MILLION-LINE FAILURE DISTRIBUTIONS FOR NARROW INTERCONNECTS

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ABSTRACT

We examine the distribution of failure times in a simple and computationally efficient, yet reasonably authentic, model of interconnect reliability that allows consideration of statistically significant samples. The model includes an approximate description of the distribution of grain sizes and texture in narrow interconnects, an effective treatment of stress evolution associated with mass transport along grain boundaries, and local relaxation of stresses due to void formation. Failure time distributions for populations of idealized structures are analyzed to aid in interpretation of model behavior.

INTRODUCTION

A central goal of materials degradation and reliability testing is the understanding of the key factors which determine details of the distribution of early failure events. These may be fundamentally different from those controlling the overall distribution of median times to failure, or the distribution of late failures. Test sample sizes are in practice limited, often forcing questionable extrapolations of failure behavior to service conditions of very large scale production devices. Major causes of failure include (i) stress voiding [1], a process by which tensile stresses are relaxed, and (ii) somewhat better characterized, electromigration damage [2] by void and hillock formation, due to current crowding in regions joining pathways for vacancy or ion diffusion of very different characteristics. The relative importance of these two mechanisms of failure, and their interplay during accelerated testing or service, remain poorly understood despite a lot of work [3].

Although long recognized, the link between microstructure and failure behavior is clearly complex. A number of experimental studies have shown, e.g., that narrowing the distributions of grain texture, grain boundary relative orientation, and grain or grain-cluster sizes, can improve interconnect lifetimes. This is related to reduction in the density of interfaces between fine- and large-grained sections of the interconnect, as well as a consequent bias towards preferred low-diffusivity grain boundaries, both of which tend to limit the strength of local flux divergences in the line. Simple models have been developed and analyzed to elucidate this behavior [3]. What seems to be missing, however, is a study that systematically correlates observed failure behavior to specific assumptions or features in a model which contains most of the competing failure mechanisms. Of central interest is their influence on the detailed shape of the distribution of failure times, on which we focus here.

MICROSTRUCTURE

We consider two-dimensional (2D), isolated line structures to illustrate failure behavior and general capabilities of our algorithm. Outer boundaries are rigid as in passivated lines. Grain misorientation and grain-boundary inclination determine the basic failure mechanisms of these lines. Generalization of the analysis, and of certain predictions, to more complex and even 3D structures, and other testing conditions is possible. It is instructive to consider the failure behavior in three limiting regimes of (i) random, (ii) annealed, and (iii) uniform grain distributions. Line segments with random grains are patterned from a 2D Voronoi tessellation, each cell within the line corresponding to a grain. For our model, typical times to reach failure with this random grain distribution are presumably lower bounds to failure times in more realistic grain structures. Annealed grain distributions are obtained from random tessellations, which subsequently undergo coarsening via elimination of the smaller cells. This process leads to an increase in the mean grain size, but also in standard deviation. Grain coarsening is implemented before line testing, although

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coarsening during longer tests or actual service is likely to occur. Many factors influence grain coarsening in real interconnects, including line geometry and packaging, correlations in grain misorientation and in the relative energies of neighboring grains, material purity, and specifics of the kinetics of grain deformation. To some extent, most of these can depend on line processing history, which adds further complexity. Here, we select a very fast algorithm based on a well-documented model of grain coarsening in 2D lines [4]. The basic ingredient is the motion of grain boundaries (which we keep straight) and triple points, driven by an effective capillarity. The latter is controlled by grain diameter. A typical annealing sequence is shown in Fig.1. The resulting grain size distribution is lognormal, as frequently found in experiment [5], but other features of the grain distribution may not match experiment. Patterning of the lines is done after annealing, see Fig.2. Finally, we also test uniformly sized hexagonal grains, so randomness in behavior originates only from the texture. In all three cases, we simplify the treatment of texture, which determines aspects of grain boundary diffusion. Specifically, we assign a random orientation to each of the grains, but note that modifications, e.g., based on experimental data, are in fact straightforward.

Figure 1. Grain coarsening in a line segment, prior to testing. Time increases tenfold for each successive frame, from top to bottom.

Figure 2. Schematic diagram of the patterning of annealed-grain lines. For comparison, on the right we show 4 μ m-wide lines of pure Al (SEM image by Joseph Michael, Sandia, New Mexico).

