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# A VALIDATION OF BAYESIAN FINITE ELEMENT MODEL UPDATING FOR LINEAR DYNAMICS

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## ABSTRACT

This work addresses the issue of statistical model updating and correlation. The updating procedure is formulated to improve the predictive quality of a structural model by minimizing out-of-balance modal forces. It is shown how measurement and modeling uncertainties can be taken into account to provide not only the correlated model but also the associated confidence levels. Hence, a Bayesian parameter estimation technique is derived and its numerical implementation is discussed. Two demonstration examples that involve test-analysis correlation with real test data are presented. First, the validation of an engine cradle model used in the automotive industry shows how the design's uncertainties can be reduced via model updating. Our second example consists of employing test-analysis correlation for identifying the degree of nonlinearity of the LANL 8-DOF testbed.

## NOMENCLATURE

The recommended "Standard Notation for Modal Testing & Analysis" proposed in Reference [1] is used throughout this paper.

## 1. INTRODUCTION

In structural dynamics, the method of obtaining a correct parametric representation of a test article is to create a finite element (FE) model of the system and correlate this model with measurement data taken from the system itself or some of its components [2]. Applied essentially to linear systems, this approach has been found quite effective when modal data are used in the correlation process, because 1) experimental procedures in the form of modal tests permit to identify these modal parameters; and 2) the same quantities can be extracted easily from a FE model when the response of interest involves the low-frequency spectrum of the dynamics. This procedure is illustrated in Figure 1 where the FE matrices or a subset of design variables are optimized until the test-analysis correlation (TAC) is found acceptable.

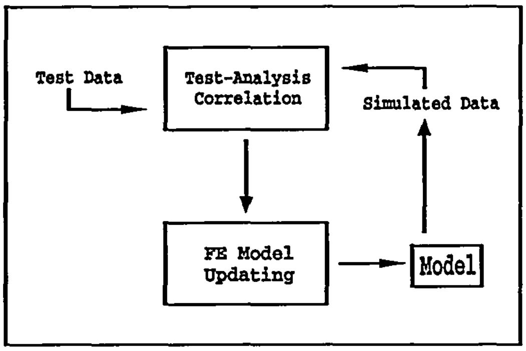


**Figure 1. Illustration of Test-Analysis Correlation and FE Model Updating.**

Since many correlation metrics can be formulated, countless model updating and damage detection techniques have been proposed and validated for the past three decades, a comprehensive review of which can be found in Reference [3]. For example, we cite TAC metrics based on the difference between identified and computed frequencies [4], the distance between test and analysis mode shapes [5], the change in mode shape curvature [6], the cross-orthogonality between test and analysis mode shapes [7], or the verification of "hybrid" equations of motion where the system's representation is given by a parametric model and the dynamics are described by measurements [8-10].

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## DISCLAIMER

As the inverse problems of health monitoring and damage detection grew increasingly popular in the past years, variability has been observed to be a major obstacle to the practical implementation of correlation and FE updating software. Measured response levels and identified modal parameters may, for example, vary according to environmental conditions, such as temperature and humidity [11]. Recently, full-scale damage detection experiments on various bridges [12-13] have shown that modal parameter variability can "hide" the effects of damage, therefore, making it very difficult to assess the structural integrity of a system based on its dynamic response.

Nevertheless, well-defined statistical tools are available for analyzing variability in test data and taking advantage of it. For example, using statistical tests it can be assessed if frequency variations are significant or not. Conversely, confidence intervals can be obtained by perturbing an experimental or a computational model [14]. Then, feeding these confidence intervals together with the baseline data to a correlation and FE model updating package might help improving the correlation by taking advantage of the statistical distribution of test data and modeling uncertainties. It is this approach, summarized in Figure 2, that this work aims at validating.

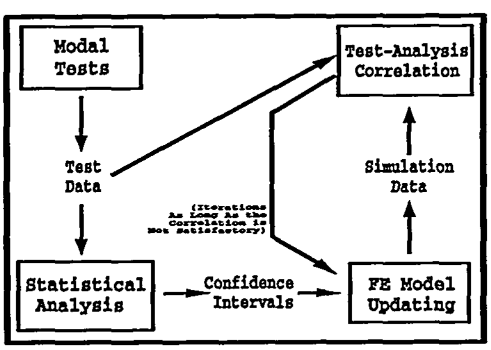


**Figure 2. Interactions Between Statistical Treatment of Test Data, Test-Analysis Correlation and FE Model Updating.**

We emphasize that the theory and implementation of the correlation technique used here have been first proposed by Alvin in Reference [15]. Rather than attempting to improve the algorithm, this work shows using two experimental testbeds how the approach illustrated in Figure 2 can help us defining our modeling rules, which is after all what FE model updating is all about.

## 2. EXPERIMENTAL TESTBEDS

A brief description of the two experiments used for validating the statistical correlation technique is provided in this Section. With the GM engine cradle testbed, our aim is to show how modeling rules can be established from a statistical-based correlation. With the LANL 8-DOF testbed, we investigate whether the covariance data provide any insight as of the degree of nonlinearity of the system tested.

