20)

For network analyzers, it is not possible to assume that k=1, hence, k must be calculated from measurements on the third standard, C. According to Eul and Schiek<sup>(6)</sup>, it will be necessary to measure C only at Port 1 of the test set. Thus, adapting Eqn.7 for standard C:

$$L_{11} + C_{11}M_{C11}L_{22} - C_{11}\Delta L + 0 + C_{21}M_{C12}kR_{22} + 0 = M_{C11}.$$
 (21)

But, since C is a one-port standard,  $C_{21} = 0$ ; thus Eqn. 21 becomes

$$L_{11} + C_{11}M_{C11}L_{22} - C_{11}\Delta L = M_{C11} \qquad (22)$$

Equation 20 can be written explicitly as

$$\begin{bmatrix} L_{11} \\ L_{22} \\ \Delta L \\ kR_{11} \\ kR_{22} \\ k\Delta R \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} + \begin{bmatrix} kY_1 \\ kY_2 \\ kY_3 \\ kY_4 \\ kY_5 \\ kY_6 \end{bmatrix}.$$
(23)

Substituting Eqn. 23 into 22 gives

$$X_{1} + kY_{1} + C_{11}M_{C11}(X_{2} + kY_{2}) - C_{11}(X_{3} + kY_{3}) = M_{C11}.$$
 (24)

Collecting terms and solving for k yields

$$k = \frac{M_{c11} - X_1 - C_{11}(X_2 M_{c11} - X_3)}{Y_1 + C_{11}(Y_2 M_{c11} - Y_3)}.$$
 (25)

As previously mentioned, the three standards required are:

Standard A = Thru 
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$
  
Standard B = Match 
$$B = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix}$$
 (26)

Standard C = Short  $C = C_{11}$ 

Standard C is used only to calculate  $k = L_{21}/R_{21}$ . With the definitions above, the matrices **N**, **G**, and **H** may be written to exclude the B transmission terms. Therefore,

$$N = \begin{bmatrix} 1 & A_{11}M_{A11} & -A_{11} & 0 & A_{21}M_{A12} & 0 \\ 0 & A_{12}M_{A11} & -A_{12} & 0 & A_{22}M_{A12} & 0 \\ 0 & A_{11}M_{A21} & 0 & 0 & A_{21}M_{A22} & -A_{21} \\ 0 & A_{12}M_{A21} & 0 & 1 & A_{22}M_{A22} & -A_{22} \\ 1 & B_{11}M_{B11} & -B_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & B_{22}M_{B22} & -B_{22} \end{bmatrix}$$
(27)

The N matrix was changed from the 8x6 term form of Eqns. 7-14 to the 6x6 form above. Eqns. 12 and 13 were eliminated because their terms are all zero. The resulting **G** and **H** matrices are

$$G = \begin{bmatrix} M_{A11} \\ 0 \\ M_{A21} \\ 0 \\ M_{B11} \\ 0 \end{bmatrix} \qquad H = \begin{bmatrix} 0 \\ M_{A12} \\ 0 \\ M_{A22} \\ 0 \\ M_{B22} \end{bmatrix}, \qquad (28)$$

where these matrices are now six-term vectors rather than the eight-term vectors of Eqn. (16).

The error vector, E, can now be used to calculate the values of a hypothetical DUT D. The measured values of the theoretical DUT D (denoted by  $M_{D11}$ ,  $M_{D12}$ ,  $M_{D21}$ , and  $M_{D22}$ ) are set to known reflection coefficient magnitudes and angles. Then, the calculated values for D are found using these measured values and the calculated error vector. The de-embedding equations written in terms of S-parameters for the DUT D are

$$\begin{bmatrix} L_{22}M_{D11} - \Delta L & 0 & kR_{22}M_{D12} & 0 \\ 0 & L_{22}M_{D11} - \Delta L & 0 & kR_{22}M_{D12} \\ L_{22}M_{D21} & 0 & kR_{22}M_{D22} - k\Delta R & 0 \\ 0 & L_{22}M_{D21} & 0 & kR_{22}M_{D22} - k\Delta R \end{bmatrix}$$