THE MODEL

We now describe our electromigration and stress-evolution model in some more detail. Here, we discuss a version where key quantities are defined only at triple points and grain boundaries. See Fig.3. This model provides useful benchmark results. In addition, average CPU times on a Sun UltraSPARC 300MHz processor do not exceed 10 seconds per failure, so service-size populations of interconnects can be efficiently tested.

At a grain boundary (gb) joining a triple point (tp), one defines the average atomic flux [3],

$$J_{gb} \approx [D_{gb}/(\Omega_{gb}k_B T)] \cdot \{ (eZ^*E) \cdot \cos(\theta_{gb}) - [\Omega(\sigma_{tp} - \sigma_{gb})/L_{gb}] \}. \quad (1)$$

The first term on the rhs of (1) is dominated by the electronic wind force. Here eZ^* , the effective valence of the diffusing ions, characterizes the momentum exchange between electrons and metal ions. $E \cdot \cos(\theta_{gb})$ is the component of the applied electric field parallel to the gb, θ_{gb} gives the orientation of the gb relative to E , and $1/\Omega_{gb}$ ($1/\Omega$) is the average atomic density at a gb (in the bulk). The second term on the rhs of (1) is the gradient in the normal stress component (σ_{tp}) in a gb of effective length L_{gb} . This term produces an atomic flux that opposes electromigration. Finally, T is the average temperature, k_B is Boltzmann's constant, and D_{gb} is the effective diffusion constant in the gb. It is modeled after experimental data for self-diffusion along symmetric tilt boundaries [6], which suggest a dependence on material parameters and line microstructure of the form

$$D_{gb} \approx A_{gb} \cdot (d_{gb}/d) \cdot \sin(\phi_{gb}/2) \cdot \cos(\psi_{gb}) \cdot \exp[-E_d/(k_B T)]. \quad (2)$$

A_{gb} includes gb information about typical attempt frequencies, entropy factors, average jump distances, atomic coordination of the diffusing species, and effects of correlations or anisotropy in the jumps. Its value (10^8 - 10^{10} Å²/s) is not well known. d_{gb} is the average gb width, d is the average linear grain size in the line, $\phi_{gb} \in (0^\circ, 60^\circ)$ is the misorientation of the grains adjacent to the gb, so $1/\sin(\phi_{gb}/2)$ is proportional to the spacing between dislocations at the gb, $\psi_{gb} \in (0^\circ, 90^\circ)$ is the angle between the gb and the current flow, and E_d an effective activation barrier for gb diffusion.

Due to differences in diffusion parameters, and thus in J_{gb} , between neighboring gb's, material builds up or depletes near tp's or inside gb's of complex structure. If $\delta V_{tp}/V_{tp}$ denotes the fraction of material accumulated or depleted at a tp, then, at each time interval δt , one has

$$\delta V_{tp}/V_{tp} \propto \Omega_{gb} \cdot \delta t \cdot [J_{gb} - J_{gb'} - J_{gb''}]. \quad (3)$$

In our model, the associated increments in the stress are approximately written as

$$\delta \sigma_{tp} \approx \Gamma \cdot \delta V_{tp}/V_{tp}. \quad (4)$$

Here, Γ is an effective elastic modulus function (e.g., $\Gamma \approx 50$ GPa for Al). For a discussion of this approximation, see [7] and the Appendix for details. If failure is assumed to occur when the stress exceeds a certain value, σ^* , then the time, τ_F , to reach failure scales as $\tau_F \sim \sigma^*/(D_{gb}\Gamma)$ [3]. This model of stress evolution can be generalized to include non-local effects. See the Appendix. Detailed test results will be presented elsewhere.

Figure 3. Schematic diagram of a pair of adjacent triple junctions, with notation defined. See text.

SIMULATION RESULTS

We implemented equations (1)-(4) at each gb and tp in the lines tested. Outer interfaces of the line carry no current in the examples discussed below. A summary of the basic steps in the algorithm is given in Fig.4. Lines are $20\mu\text{m}$ long, $2\mu\text{m}$ wide, and $0.25\mu\text{m}$ thick (unless otherwise stated). Tests were done at 200°C , using current densities of the order of $10\text{MA}/\text{cm}^2$. We chose $Z^*=20$, $\Omega_{\text{gb}}=25\text{\AA}^3\approx 0.7\Omega$, $\delta_{\text{gb}}=10\text{\AA}$, $d\approx 1\mu\text{m}$, and $E_d=0.55\text{eV}$. Initial stresses are spatially uniform.