### 2.1 The GM Engine Cradle Testbed

Our first testbed is a simplified model of engine support constructed and tested by GM. The structure consists of two tubular longerons welded to two transverse beams. A very crude FE model is illustrated in Figure 3: it involves essentially 116 Euler-Bernoulli beam elements, 4 rotation and translation springs and it features a total of 672 active DOFs. The most uncertain aspects of the model are the four welded connections and the associated joint compliance.

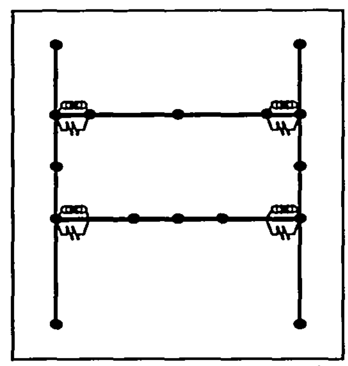

**Figure 3. Geometry, Modeling and Measurement Locations of the GM Engine Cradle Testbed.**

For TAC purposes, we select the subset of 16 nodes indicated in Figure 3 by the solid dots. Three translation measurements are available at each instrumented node. The structure is tested in a free-floating configuration and a description of the first three non-rigid identified modes is given in Table 1.

**Table 1. Description of Identified Mode Shapes of the Engine Cradle Support.**

| Identified Frequency | Description of Mode Shape Vector |
|---|---|
| 79.0 Hz | Out-of-phase Bending-1 of Longerons |
| 170.6 Hz | In-phase Bending-2 of Longerons |
| 174.5 Hz | Out-of-phase Bending-2 of Longerons |

Table 2 shows a typical correlation attempted between the test data and the FE model prior to any parametric adjustment. Despite our simplistic modeling (no offsets, for example, are introduced to account for the sensors and their off-centered measurement points), the lowest end of the frequency spectrum is captured with reasonable accuracy. It can be observed in Table 2 that no systematic error seems to have been introduced in the FE model since the first four FE modes are too stiff, compared to the identified dynamics, while the next three are too flexible.

**Table 2. TAC Before FE Model Updating (Identified System Vs. Nominal FE Model).**

| Identified Frequency | FE Model Frequency | Frequency Error | MAC |
|---|---|---|---|
| 79.0 Hz | 89.4 Hz | 13.2% | 98.9% |
| 170.6 Hz | 178.4 Hz | 4.6% | 96.9% |
| 174.5 Hz | 180.8 Hz | 3.6% | 98.4% |
| 214.7 Hz | 224.8 Hz | 4.7% | 97.8% |
| 250.9 Hz | 200.6 Hz | -20.0% | 98.6% |
| 312.2 Hz | 251.1 Hz | -19.6% | 97.1% |
| 315.8 Hz | 288.5 Hz | -8.6% | 96.3% |

Further details on the GM engine cradle testbed are available from Reference [15]. A thorough investigation is presented where the author correlates unambiguously the first 14 identified modes using the same Bayesian updating method and a more sophisticated FE model.

### 2.2 The LANL 8-DOF Testbed

The LANL 8-DOF (which stands for Los Alamos National Laboratory eight degrees of freedom) testbed consists of eight masses connected by linear springs. The masses are free to slide along a center rod that provides support for the whole system. Boundary conditions are unrestrained. Figure 4 shows the experimental testbed that is instrumented with eight accelerometers and where excitation is provided using either a hammer or a shaker. Modal tests are performed on the nominal system and on a damaged version where the stiffness of the fifth spring is reduced by 14%.

**Figure 4: LANL 8-DOF Testbed.**

Table 3 compares the seven flexible modes identified with the nominal and damaged systems. Hammer excitations and data averaging are used for these series of tests. Large damping ratios can be observed. It suggests that the sliding mechanism and the friction it generates play an important role in the dynamics measured. Also, the reduction of stiffness translates, as expected, into a reduction of modal frequencies.

**Table 3. Identified Modal Parameters For the Nominal and Damaged Systems.**

| Nominal Frequency | Modal Damping | Damaged Frequency | Modal Damping |
|---|---|---|---|
| 22.6 Hz | 8.5% | 22.3 Hz | 13.6% |
| 44.5 Hz | 4.3% | 43.9 Hz | 5.0% |
| 65.9 Hz | 3.3% | 64.8 Hz | 3.5% |
| 86.6 Hz | 5.0% | 85.9 Hz | 5.9% |
| 99.4 Hz | 2.6% | 99.7 Hz | 3.6% |
| 113.0 Hz | 1.5% | 113.2 Hz | 2.0% |
| 133.2 Hz | 0.7% | 131.9 Hz | 1.8% |

Correlation results are shown in Table 4 between the identified modal parameters of the damaged system and results obtained with the nominal (undamaged) FE model. The modal assurance criterion (MAC) illustrates the excellent agreement between test and model vectors since most values are above 90%, despite the unmodeled stiffness reduction and the effect of friction.