$$\begin{bmatrix} D_{11} \\ D_{12} \\ D_{21} \\ D_{22} \end{bmatrix} = \begin{bmatrix} M_{D11} - L_{11} \\ kM_{D12} \\ M_{D21} \\ kM_{D22} - kR_{11} \end{bmatrix}$$
(29)

or, in matrix form

$$\mathbf{M} \, \mathbf{D} \, = \, \mathbf{H}' \tag{30}$$

Therefore, using the elements of the error term vector,  $\mathbf{E}$ , and the measurements of the DUT,  $M_{D11}$ , etc., the calculated "true" values of the DUT parameters,  $\mathbf{D}$ , are found from

$$\mathbf{D} = \mathbf{M}^{-1} \mathbf{H}^{\prime} \tag{31}$$

where  $M^{-1}$  is the inverse of the M matrix.

# Incorporating uncertainty of standards

To include the uncertainties in the standards (the NIST certified airlines for the TRL method and the NIST certified load or matches for the TMS method), a Monte Carlo technique<sup>(8)</sup> is utilized. In this technique, a normal distribution is assumed for the uncertainties of the standards. Using the Box-Muller transformation<sup>(8)</sup>, a value is selected for the standard. This value is then used in the computation of the error box terms (TRL) or the error coefficients (TMS). The mean value and standard deviation of 101 computations is calculated for the errors given by

$$E_{mag} = \left\| \Gamma_{true} \right\| - \left| \Gamma_{calc} \right\| \tag{32}$$

$$E_{angle} = Arg(\Gamma_{true} - \Gamma_{calc})$$
(33)

where  $\Gamma_{true}$  is the expected value of the reflection coefficient and  $\Gamma_{calc}$  is the calculated value.  $E_{mag}$  and  $E_{angle}$  are the errors in the reflection coefficient magnitude and phase angle, respectively. The standard deviations of  $E_{mag}$  and  $E_{angle}$  are calculated to be Type A standard uncertainties<sup>(9)</sup>. The final uncertainties include a coverage factor which is usually k=3<sup>(10)</sup>.

# **Results - Uncertainty Summary**

The previous two sections briefly developed the theory behind the methods used and the calculations executed to estimate the uncertainties for the reflection coefficient measurements using network analyzers. This section will present a few examples of the uncertainties obtained using these techniques. The entire process, from setup, data acquisition, calculation, to printing of the calibration certificate, is controlled via a computer program<sup>(11)</sup>.

Table 1 is an example of the uncertainties calculated by the two methods previously described. The uncertainties for  $|S_{11}|$  and  $Arg(S_{11})$  are shown for the 3.5 mm connector on a HP 8753C Network Analyzer<sup>(7)</sup>. The uncertainties at frequencies 0.3 through 10 MHz were calculated using the TMS method while the remaining uncertainties were calculated using the TRL method.

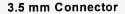
Figure 3 plots  $|S_{11}|$  uncertainties from Table 1 at reflection coefficient magnitude ( $\rho$ ) values of 0.003, 0.1 and 0.5 as a function of frequency. The data at each  $\rho$  value follow the same trend with the largest uncertainty occurring at 500 MHz. Data up to 10 MHz were obtained using the TMS method while the data at higher frequencies used the TRL method to estimate the uncertainties.

Figure 4 displays uncertainty data at  $\rho$  values of 0.003, 0.1, and 0.5 for 7 mm connectors measured on the HP 8753C network analyzer. Again, uncertainties at frequencies up to 10 MHz were found using the TMS method. The uncertainties above 10 MHz (from TRL) are less than those calculated below 10 MHz which is opposite to the uncertainties for the 3.5 mm connector.