Figure 4. Left: Flowchart of the algorithm used in the simulations. Our code was adapted from a program developed at Hewlett-Packard (see P.J. Marcoux, P.P. Merchant, V. Naroditsky, and W.D. Rehder, June 1989 Hewlett-Packard Journal, p.79). A new ingredient is a treatment of stress evolution as mass flows down gb's, or voids open. We assume rapid heat dissipation to the line surroundings, which simplifies the analysis of the temperature distribution. Right: Two extreme examples of line structures that failed early (top) and late (bottom) compared to the average. One noticeable difference between these lines is in the density of gb's nearly parallel to the applied field (along the line) which join gb's nearly perpendicular to the field.

In the absence of initial voids and void migration, failure in our model results from percolation of voids that nucleate and grow at gb's. In the model, voids nucleate when the local atomic density decreases below a pre-set value, and grow in the direction of the atomic flux in the gb, starting at the tp's. Fig.5 shows voided structures, and corresponding early-time behavior of the electrical resistance, reminiscent of experiment [8].

Figure 5. Simulation results for electromigration-induced electrical resistance changes, $\delta R/R$, versus time, t . Two tested lines are shown. Sudden fluctuations in $\delta R/R$, as in (b), are usually assigned to sequences of void formation and recovery events near quasi-bamboo line sections [8].

A basic question relates to the minimum number of lines needed for a reasonably reliable assessment of early-time failure behavior. Fig.6 addresses this issue for line segments with random grains. Results suggest a minimum of 1000 lines may be typically required to determine the probability of failures below about one tenth of the mean failure time. Similar results are anticipated for other grain structures.

Figure 6. Failure distributions for increasing number of failures, N , in segments with random grain sizes and texture. (a) $N=100$, (b) $N=1000$, and (c) $N=10000$ lines.

A few statistical distributions of similar shape have been proposed to analyze time-to-failure data, or to forecast interconnect reliability [9]. They can differ in predictions for early-time failures. Although never rigorously derived for electromigration or stress-voiding, and despite some problems fitting experiment [10], the lognormal distribution remains the most popular. Lognormality is typically justified in problems where a complex outcome depends on the completion of a chain of events [11]. This seems natural here, since increments in atomic fluxes or stress gradients at gb's are themselves portions of the instantaneous fluxes and stress gradients in the line, the randomness originating in features of the microstructure and the kinetics of mass transport in gb's. See equations (1)-(4). However, our failure distributions show deviations from lognormality, particularly at early-times, for lines with non-random grains. See Figs.7 and 8.

Figure 7. Failure distributions for several thousand segments with (a) random, (b) annealed, and (c) hexagonal grains. Lognormal distributions would look Gaussian in these plots. Instead, the distributions are skewed, with longer early-time tails in (a) and (b), and a (expected) sharp onset of failure in (c).

Figure 8. Cumulative time-to-failure distributions for line segments with (a) random, (b) annealed, and (c) hexagonal grains. Straight lines are guides to the eye. The inset in (a) shows the distribution for a 0.5μm-wide line. Note the "incubation" time [12] in (c), and the fact that all distributions show Pareto-type power tails [13] in the high percentiles of the failure population.

CONCLUSION

Our simulation results estimate that a few thousand lines need be tested to reliably assess the probability of failures below about one tenth of the mean time to failure. All results show deviations from lognormality, especially for early failure events, the extent depending on details of the grain structure (for fixed texture). Several features are missing in the model, that can affect

failure behavior described above. Non-local stress relaxation, e.g., due to void growth and migration, can actually delay failure. For quasi-bamboo lines, interface defects (such as debonding or residual contaminants), as well as grain coarsening and precipitation of impurities at gb's during testing, rather than topological inhomogeneities in the grain structure, may determine failure behavior. We are in the process of studying such effects.