**Table 4. TAC Before FE Model Updating (Damaged System Vs. Nominal FE Model).**

| Identified Frequency | FE Model Frequency | Frequency Error | MAC |
|---|---|---|---|
| 22.3 Hz | 21.8 Hz | -2.3% | 99.7% |
| 43.9 Hz | 43.0 Hz | -2.0% | 99.4% |
| 64.8 Hz | 63.0 Hz | -2.8% | 99.4% |
| 85.9 Hz | 80.8 Hz | -6.0% | 93.2% |
| 99.7 Hz | 95.6 Hz | -4.1% | 98.5% |
| 113.2 Hz | 110.3 Hz | -2.5% | 93.1% |
| 131.9 Hz | 116.8 Hz | -11.5% | 77.9% |

It is interesting to notice that this FE model is already too flexible compared to the damaged test article (FE frequencies in Table 2 are all smaller than identified frequencies) even though structural damage has not been introduced in the model at this stage. Therefore, prior to our damage detection experiment, the model is correlated with the nominal (undamaged) system in an attempt to increase its stiffness and provide a better starting point.

The LANL 8-DOF testbed is designed for validating modeling and TAC techniques for nonlinear, transient dynamics. To this effect, a contact mechanism can be added that makes two masses impact each other during the modal tests. In this work, however, only linear (no impact) configurations are investigated, whether they feature damage (in the form of stiffness reduction) or not.

## 3. DETERMINISTIC FE MODEL UPDATING

In this Section, we present the deterministic FE model updating technique developed in References [9] and [15-16]. For simplicity, we restrict ourselves to the case where only undamped modal data are considered. However, generalization to arbitrary data and inclusion of dissipative modeling offer no difficulty other than programming challenges.

### 3.1 Formulation of FE Model Updating

The method consists of minimizing the modal residue vectors defined as out-of-balance forces acting on the FE model when the reference dynamics are represented by a set of identified modal parameters $\left(\omega^2;\{\phi\}\right)$. The residue vectors are obtained by satisfying

$$[K(p)]\{\phi\} = \omega^2[M(p)]\{\phi\} + \{R_f(p,\omega)\} \qquad (1)$$

where $[M(p)]$ and $[K(p)]$ represent the model's mass and stiffness matrices, respectively. These depend on design variables $\{p\}$ which express the parametric nature of FE representations. Here, we define model updating as the procedure by which these variables $\{p\}$ are optimized to minimize the distance between test data and FE simulations.

Clearly, the out-of-balance residues exhibit the largest entries at DOFs where the equilibrium is violated the most. (In the best case scenario, the numerical model is a perfect representation of the structure, no measurement noise is affecting the test data, and the residue vector in equation (1) is equal to zero because the equation of vibration must be satisfied.) Hence, the source of modeling error can be isolated by reviewing these out-of-balance forces and by investigating the elements connected to DOFs where they are the largest. This approach is referred to as force-based modal updating since entries of residues $\{R_f(p,\omega)\}$ in equation (1) are consistent with forces. Then, an objective function $J(p)$ is defined that represents the 2-norm (Euclidean norm) of our residue vectors

$$J(p) = \left\|R_f(p,\omega)\right\| + \alpha\left\|p - p_0\right\| \qquad (2)$$

The objective function includes a minimum change term, or regularization term, that helps reducing the numerical ill-conditioning characteristic of inverse problems. From an engineering point-of-view, it simply means that an optimum design $\{p\}$ is sought after that brings the least possible change to the original design $\{p_0\}$.

Finally, optimization algorithms are used for minimizing the objective function while satisfying constraints on the design. Typical choices are order-zero algorithms (the simplex method) and order-one algorithms (Gauss-Newton, BFGS and Levenberg-Marquardt, for which documentation can be found in Reference [17]). Gradients are required with order-one optimization methods. They can either be calculated with a centered finite difference scheme, which is accurate but becomes computationally intensive with large dimensional FE models or they can be estimated analytically based on definition (1), which is computationally efficient but may yield convergence difficulties if the mode shapes are highly sensitive to design changes.

### 3.2 Matching Test and FE Discretizations

The reason why the objective function (2) can not be minimized directly using, for example, a first-order Taylor's expansion comes from the fact that, usually, measurements available are incomplete. Therefore, a mismatch appears in definition (1) between the identified mode shape $\{\phi\}$ (known at measurement locations only) and the full-size FE matrices. Either model reduction or vector expansion must be implemented prior to calculating the modal residues. In the first case, reduced-order FE matrices become nonlinear functions of the design variables; in the second case, it is the expanded vectors that are made implicitly dependent on the design.

To preserve the ability to locate modeling errors using out-of-balance forces at each DOF of the discretization, we chose to expand the test vectors. Hence, missing mode shape components are calculated by solving a system of linear equations obtained by differentiating the objective function (2) with respect to any non-measured mode shape DOF.

## 4. BAYESIAN PARAMETER ESTIMATION

Prior to applying the concepts of statistical inference to our inverse problem, the basics of Bayesian estimation are presented in Section 4.1, the solution procedure is derived in Section 4.2 and a key issue is discussed in Section 4.3.