Table 1. Example of the uncertainties ( $k=3$ ) calculated for S <sub>11</sub> for 3.5 mm connectors on a HP
8753C Network Analyzer <sup>(7)</sup> . The measured reflection coefficient magnitude ( $\rho$ ) values are listed
in the left column. The uncertainties at frequencies of 0.3 through 10 MHz were produced using
the TMS method while the TRL method was used for the remaining frequencies.

S <sub>11</sub>   (linear magnitude)										
	Frequency (MHz)									
ρ	0.3	1	5	10	50	100	250	500	1500	2500
0.0030	0.0051	0.0054	0.0053	0.0054	0.0090	0.0092	0.0095	0.0113	0.0063	0.0060
0.0100	0.0056	0.0061	0.0060	0.0061	0.0159	0.0163	0.0167	0.0177	0.0095	0.0077
0.0150	0.0057	0.0061	0.0060	0.0061	0.0172	0.0178	0.0183	0.0199	0.0096	0.0078
0.0200	0.0057	0.0061	0.0060	0.0061	0.0175	0.0182	0.0188	0.0206	0.0097	0.0078
0.0300	0.0057	0.0061	0.0060	0.0061	0.0177	0.0184	0.0190	0.0210	0.0097	0.0078
0.0500	0.0057	0.0061	0.0060	0.0061	0.0178	0.0185	0.0192	0.0212	0.0097	0.0078
0.0700	0.0057	0.0061	0.0060	0.0061	0.0178	0.0186	0.0192	0.0212	0.0097	0.0077
0.1000	0.0057	0.0061	0.0060	0.0061	0.0178	0.0185	0.0192	0.0212	0.0096	0.0077
0.1500	0.0056	0.0061	0.0060	0.0061	0.0178	0.0185	0.0191	0.0210	0.0095	0.0074
0.2000	0.0056	0.0060	0.0059	0.0060	0.0177	0.0183	0.0189	0.0208	0.0092	0.0071
0.3000	0.0055	0.0059	0.0058	0.0059	0.0173	0.0180	0.0186	0.0202	0.0086	0.0062
0.4000	0.0053	0.0057	0.0056	0.0057	0.0169	0.0174	0.0180	0.0194	0.0078	0.0050
0.5000	0.0051	0.0054	0.0054	0.0055	0.0164	0.0168	0.0173	0.0183	0.0068	0.0035
0.9999	0.0034	0.0032	0.0036	0.0036	0.0126	0.0119	0.0123	0.0103	0.0063	0.0112
Arg(S <sub>11</sub> ) (degrees)										
0.0030	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00	180.00
0.0100	32.27	34.87	34.35	34.76	180.00	180.00	180.00	180.00	59.60	46.67
0.0150	21.34	23.02	22.68	22.94	180.00	180.00	180.00	180.00	38.45	30.45
0.0200	15.96	17.21	16.96	17.15	53.87	56.53	58.91	180.00	28.55	22.69
0.0300	10.62	11.45	11.28	11.41	34.48	36.02	37.33	41.92	18.92	15.08
0.0500	6.37	6.87	6.77	6.84	20.33	21.21	21.93	24.40	11.35	9.07
0.0700	4.55	4.91	4.84	4.89	14.47	15.09	15.60	17.32	8.14	6.51
0.1000	3.19	3.44	3.39	3.43	10.12	10.56	10.91	12.11	5.75	4.62
0.1500	2.14	2.31	2.27	2.30	6.77	7.07	7.30	8.12	3.92	3.17
0.2000	1.62	1.75	1.72	1.74	5.10	5.34	5.51	6.15	3.03	2.48
0.3000	1.10	1.19	1.17	1.18	3.46	3.63	3.75	4.22	2.18	1.84
0.4000	0.85	0.92	0.90	0.91	2.66	2.80	2.89	3.29	1.80	1.57
0.5000	0.70	0.77	0.75	0.76	2.19	2.32	2.40	2.76	1.61	1.46
0.9999	0.45	0.51	0.48	0.49	1.39	1.51	1.56	1.93	1.46	1.54

The comparable data for a Type N connector are shown in Figure 5. The uncertainties plotted here are similar to the data for the 7 mm connector in terms of the largest uncertainty. However, the frequency dependence of the calculated uncertainties differs substantially among the different connector types.