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REFERENCES

1. J.R. Black, IEEE: New York (1967) p.148; IEEE Trans. Elect. Devices **ED-16** (1969) p.338.
2. J.T. Yue, W.P. Funsten, R.V. Taylor, IEEE: New York (1985) p.126.
3. A.H. Verbruggen, IBM J. Res. Develop. **32** (1988) p.465; C.-K. Hu *et al.*, *ibid.* **39** (1995) p.465; P.S. Ho and T. Kwok, Rep. Prog. Phys. **52** (1989) p.301; A. Scorzoni *et al.*, Mat. Sci. Rep. **7** (1991) p.143; H. Okabayashi, Mater. Sci. Eng. **5** (1993) R11; M.D. Thouless, Annu. Rev. Mater. Sci. **25** (1995) p.69; T.D. Sullivan, *ibid.* **26** (1996) p.333.
4. C.S. Smith, Metal Interfaces (Am. Soc. Metals, 1952), p.65; H.V. Atkinson, Acta Metall. **36** (1988) p.469; J.A. Glazier and D. Weaire, J. Phys.: Condensed Matter **4** (1992) p.1867; N. Rivier, in Disorder and Granular Media, edited by D. Bideau and A. Hansen (Elsevier, 1993); see also various contributions in Modeling of Coarsening and Grain Growth, edited by S.P. Marsh and C.S. Pande (The Minerals, Metals & Materials Society, Warrendale, PA 1993).
5. P. Feltham, Acta Metall. **5** (1957) p.97; N.P. Louat, *ibid.* **22** (1974) p.721; S.K. Kurtz and F.M.A. Carpay, J. Appl. Phys. **51** (1980) p.5725; **51** (1980) p.5745; F.N. Rhines and B.R. Patterson, Metall. Trans. **13A** (1982) p.985; M.P. Anderson *et al.*, Acta Metall. **32** (1984) p.783; **33** (1985) p.2233; C.S. Pande, *ibid.* **35** (1987) p.2671; **36** (1988) p.2161; J.E. Palmer *et al.*, J. Appl. Phys. **62** (1987) p.2492; C.S. Pande and E. Dantsker, Acta Met. **38** (1990) p.945; H.J. Frost, C.V. Thompson, and D.T. Walton, *ibid.* **38** (1990) p.1455.
6. Fundamentals of Grain and Interphase Boundary Diffusion, I. Kaur, Y. Mishin, and W. Gust (John Wiley, 1995); Interfaces in Crystalline Materials, A.P. Sutton and R.W. Balluffi (Oxford Science Publishers, 1996).
7. M.A. Korhonen, P. Børgesen, K.N. Tu, and C.-Y. Li, J. Appl. Phys. **73**, p.3790 (1993).
8. J. Niehof, H.C. de Graaff, and J.F. Verwey, MRS Proc. **309** (1993) p.295; U.E. Möckl, J.R. Loyd, and E. Arzt, *ibid.* **309** (1993) p.301; S. Shingubara *et al.*, MRS Proc. **338** (1994) p.441; G.B. Alers, A.S. Oates, and N.L. Beverly, Appl. Phys. Lett. **66** (1995) p.3600; J.R. Kraayeveld *et al.*, *ibid.* **67** (1995) p.1226.
9. M. Shatzkes and J.R. Lloyd, J. Appl. Phys. **59** (1986) p.3890; L.P. Muray, L.C. Rathbun, and E.D. Wolf, Appl. Phys. Lett. **53** (1988) p.1414; J. Cho and C. V. Thompson, *ibid.* **54** (1989) p.2577; J.R. Lloyd, SPIE **1596** (1991) p.106; J.R. Lloyd and J. Kitchin, J. Appl. Phys. **69** (1991) p. 2117; J. Kitchin and J.R. Lloyd, MRS **225** (1991) p.27.
10. M.J. Attardo, R. Rutledge, and R.C. Jack, J. Appl. Phys. **42** (1971) p.4343.
11. P.W. Anderson *et al.*, Phys. Rev. B **22** (1980) p.3519; A. Douglas Stone, J.D. Joannopoulos, and D.J. Chadi, *ibid.* **24** (1981) p.5583; S. Havlin, J.E. Kiefer, and G.H. Weiss, *ibid.* **38** (1988) p.4761; L. Baker, A.J. Giancola, and F. Allahdadi, J. Appl. Phys. **72** (1992) p.2724; O. Sotolongo-Costa *et al.*, Phys. Rev. Lett. **76** (1992) p.42; R. Delannay, G. Le Caër, and R. Botet, J. Phys. A **29** (1996) p.6693; T. Antal and Z. Rácz, Phys. Rev. E **54** (1996) p.2256.
12. R.G. Filipi, G.A. Biery, and R.A. Wachnik, Appl. Phys. Lett. **66** (1995) p.1897.
13. E.W. Montroll and M.F. Shlesinger, J. Stat. Phys. **32** (1983) p.209.