### 4.1 Theoretical Background

The linear problem of parameter estimation consists of determining the optimal parameters $\{p\}$ such that the following system of (linear) equations is satisfied

$$\{y\}=\{y_0\}+[A(p_0)](\{p\}-\{p_0\})+\{e\} \qquad (3)$$

In equation (3), vectors $\{y\}$ and $\{e\}$ would typically represent measurements obtained through the instrumentation of a physical system and the associated measurement noise, respectively. Matrix $[A(p)]$ is the parametric model used for best-fitting the test data. We can see that, to obtain the best possible curve fit, it is desirable to minimize the error vector $\{e\}$. Dealing with nonlinear parametric models (which is what we are eventually interested in for our structural dynamics application) basically consists of solving similar systems of linearized equations where matrix $[A(p)]$ represents the gradient, with respect to parameters $\{p\}$, of the nonlinear model

$$\{y\}=\{F(p)\}+\{e\}, \quad [A(p)]=\left[\frac{\partial F}{\partial p}(p)\right] \qquad (4)$$

Hence, the solution procedure for nonlinear models can be summarized as follows: 1) Calculate the gradient matrix $[A(p_0)]$ at current design $\{p_0\}$; 2) Solve the linearized estimation problem (3); and 3) Advance the solution and keep iterating until convergence. Clearly, the core problem of parameter estimation, whether linear or not, is represented by the linear system of equations (3).

When it is assumed that the random variables are normally distributed, it can be shown that many popular statistical estimators all provide the same solution. For example, we cite the least-squares, the maximum likelihood, the minimum mean square error and the best linear unbiased (BLUE) estimators. All of these are obtained by minimizing different objective functions but they yield the same estimator that we refer to as the "Bayesian estimator" in the remainder. The reader is referred to Reference [18] for further details and a comprehensive list of publications where these theories are explicated. Note that, for engineering applications, the assumption of normal distributions is the usual assumption when dealing with test data and it also applies to material and geometry uncertainties to a great extent.

### 4.2 Practical Solution Procedure

Since many objective functions can be used as starting points, we define for clarity the Bayesian estimator as the optimal set of parameters $\{p\}$ that minimize the cost function $J(p)$ defined by

$$J(p)=\{e\}^T[S_{ee}]^{-1}\{e\} + (\{p\}-\{p_0\})^T[S_{pp}]^{-1}(\{p\}-\{p_0\}) \qquad (5)$$

In equation (5), matrices $[S_{ee}]$ and $[S_{pp}]$ represent covariance matrices of the error and parameter change terms, respectively. If left constant throughout the optimization, the procedure simply becomes a generalized least-squares minimization. However, in the general case, the solution procedure consists of solving the system of equations obtained by writing the necessary Kuhn-Tucker condition $\delta J(p)=0$. It can be verified that its solution is

$$\{p\}=\{p_0\}+[S_{rr}]^{-1}[A(p_0)]^T[S_{ee}]^{-1}\{y\} - [S_{rr}]^{-1}[A(p_0)]^T[S_{ee}]^{-1}[A(p_0)]\{p_0\} \qquad (6)$$

where

$$[S_{rr}]=[S_{pp}]^{-1}+[A(p_0)]^T[S_{ee}]^{-1}[A(p_0)] \qquad (7)$$

Equations (6-7) show that many matrix inversions are required to compute the solution: this is a practical reason why covariance matrices are usually approximated and kept as simple as possible. For example, the covariance matrix of the optimized design can be estimated as

$$[S_{pp}]^{(new)}=\left([S_{pp}]^{-1}+\left[\frac{\partial e}{\partial p}\right]^T[S_{ee}]^{-1}\left[\frac{\partial e}{\partial p}\right]\right)^{-1} \qquad (8)$$

Equation (8) is based on a first-order approximation where it is assumed that the original covariance of the model $\left[S_{pp}\right]$ is "small" compared to the covariance of the error $\left[S_{ee}\right]$, which happens when the model is a fair representation of the system. Another approximation commonly used is to consider that covariance matrices $\left[S_{ee}\right]$ and $\left[S_{pp}\right]$ are diagonal. This keeps the matrix inversions in equations (6-8) to their simplest possible expressions and yields significant computational savings.

Nevertheless, we emphasize that our main interest in the Bayesian estimation procedure outlined previously is that it provides an update of variances $S_{PP}(i,i)$ as the model is optimized. These variances characterize the statistical distribution of the design $\{p\}$. An illustration is given in equation (8): even though approximations are made, the diagonal entries of the inverse covariance matrix $\left[S_{pp}\right]^{-1}$ increase because a positive semi-definite matrix is added. In other words, variances $S_{PP}(i,i)$ of design variables $p(i)$ decrease, which basically means that the Bayesian estimation can only improve our knowledge of the system.

### 4.3 Approximation of Covariance Matrices

As mentioned previously, a critical issue of Bayesian estimation is to approximate the various covariance matrices. The procedure we have found general and effective is to implement a first-order Taylor's expansion as discussed briefly in this Section.