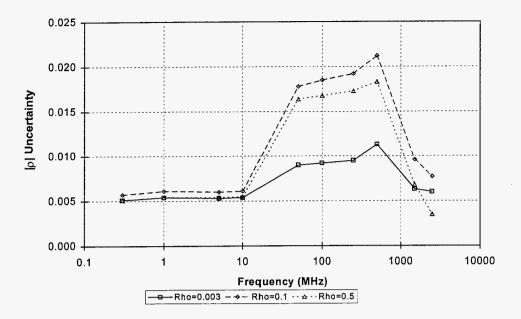


Figure 3. Selected reflection coefficient magnitude uncertainties from Table 1 plotted as a function of frequency.

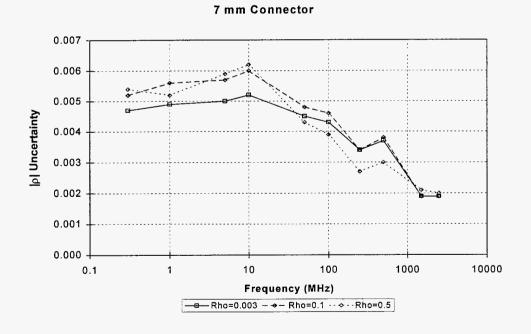


Figure 4. Selected reflection coefficient magnitude uncertainties for a 7 mm connector measured from 0.3 to 3000 MHz.

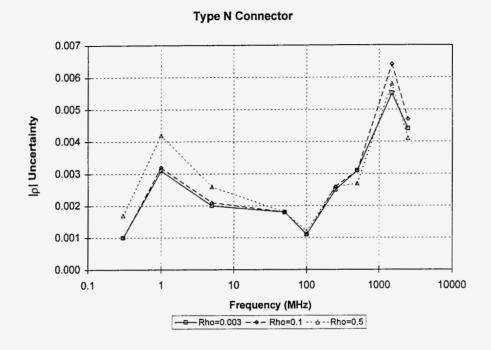


Figure 5. Selected reflection coefficient magnitude uncertainties for a Type N connector measured from 0.3 to 3000 MHz.

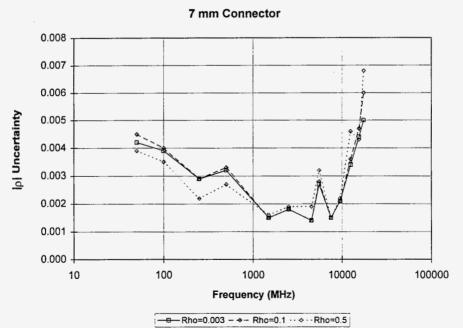
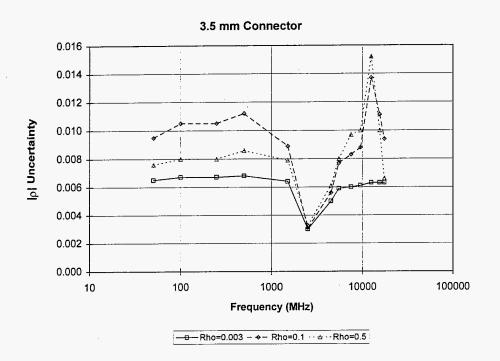
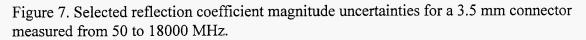


Figure 6. Selected reflection coefficient magnitude uncertainties for a 7 mm connector measured from 50 to 18000 MHz.