APPENDIX

GENERALIZED KORHONEN ANALYSIS OF STRESS EVOLUTION - We assume that mass transport along gb's produces inelastic strains in the line. The goal is to then derive a relation between these strains and the resulting stresses. The standard relation between mass displacements, $\mathbf{u}(\mathbf{r})$, and the associated strains, $\mathbf{e}(\mathbf{r})$, at position $\mathbf{r}$, is written as $\mathbf{C}_{ijkl}(\mathbf{u}_{k,lj} - \mathbf{e}_{kl,j}) = 0$, where $\mathbf{C}$ is the elastic tensor of the line. Our notation includes a sum over repeated indices, and the usual abbreviation for spatial derivatives. We augment this relation with the boundary condition $Z_{ik} \mathbf{u}_k = \mathbf{C}_{ijkl}(\mathbf{u}_{k,l} - \mathbf{e}_{kl}) \mathbf{n}_j(\mathbf{r})$, at the interface between the line and the substrate or passivation. Z is a modulus function for the material surrounding the line, and $\mathbf{n}$ is a unit vector normal to the interface. Formally, one can introduce the Green's function (tensor), $\mathbf{G}(\mathbf{r}, \mathbf{r}_0)$, which obeys $\mathbf{C}_{ijkl} \mathbf{G}_{km,jl}(\mathbf{r}, \mathbf{r}_0) + \delta_{im} \delta(\mathbf{r} - \mathbf{r}_0) = 0$, and the boundary condition $Z_{in} \mathbf{G}_{im}(\mathbf{r}, \mathbf{r}_0) = \mathbf{C}_{njkl} \mathbf{G}_{km,l}(\mathbf{r}, \mathbf{r}_0) \mathbf{n}_j$, so that, in 2D, $\mathbf{u}_m(\mathbf{r}_0) = \mathbf{C}_{ijkl} \int d^2 \mathbf{r} \mathbf{G}_{im,j}(\mathbf{r}, \mathbf{r}_0) \mathbf{e}_{kl}(\mathbf{r})$. Then $\sigma_{pq}(\mathbf{r}_0) = \mathbf{C}_{pqmn} \mathbf{u}_{m,n} = \mathbf{C}_{pqmn} \mathbf{C}_{ijkl} \int d^2 \mathbf{r} \mathbf{G}_{im,jn}(\mathbf{r}, \mathbf{r}_0) \mathbf{e}_{kl}(\mathbf{r})$ gives the stress components, σ_{pq} . To separate *local* ($\mathbf{r} = \mathbf{r}_0$) from *non-local* ($\mathbf{r} \neq \mathbf{r}_0$) contributions, we deform the domain of integration, so that $\sigma_{pq}(\mathbf{r}_0) = \Gamma_{pqkl}^{(1)}(\mathbf{r}_0) \mathbf{e}_{kl}(\mathbf{r}_0) + P \int d^2 \mathbf{r} \Gamma_{pqkl}^{(2)}(\mathbf{r}, \mathbf{r}_0) \mathbf{e}_{kl}(\mathbf{r})$, where $\Gamma^{(1)}$ and $\Gamma^{(2)}$ are elastic response functions related to $\mathbf{C}$ and $\mathbf{G}$. E.g., one has $\Gamma_{pqkl}^{(2)}(\mathbf{r}, \mathbf{r}_0) = \mathbf{C}_{pqmn} \mathbf{C}_{ijkl} \mathbf{G}_{im,jn}(\mathbf{r}, \mathbf{r}_0)$. Here, P denotes Cauchy's principal value. These results reduce to Korhonen's model [7], $\sigma_{pq}(\mathbf{r}_0) \approx \Gamma_{pqkl}^{(1)}(\mathbf{r}_0) \mathbf{e}_{kl}(\mathbf{r}_0)$, when non-local effects are neglected.