The covariance between random variables $\{a\}$ and $\{b\}$ is defined as

$$\left[S_{ab}\right] = \mathrm{E}[(a-\mathrm{E}[a])(b-\mathrm{E}[b])^T] \qquad (9)$$

where $\mathrm{E}[]$ denotes the expected value. In our structural dynamics application, random variables such as $\{a\}$ depend on measured quantities that feature covariance matrices of their own. For example, the error $\{e\}$ can be defined as the difference between test and analysis eigenvalues and modal tests usually provide accurate estimations of the measured eigenvalue's variances. For simplicity, we denote these (measured) variables as $\{q\}$ and the corresponding covariance matrix as $\left[S_{qq}\right]$. Then, a first-order Taylor's expansion of a random variable $\{a\}$ about the current set of modal parameters $\{q_0\}$ can be written as

$$\begin{aligned}\{a\}-\mathrm{E}[a] &= \{a(q)\}-\{a(q_0)\} \\ &= \left[\frac{\partial a}{\partial q}(q_0)\right](\{q\}-\{q_0\})\end{aligned} \qquad (10)$$

Finally, substituting the previous expansion in definition (9) provides a first-order approximation for the covariance matrix between random variables $\{a\}$ and $\{b\}$ where gradients are obtained (usually, explicitly) from the equation of motion and the TAC metrics

$$\left[S_{ab}\right] = \left[\frac{\partial a}{\partial q}(q)\right]\left[S_{qq}\right]\left[\frac{\partial b}{\partial q}(q)\right]^T \qquad (11)$$

In our numerical implementation, covariance matrices are approximated using this procedure, then, they are reduced to their main diagonal for providing cheap storage and matrix inversion. Practically, a zero covariance at an off-diagonal entry $(i,j)$ means that random variables $a(i)$ and $b(j)$ are uncorrelated, which is generally the assumption made when dealing with measurement noise or modeling uncertainties (and when no systematic source of error can be justified).

## 5. BAYESIAN FE MODEL UPDATING

After having discussed the general theory of Bayesian parameter estimation, it is now applied to FE model updating. Rather than showing how to derive the entire procedure, we provide a brief summary in Section 5.1 and the key issue of mode shape expansion is discussed in Section 5.2. The reader is referred to Reference [15] for more details.

### 5.1 Formulation of the Inverse Problem

In the light of Section 4, generalizing our FE model updating procedure to include the Bayesian estimation concept is rather trivial. The objective function (5) is minimized using error vectors $\{e\}$ defined as our modal residues (1).

One major difference with the linear case described in Section 4 is that the state equation that relates the error $\{e\}$ to variables $\{p\}$ is no longer linear. Measurement incompleteness introduces an implicit relationship between expanded test vectors and design variables. To minimize the objective function, we must therefore employ the same optimization algorithms as before (see Section 3) and centered finite differences are used whenever gradients are needed.

The last remaining difficulty is the calculation of covariance matrices for our modal residue vectors. According to the procedure outlined in Section 4.3, they are first estimated using order-one expansions, then reduced to their main diagonal. With modal data, the only two variables to consider in the generic solution (11) for a given mode are the identified eigenvalue $\omega^2$ and mode shape $\{\phi\}$. Their variance/covariance data are denoted by $s_{\omega\omega}$ and $[S\phi\phi]$, respectively. Then, the covariance of modal residues is approximated from equation (11) as

$$\begin{aligned}[S_{RR}] &= s_{\omega\omega}[M][P]\left(\{\phi\}\{\phi\}^T\right)[P]^T[M]^T \\ &+[Z][P][S\phi\phi][P]^T[Z]^T\end{aligned} \tag{12}$$

where $[Z]=\left([K]-\omega^2[M]\right)$ is the dynamic stiffness matrix and where the notation is simplified to $[M]=[M(p)]$ and $[K]=[K(p)]$ for clarity. In equation (12), matrix $[P]$ represents the modal expansion: it is a transformation matrix that projects the measured mode shape $\{\phi\}$ into the full-order FE space. Many different expansion techniques can be implemented, which is why matrix $[P]$ is left somewhat arbitrary here. For more details, Reference [19] provides a description of popular expansion matrices that can be used.

This concludes our presentation of the Bayesian estimation technique applied to FE model updating. In the next Section, we emphasize the coupling between covariance matrices and modal expansion because it is, to our opinion, the key issue of the computational procedure.

### 5.2 Incomplete Measurement Coupling

To show how coupling is introduced between covariance matrices and modal expansion, we need to explicit the expansion matrix.

For simplicity, we partition symbolically the expansion matrix according to the measured and non-measured DOFs. Usually, the part of the projection matrix acting on instrumented DOFs is simply equal to the identity because the expanded mode shape is required to reproduce the test data at measurement points. As mentioned previously, modal expansion results from solving the system of linear equations obtained by differentiating the objective function (5) with respect to non-measured DOFs. By doing so, it can be verified that, for a given mode, the expansion matrix consistent with our cost function is equal to

$$[P]=\begin{bmatrix}[I] \\ -\left([Z_2]^T[S_{RR}]^{-1}[Z_2]\right)^{-1}\left([Z_2]^T[S_{RR}]^{-1}[Z_1]\right)\end{bmatrix} \tag{13}$$

In equation (13), notations $[Z_1]$ and $[Z_2]$ represent partitions of the dynamic stiffness matrix: the first one is a restriction to the columns of instrumented DOFs while the second one is a restriction to the columns of non-measured DOFs. Comparing equations (12) and (13) shows that the projection and covariance matrices are required to calculate each other. Clearly, solving the system of nonlinear equations (12-13) for $[P]$ and $[S_{RR}]$ is out of the question, even with small FE models. Instead, it is resolved in a staggered way: modal expansion (13) is performed using the covariance matrix from previous iteration; then, variances are updated with equation (12) using the newly expanded test data.