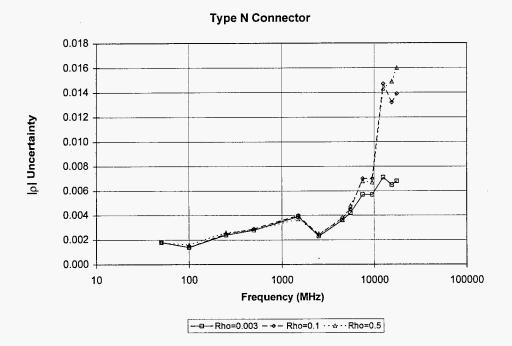


Figure 8. Selected reflection coefficient magnitude uncertainties for a Type N connector measured from 50 to 18000 MHz.

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Figures 6 through 8 plot uncertainty data (k=3) at  $\rho$  values of 0.003, 0.1, and 0.5 for the HP 8510C Network Analyzer<sup>(7)</sup> at connector types of 7 mm, 3.5 mm, and Type N, repectively. All uncertainties here were obtained using the TRL method. The 3.5 mm connector exhibits the largest uncertainty over the whole frequency range. However, the Type N connector does have larger uncertainties at frequencies above 10 GHz. The general trends are consistent within a particular connector, but not similar when compared to a different connector. For example, the Type N results show nearly the same uncertainty estimate up to 2.5 GHz. After this frequency, the uncertainties for this connector are again almost the same up to a frequency of 1.5 GHz. At 2.5 GHz, the calculated uncertainty dips to a minimum value that is approximately the same for all  $\rho$  values. The uncertainty values rapidly increase after this point. The larger uncertainties associated with the 3.5 mm connector may be attributable to the larger relative uncertainties of the airline standards used for the calculations.

# Conclusion

This paper has presented two methods to estimate the residual uncertainties for network analyzer measurements of the reflection coefficients of microwave devices. The theory behind the Thru-Reflect-Line (TRL) and the Thru-Match-Short (TMS) methods were presented. Some examples of calculations of the uncertainties using these methods were shown for 3.5 mm, 7 mm, and N connector types.

The uncertainties calculated by these methods are included as Type A errors when measuring a device. The expanded uncertainty for a measurement on a calibrated network analyzer is

$$U_{\rm exp} = k \cdot \sqrt{U_m^2 + U_v^2} ,$$

where  $U_{exp}$  is the expanded uncertainty, k is the coverage factor,  $U_m$  is the Type A measurement uncertainty observed when the device is measured, and  $U_v$  is the VNA uncertainty calculated from the methods described in this paper. Proper network analyzer calibration, multiple measurement on a DUT, and including the estimate of the residual VNA errors described here provide a plausible estimate of the final DUT measurement error.

# References

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3. The TRL method is preferred at higher frequencies because the NIST uncertainties on the airlines are much lower than the uncertainties of the standards required in the TMS method.

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7. Reference to a commercial product is included for completeness only and implies neither endorsement by Sandia National Laboratories or the Department of Energy nor lack of a suitable substitute.

8. Beck, James V. and Arnold, Kenneth J., Parameter Estimation in Engineering and Science, John Wiley and Sons, New York, 1977.

9. Taylor, Barry N. and Kuyatt, Chris E., "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results," NIST Technical Note 1297, September 1994.

10. The symbol, k, used here denotes the coverage factor. It multiplies the combined standard uncertainty to obtain the expanded uncertainty.

11. The software program used is named CERTVANA. It is an HP Basic program (that may be run under either HP Basic or HTBasic) that calculates the uncertainties for all the complex scattering parameters measured by the network analyzers. Besides the reflection coefficient ( $S_{11}$  and  $S_{22}$ ), the program also calculates uncertainties for  $|S_{21}|$ ,  $|S_{12}|$ , and  $Arg(S_{21})$ , and  $Arg(S_{12})$ .

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