EVALUATION OF THE ELASTIC RESPONSE FUNCTIONS - We used finite element method (FEM) calculations to study the elastic response functions defined above. Our strategy was to consider limiting situations where strains are applied only at a point $\mathbf{r}^*$, so $\mathbf{e}(\mathbf{r}^*) \neq 0$ but $\mathbf{e}(\mathbf{r} \neq \mathbf{r}^*) = 0$. Then, at any $\mathbf{r}_0 \neq \mathbf{r}^*$, one has $\sigma_{pq}(\mathbf{r}_0) = \Gamma_{pqkl}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{kl}(\mathbf{r}^*)$, so the components of $\Gamma^{(2)}$ can be obtained given the stresses. We illustrate this procedure for the thermal expansion of a cylinder in an infinite plate, a situation that mimics the development of stresses during diffusion through gb's. We assume an isotropic material, so $\Gamma_{ijkl}^{(2)} = \Gamma_{jikl}^{(2)} = \Gamma_{ijlk}^{(2)} = \Gamma_{klij}^{(2)}$, and all shear components, $\Gamma_{ijkl}^{(2)}$ with $i \neq j$ or $k \neq l$, vanish. In cylindrical coordinates (r, θ, z) , there are then only six non-zero components of $\Gamma^{(2)}$. For plane strain ($\mathbf{e}_{zz}(\mathbf{r}^*) = 0$), $\sigma_{rr}(\mathbf{r}_0) = \Gamma_{rrrr}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{rr}(\mathbf{r}^*) + \Gamma_{rr\theta\theta}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{\theta\theta}(\mathbf{r}^*)$, $\sigma_{\theta\theta}(\mathbf{r}_0) = \Gamma_{\theta\theta\theta\theta}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{\theta\theta}(\mathbf{r}^*) + \Gamma_{\theta\theta rr}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{rr}(\mathbf{r}^*)$, $\sigma_{zz}(\mathbf{r}_0) = \Gamma_{zzzz}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{zz}(\mathbf{r}^*) + \Gamma_{zz\theta\theta}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) \mathbf{e}_{\theta\theta}(\mathbf{r}^*)$. Uniform, uni-directional strains are applied by prescribing orthotropic coefficients of thermal expansion. So, if the net strain is $\mathbf{e}_{rr}(\mathbf{r}^*) \neq 0$ and $\mathbf{e}_{\theta\theta}(\mathbf{r}^*) = 0$, then one has, e.g., $\Gamma_{rrrr}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) = \sigma_{rr}(\mathbf{r}_0) / \mathbf{e}_{rr}(\mathbf{r}^*)$, $\Gamma_{rr\theta\theta}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) = \sigma_{\theta\theta}(\mathbf{r}_0) / \mathbf{e}_{rr}(\mathbf{r}^*)$, and $\Gamma_{zzzz}^{(2)}(\mathbf{r}^*, \mathbf{r}_0) = \sigma_{zz}(\mathbf{r}_0) / \mathbf{e}_{rr}(\mathbf{r}^*)$. Similarly for other components of $\Gamma^{(2)}$. Fig.9 shows results for Al. We check these results by applying arbitrary components of strain at $\mathbf{r}^* = 0$, and measuring the stresses in a FEM analysis. The results are identical to those obtained using the calculated $\Gamma^{(2)}$. For a finite Al plate with rectangular cross-

section and SiO_2 passivation, preliminary results for an element heated near the metal-oxide interface suggest $\Gamma^{(2)}$ values 20-50% different from those in Fig.9, with the larger differences at the corners of the line.

Figure 9 - Non-zero components of $\Gamma^{(2)}$ for a heated cylindrical volume in an infinite plate. $\Delta T = 100^\circ \text{C}$; $Y \approx 70 \text{ GPa}$; $\nu \approx 0.3$; $\alpha_r \approx 23.5 \text{ ppm}$, $\alpha_\theta \approx 1 \text{ ppm} \approx 0$, $\alpha_z \approx 0$. a- $\Gamma_{\theta\theta\theta\theta}^{(2)}$, b- $\Gamma_{\theta\theta zz}^{(2)}$, c- $\Gamma_{rr\theta\theta}^{(2)}$, d- $\Gamma_{rrzz}^{(2)}$, e- $\Gamma_{rrrr}^{(2)}$. r_0 is the distance to the cylinder axis ($\mathbf{r}^* = 0$), in units of the cylinder radius. For large r_0 , all components of $\Gamma^{(2)}$ decay like $1/r_0^2$.

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