As pointed out in Reference [18], this computational procedure is similar to the predictor-corrector steps of Kalman filtering. Nevertheless, we have witnessed the high computational requirement of handling the coupling (12-13). Also, numerical difficulties may arise during modal expansion (13) where ill-conditioned systems of equations must be inverted.

## 6. ILLUSTRATION OF MODEL IMPROVEMENT WITH THE GM ENGINE CRADLE TESTBED

The Bayesian parameter estimation procedure is first applied to the GM engine cradle testbed. Our purpose is to assess by how much our very simple FE model can be improved and to estimate the degree of confidence the analyst can have in the updated model.

The updating discussed here consists of using only the first three identified modes to refine the model. These are provided at 48 measured translations, meaning that 7% of the model's DOFs are measured, a ratio of measurement points typical of those encountered in the automotive industry. The modulus of elasticity of each element is allowed to be modified but reductions smaller than 50% of increases greater than 200% are not permitted. This approach is chosen, rather than grouping elements that belong to the same component together, for checking how consistent corrections brought to the model are. We also mention that our software is currently being validated under the form of a Matlab toolbox for FE modeling, parametric adjustment and optimization of structural dynamics models.

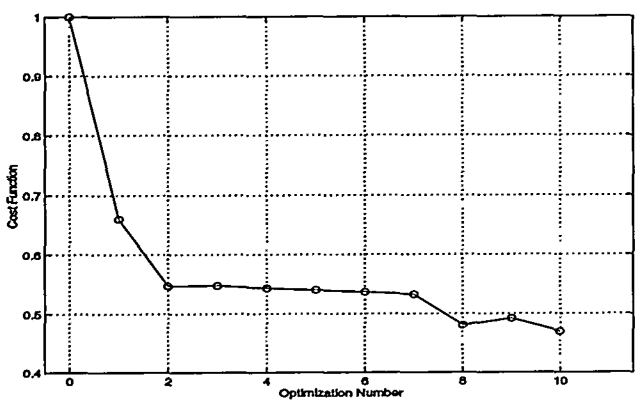


**Figure 5. Convergence of the Updating.**

Figure 5 illustrates a typical convergence curve obtained when the GM engine cradle model is updated. A total of ten optimizations are performed using the gradient-based BFGS algorithm. Usually, a single optimization is sufficient to identify most of the modeling error, as shown in Figure 5 by the steep slope of the curve during the first optimizations.

We have learned from this application the high cost of coupling FE model updating to Bayesian estimation. Up to one hour of CPU time is spent per optimization, when running Matlab 5.1 on a Silicon Graphics R10,000 processor. One reason is that Matlab is an interpreted rather than compiled language. Another is that singular systems of equations must be inverted when covariance matrices are updated, which requires special algorithms that are not available for sparse matrices in Matlab's core environment. These are therefore custom-made or replaced with full-matrix algebra, both solutions being very expensive in terms of CPU time.

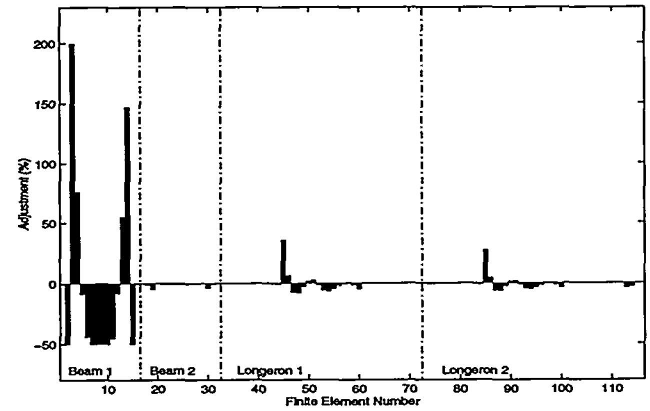


**Figure 6. Updating of the GM Engine Cradle FE Model.**

Figure 6 shows the updating results. Bars represent the percentage of change brought to each element of the model. Modifications are brought essentially to the first transverse beam: stiffnesses are increased by 120% in average near the two joints and decreased by half in the center. The second transverse beam is not modified significantly. Although it is not enforced during the updating, Figure 6 shows that the two longerons receive the same correction, which is actually expected due to the symmetry of the geometry and the mode shapes used during the updating (refer to Table 1).

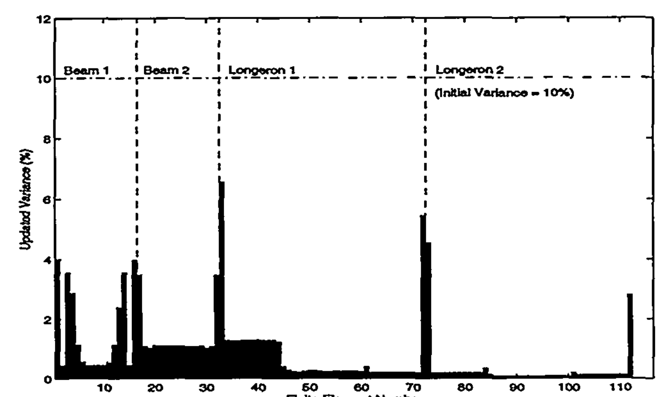


**Figure 7. Final Variance Data For the Adjusted GM Engine Cradle FE Model.**

Variance data after the tenth optimization are illustrated in Figure 7. The Bayesian estimation is initialized with a uniform 10% variance of all moduli of elasticity. Figure 7 shows that most values are reduced to less than 1%. This is especially true for the two longerons, which basically means that the family of models that could be used for representing the longerons exhibits very little dispersion. The larger variance levels can be seen as an indication of where the modeling is still faulty. For example, the first longeron exhibits slightly more dispersion than the other one. These higher variance results are associated to FEs located where the first transverse beam is welded, which is consistent with the significant refinement brought to the first beam.

**Table 5. TAC After FE Model Updating (Identified System Vs. Adjusted FE Model).**

| Identified Frequency | FE Model Frequency | Frequency Error | MAC |
|---|---|---|---|
| 79.0 Hz | 80.6 Hz | 2.0% | 98.5% |
| 170.6 Hz | 171.8 Hz | 0.7% | 96.8% |
| 174.5 Hz | 173.3 Hz | -0.6% | 98.2% |
| 214.7 Hz | 210.3 Hz | -2.1% | 97.5% |
| 250.9 Hz | 239.6 Hz | -4.5% | 96.6% |
| 312.2 Hz | 276.1 Hz | -11.6% | 95.5% |
| 315.8 Hz | 359.7 Hz | 13.9% | 96.3% |

Finally, Table 5 gives the TAC between the updated FE model and test data. It should be compared to Table 2 before updating. Clearly, the correlation is improved significantly. The first mode that was 13% too stiff before updating is now predicted with an acceptable 2% frequency error. Corrections brought to the first transverse beam are consistent with this improvement: the first mode features out-of-phase bending of the longerons and, therefore, it is expected to be very sensitive to modifications of the transverse beam component.

In conclusion, we measure the success of the update by the fact that modes 4-7 that were not included in the correlation procedure are predicted with better accuracy by the updated FE model. Furthermore, variance data in Figure 7 can be used to indicate where further improvement is needed. We emphasize that, without Bayesian estimation, such information would simply not be available.

## 7. ILLUSTRATION OF NONLINEARITY ASSESSMENT WITH THE LANL 8-DOF TESTBED

In this Section, we discuss an application of Bayesian estimation to the updating of a simple model that does not account for a source of nonlinearity otherwise present in the test data. Our goal is to suggest another potential use of variance information, namely, the detection of nonlinearity sources.

As previously, only the first three identified modes are used during model refinement. These are provided at all translation DOFs, therefore, alleviating any type of modal expansion of FE matrix reduction. The axial stiffness of each spring is allowed to be modified. It is recalled that the experiment consists of identifying a 14% reduction at spring number 5 and that friction is not included in the modeling.

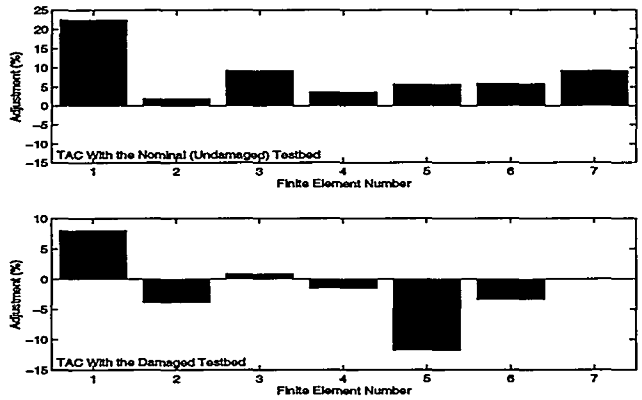


**Figure 8. Updating of the LANL 8-DOF FE Model (Top: TAC With Undamaged Data; Bottom: TAC With Damaged Data).**

Results of two updates are illustrated in Figure 8. First, the model is adjusted to match the nominal, undamaged system (top of Figure 8). It can be noticed that a stiffer system is produced as a result of this correlation. The first spring stiffness is changed by nearly 22%: we attribute this large variation to the driving point measurement and its attachment system that are not modeled, although the total mass is correct. In a second stage, this adjusted model is updated again, this time using the damaged test data (bottom of Figure 8). It can be observed that the location and extent of damage are predicted with reasonable accuracy.

**Table 6. TAC After FE Model Updating (Damaged System Vs. Adjusted FE Model).**

| Identified Frequency | FE Model Frequency | Frequency Error | MAC |
|---|---|---|---|
| 22.3 Hz | 22.3 Hz | 0.2% | 99.8% |
| 43.9 Hz | 44.3 Hz | 1.0% | 99.7% |
| 64.8 Hz | 65.7 Hz | 1.4% | 99.7% |
| 85.9 Hz | 83.9 Hz | -2.4% | 96.9% |
| 99.7 Hz | 99.6 Hz | -0.4% | 99.7% |
| 113.2 Hz | 110.1 Hz | -2.7% | 92.1% |
| 131.9 Hz | 117.6 Hz | -10.9% | 75.2% |

Final results of this correlation are presented in Table 6. As before, a significant amelioration over figures presented in Table 4 (before updating) is noticeable. While MAC values remain essentially unchanged, the prediction of the first five modes is improved which is encouraging considering that only modes 1-3 were used during updating.

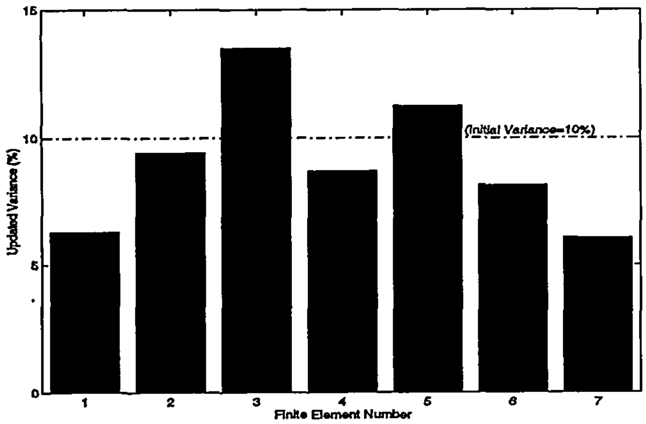


**Figure 9. Final Variance Data For the Adjusted LANL 8-DOF FE Model.**

Variance data obtained after the optimization are pictured in Figure 9. The contrast with results of Section 6 is striking: although the updating is clearly successful (the structural damage is identified and the TAC is improved), variance data remain close to their original 10% level throughout the model. (The reader should not be mislead: the variance does not increase after updating, as we have seen that it is mathematically impossible. It is the ratio of updated variance to updated stiffness that increases for some springs.) Although this example does not constitute a formal proof, we believe that these residual high dispersions point to a systematic modeling error: the friction not introduced in our linearized FE model.

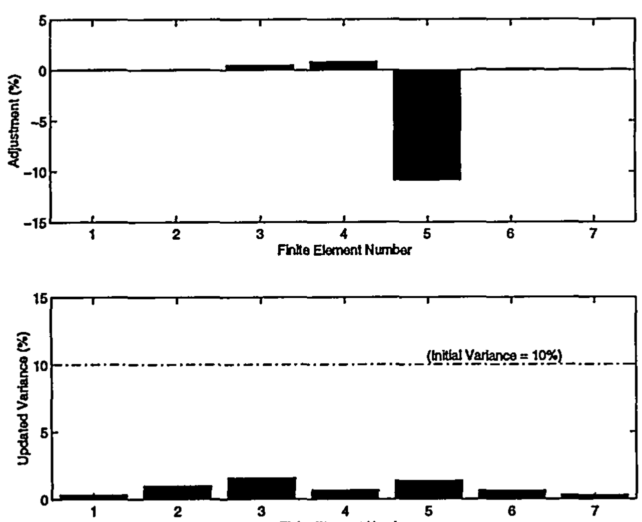


**Figure 10. Updating of the LANL 8-DOF FE Model and Final Variance Data Obtained Using Numerically Simulated "Test" Data.**

To verify this hypothesis, one final test is performed. Using our nominal FE model, "test" data are simulated numerically with a reduction of 14% of the fifth spring stiffness. Updating is then performed with the numerical data instead of the experimental data. The first one of Figure 10 shows the updating result and the second one illustrates the updated variance. The conclusion is that variances obtained with the test data are much higher than those obtained with the simulated data. In both cases, the sensing configuration is the same, modes 1-3 are used for the update and measurement noise is not an issue. We conclude that the parametric form of the FE model is inappropriate to capture the measured dynamics while it is obviously correct in the case of simulated data.

## 8. CONCLUSION

We present an application of Bayesian parameter estimation concepts to finite element model updating. The updating procedure is formulated to improve the predictive quality of a structural model by minimizing out-of-balance modal forces. The critical issues of modal expansion and covariance matrix updating are addressed.

Practically, this procedure enables measurement and modeling uncertainties to be considered not as "nuisance" any more but as additional inputs provided to the correlation procedure. In return, confidence levels associated to the correlated model can be assessed. Demonstration examples using real test data, linear and nonlinear systems are discussed to illustrate the benefits of this approach. Although this work remains a first step in the overall assessment of the method, we conclude to the usefulness of Bayesian model updating provided that some numerical difficulties can be resolved.

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